

# Automata Learning with an Incomplete Teacher

ECOOP 2023

Mark Moeller *Cornell University*

Thomas Wiener *Cornell University*

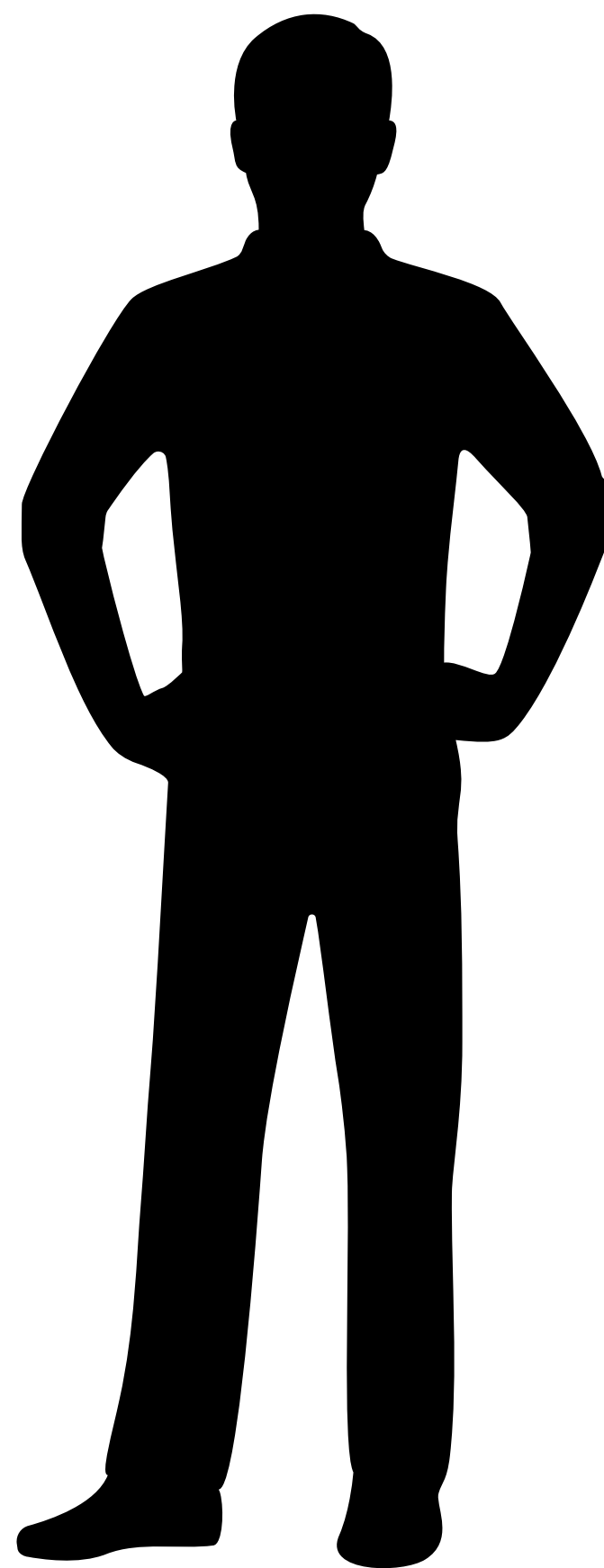
Alaia Solko-Breslin *University of Pennsylvania*

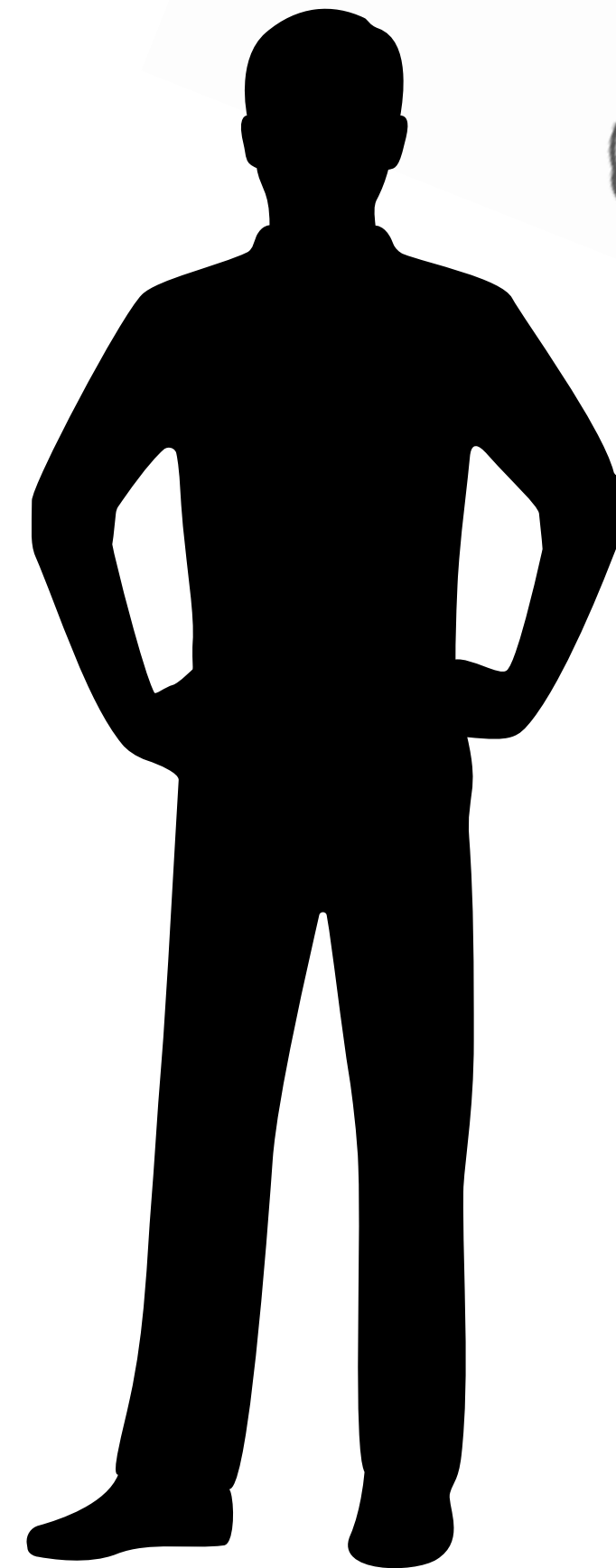
Caleb Koch *Stanford University*

Nate Foster *Cornell University*

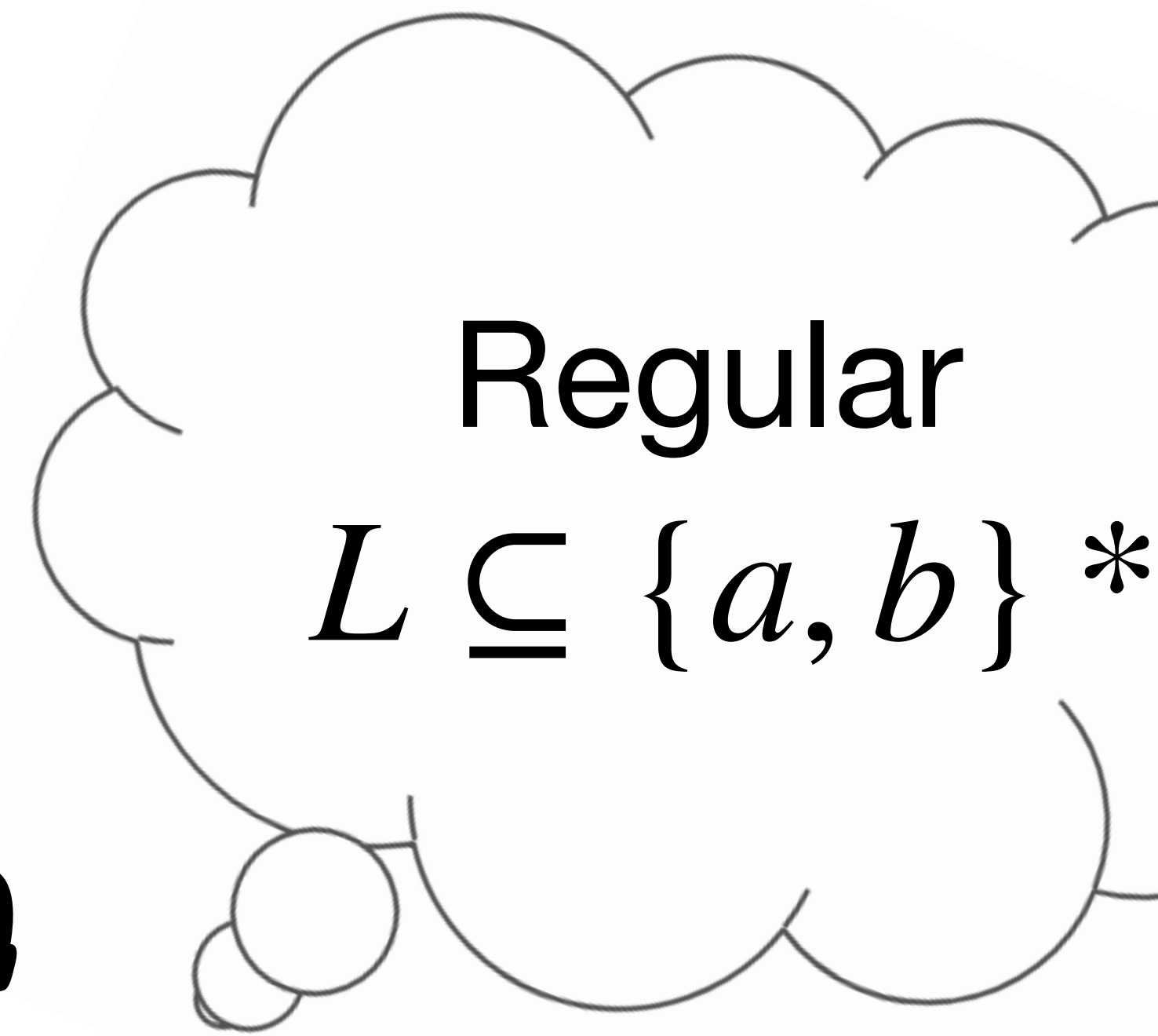
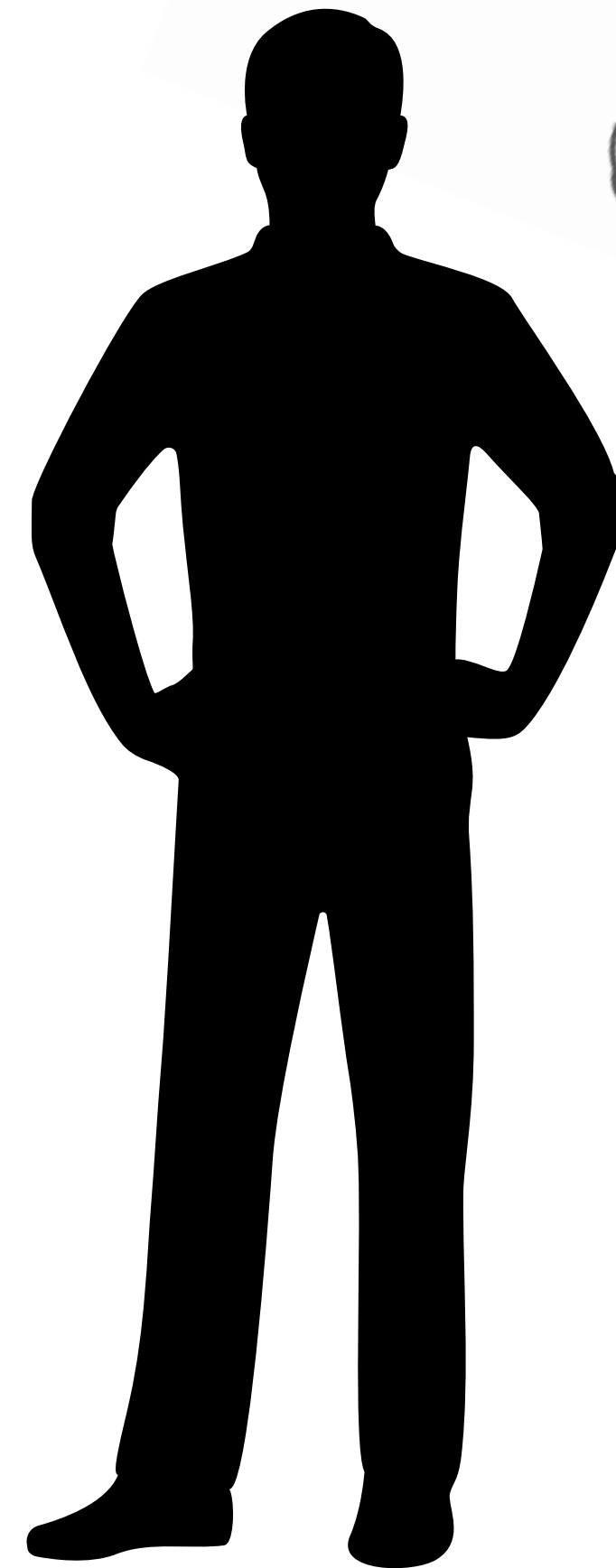
Alexandra Silva *Cornell University*

19 July, 2023

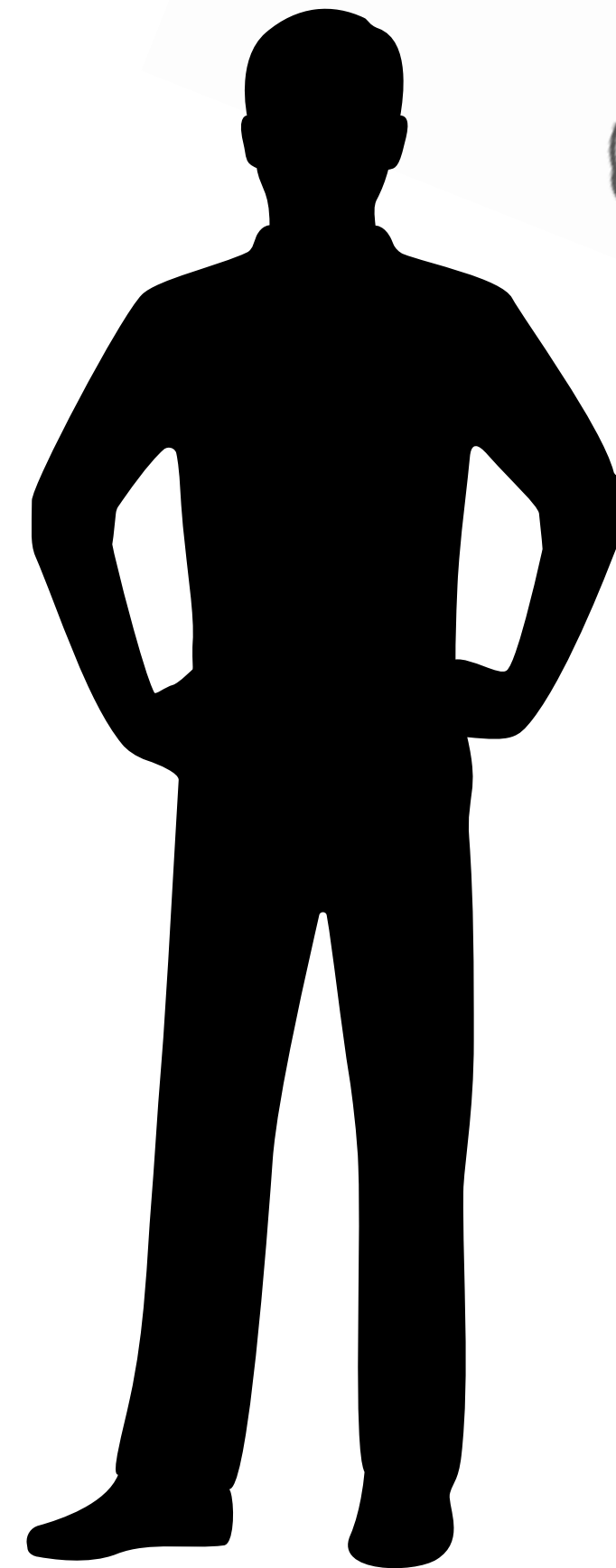




Regular  
 $L \subseteq \{a, b\}^*$



1. You can ask if a given string is in  $L$



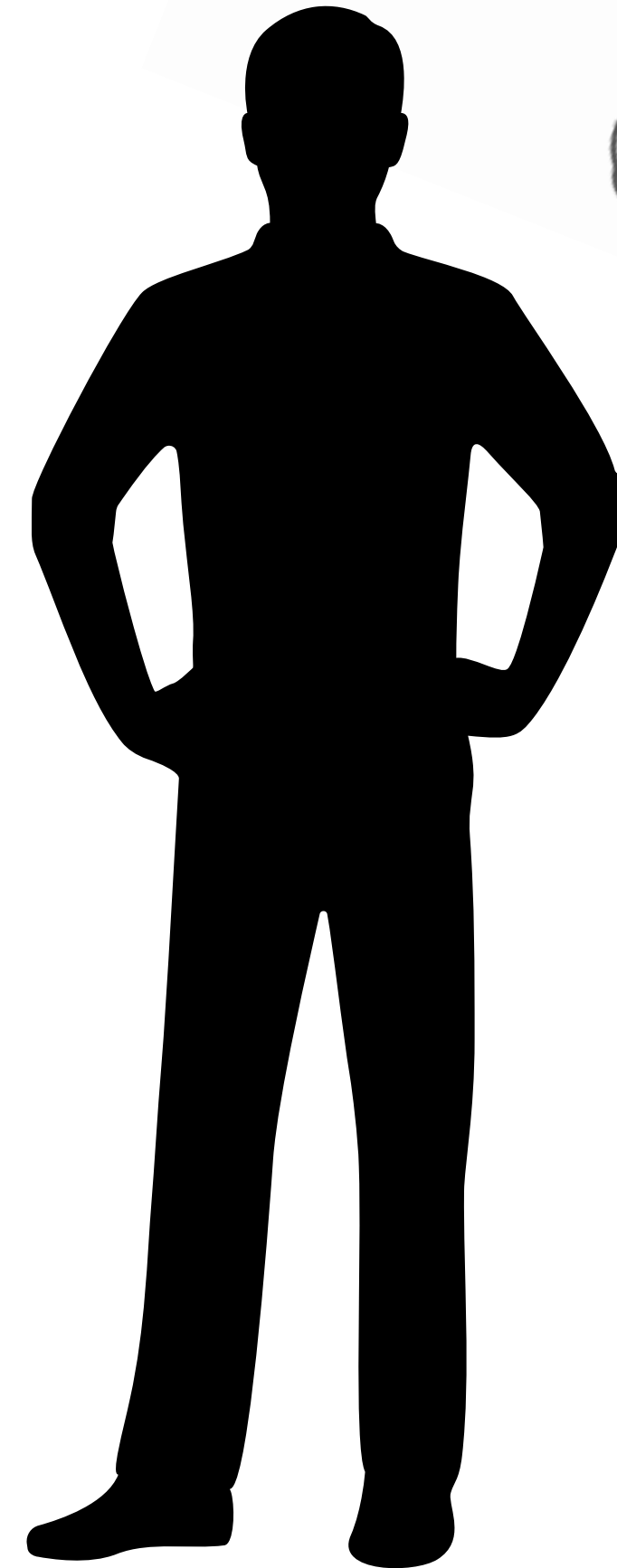
Regular  
 $L \subseteq \{a, b\}^*$

1. You can ask if a given string is in  $L$
2. You can guess a DFA for  $L$

$\varepsilon \in L?$



Regular  
 $L \subseteq \{a, b\}^*$

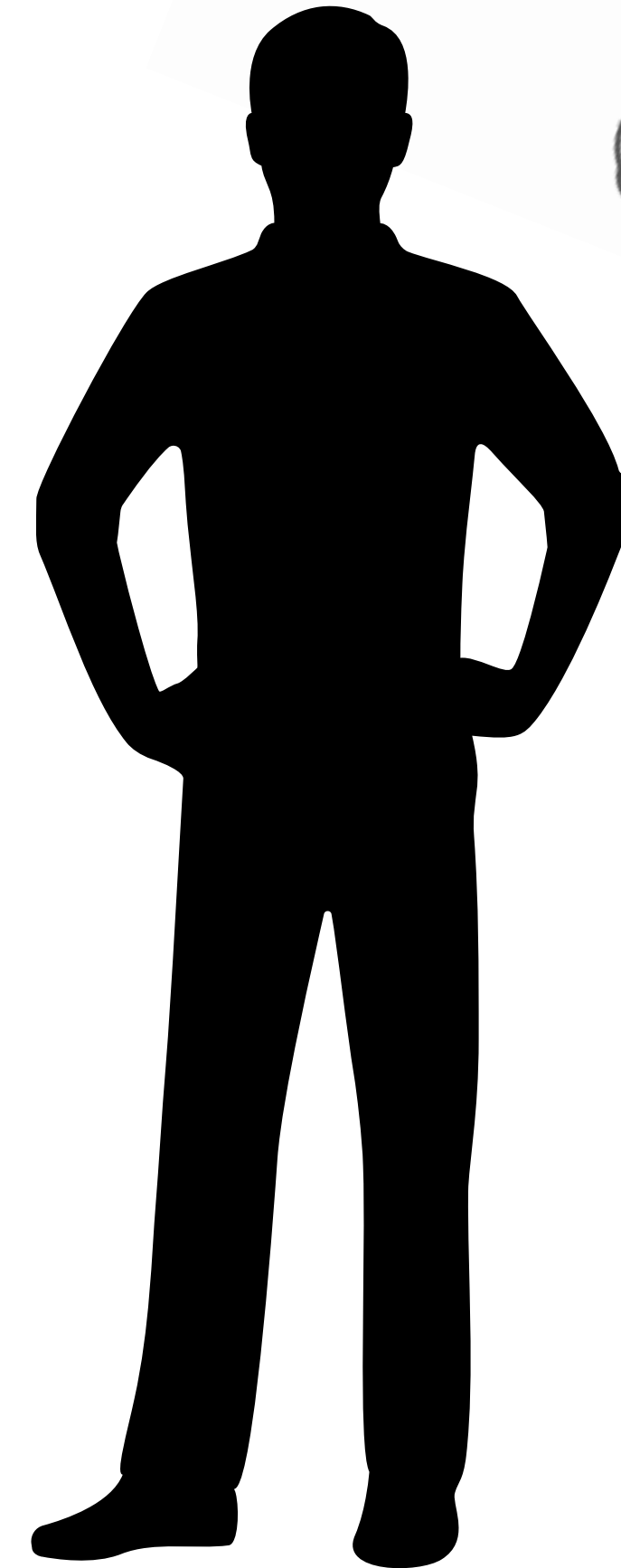


1. You can ask if a given string is in  $L$
2. You can guess a DFA for  $L$

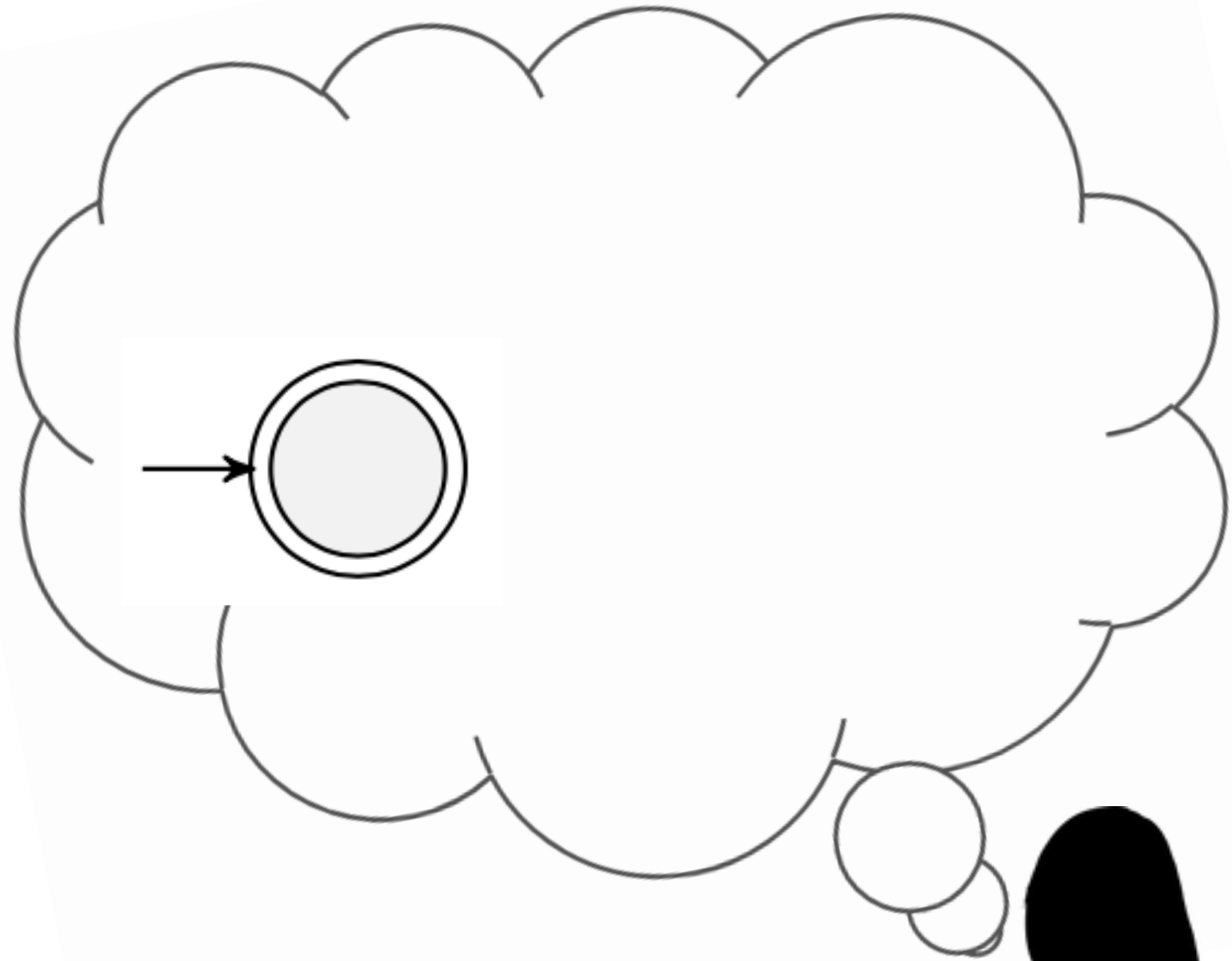
$\varepsilon \in L?$

Yes

Regular  
 $L \subseteq \{a, b\}^*$

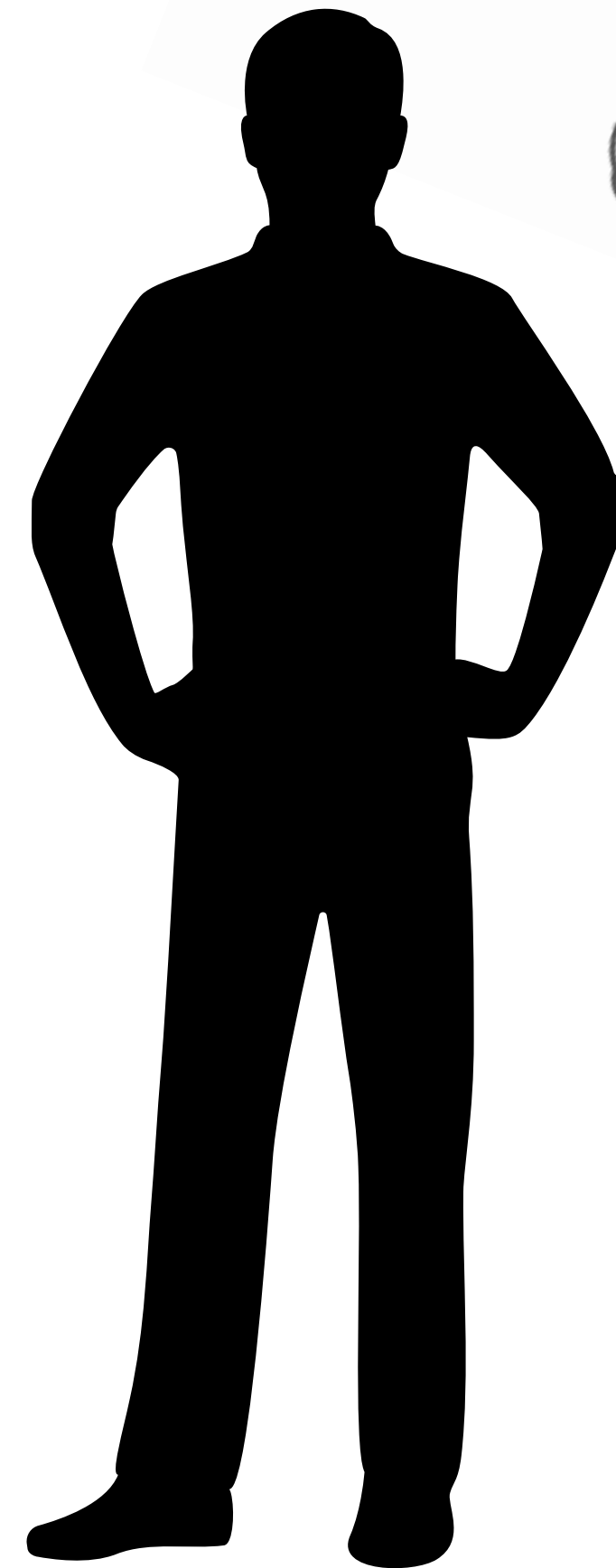
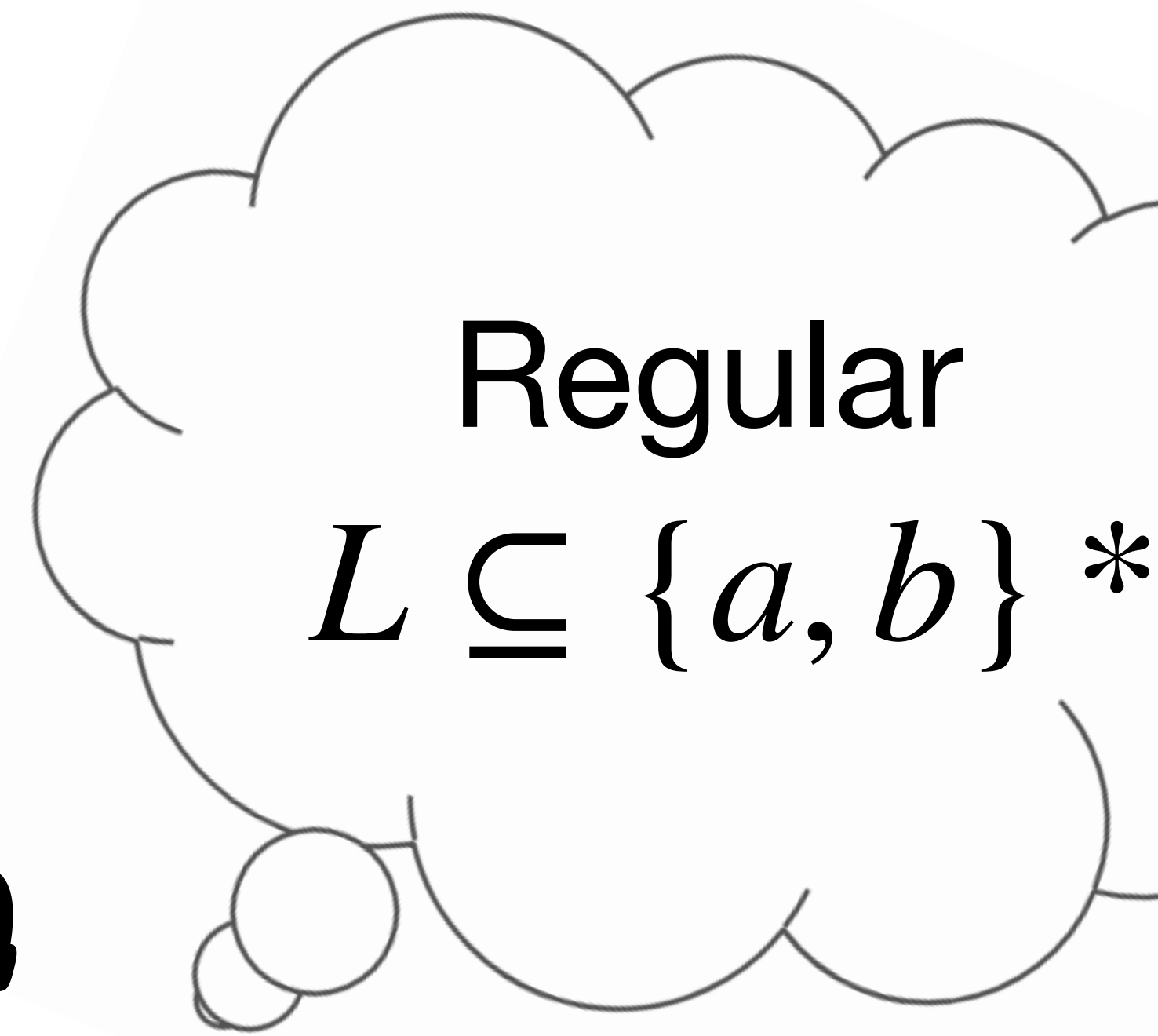


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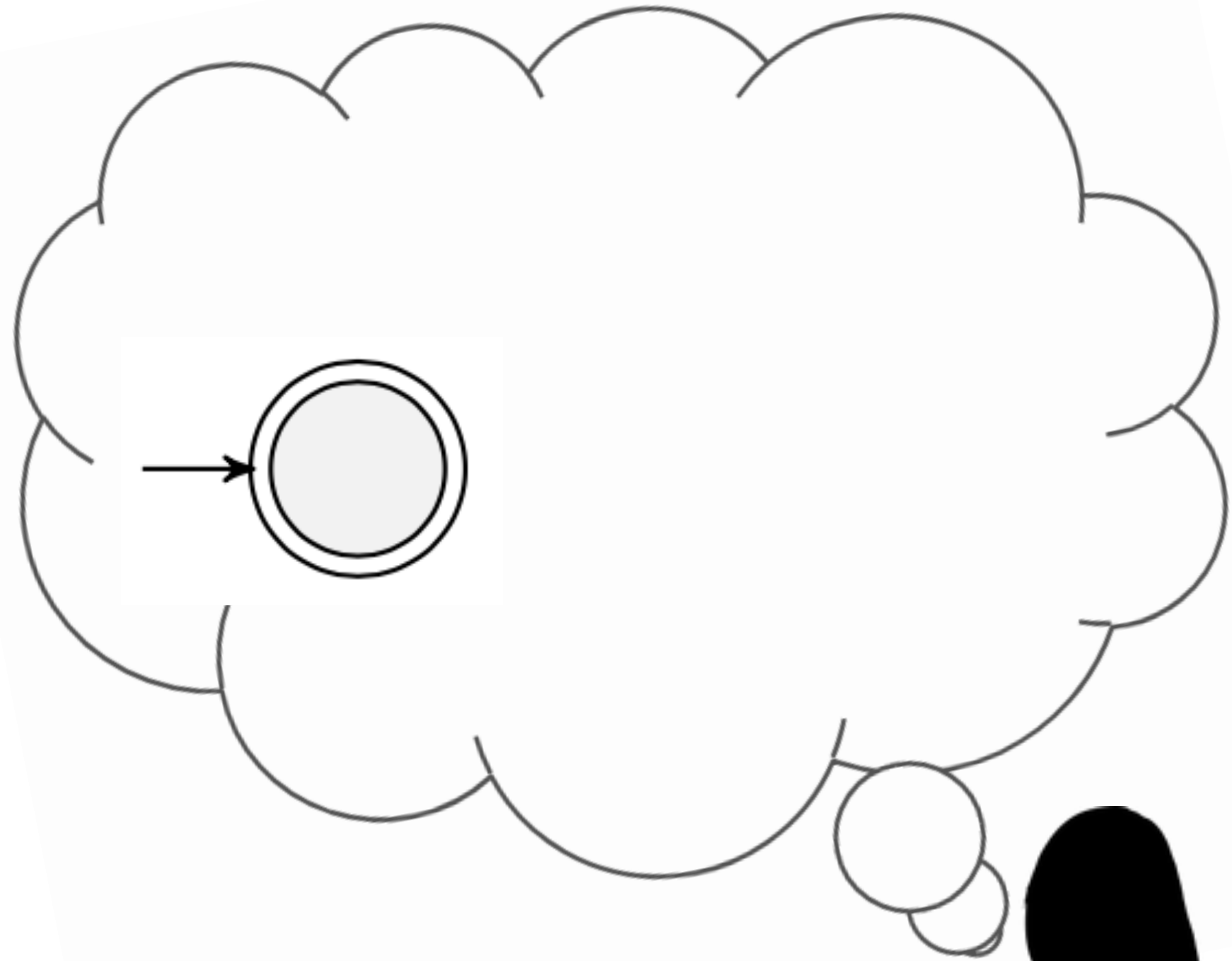
$\epsilon \in L?$

Yes



1. You can ask if a given string is in  $L$
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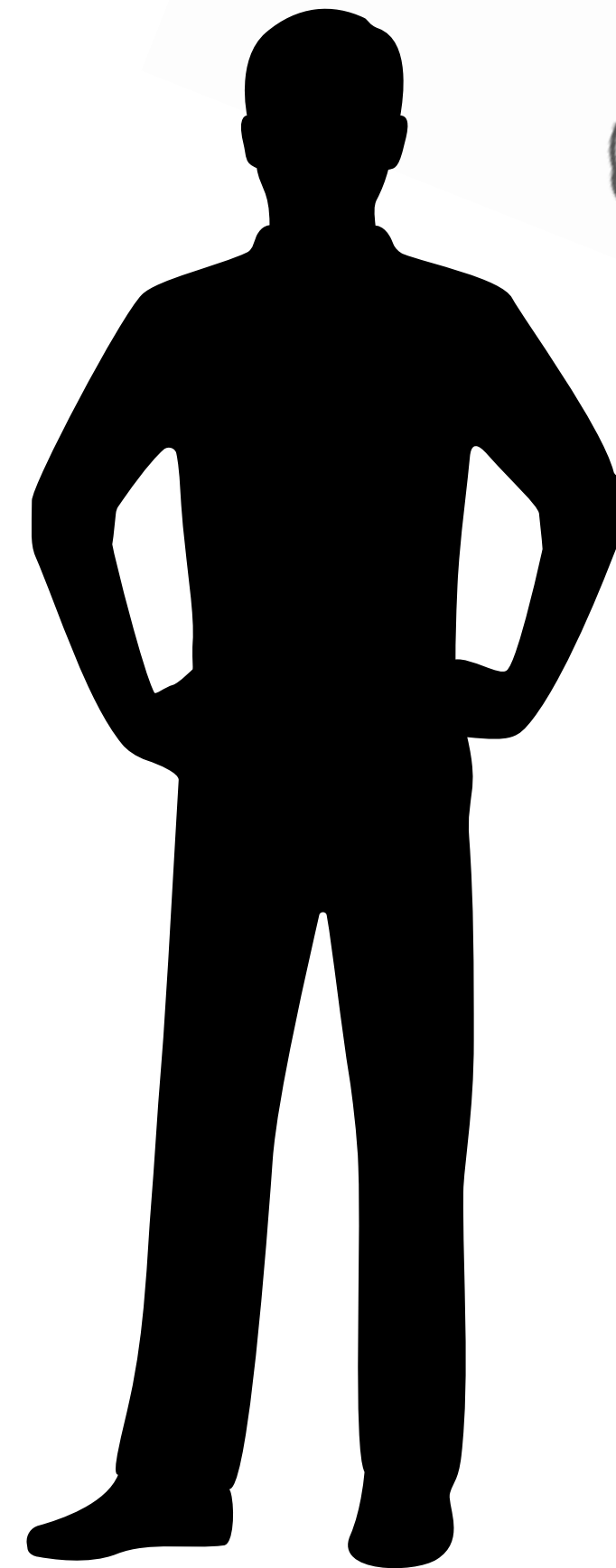
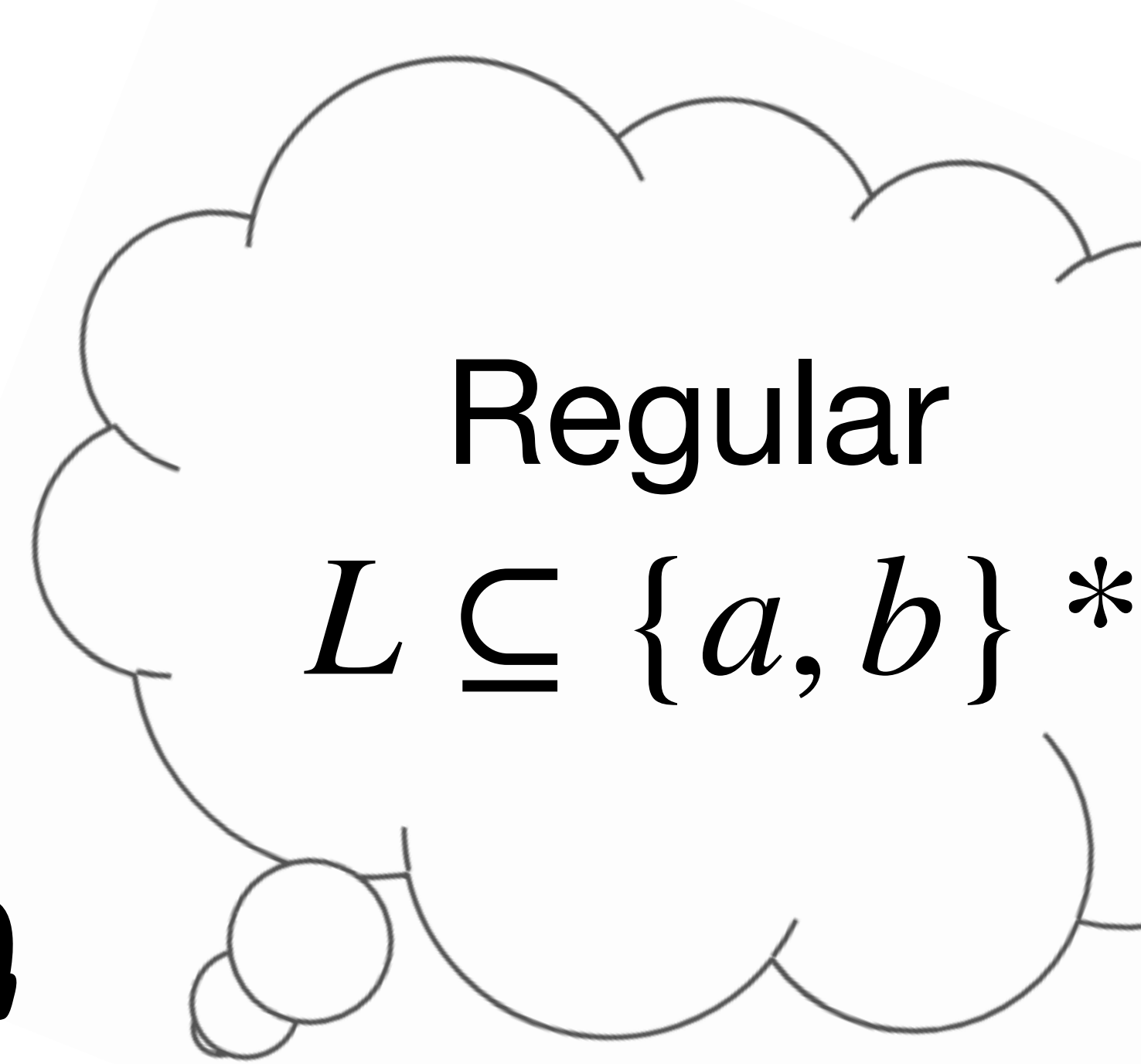




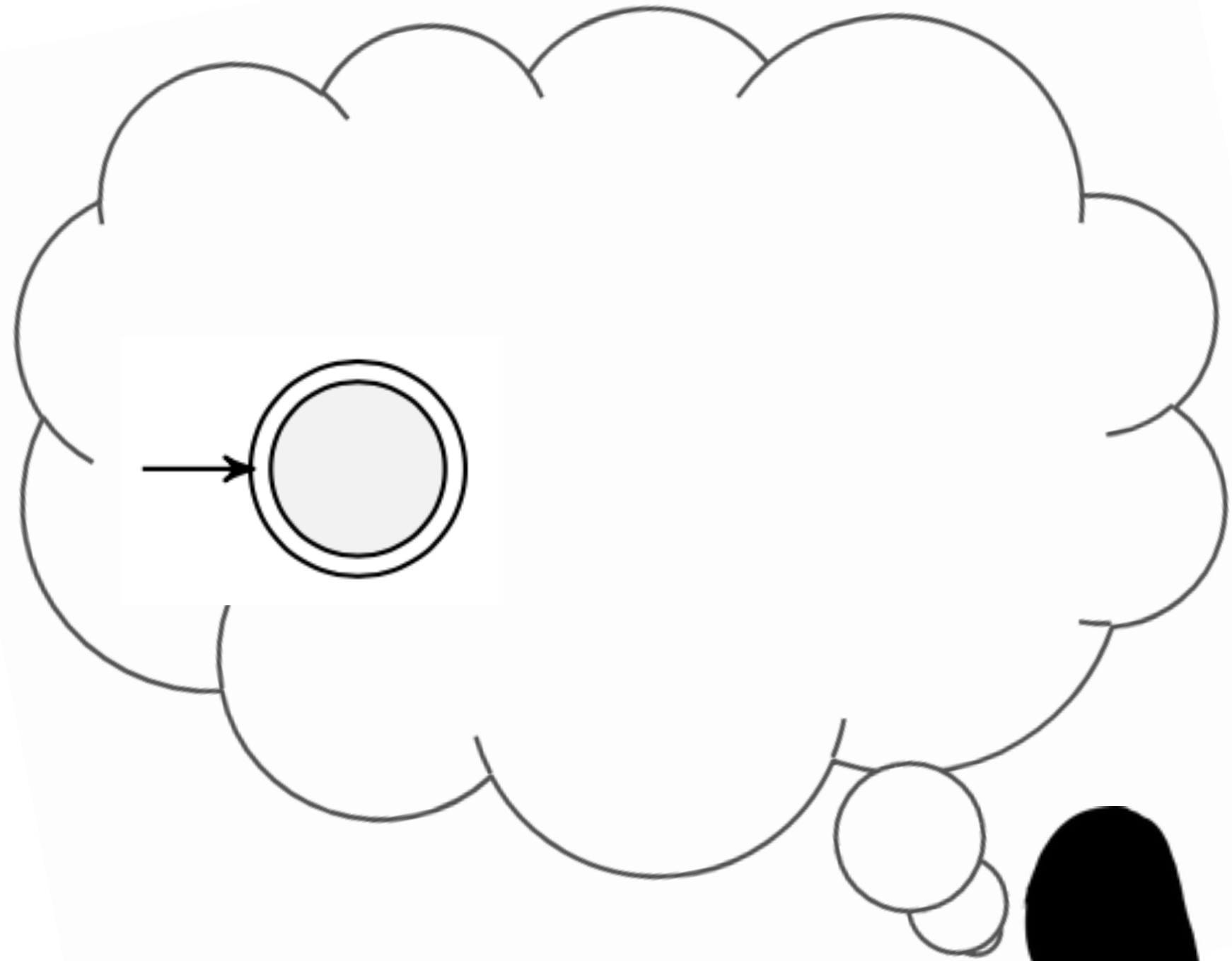
$\epsilon \in L?$

Yes

$a \in L?$



1. You can ask if a given string is in L
2. You can guess a DFA for L

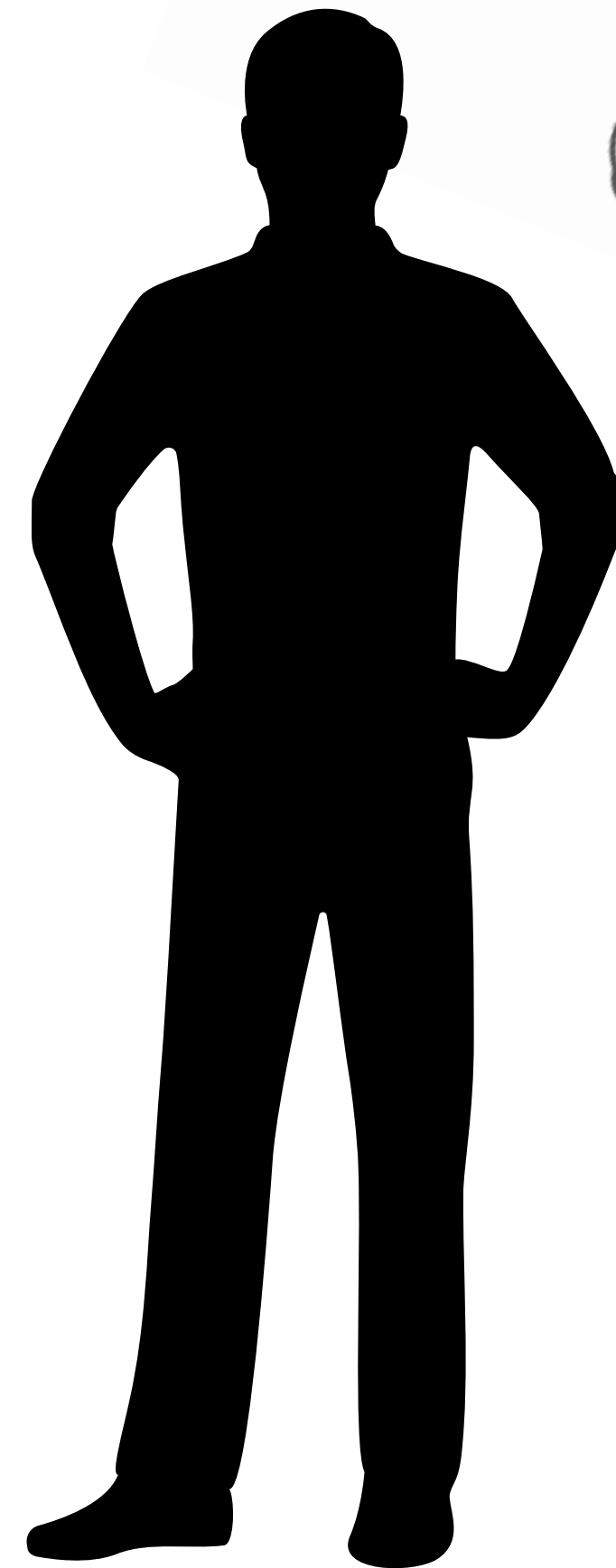
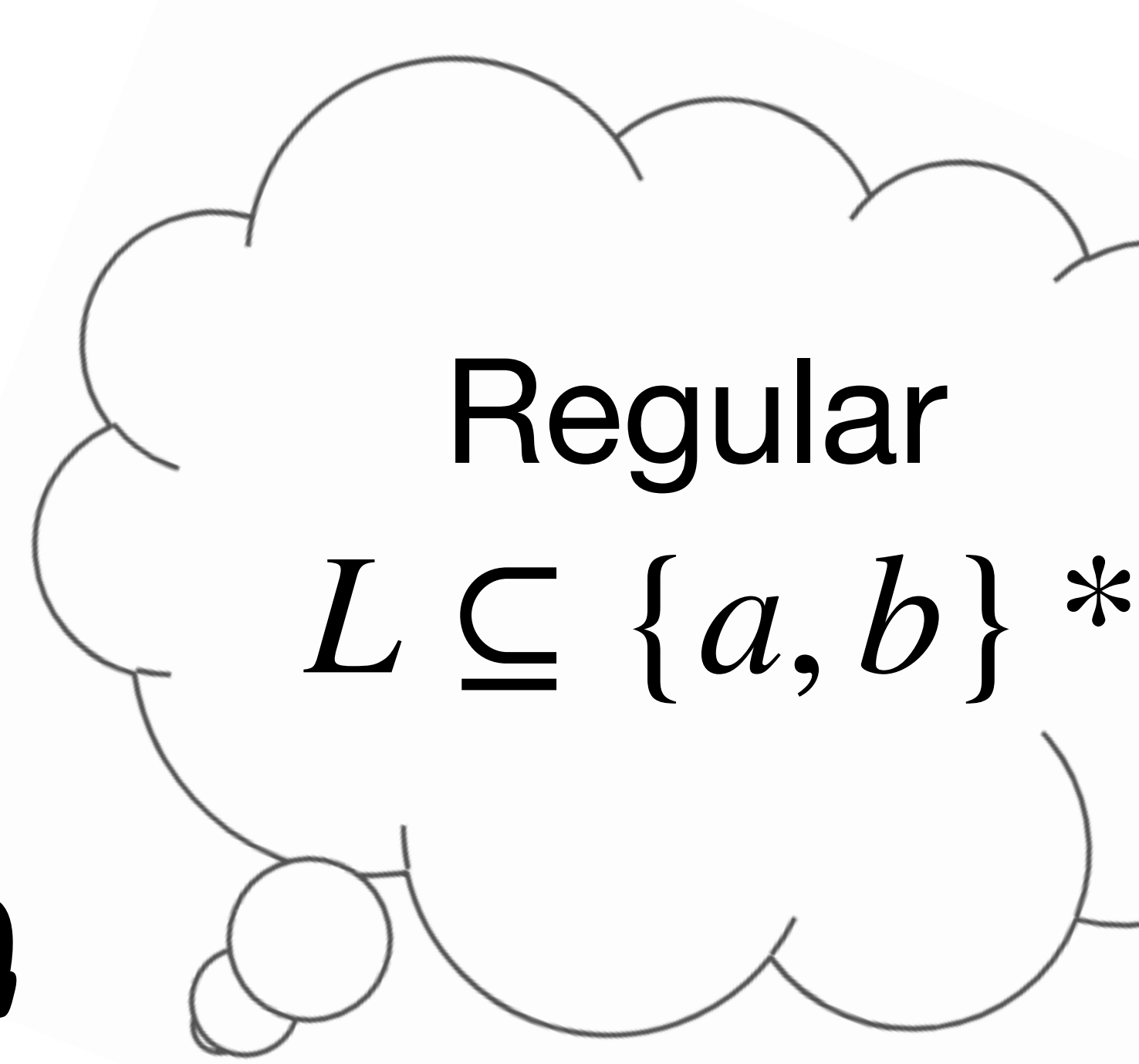


$\epsilon \in L?$

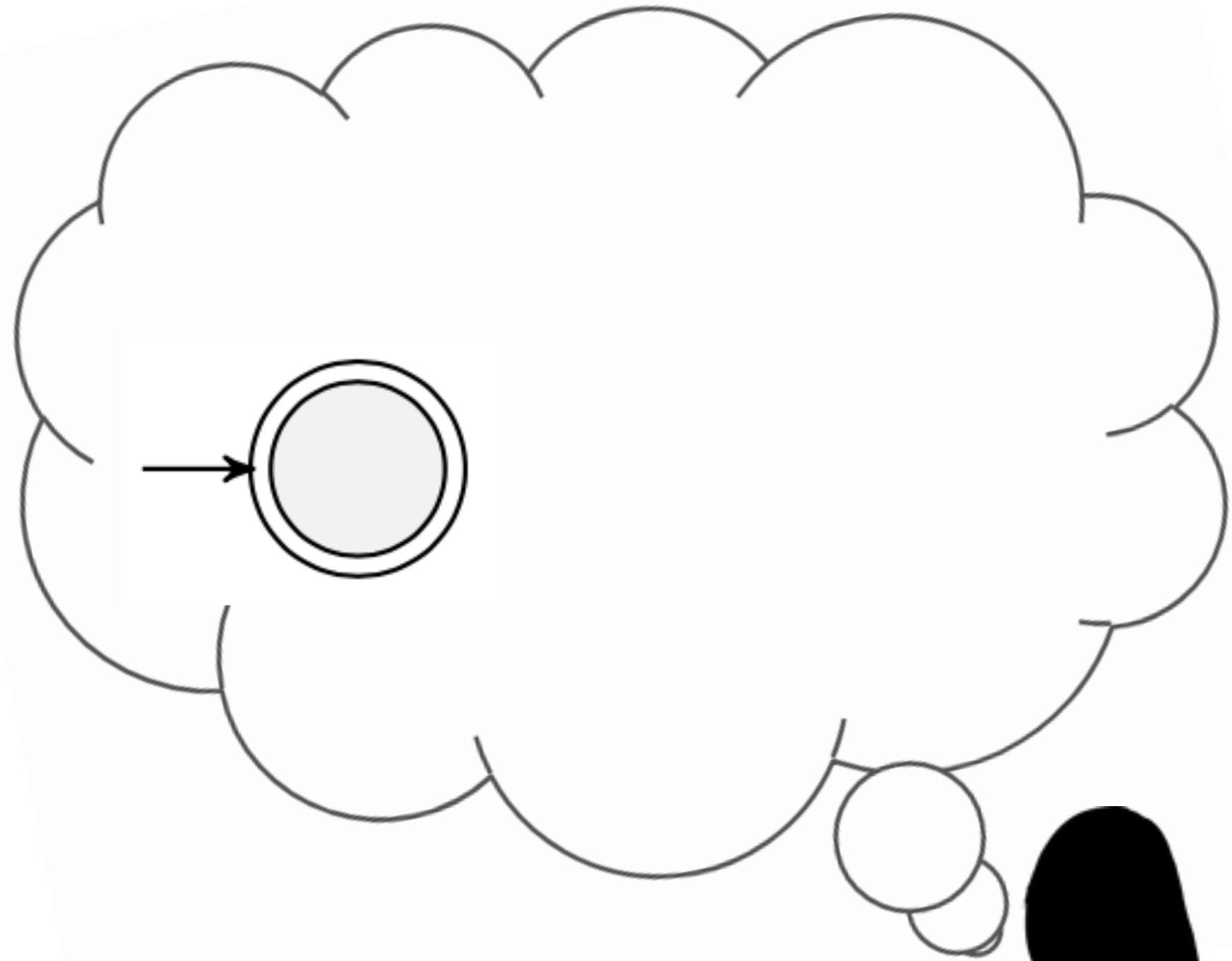
Yes

$a \in L?$

Yes



1. You can ask if a given string is in L
2. You can guess a DFA for L



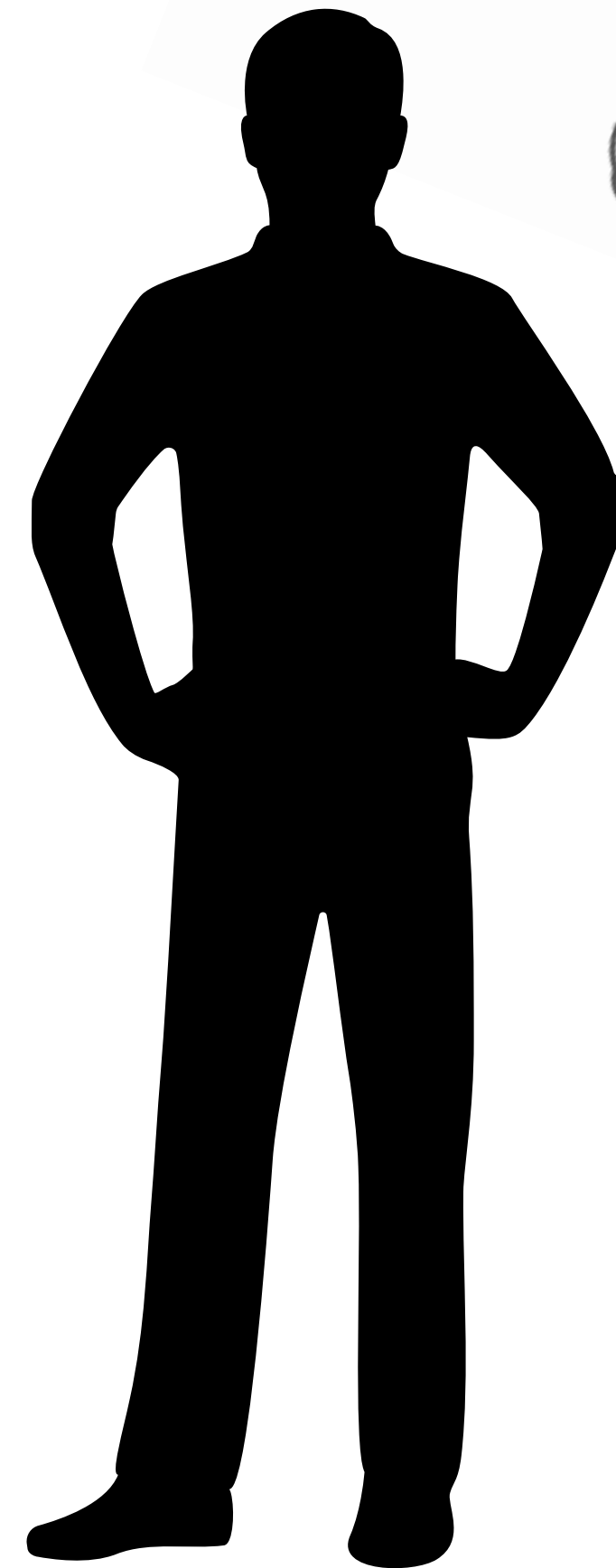
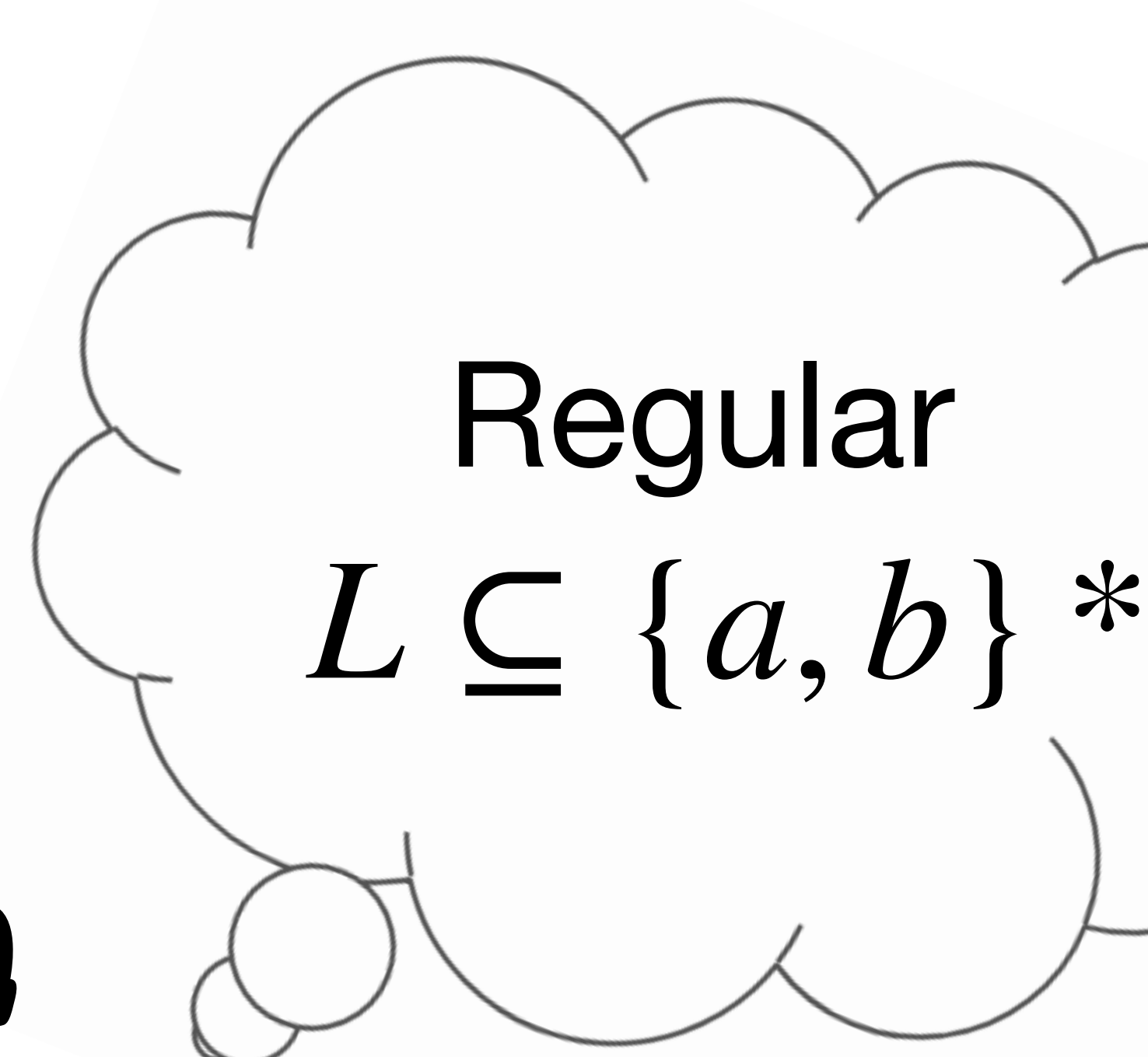
$\epsilon \in L?$

Yes

$a \in L?$

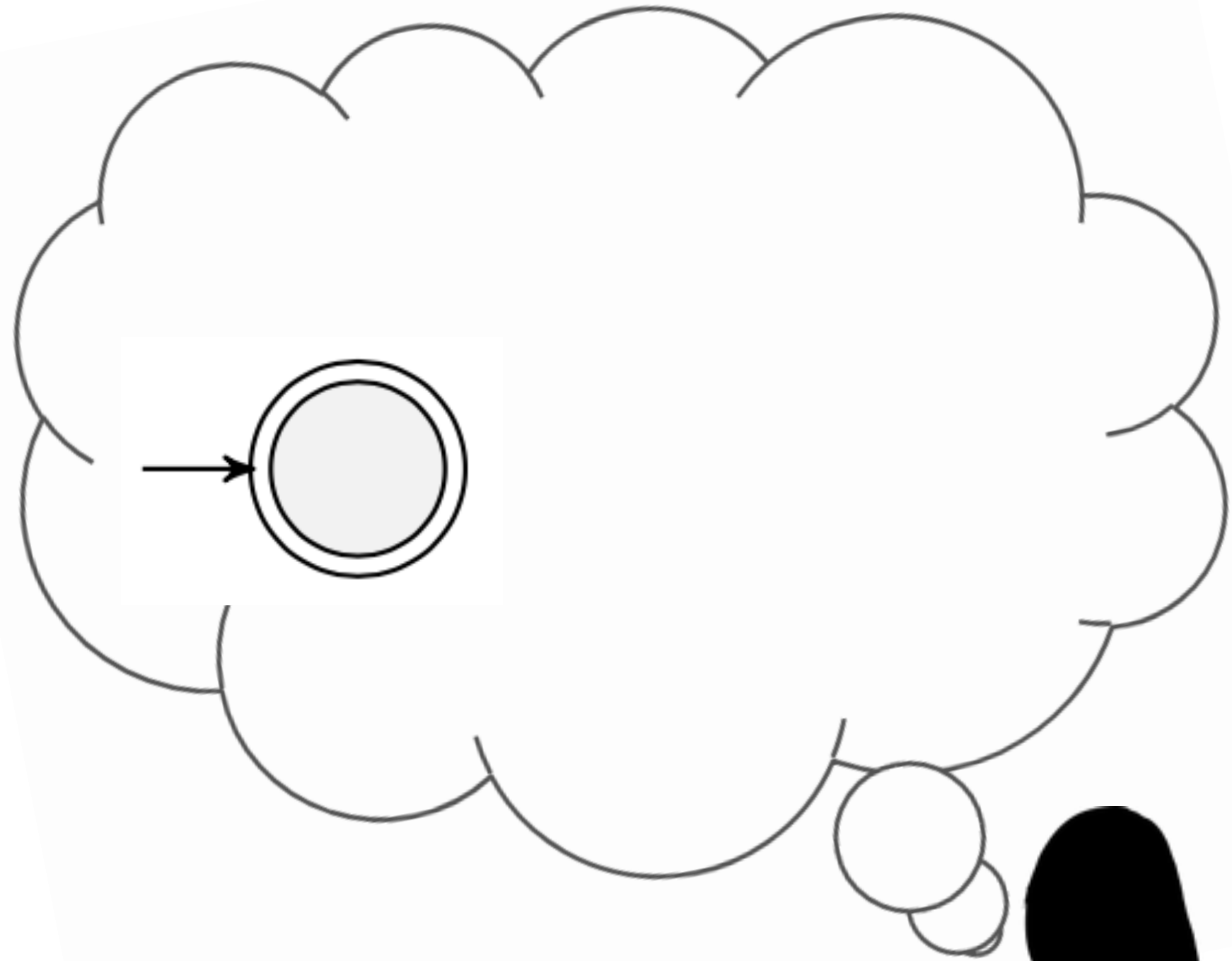
Yes

$b \in L?$

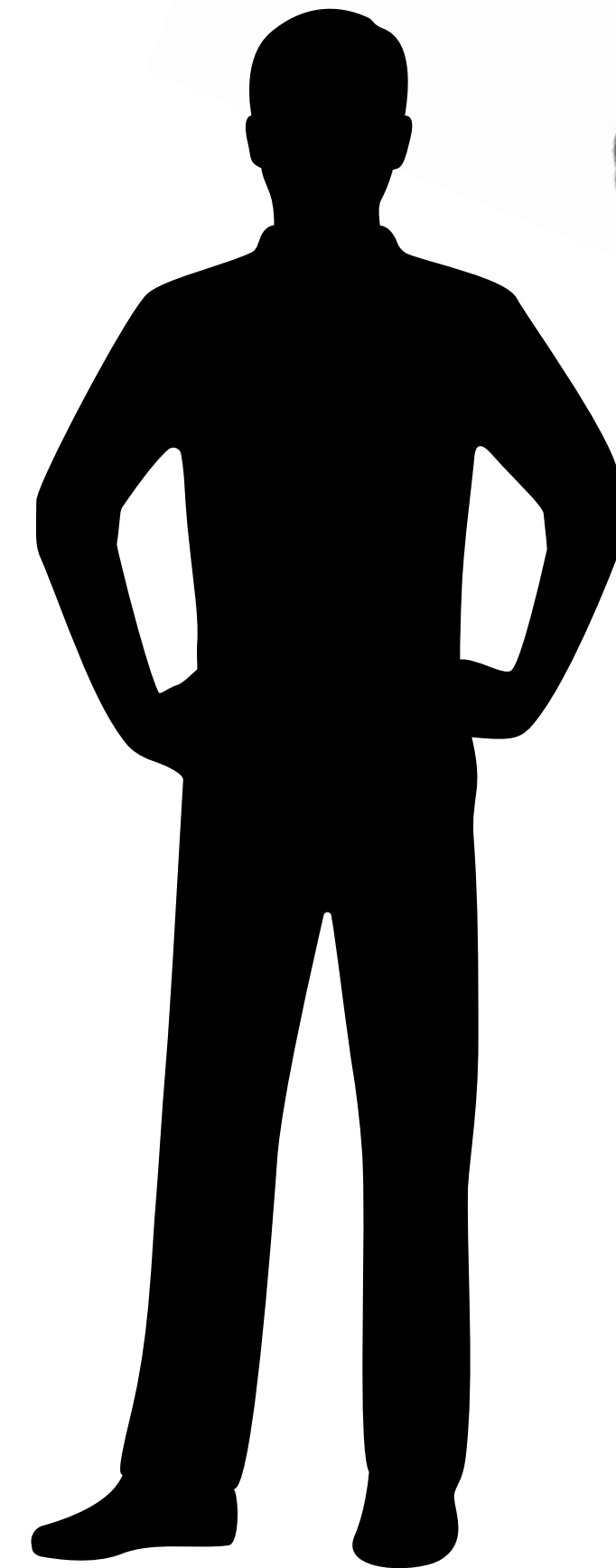
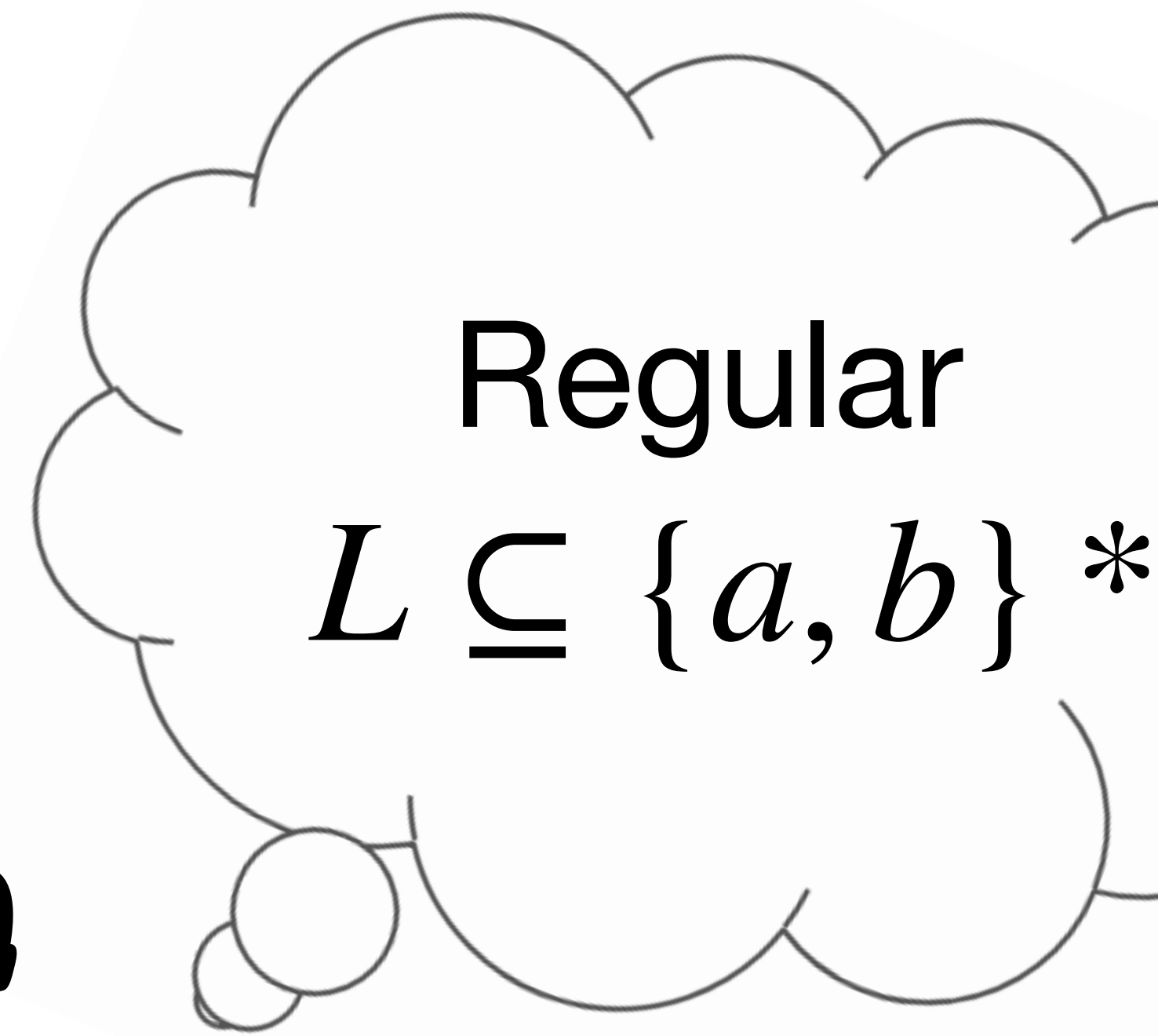


1. You can ask if a given string is in L

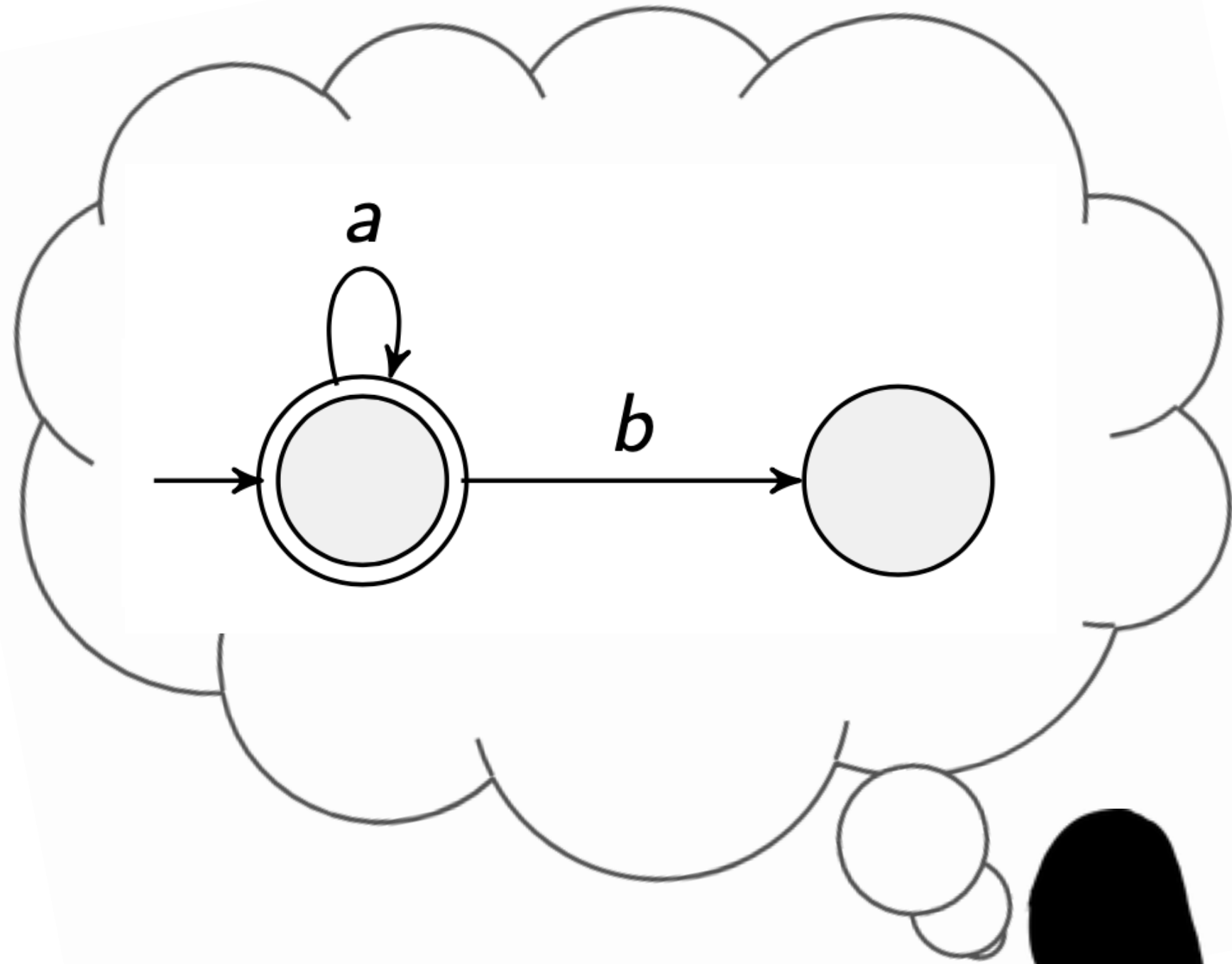
2. You can guess a DFA for L



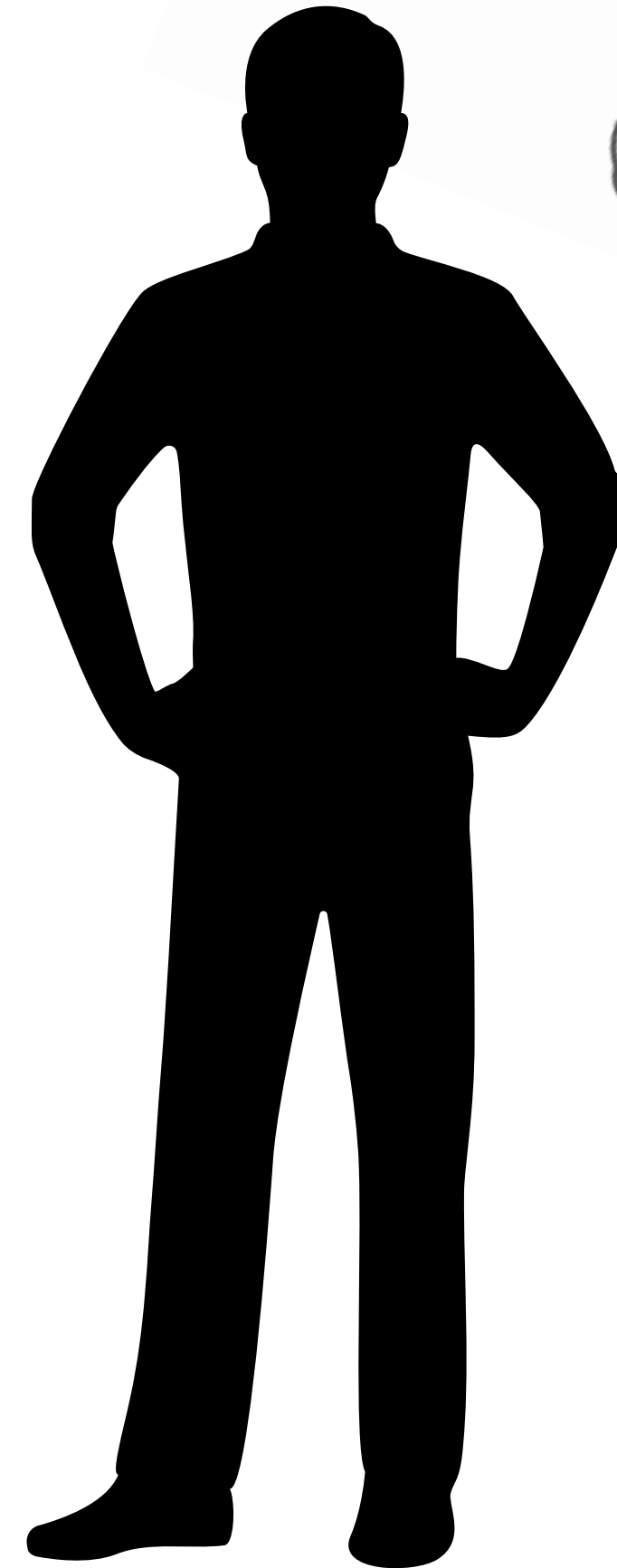
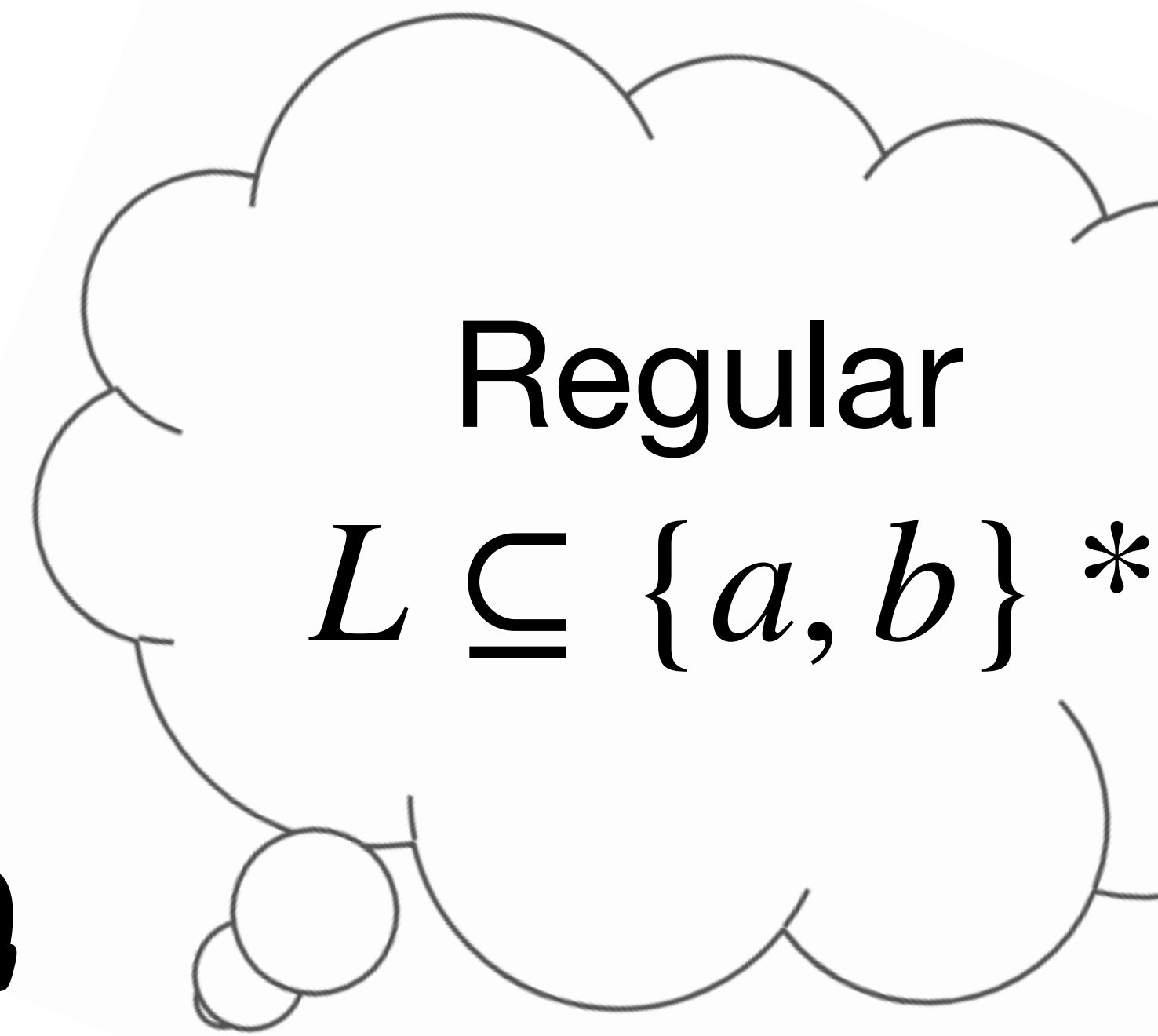
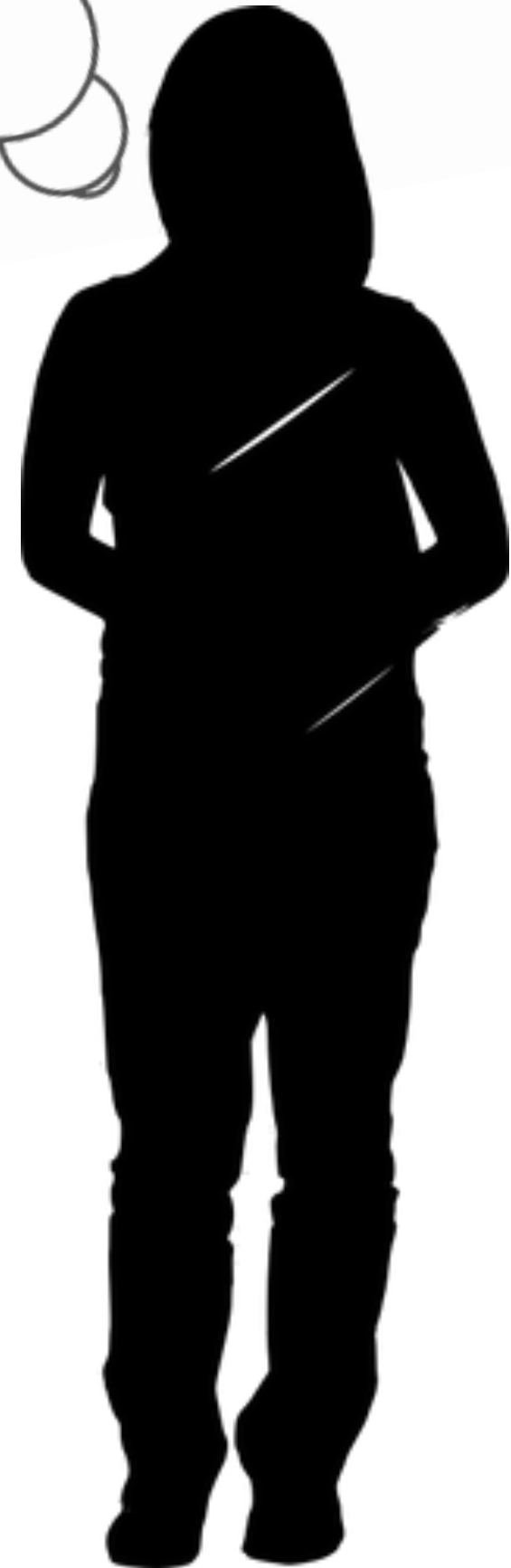
$\epsilon \in L?$       Yes  
 $a \in L?$       Yes  
 $b \in L?$       No



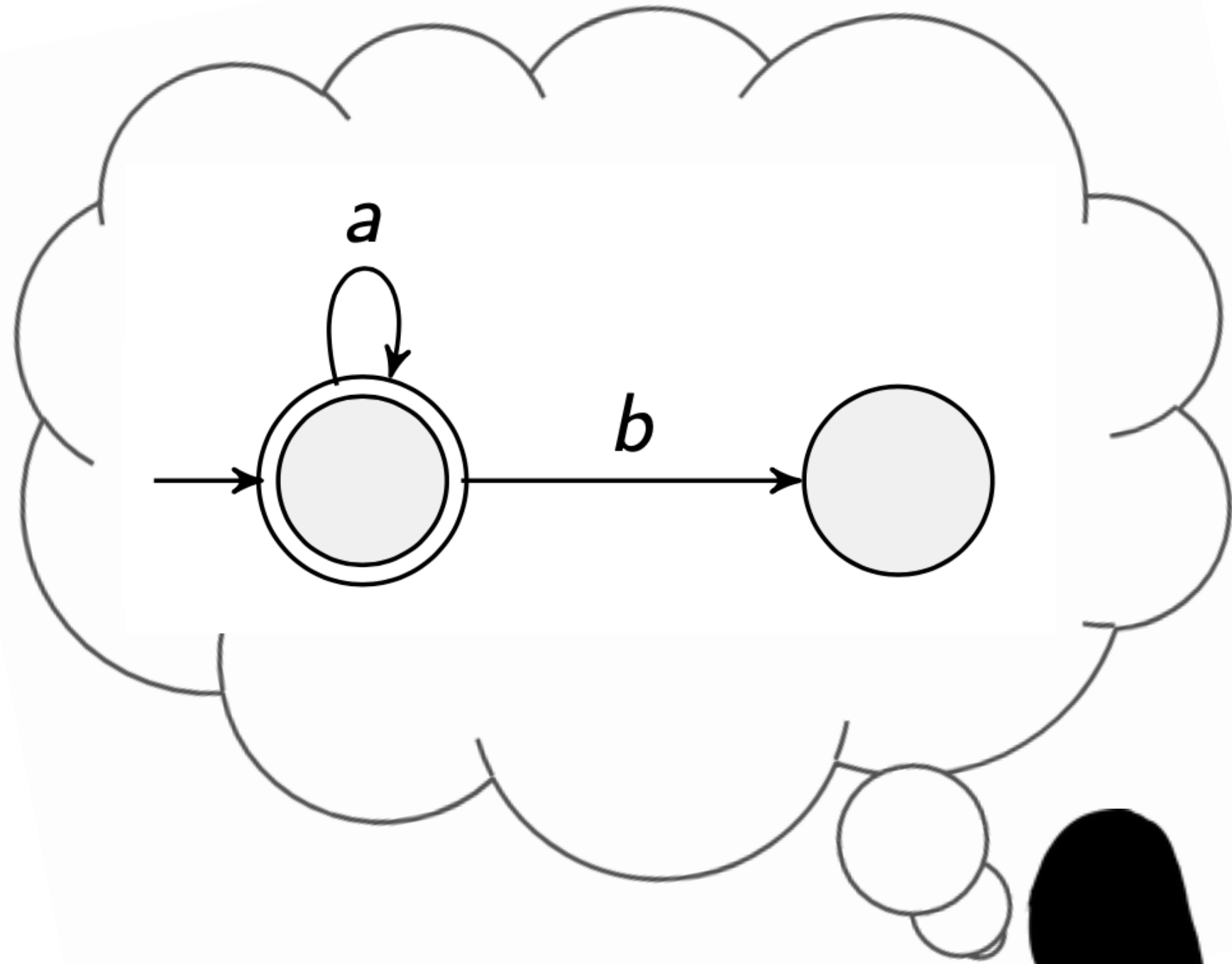
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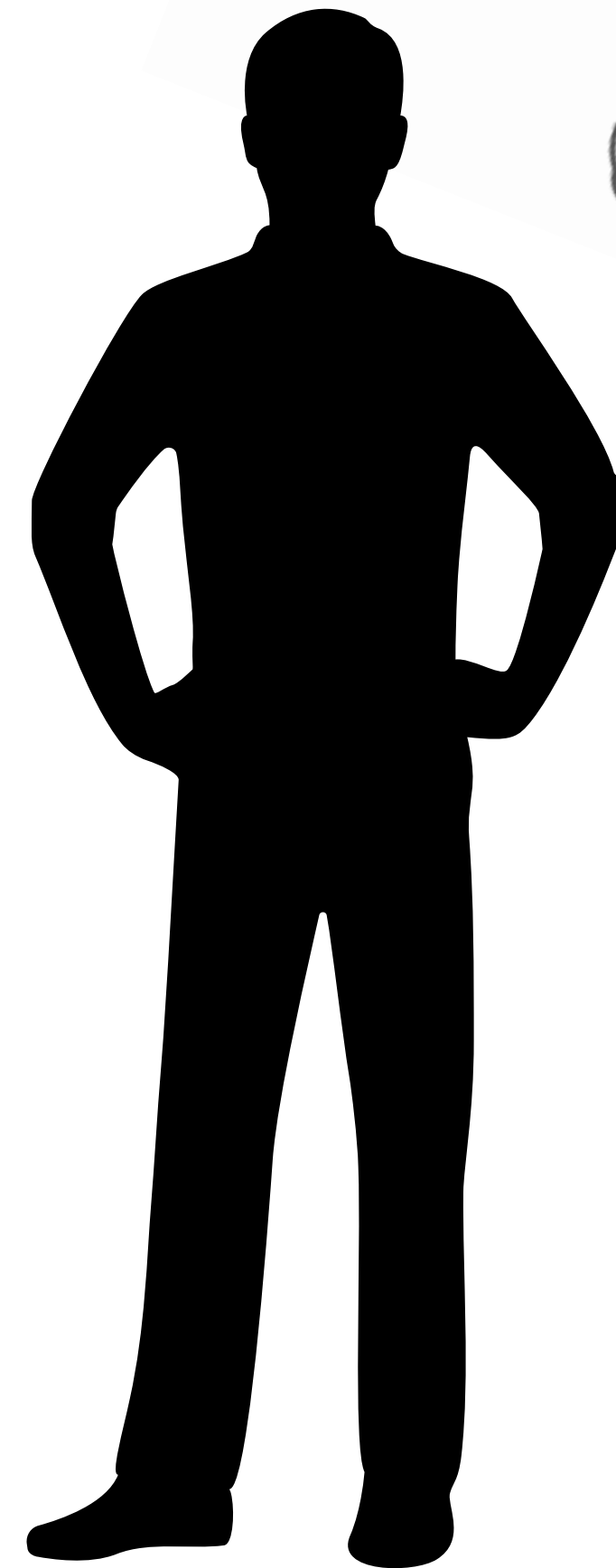
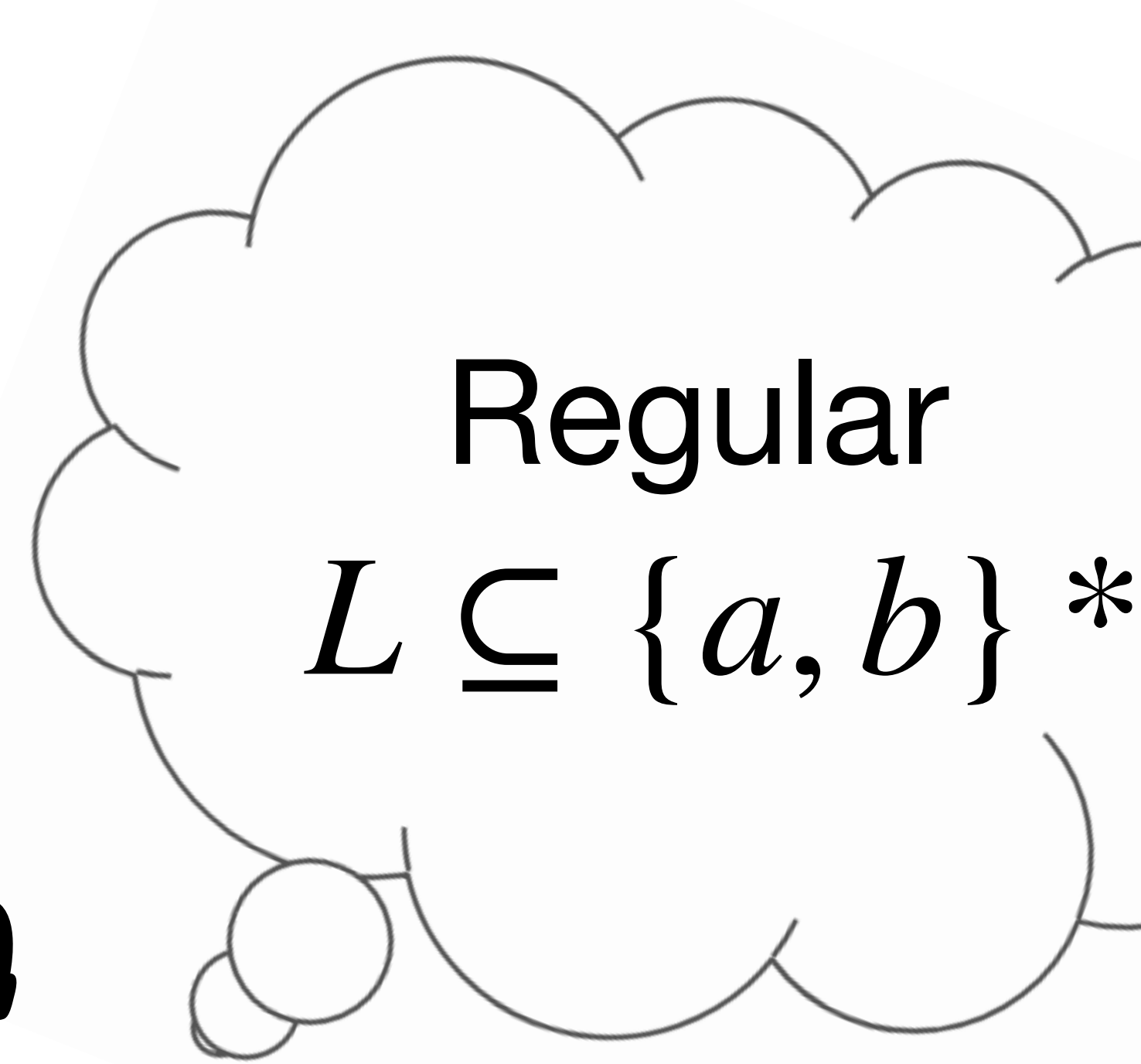
$\epsilon \in L?$  Yes  
 $a \in L?$  Yes  
 $b \in L?$  No



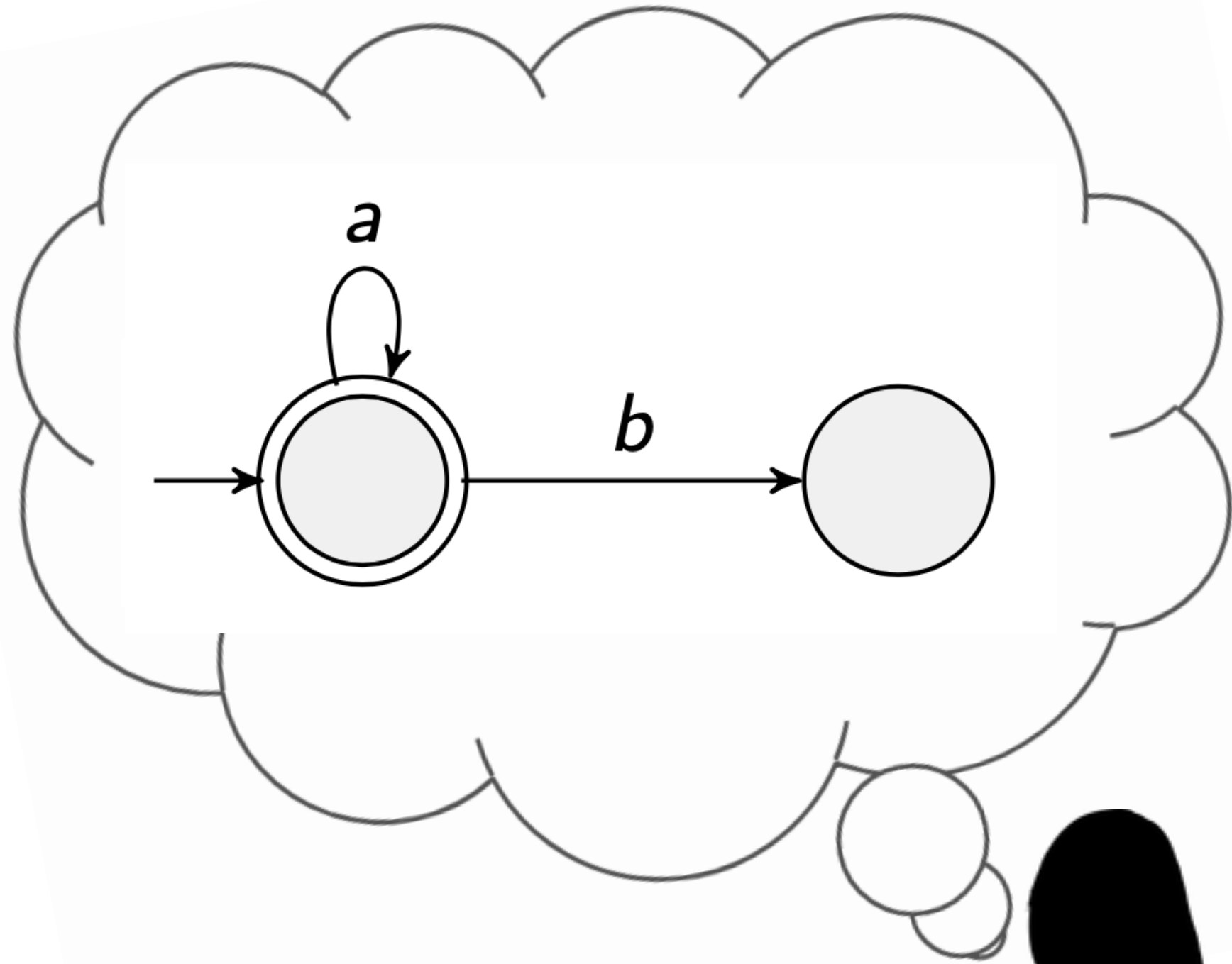
1. You can ask if a given string is in L
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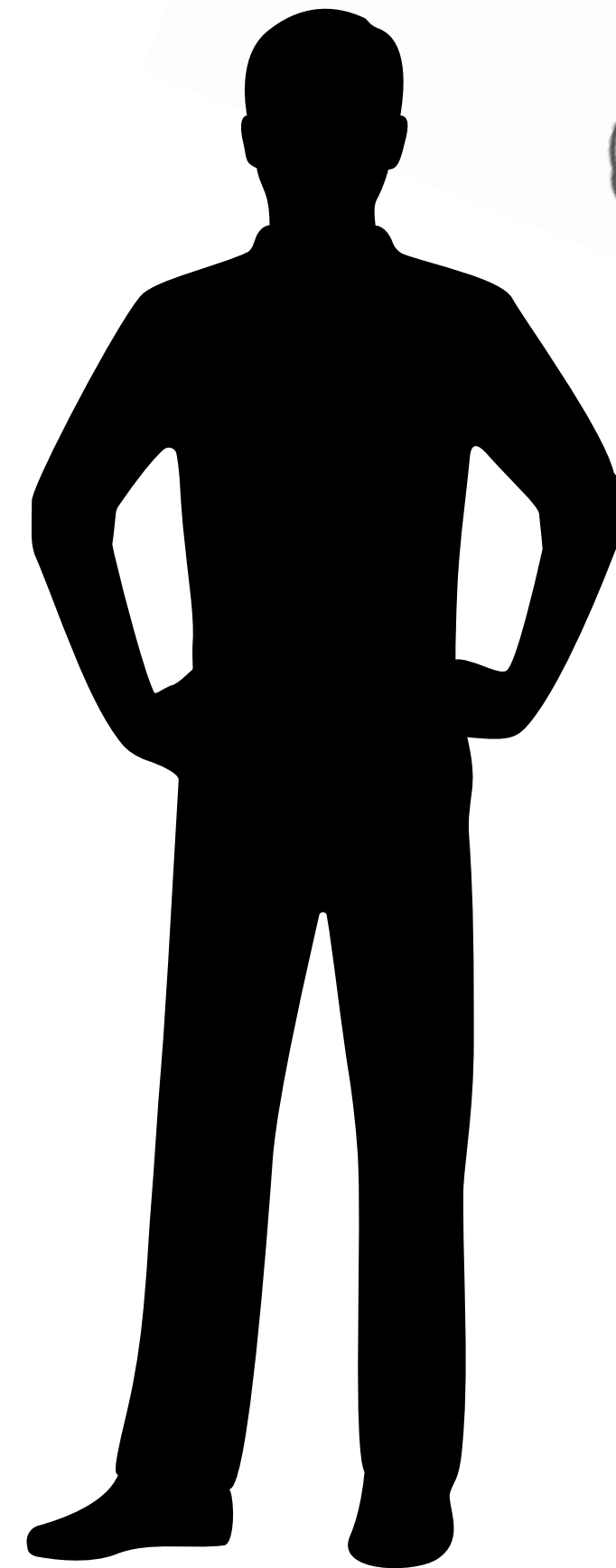
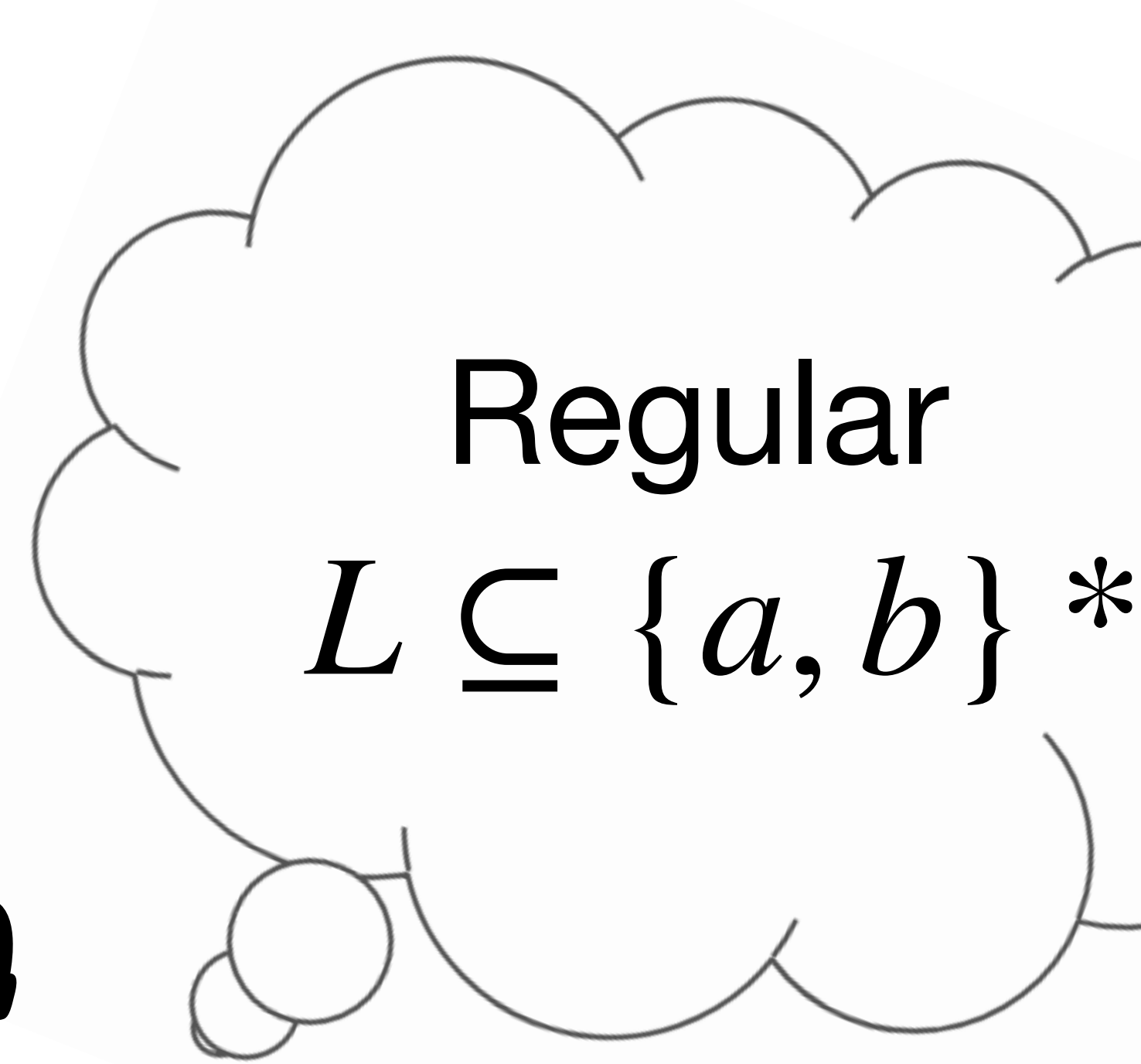
$\epsilon \in L?$       Yes  
 $a \in L?$       Yes  
 $b \in L?$       No  
 $ba \in L?$



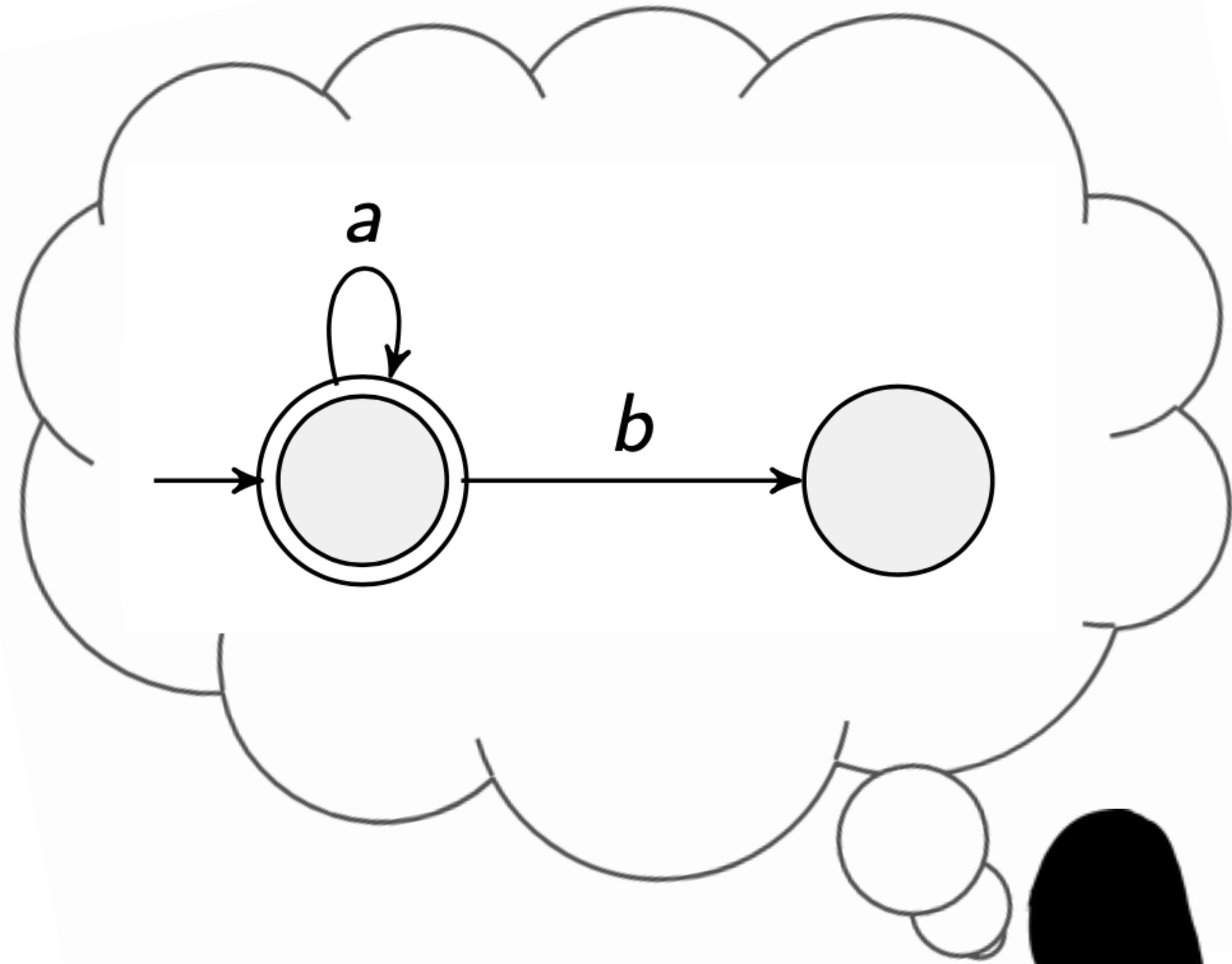
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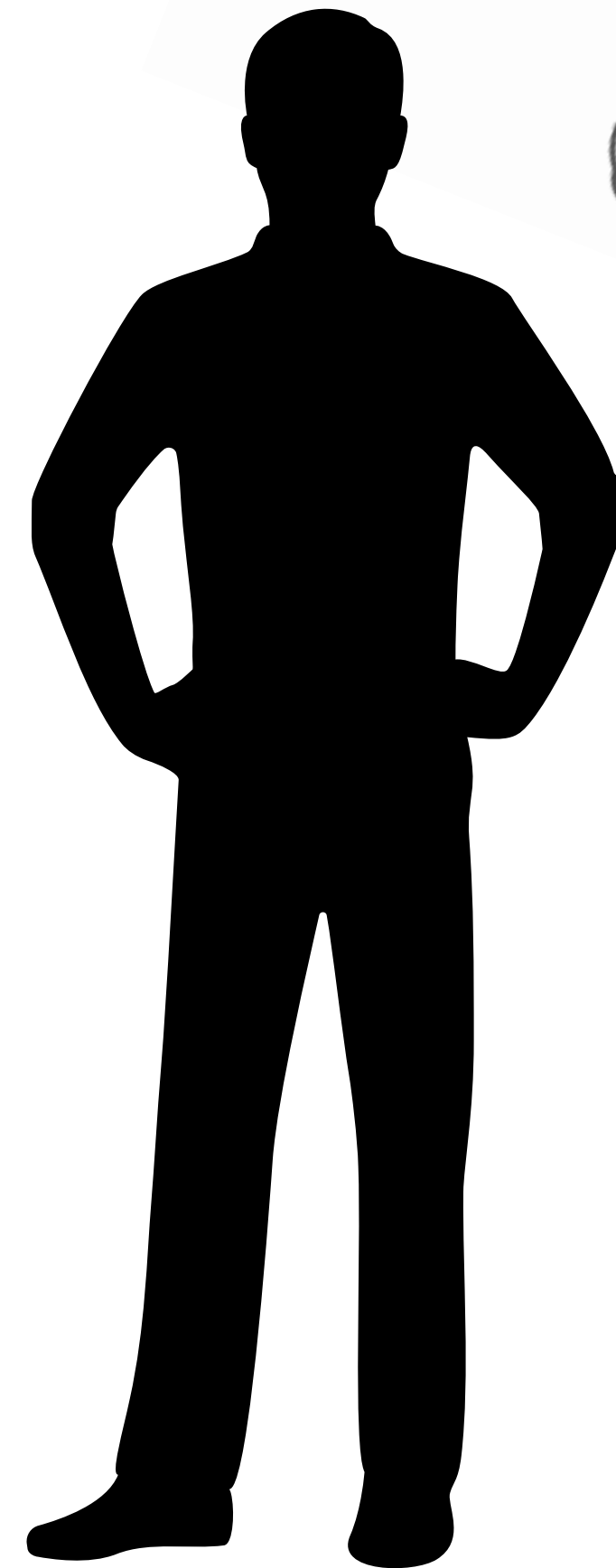
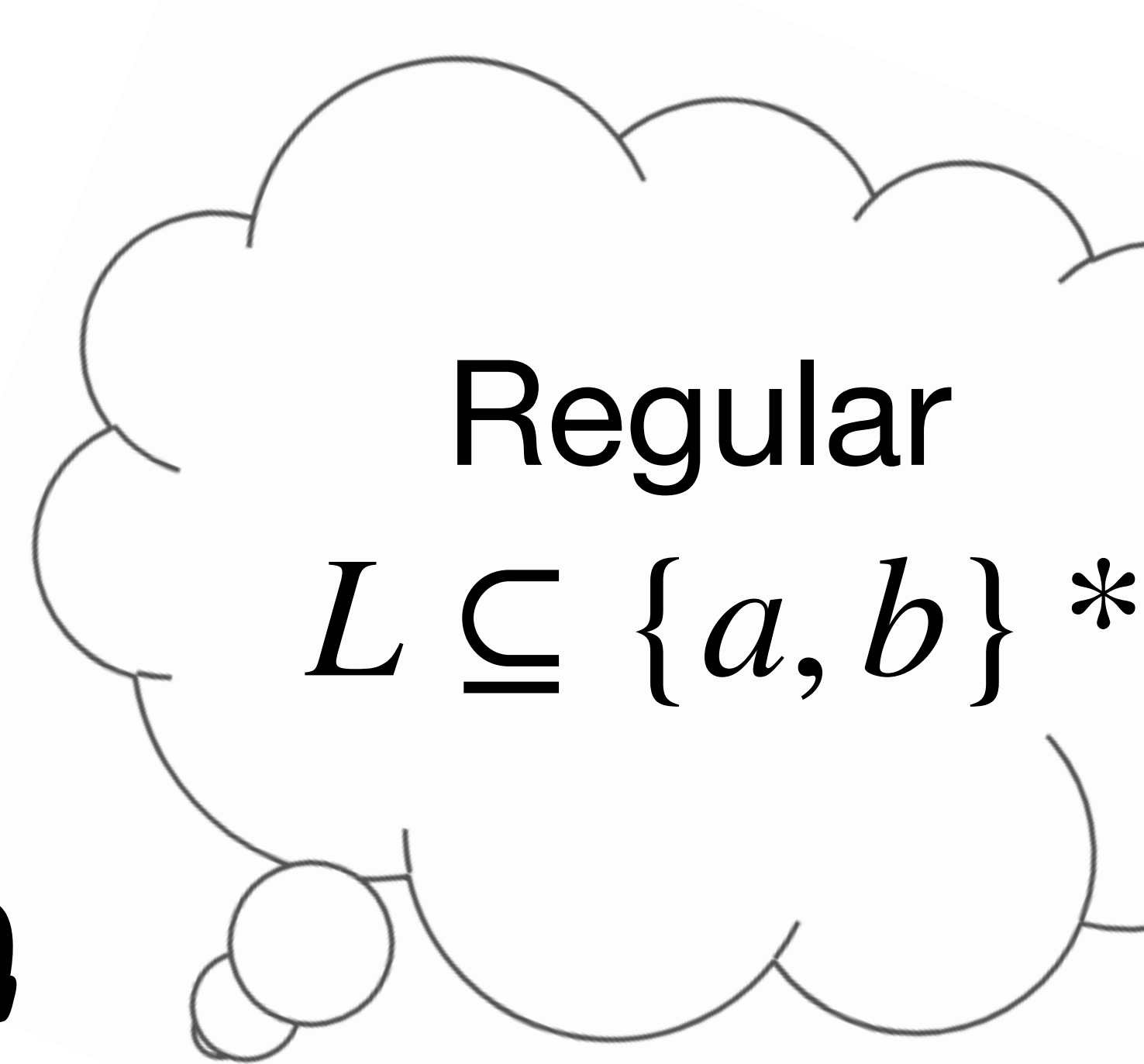
$\epsilon \in L?$       Yes  
 $a \in L?$       Yes  
 $b \in L?$       No  
 $ba \in L?$       Yes



1. You can ask if a given string is in L
2. You can guess a DFA for L

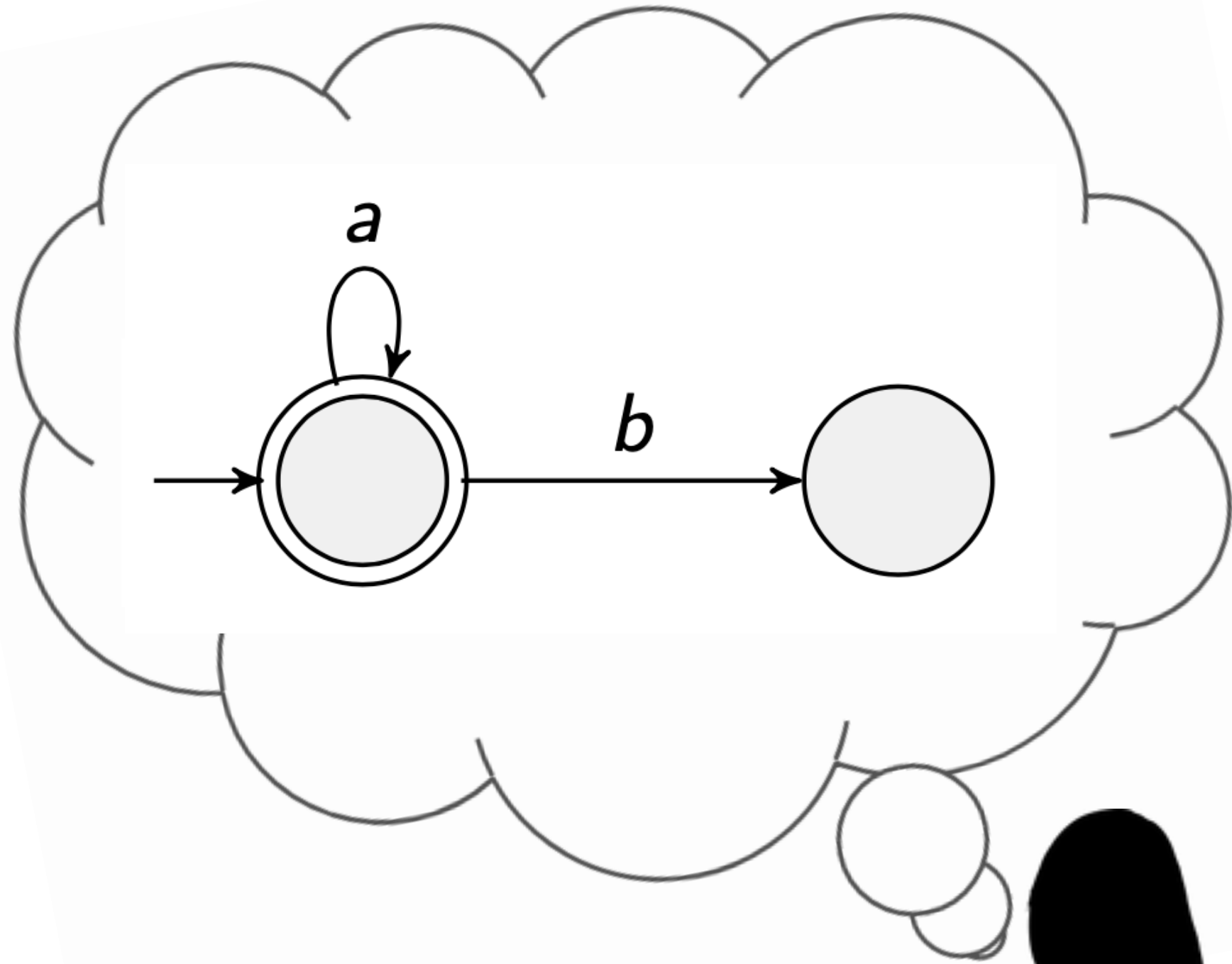


$\epsilon \in L?$       Yes  
 $a \in L?$       Yes  
 $b \in L?$       No  
 $ba \in L?$       Yes  
 $bb \in L?$

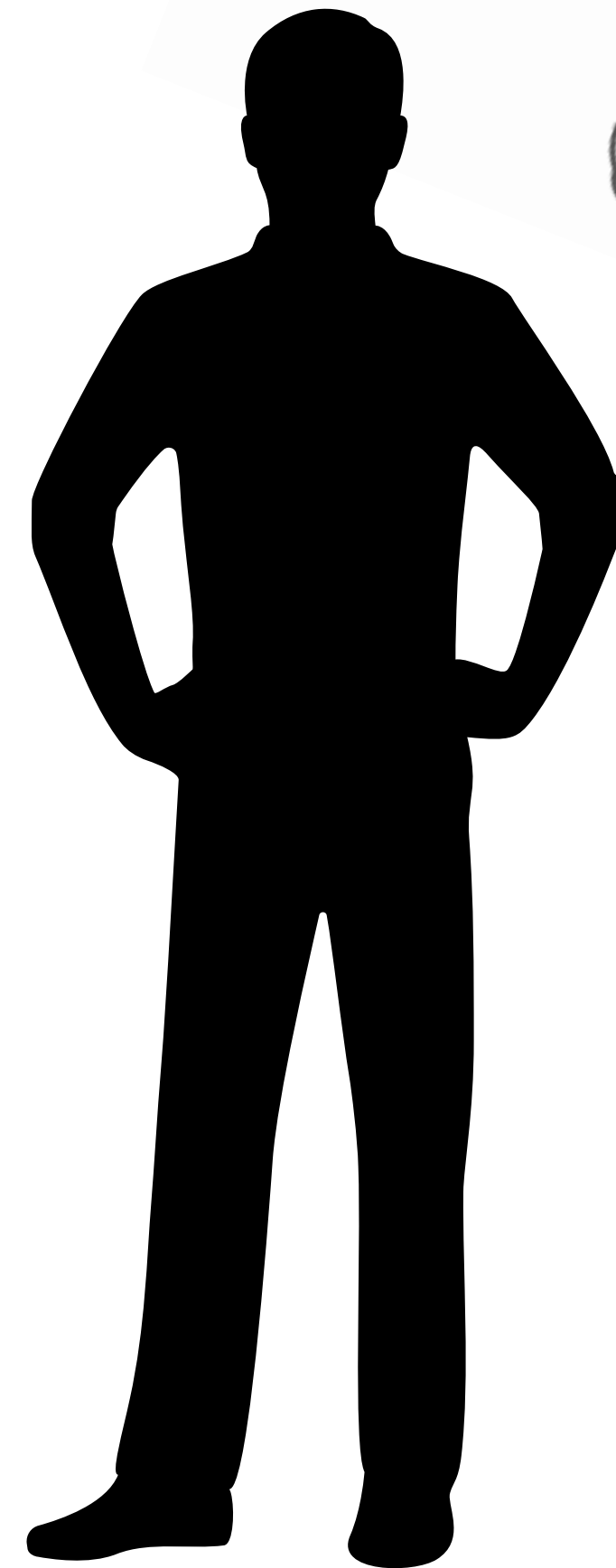
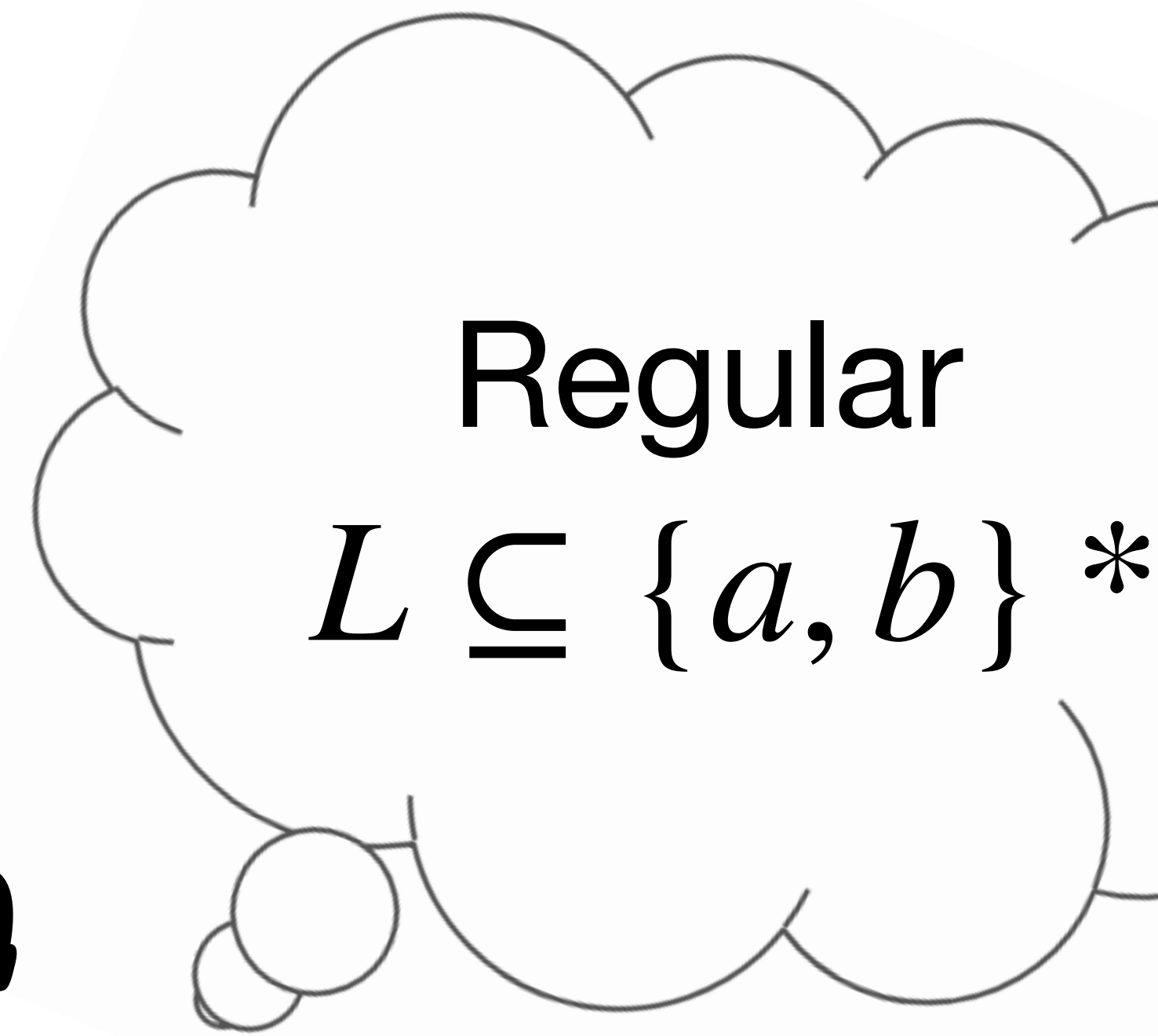


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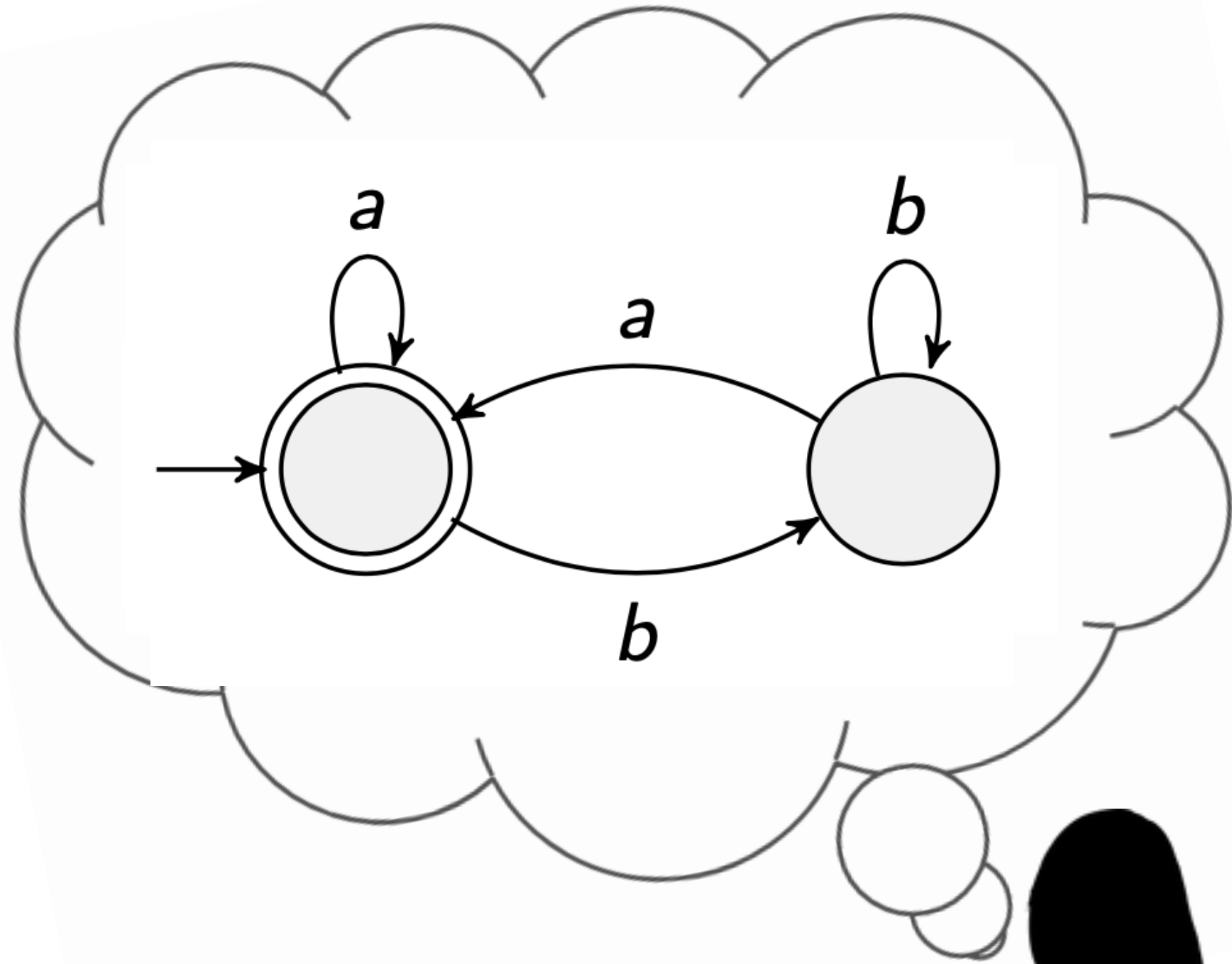




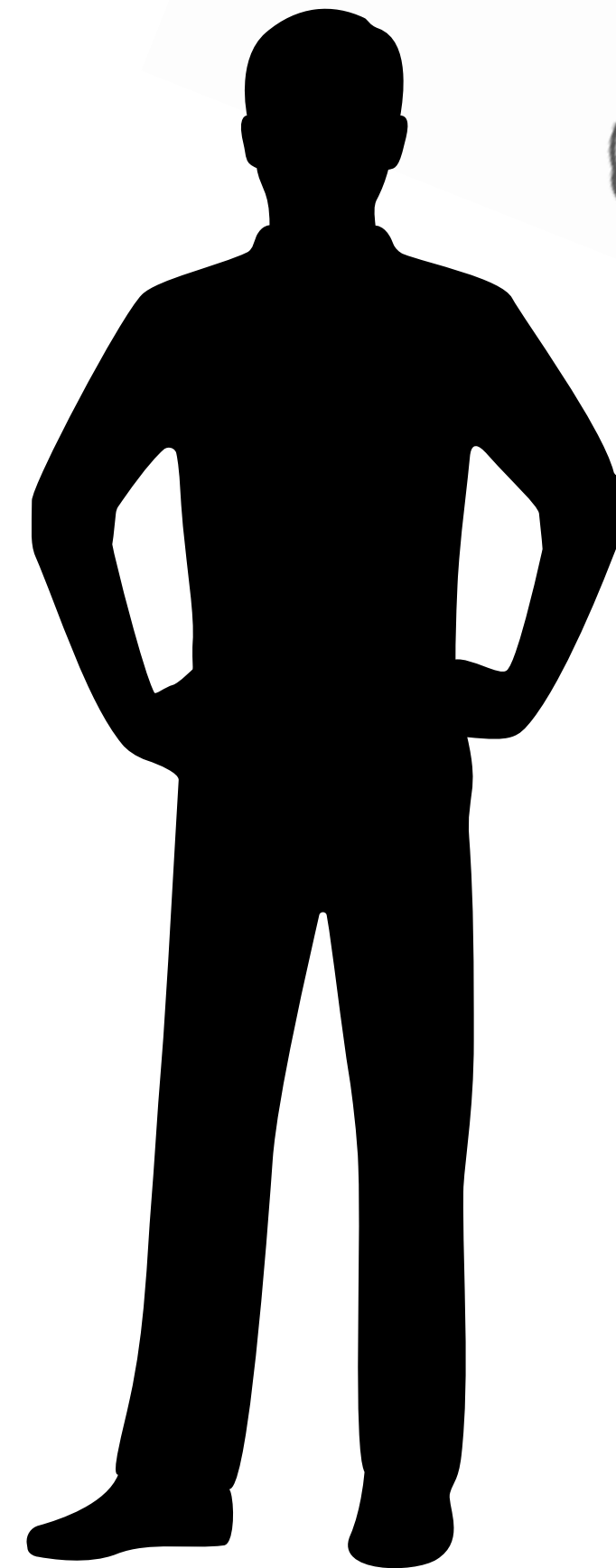
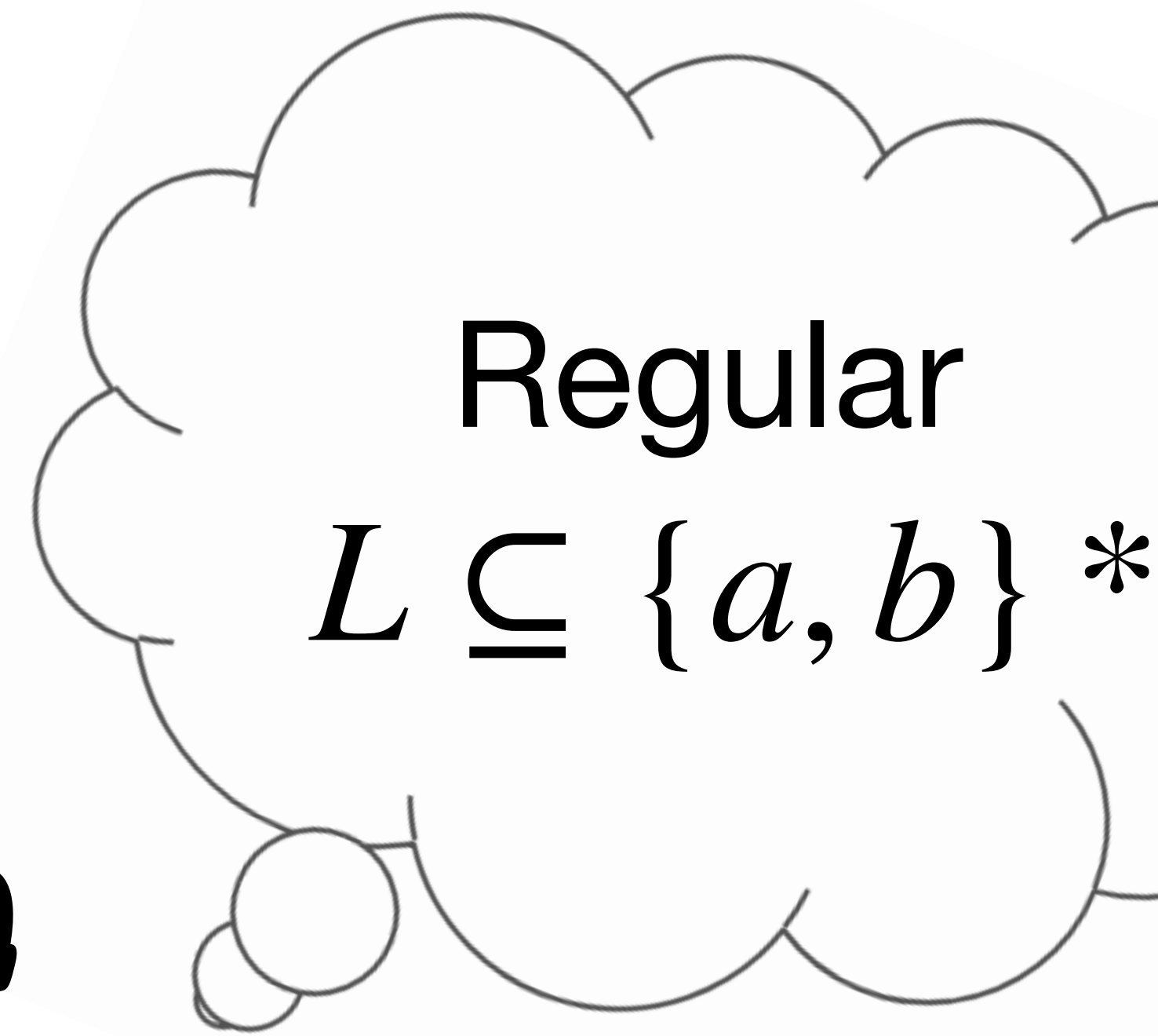
$\epsilon \in L?$	Yes
$a \in L?$	Yes
$b \in L?$	No
$ba \in L?$	Yes
$bb \in L?$	No



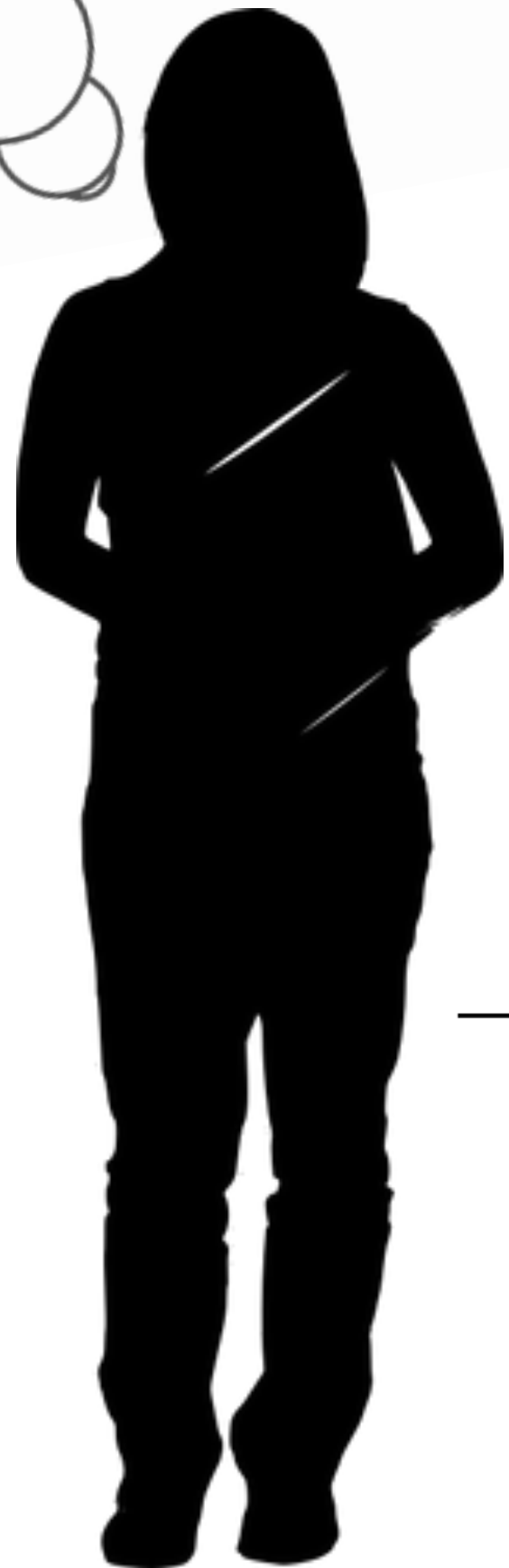
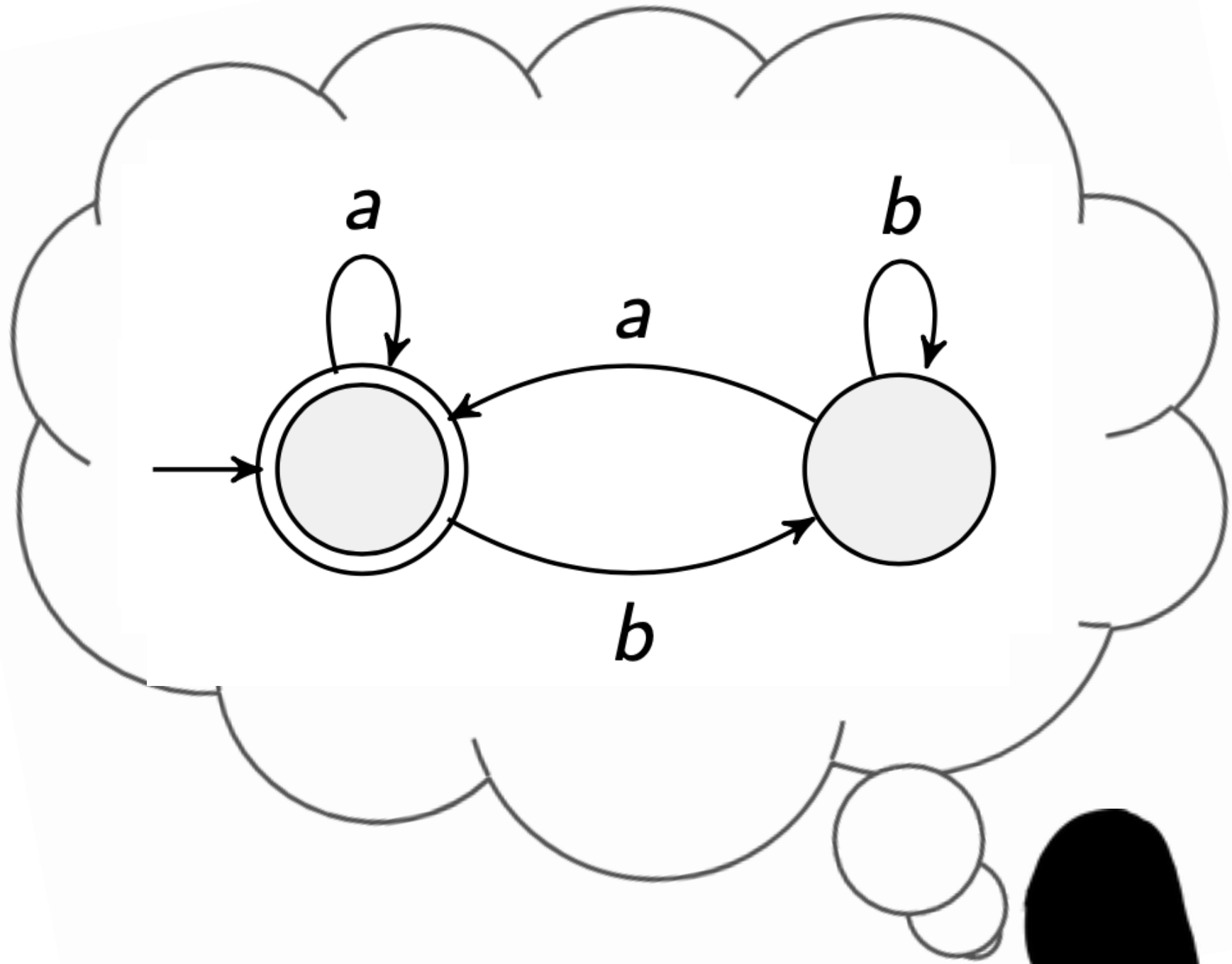
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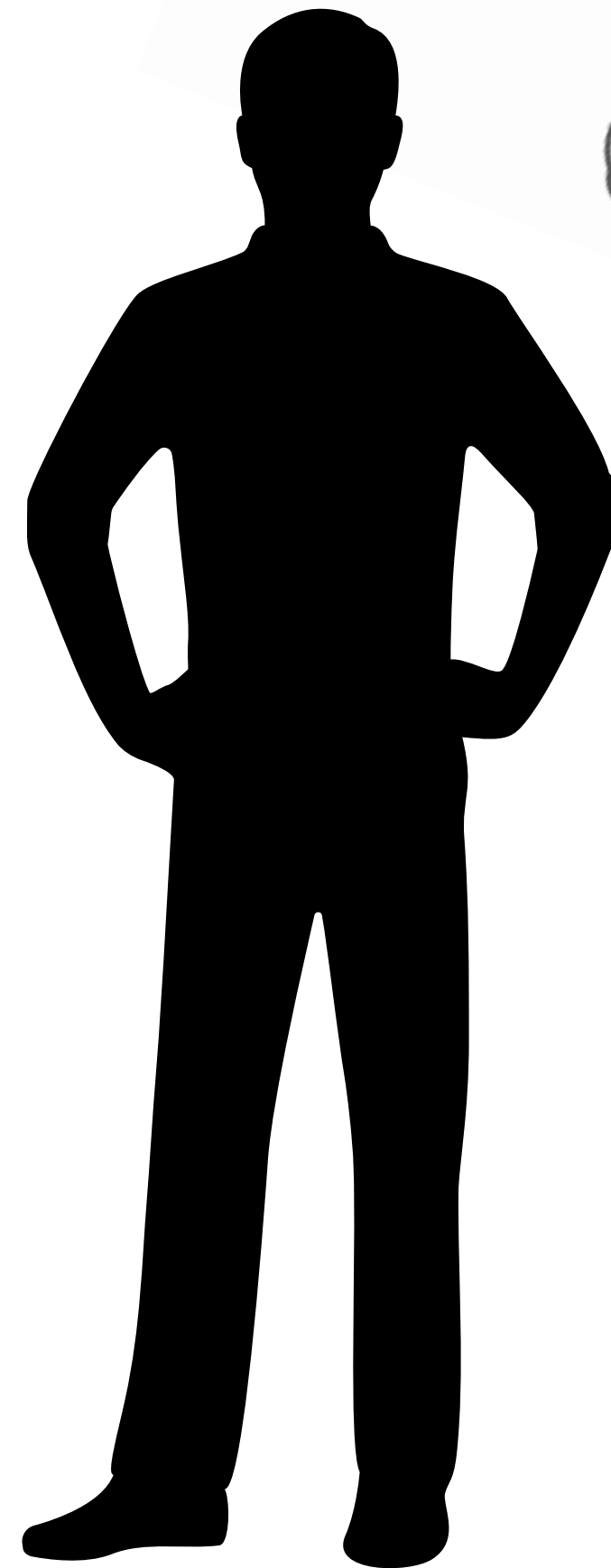
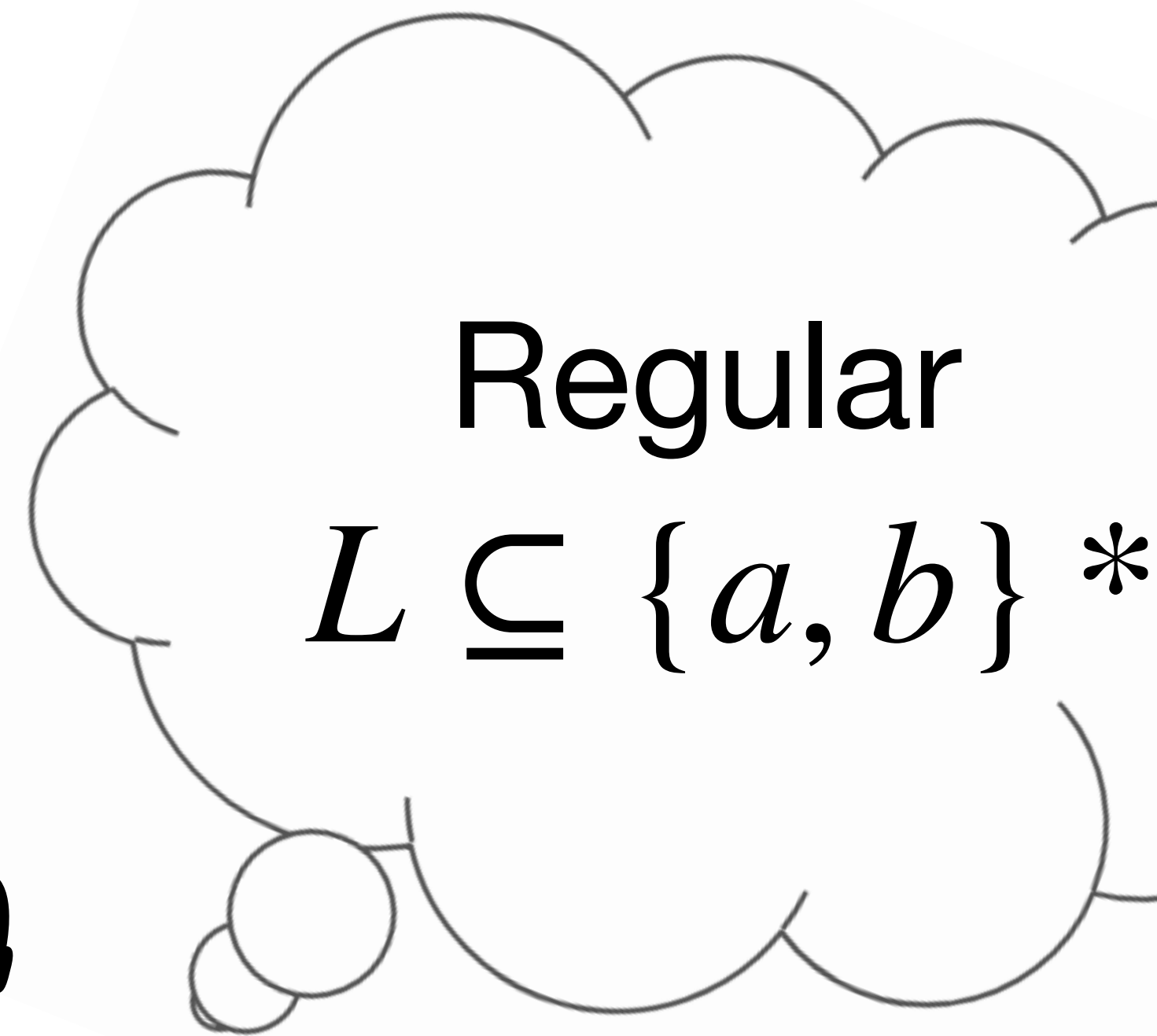
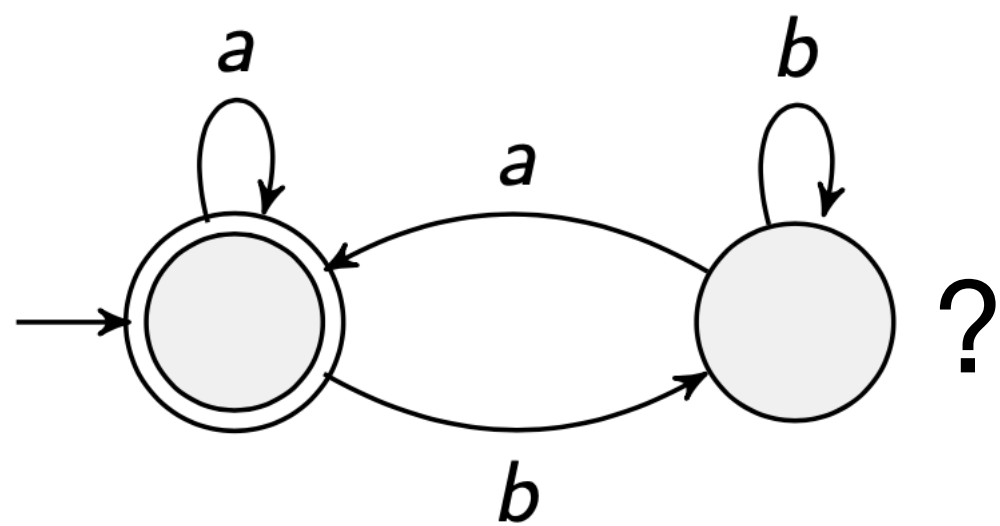
$\epsilon \in L?$	Yes
$a \in L?$	Yes
$b \in L?$	No
$ba \in L?$	Yes
$bb \in L?$	No



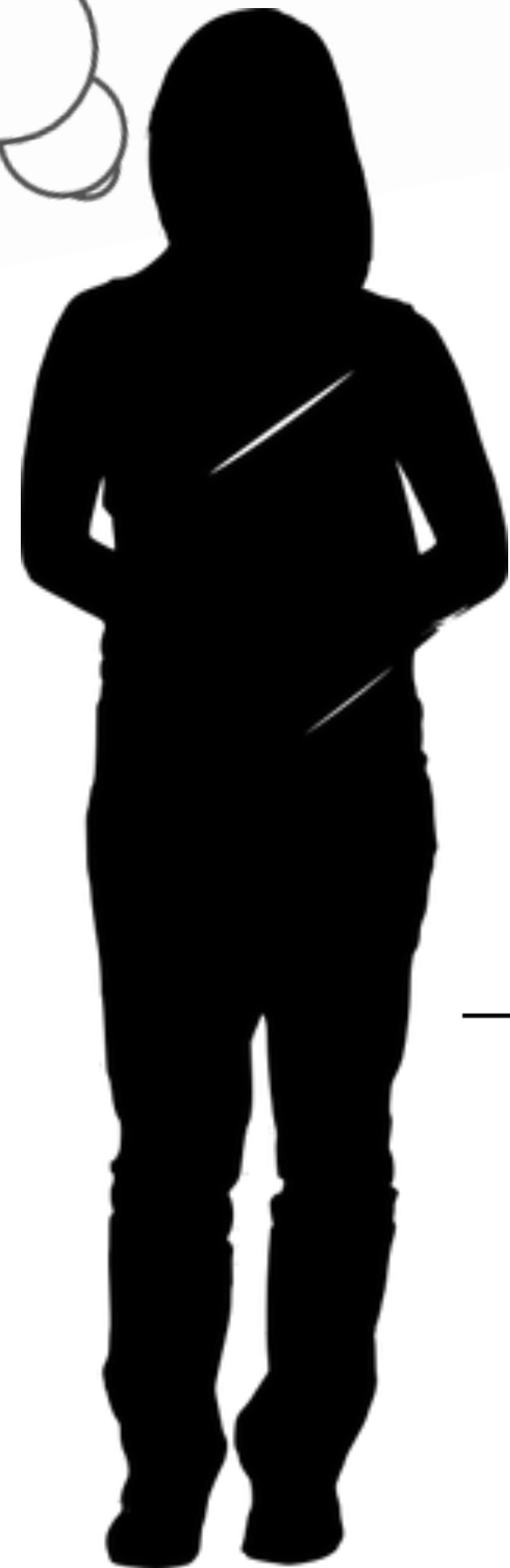
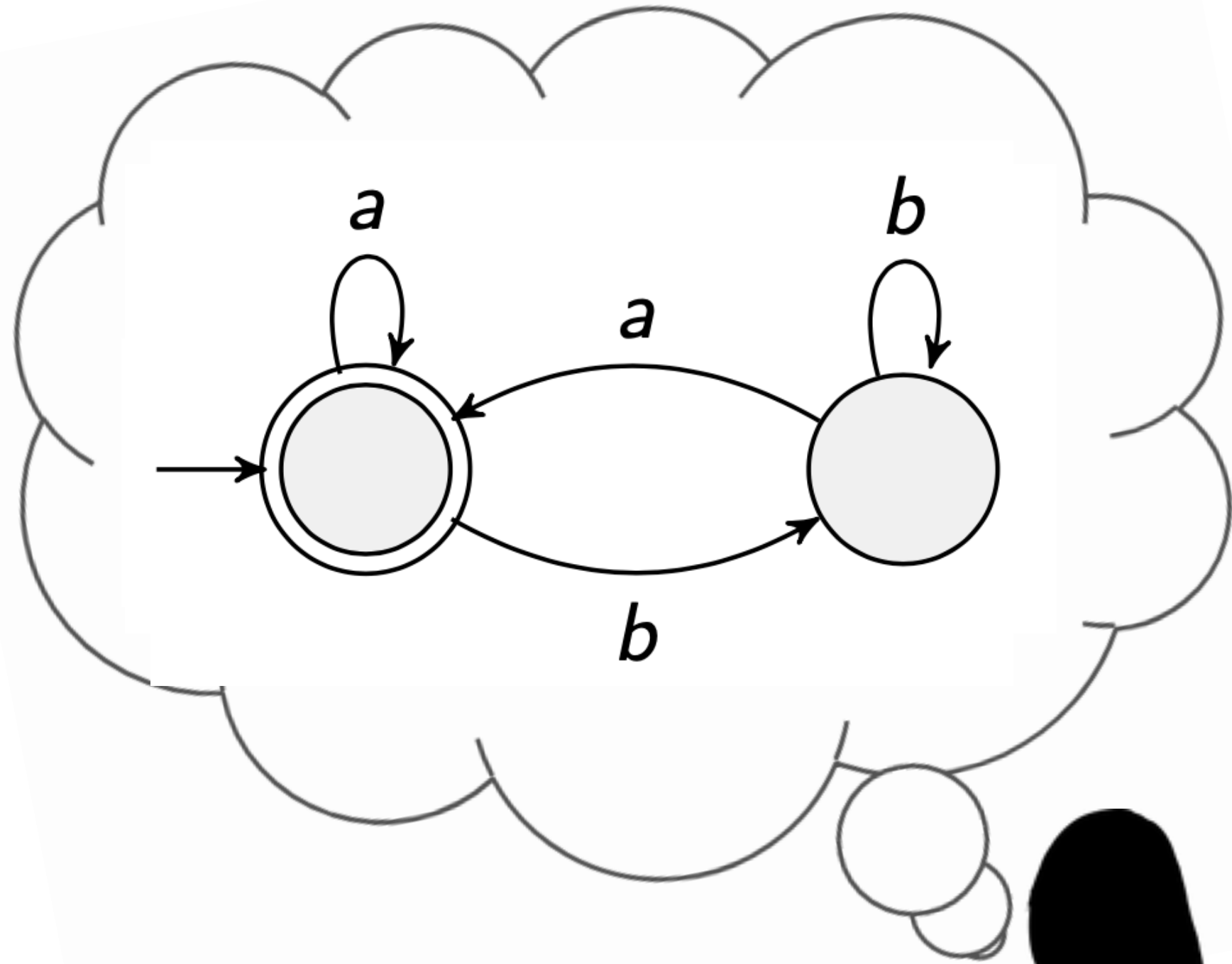
1. You can ask if a given string is in L
2. You can guess a DFA for L



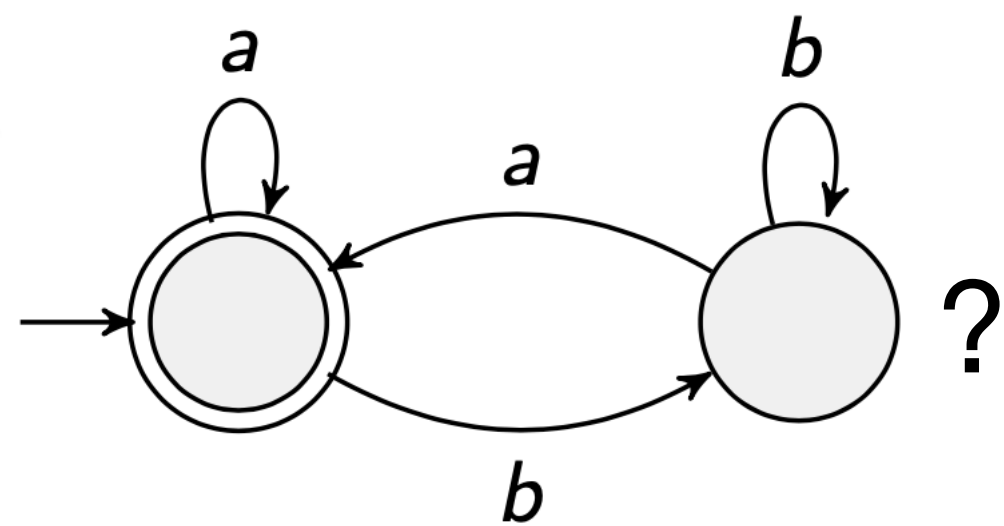
$\epsilon \in L?$  Yes  
 $a \in L?$  Yes  
 $b \in L?$  No  
 $ba \in L?$  Yes  
 $bb \in L?$  No



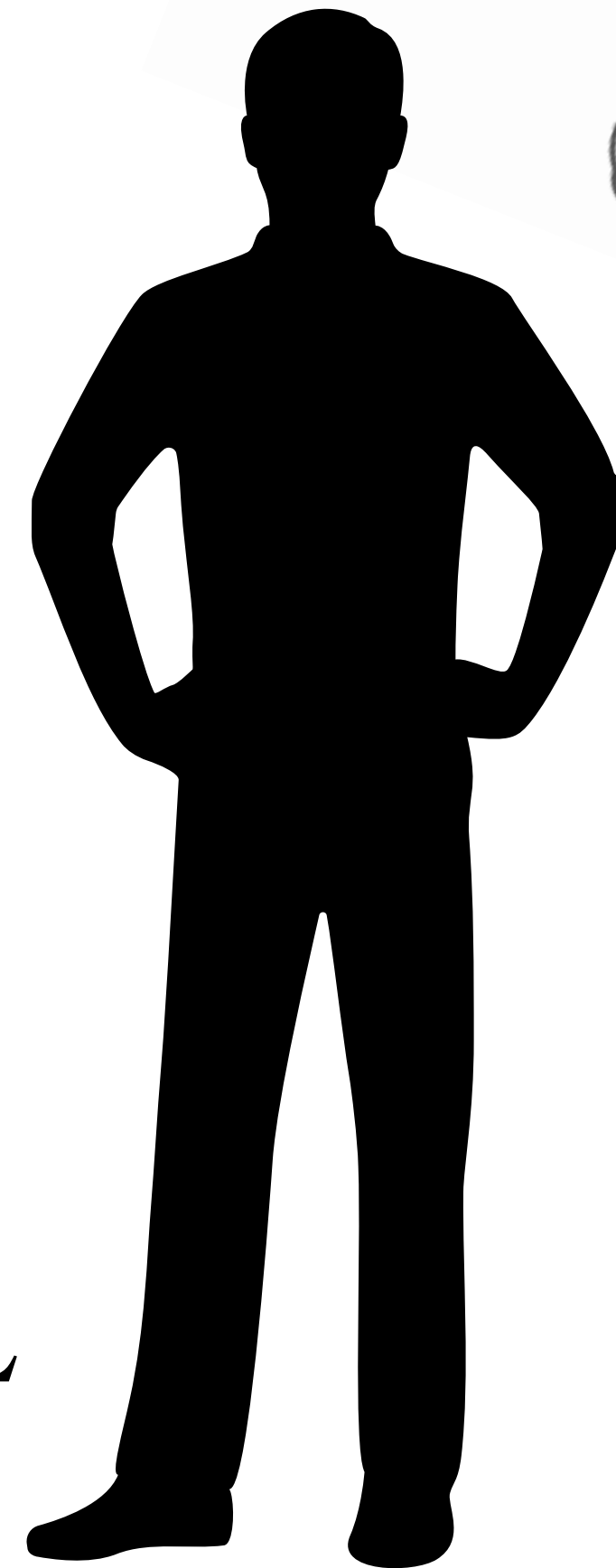
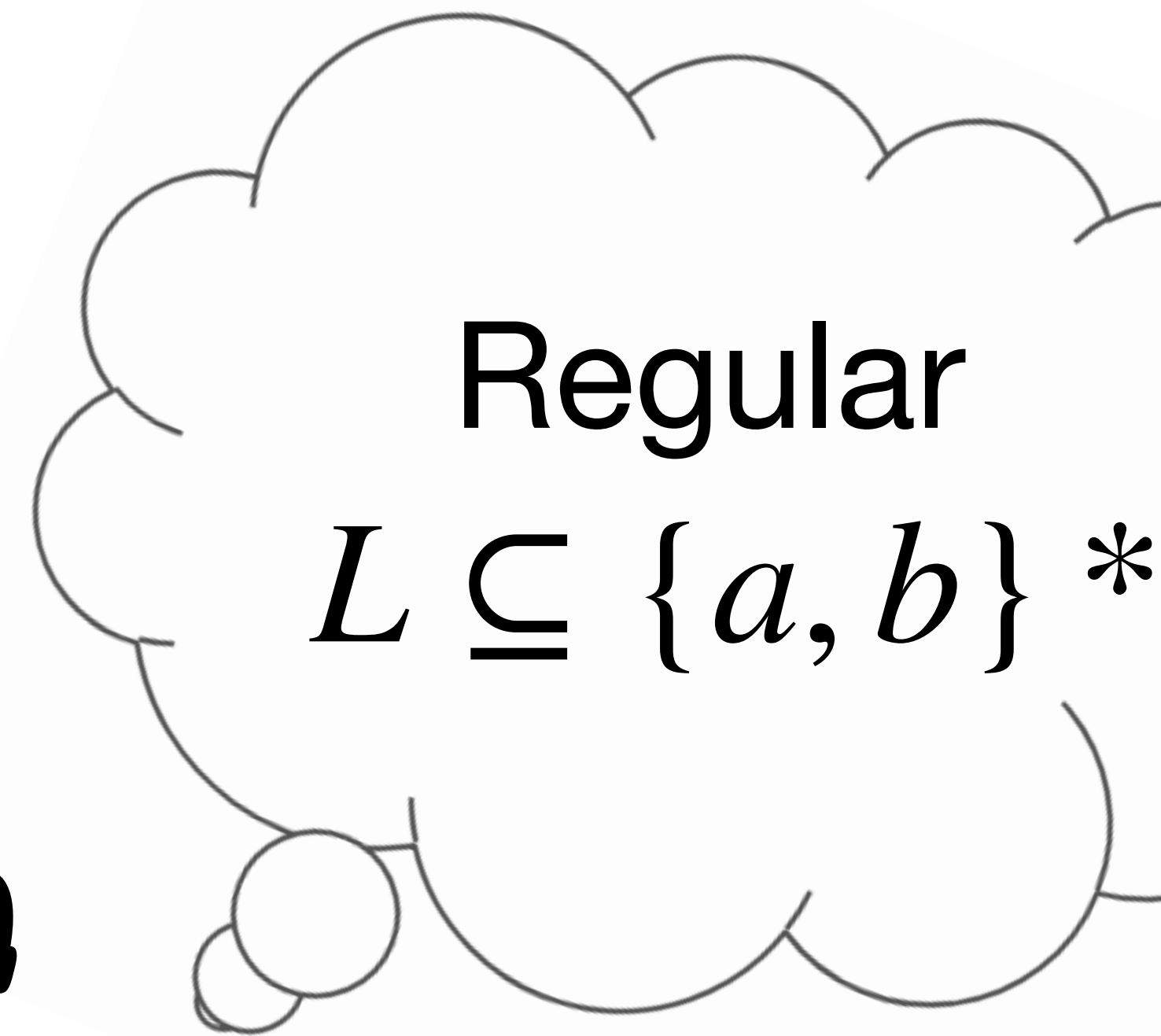
1. You can ask if a given string is in L
2. You can guess a DFA for L



$\epsilon \in L?$	Yes
$a \in L?$	Yes
$b \in L?$	No
$ba \in L?$	Yes
$bb \in L?$	No



No,  $bbb \in L$



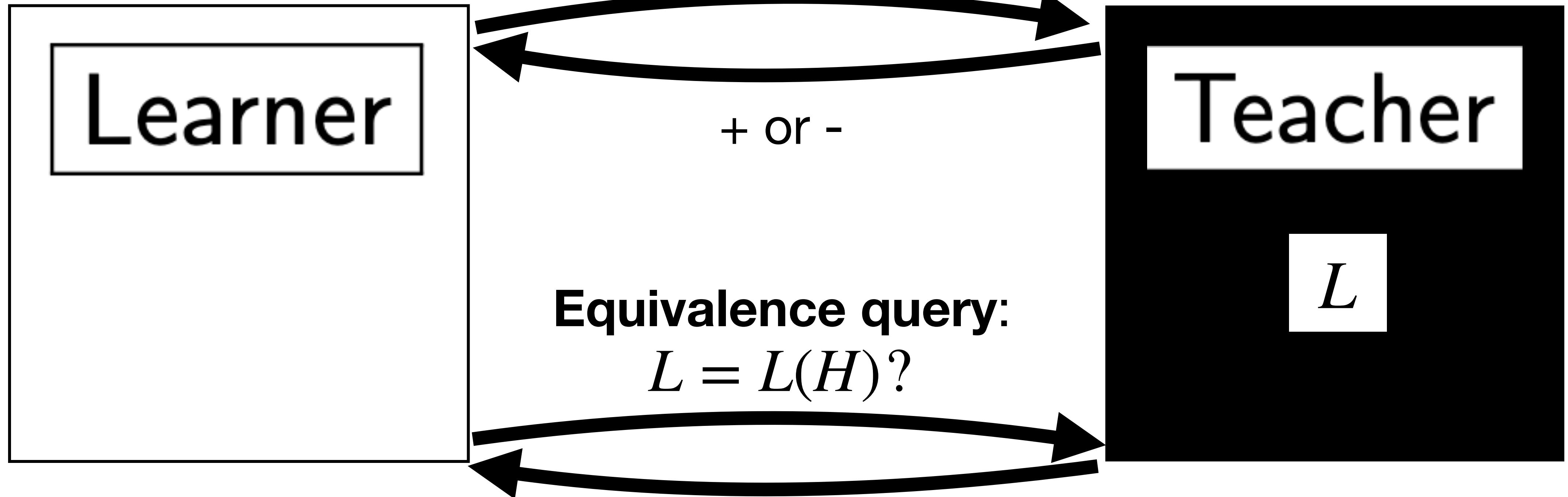
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# Minimally Adequate Teacher (MAT) Framework

(Angluin, 1987)

Membership query:

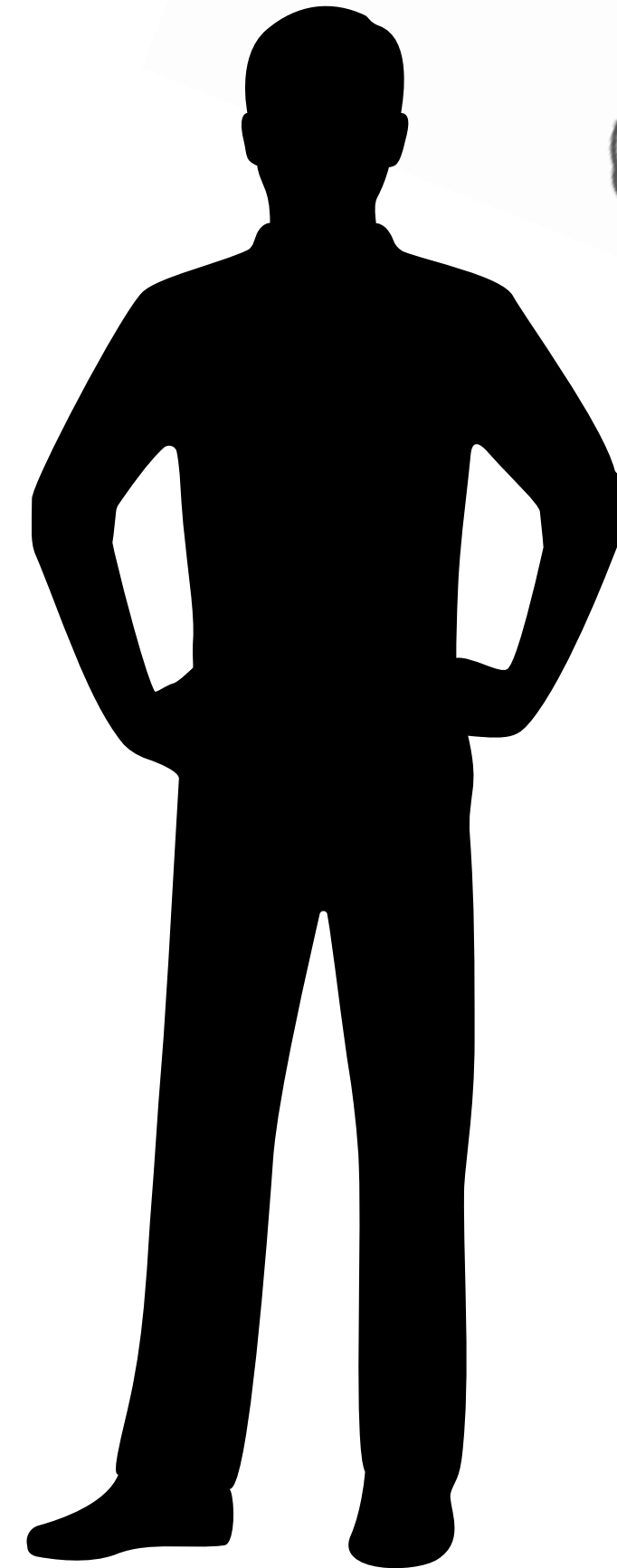
$$w \in L?$$

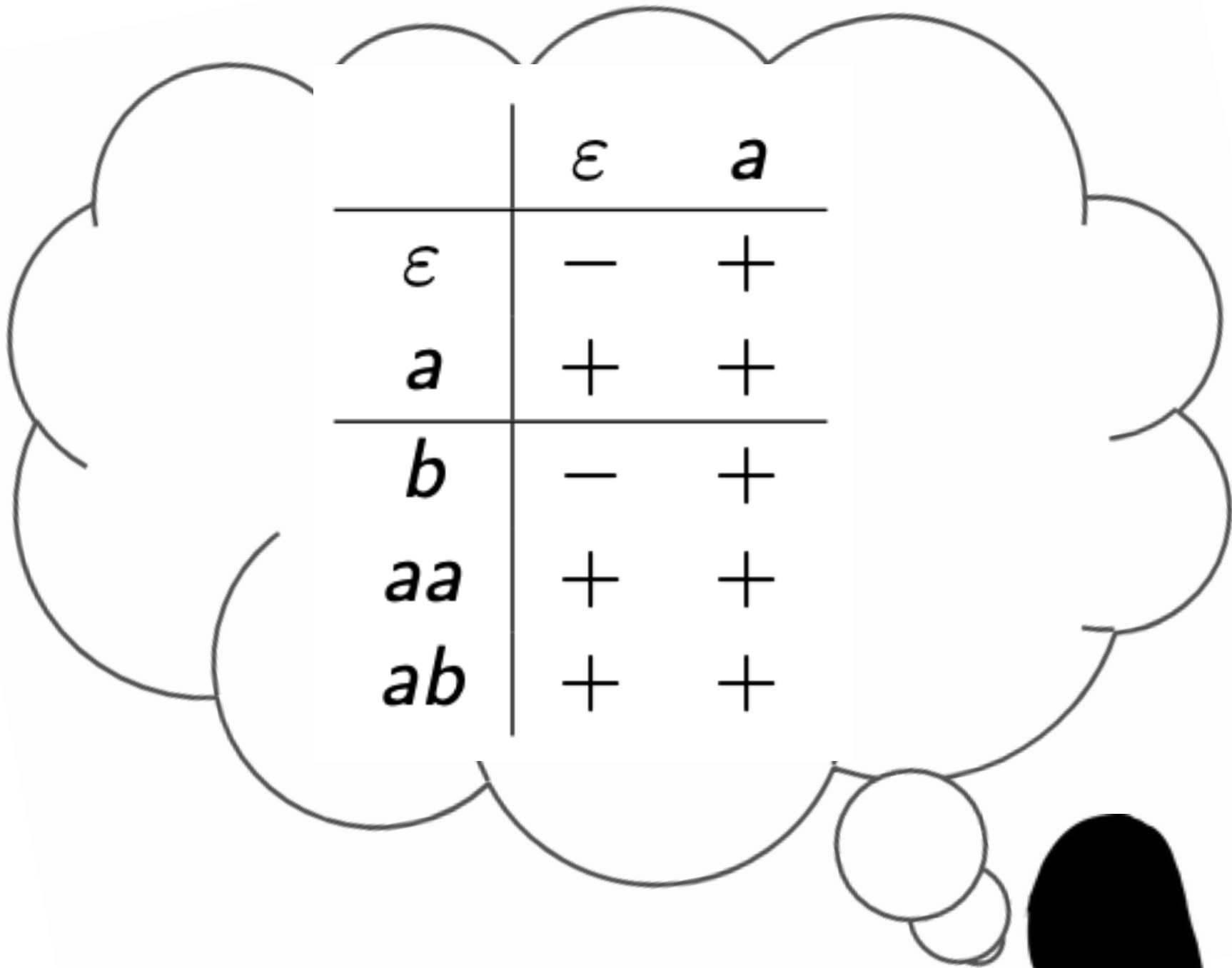


# Angluin's $L^*$

	$\varepsilon$	$a$
$\varepsilon$	-	+
$a$	+	+
$b$	-	+
$aa$	+	+
$ab$	+	+

Regular  
 $L \subseteq \{a, b\}^*$

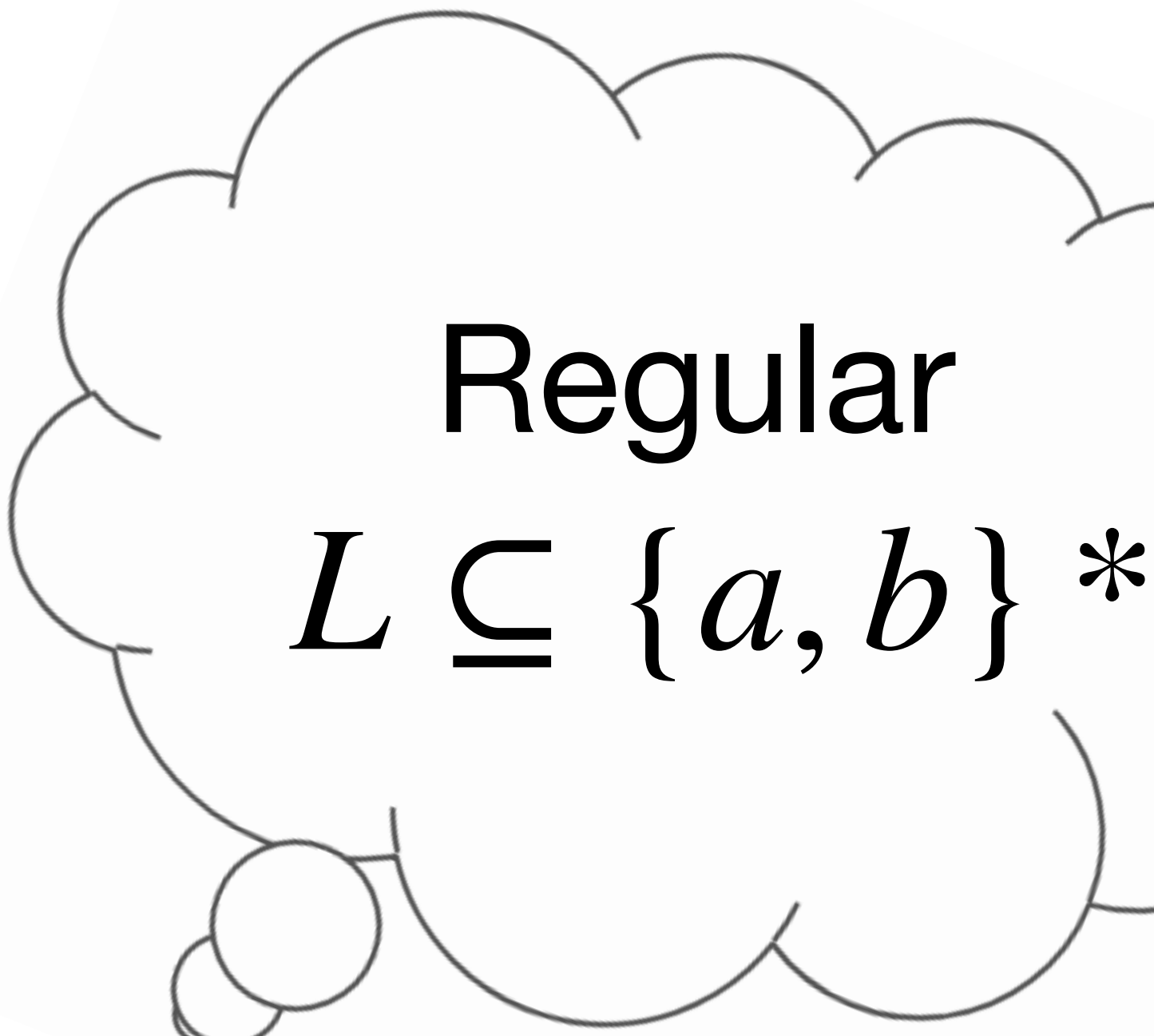




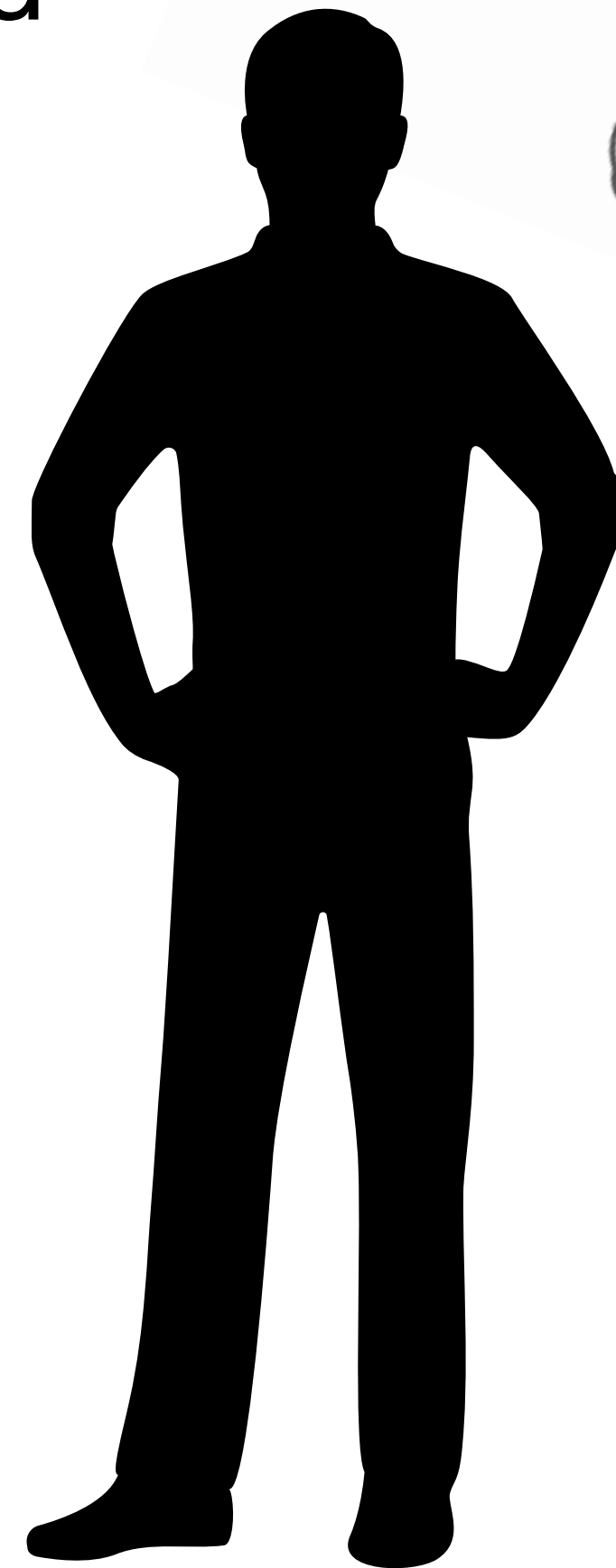
	$\varepsilon$	$a$
$\varepsilon$	-	+
$a$	+	+
$b$	-	+
$aa$	+	+
$ab$	+	+

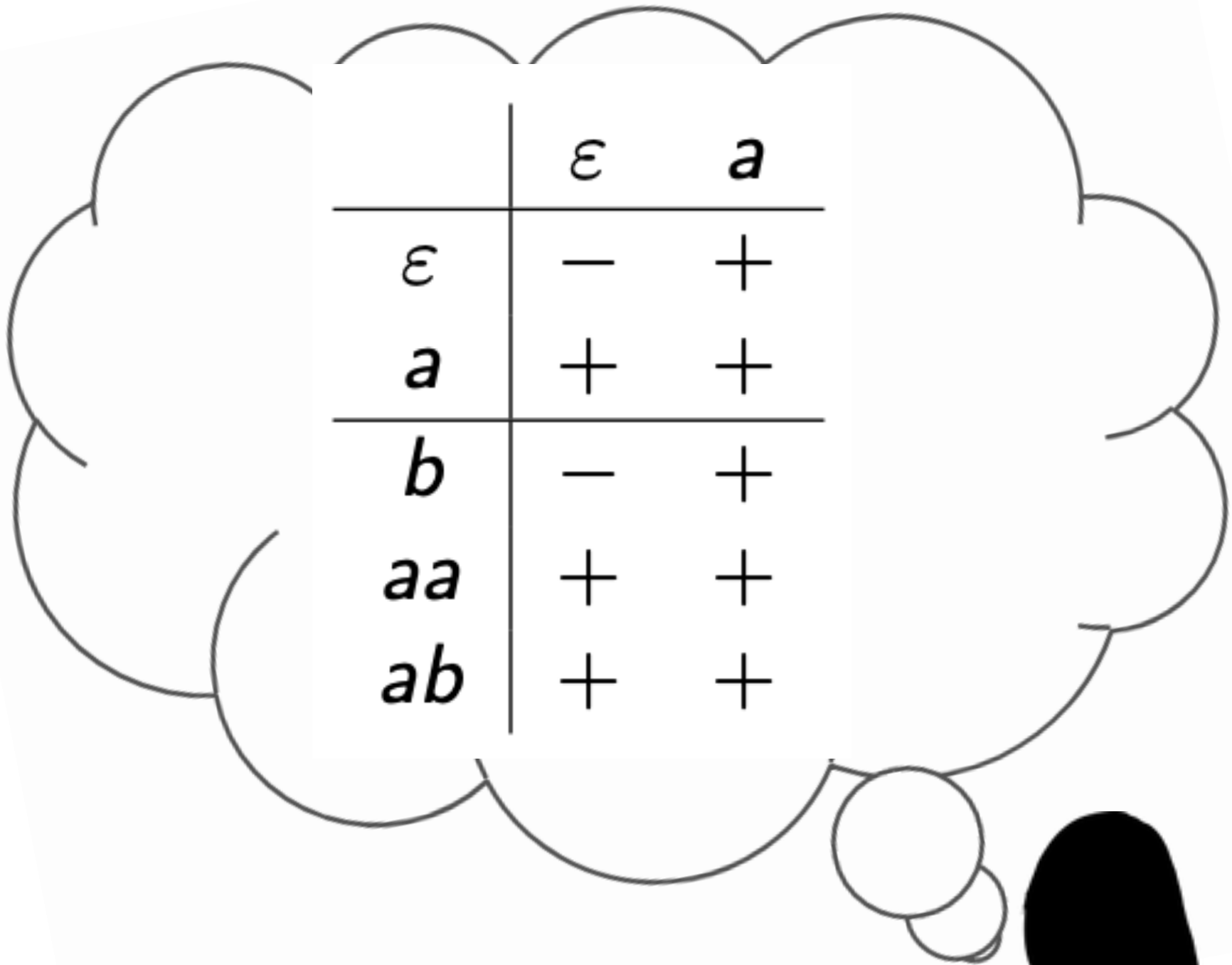
# Angluin's $L^*$

- Uses an *observation table* to guide queries and conjectures



Regular  
 $L \subseteq \{a, b\}^*$

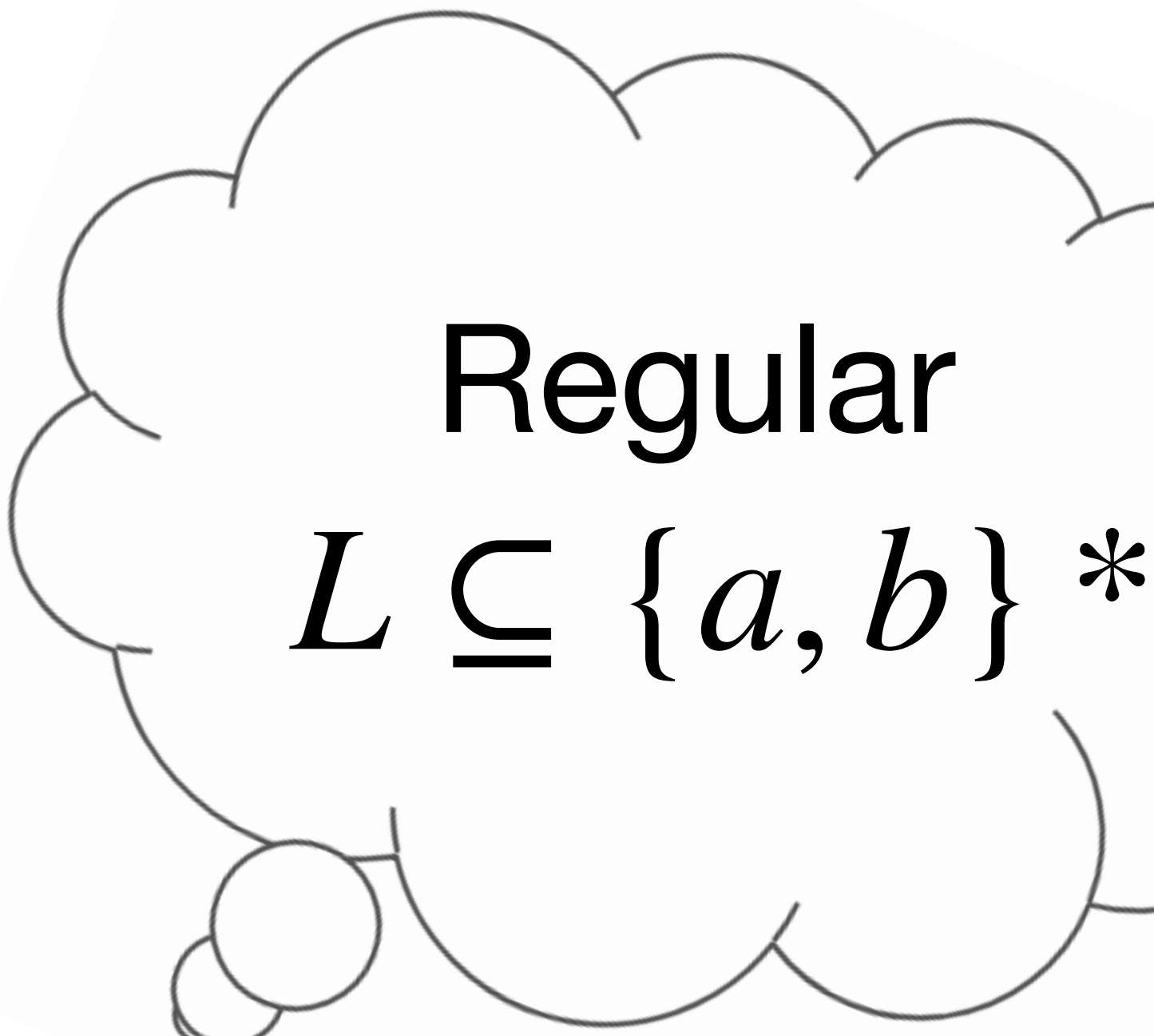




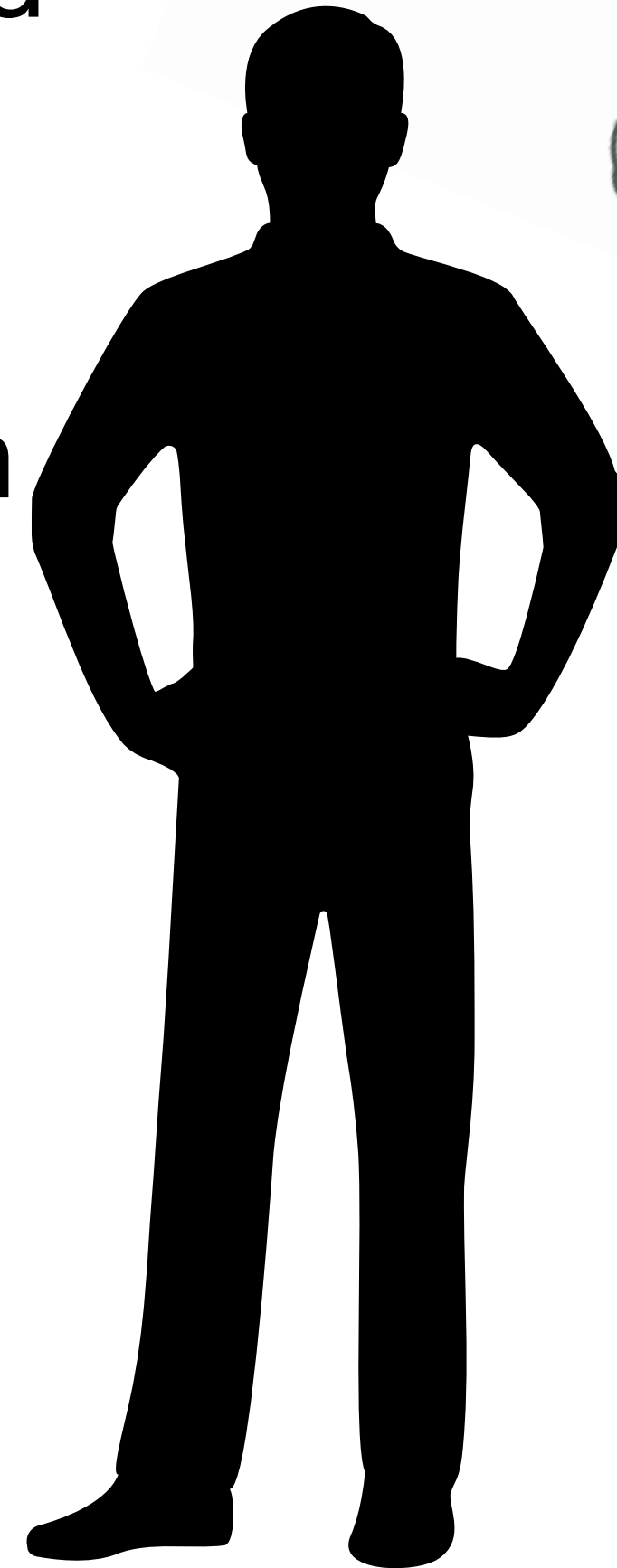
	$\varepsilon$	$a$
$\varepsilon$	-	+
$a$	+	+
$b$	-	+
$aa$	+	+
$ab$	+	+

# Angluin's $L^*$

- Uses an *observation table* to guide queries and conjectures
- Guaranteed to learn *minimal* DFA in polynomial queries



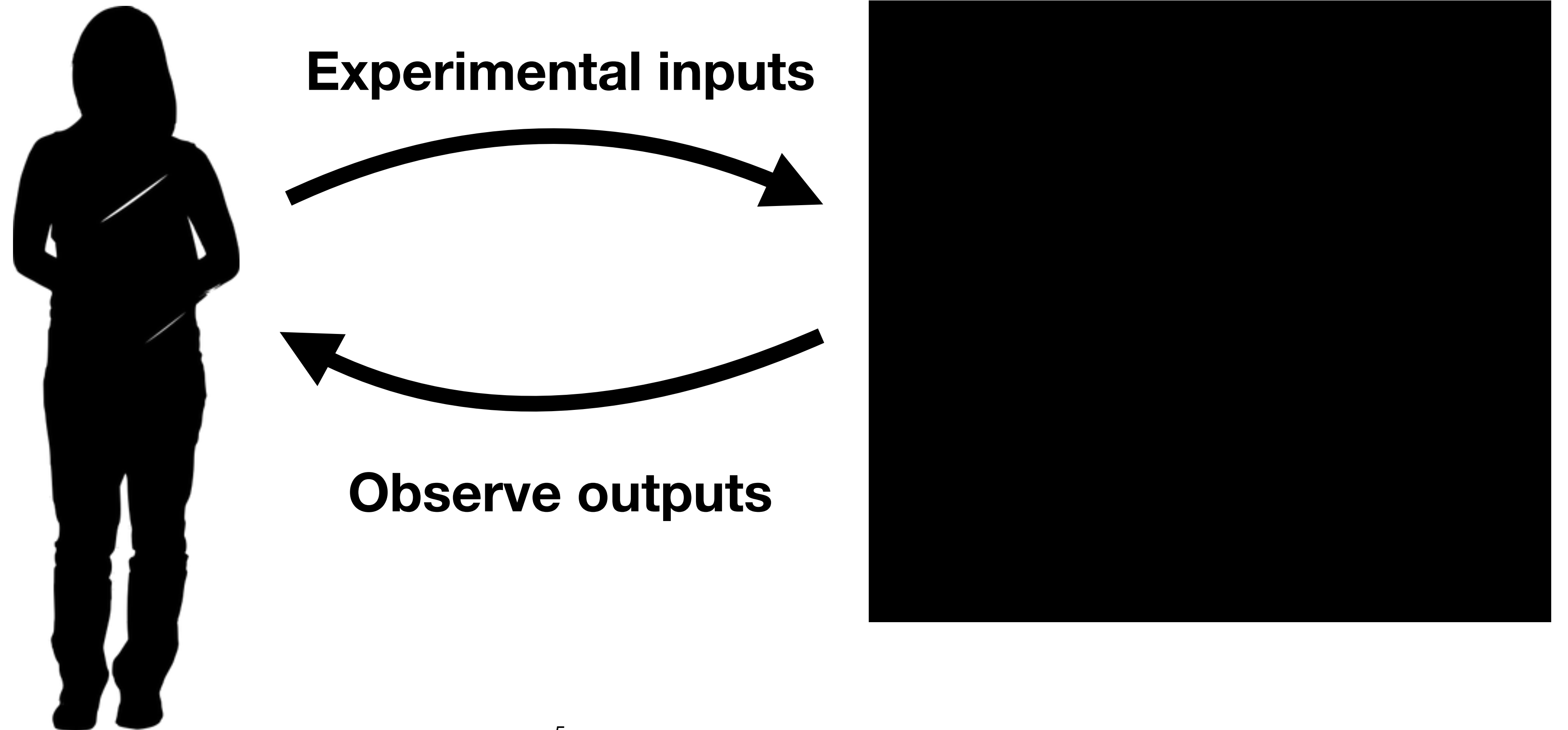
Regular  
 $L \subseteq \{a, b\}^*$





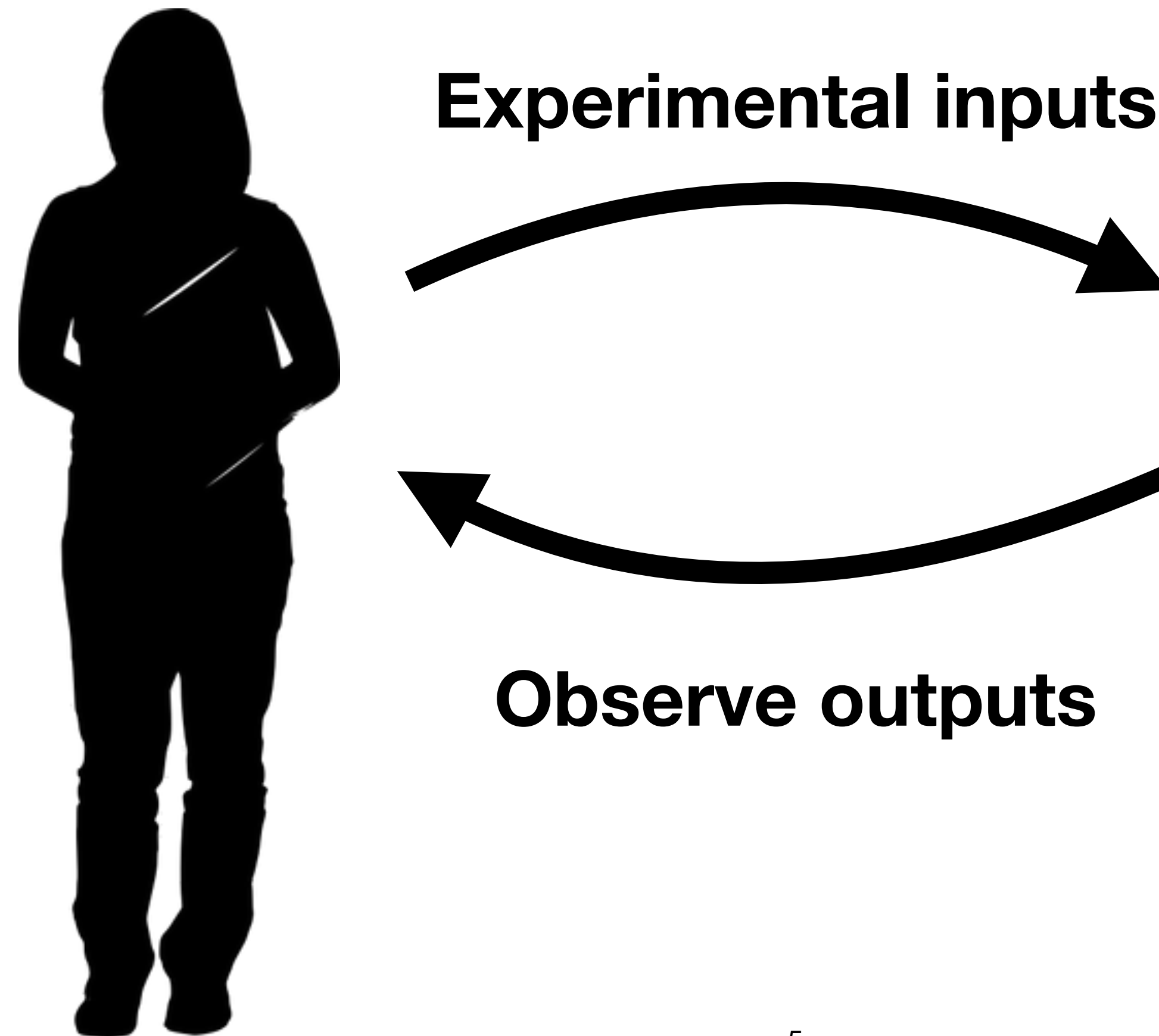
# Closed-box learning

$L^*$  in practice



# Closed-box learning

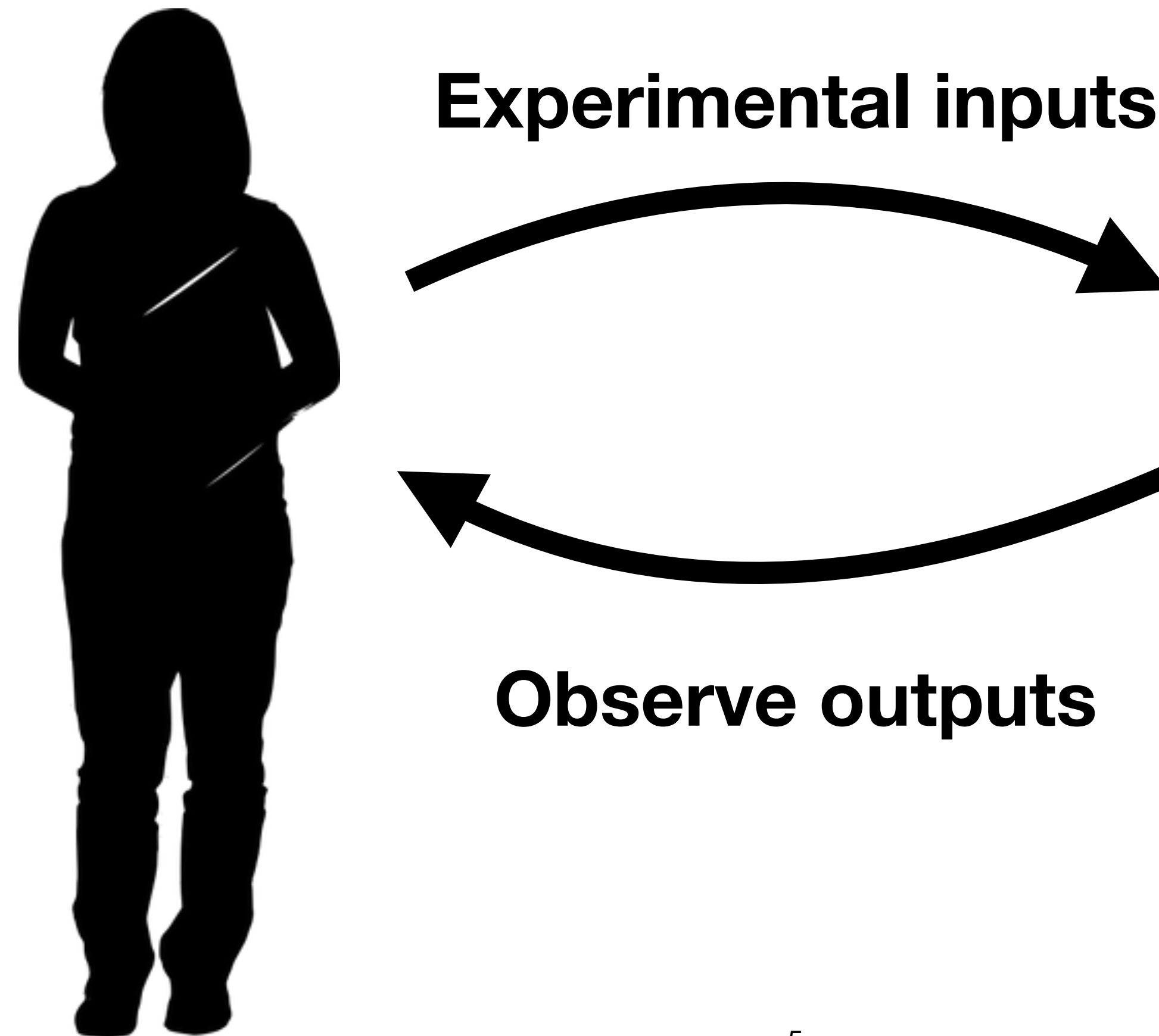
$L^*$  in practice



- TCP  
(Fiterău-Broștean et al, 2014)

# Closed-box learning

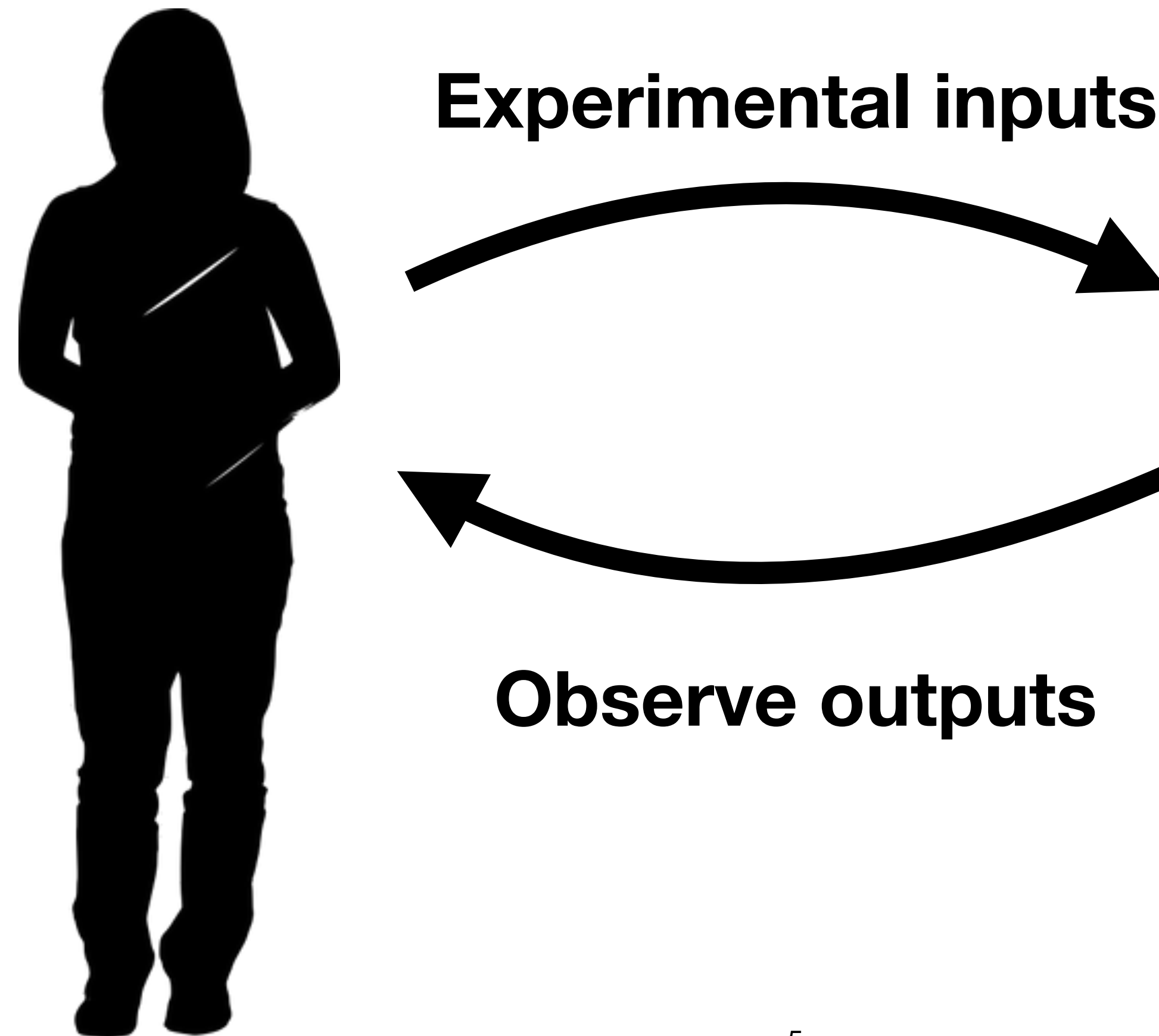
$L^*$  in practice



- TCP  
([Fiterău-Broștean et al, 2014](#))
- Smartcard Reader  
(Chalupar et al, 2014)

# Closed-box learning

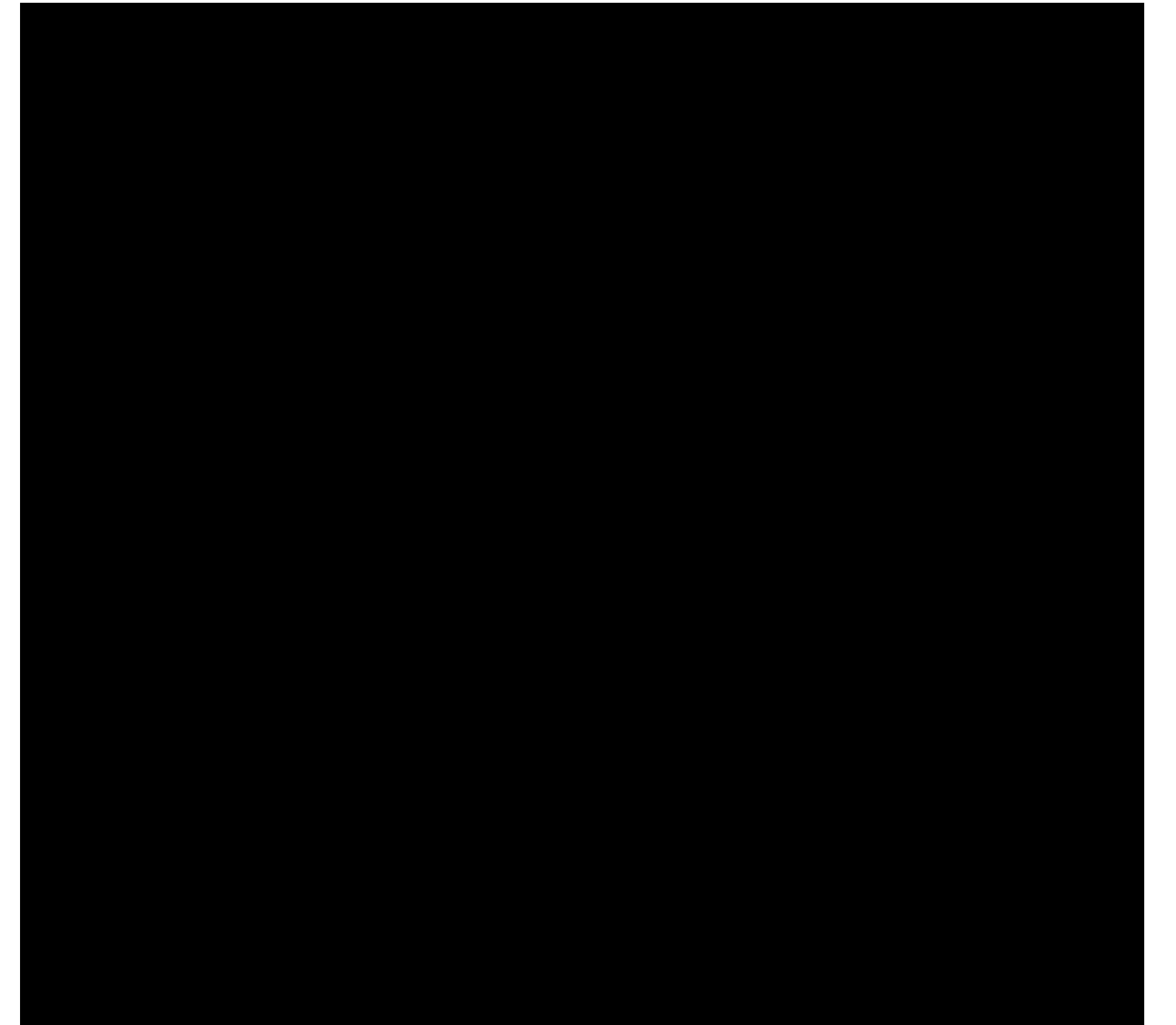
$L^*$  in practice



- TCP  
([Fiterău-Broștean et al, 2014](#))
- Smartcard Reader  
(Chalupar et al, 2014)
- Java interface specifications  
(Alur et al, 2005)

# Closed-box learning

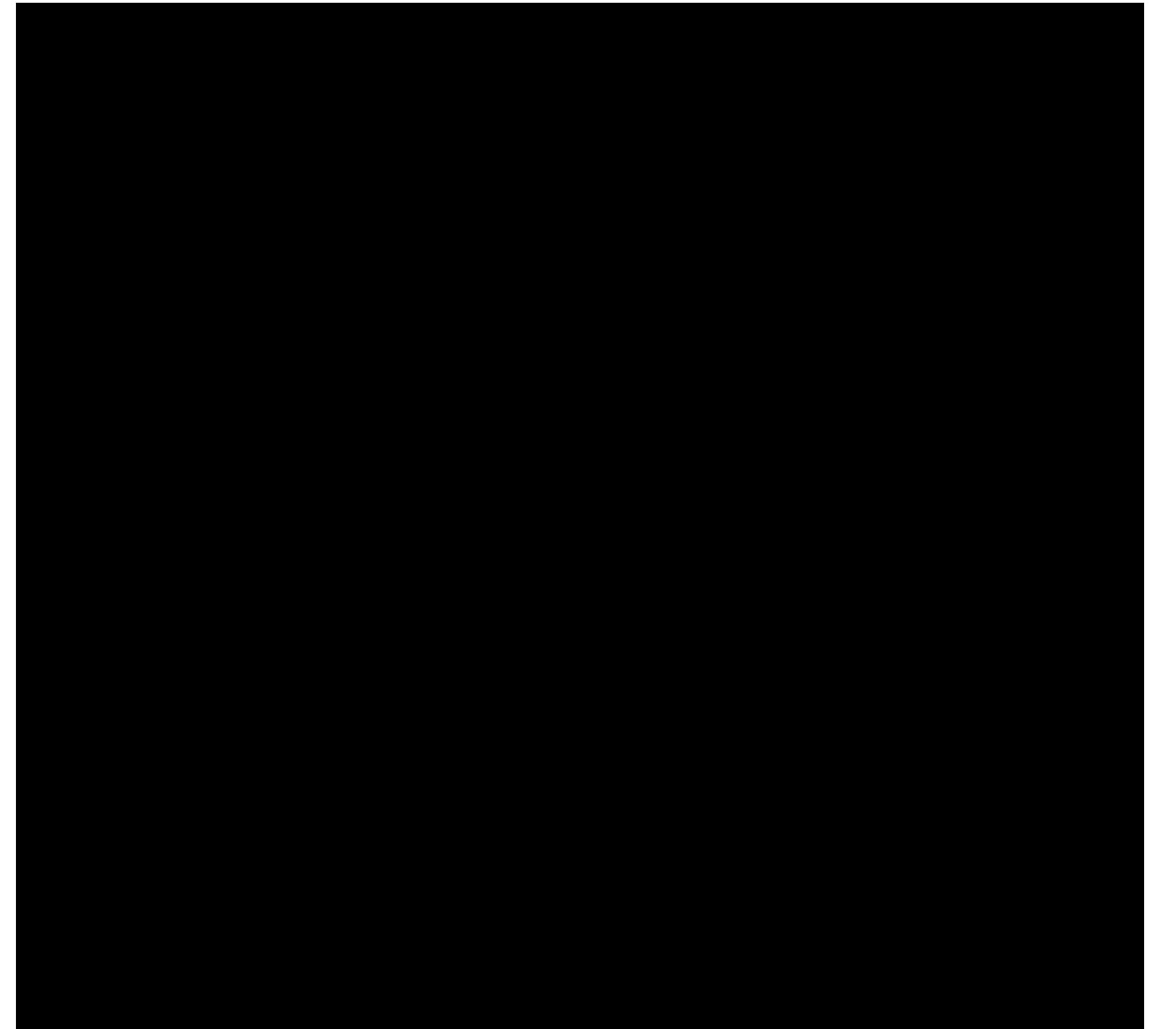
$L^*$  in practice



# Closed-box learning

$L^*$  in practice

$\varepsilon \in L?$

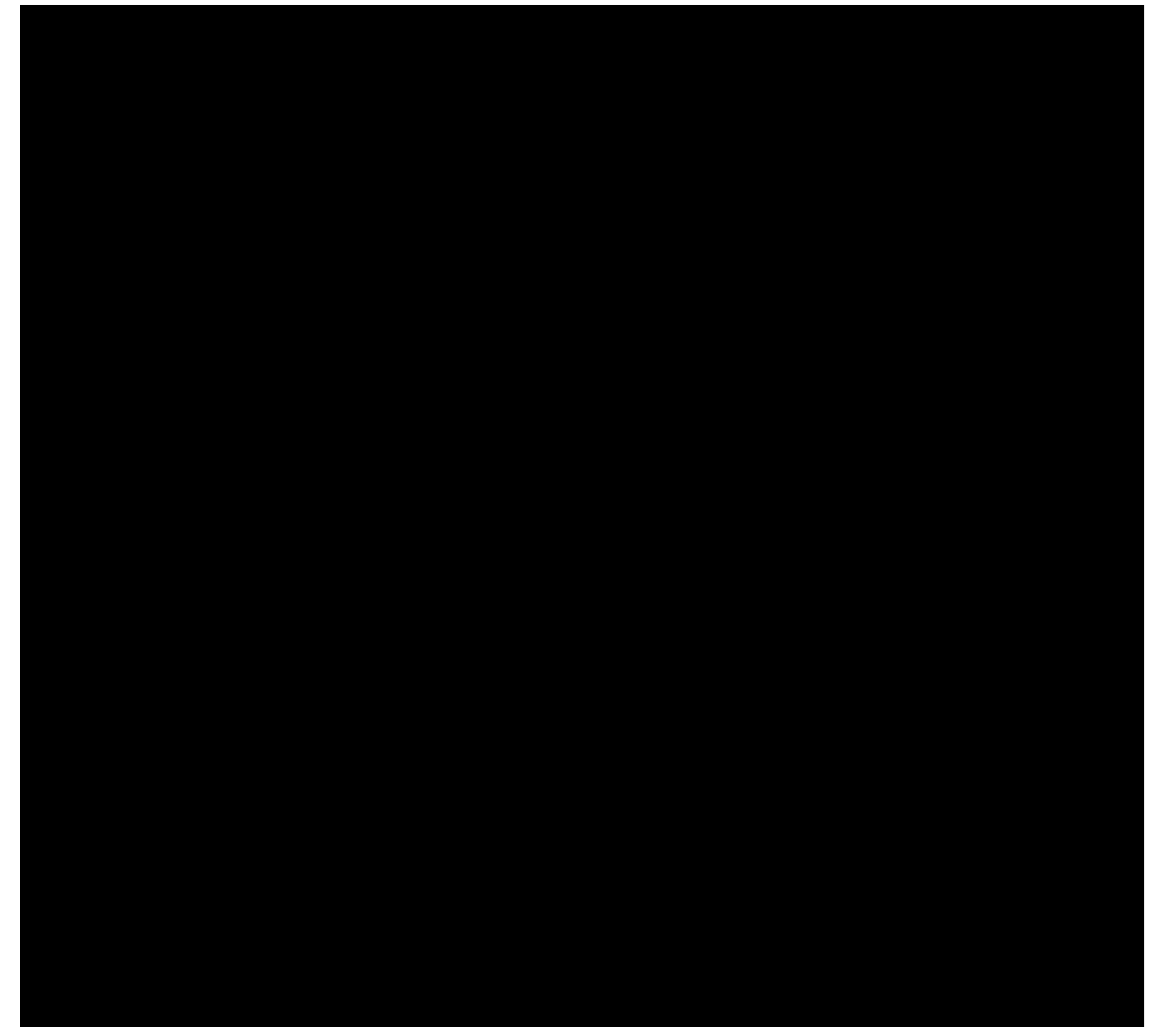


# Closed-box learning

$L^*$  in practice

$\varepsilon \in L?$

Yes



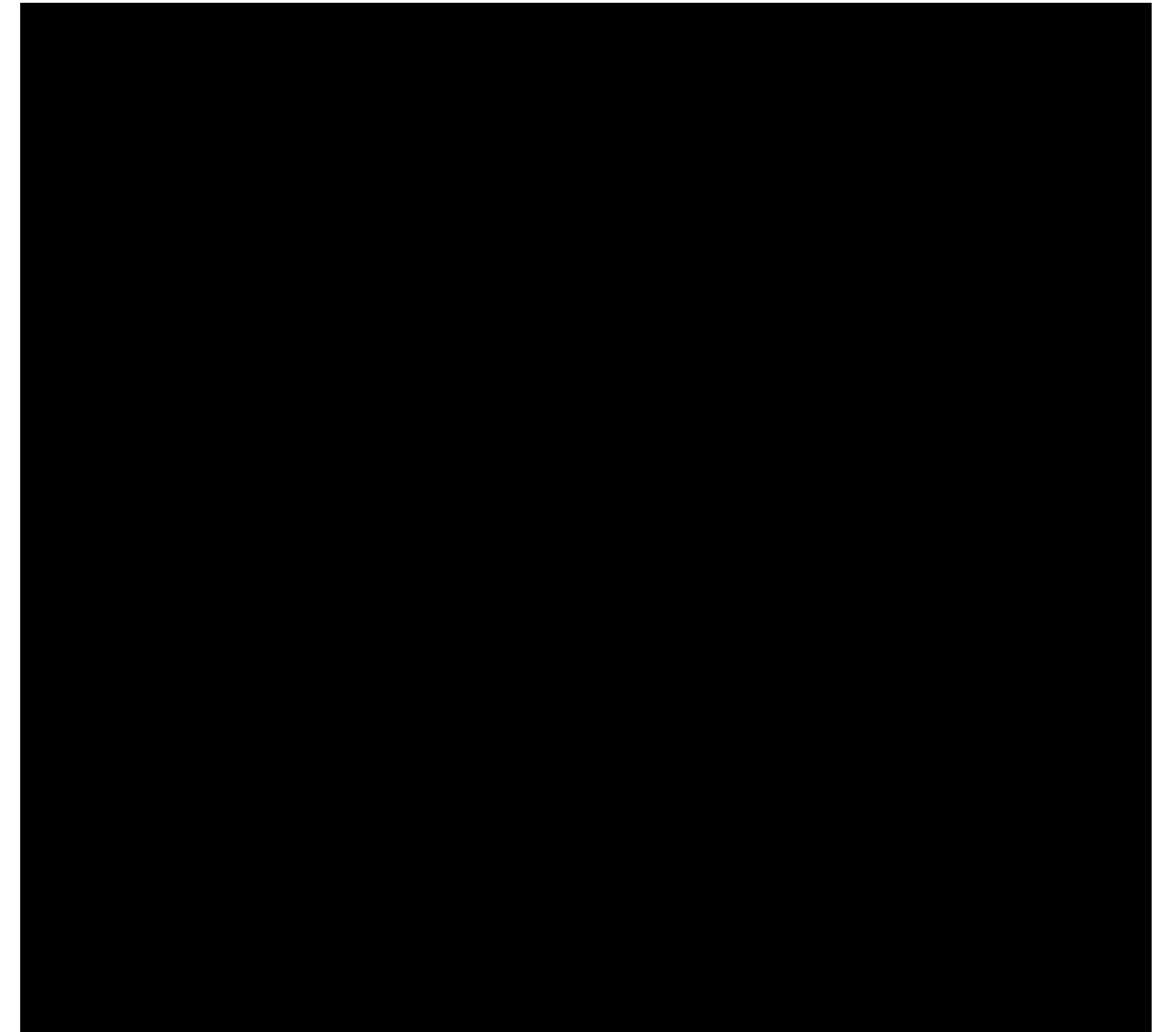
# Closed-box learning

$L^*$  in practice

$\varepsilon \in L?$

Yes

$a \in L?$



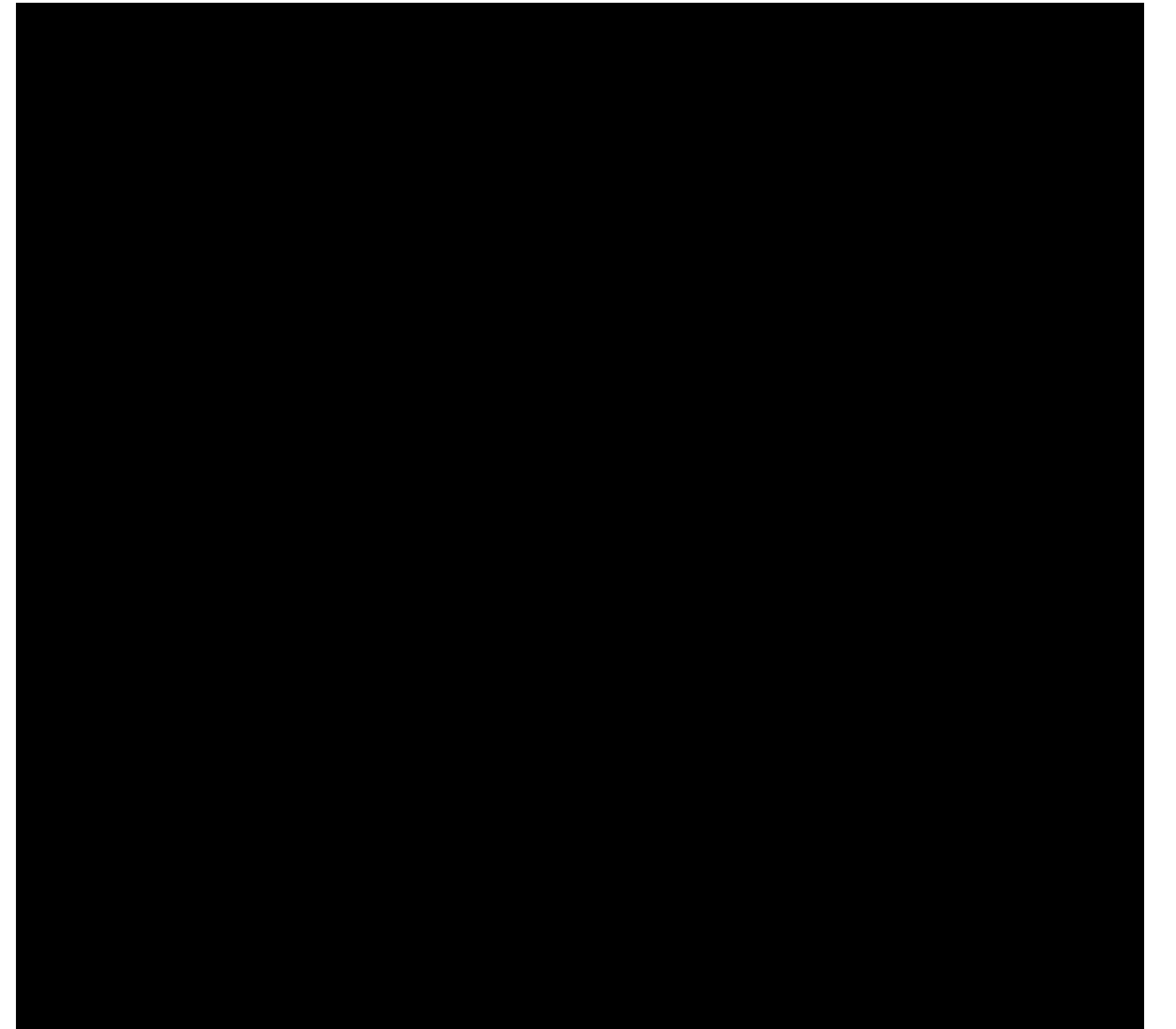


# Closed-box learning

$L^*$  in practice

$\varepsilon \in L?$  Yes

$a \in L?$  No



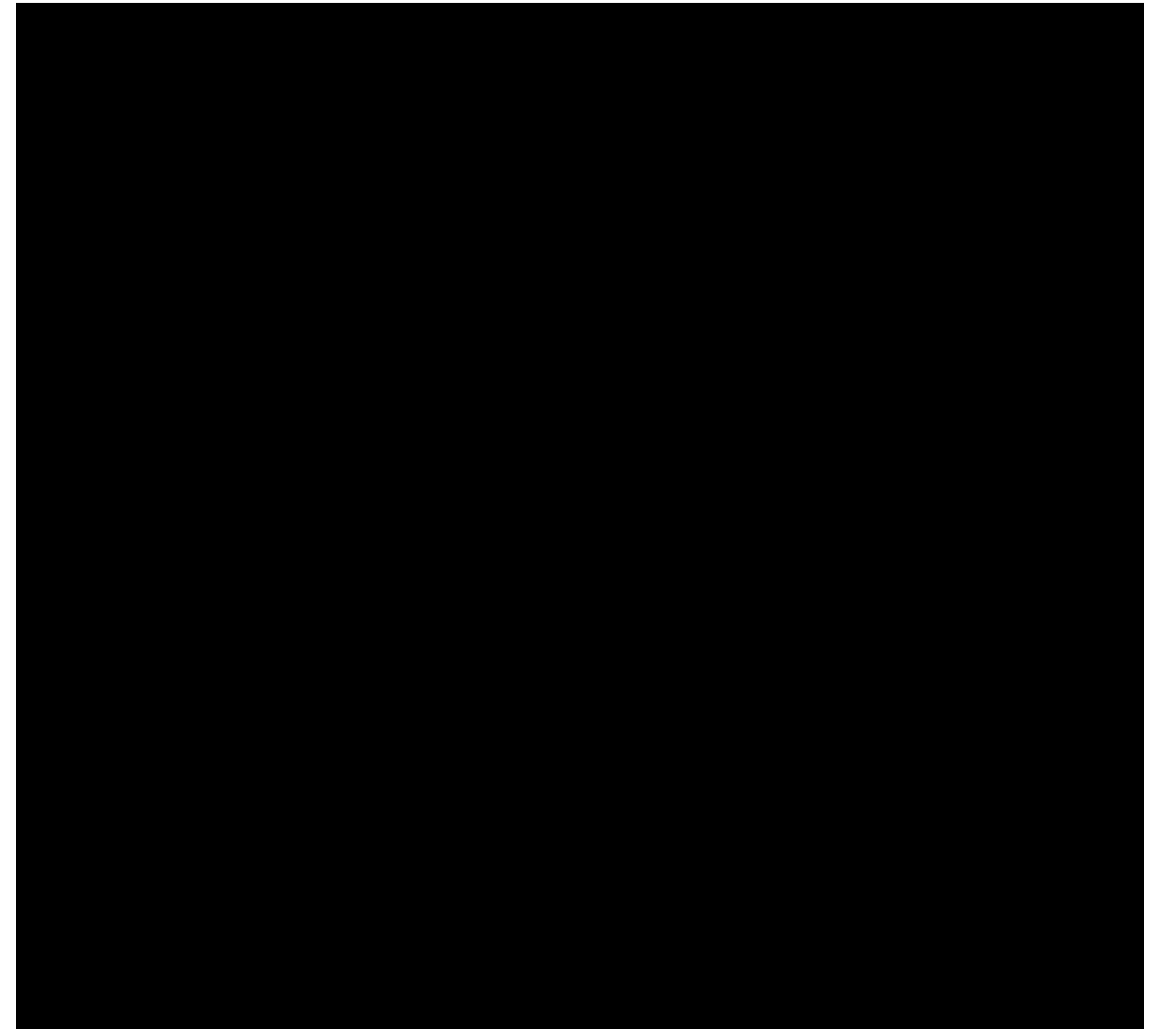
# Closed-box learning

$L^*$  in practice

$\varepsilon \in L?$  Yes

$a \in L?$  No

$b \in L?$



# Closed-box learning

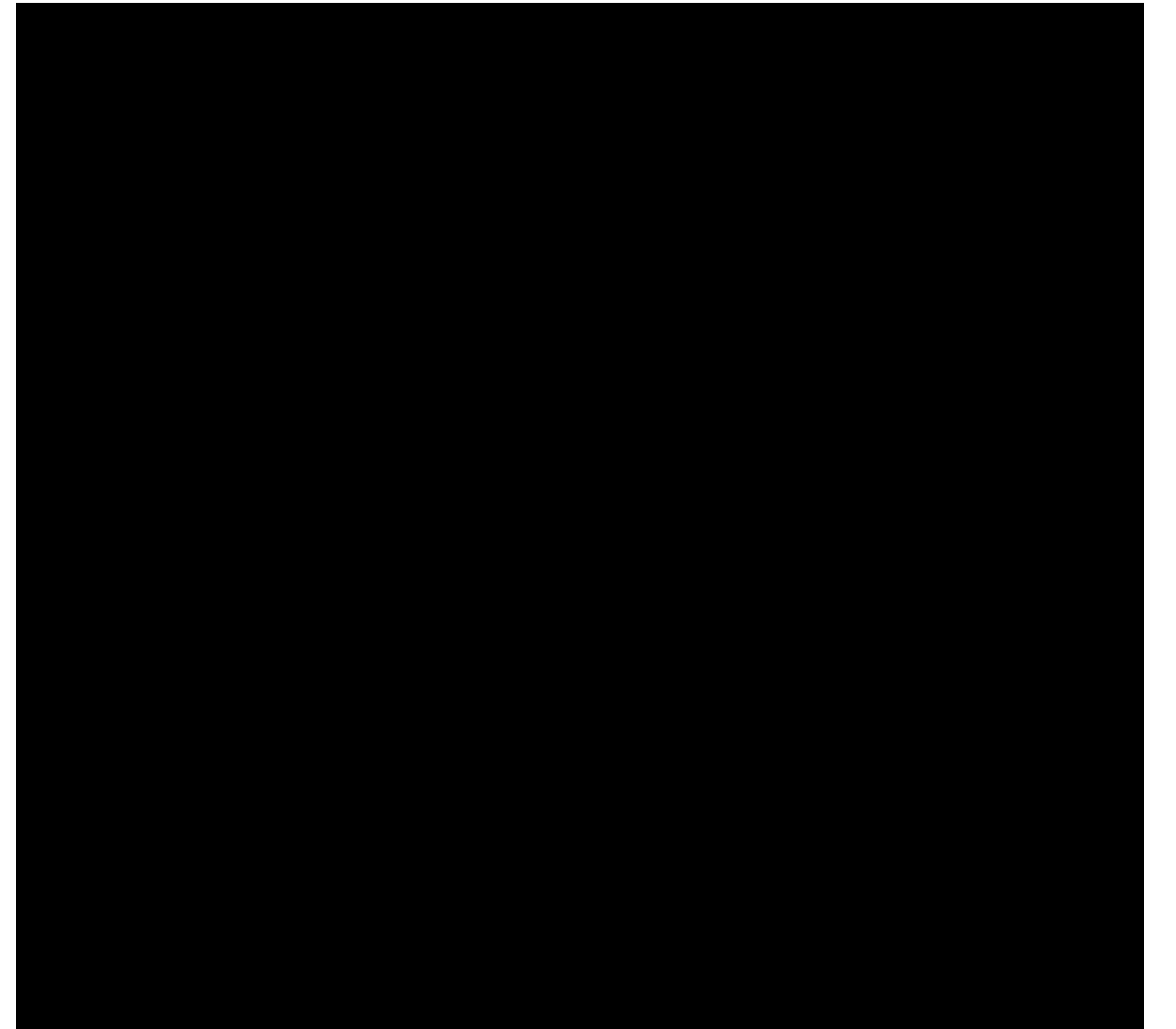
$L^*$  in practice

$\varepsilon \in L?$  Yes

$a \in L?$  No

$b \in L?$

$ba \in L?$



# Closed-box learning

$L^*$  in practice



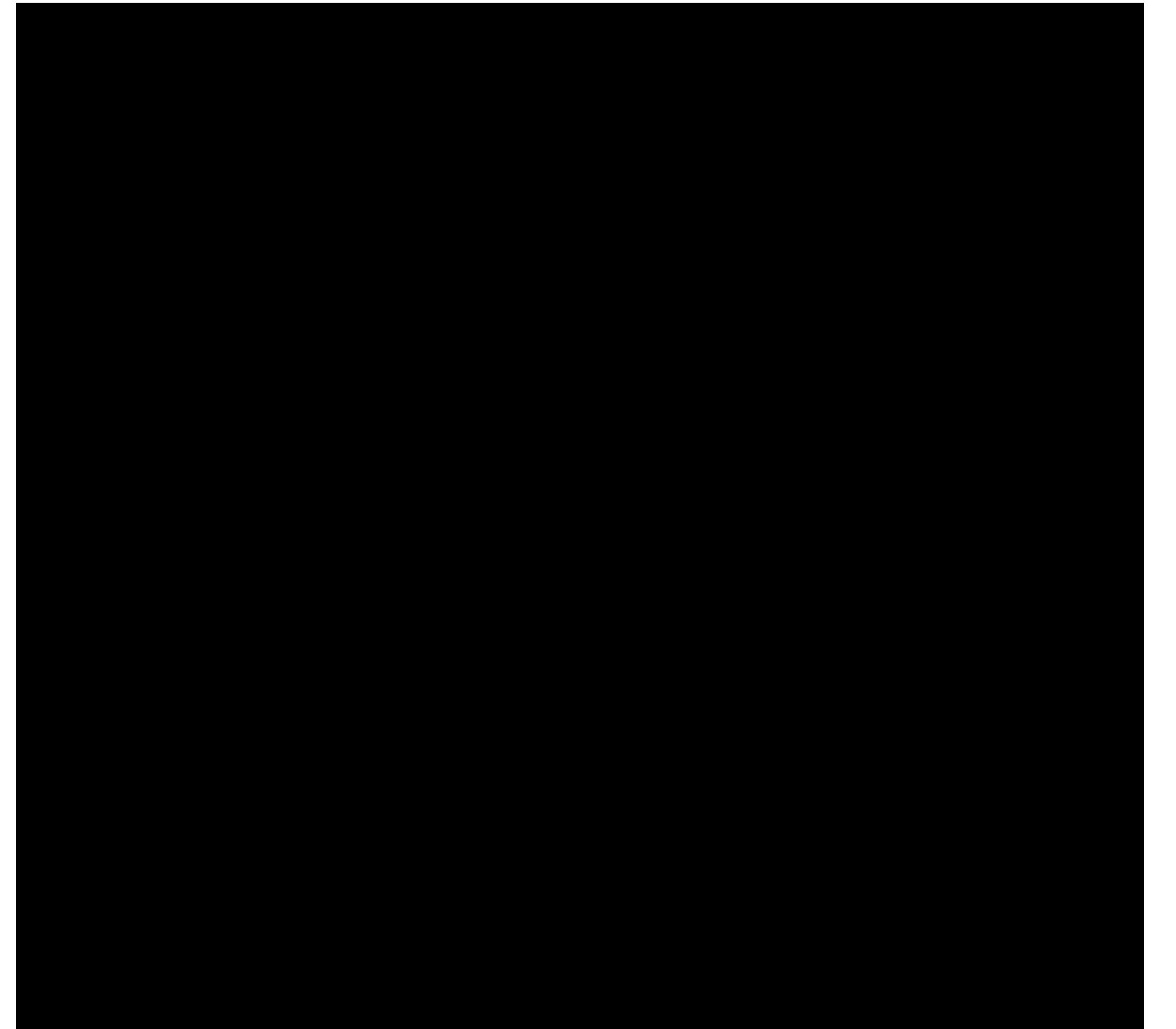
$\varepsilon \in L?$  Yes

$a \in L?$  No

$b \in L?$

$ba \in L?$

$bb \in L?$



# Closed-box learning

$L^*$  in practice



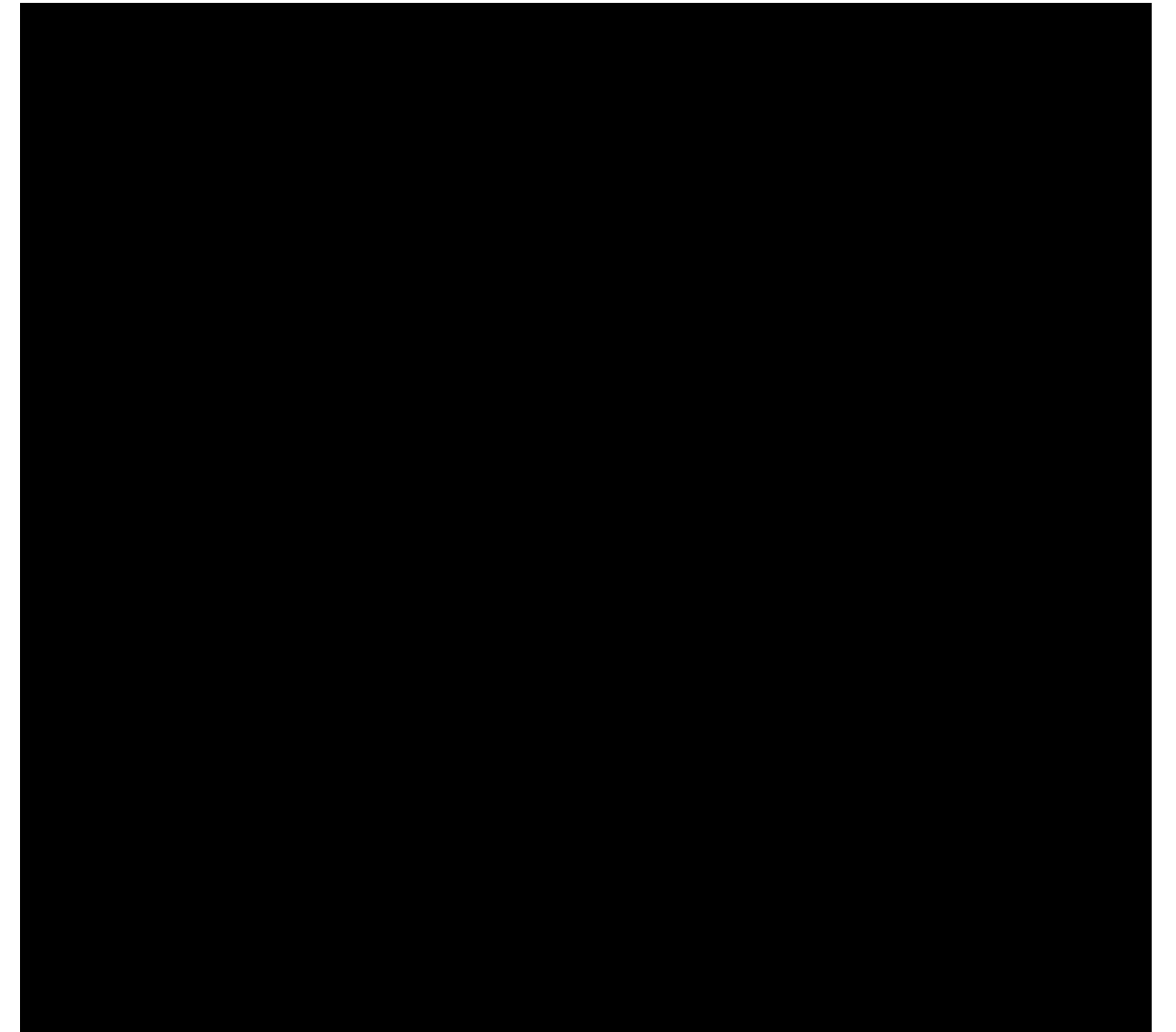
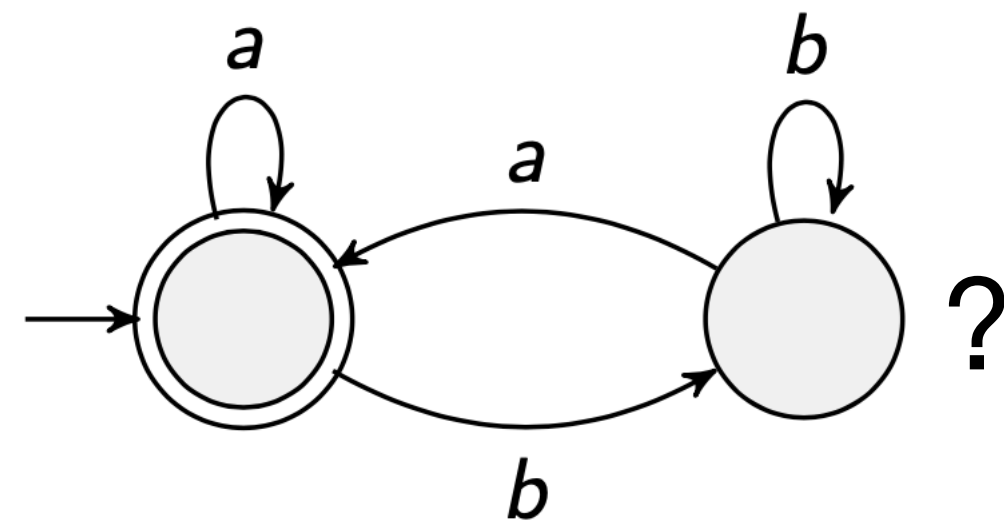
$\varepsilon \in L?$  Yes

$a \in L?$  No

$b \in L?$

$ba \in L?$

$bb \in L?$



# Closed-box learning

$L^*$  in practice

Challenges:

- System *might not* answer every membership query



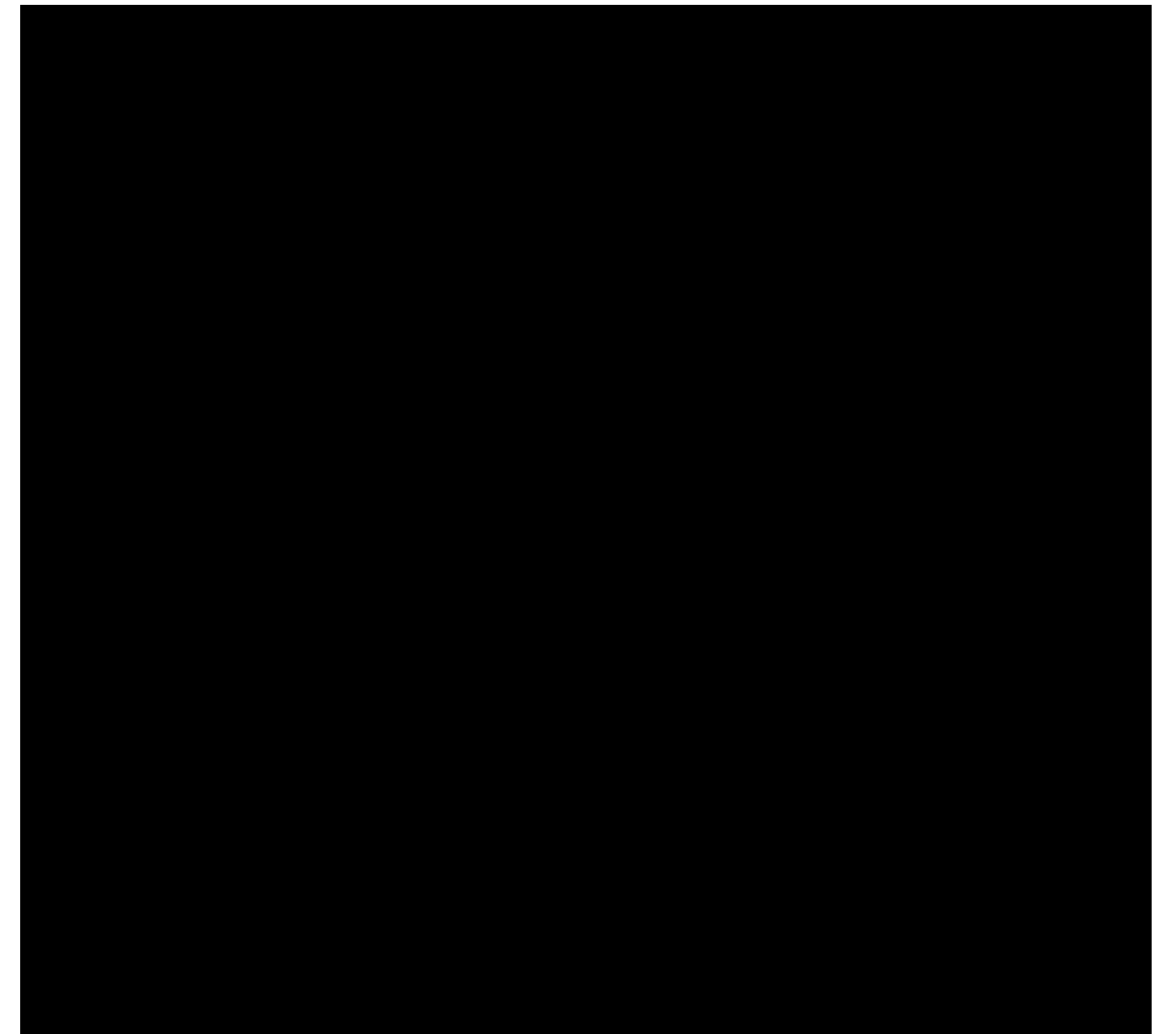
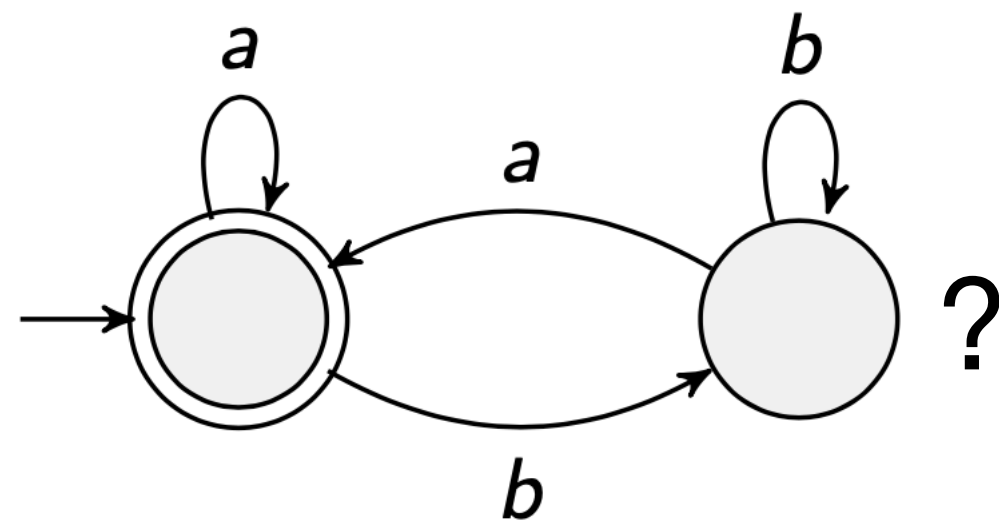
$\varepsilon \in L?$  Yes

$a \in L?$  No

$b \in L?$

$ba \in L?$

$bb \in L?$



# Closed-box learning

$L^*$  in practice

Challenges:

- System *might not* answer every membership query
- System certainly does not answer equivalence query



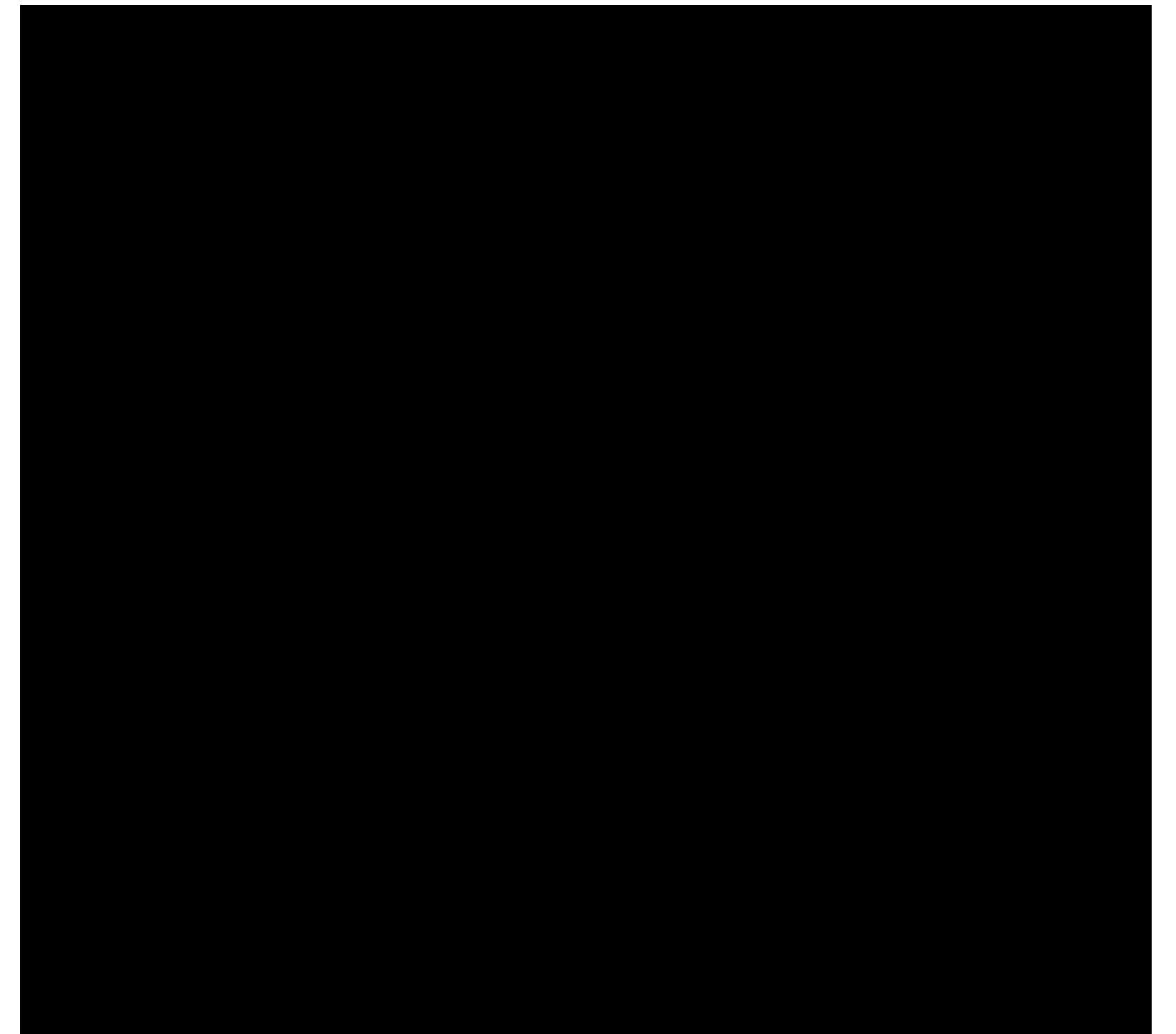
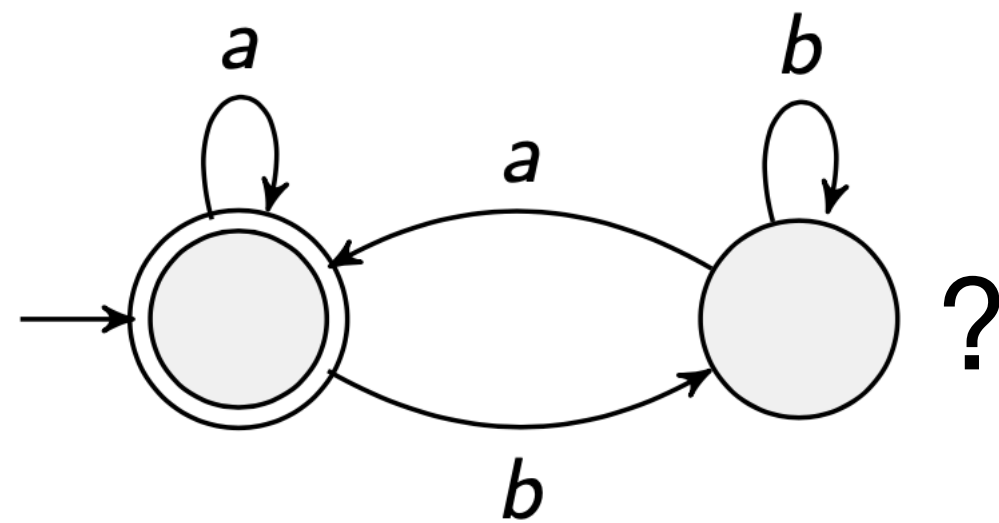
$\varepsilon \in L?$  Yes

$a \in L?$  No

$b \in L?$

$ba \in L?$

$bb \in L?$

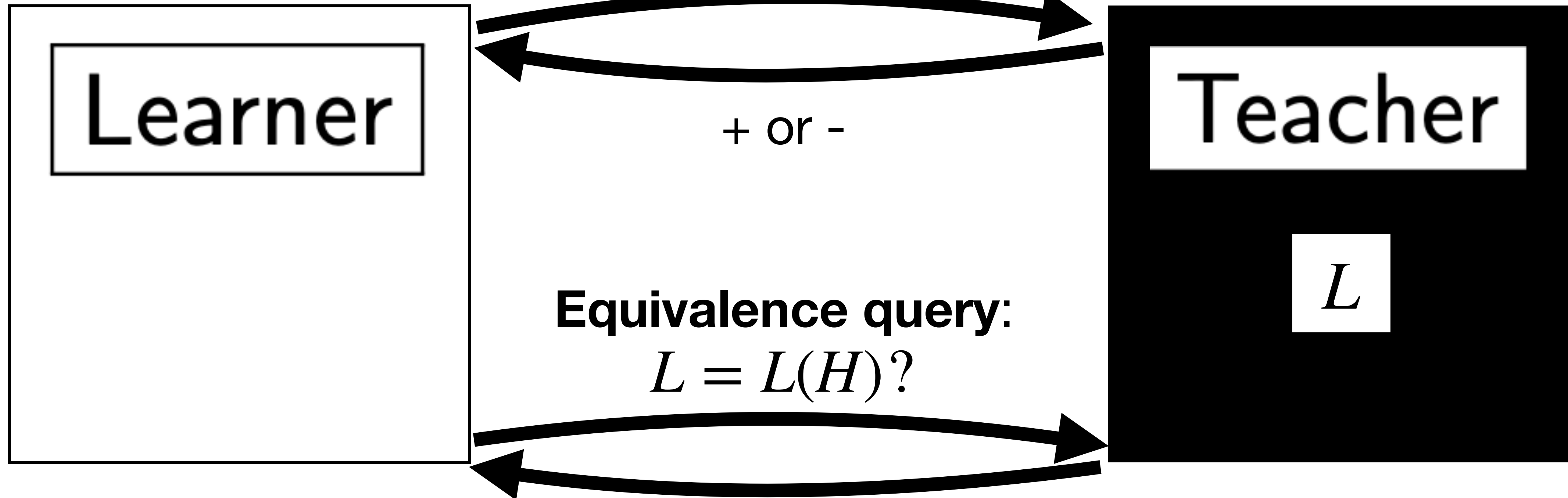


# Minimally Adequate Teacher (MAT) Framework

(Angluin, 1987)

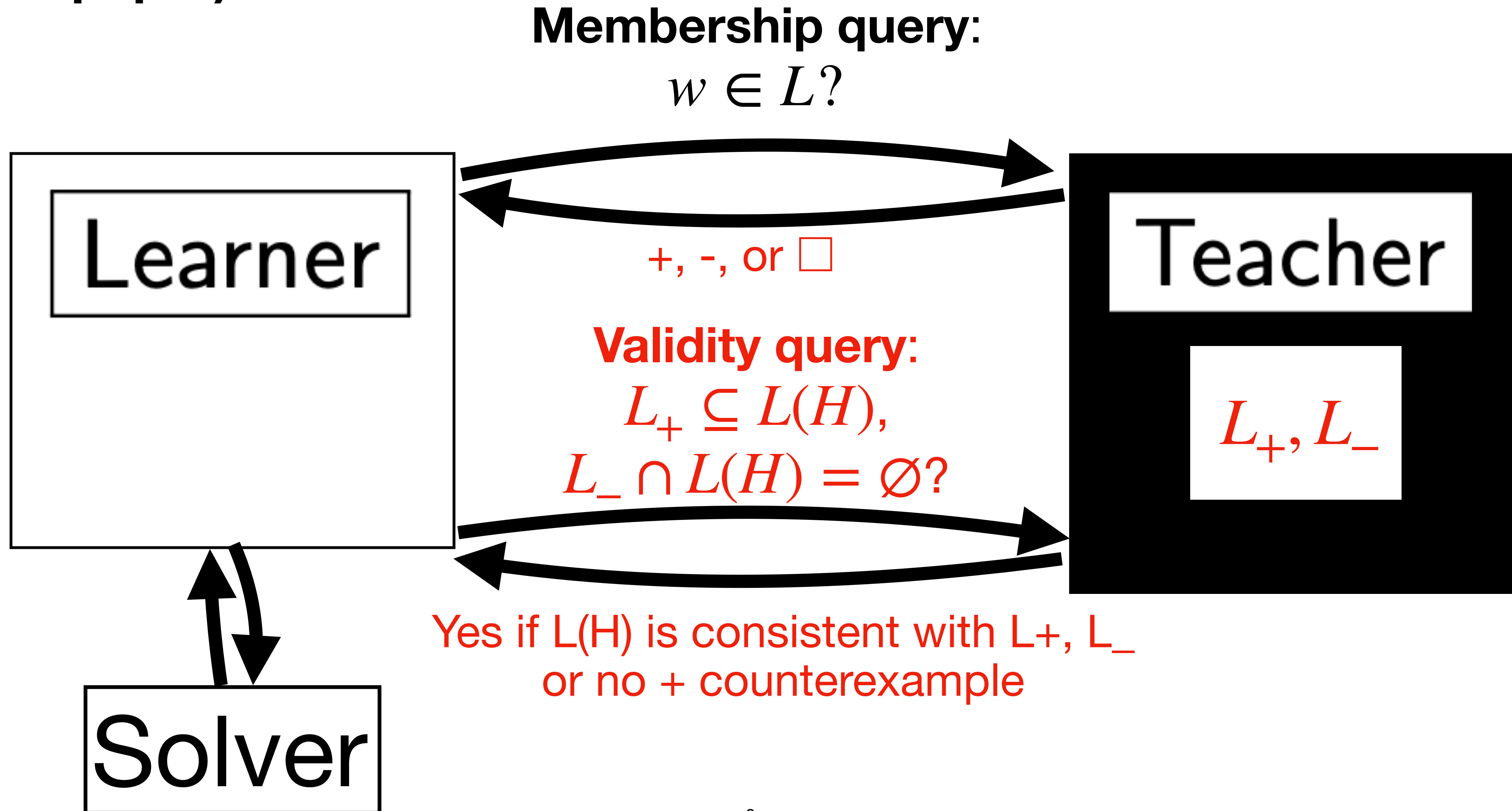
Membership query:

$$w \in L?$$



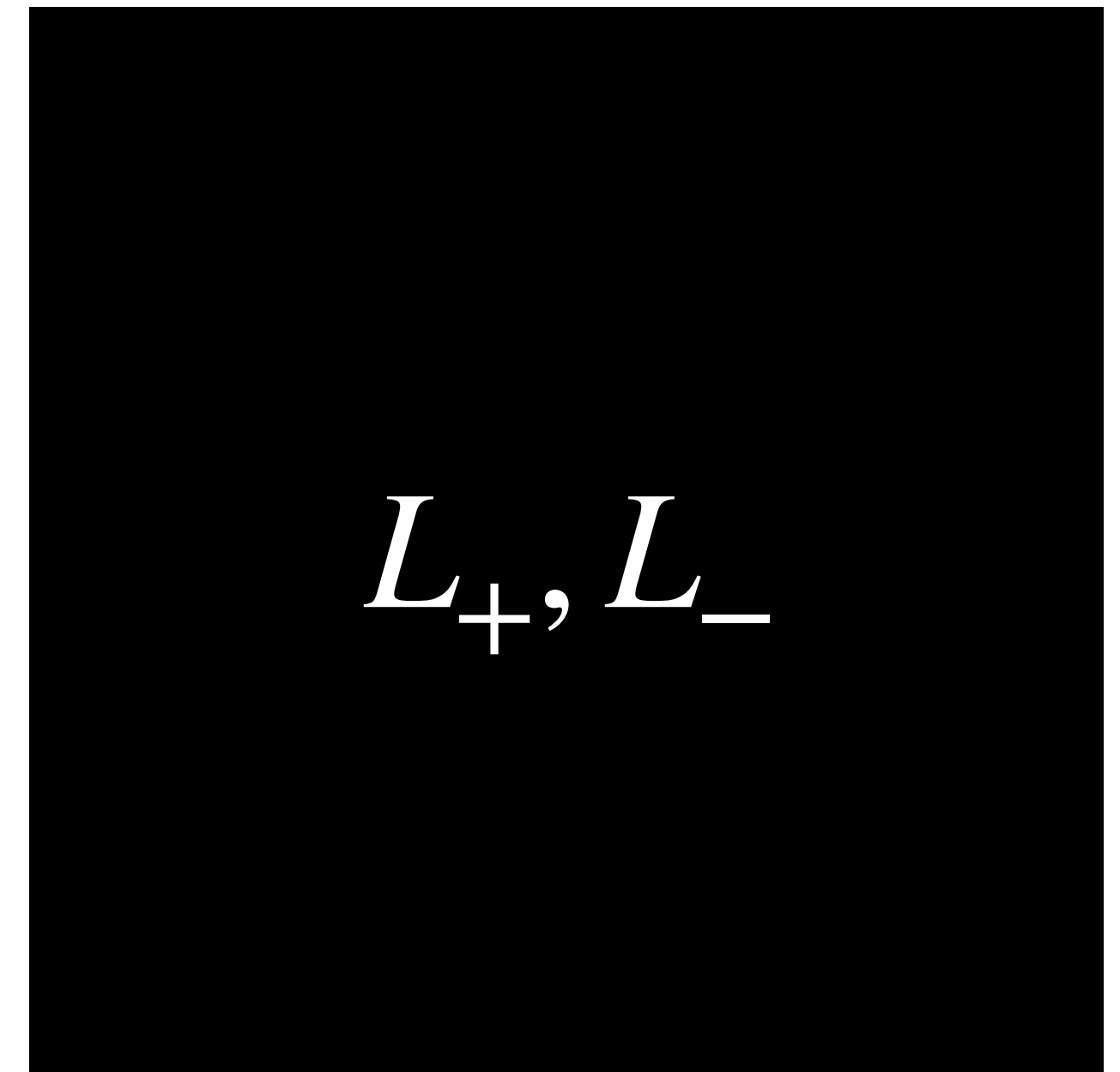


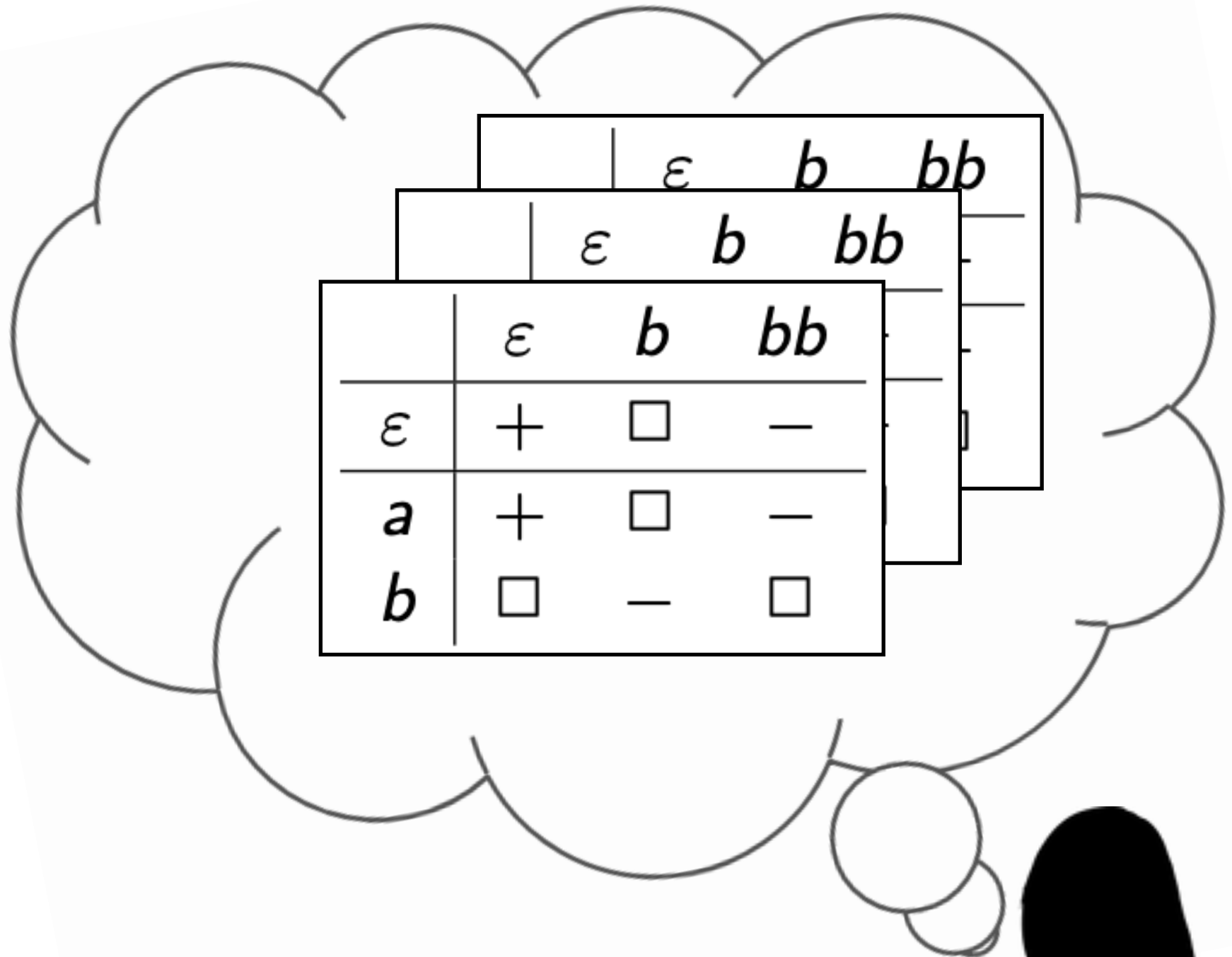
# Incomplete Minimally Adequate Teacher (iMAT) Framework (This paper)



**Our solution:  $L_{\square}^{\star}$**

	$\varepsilon$	$b$	$bb$
$\varepsilon$	+	$\square$	-
$a$	+	$\square$	-
$b$	$\square$	-	$\square$





	$\epsilon$	$b$	$bb$
$\epsilon$	+	-	-
$a$	+	-	-
$b$	-	-	-

**Our solution:**  $L_{\square}^{\star}$

- Uses a worklist of *observation tables with blanks*

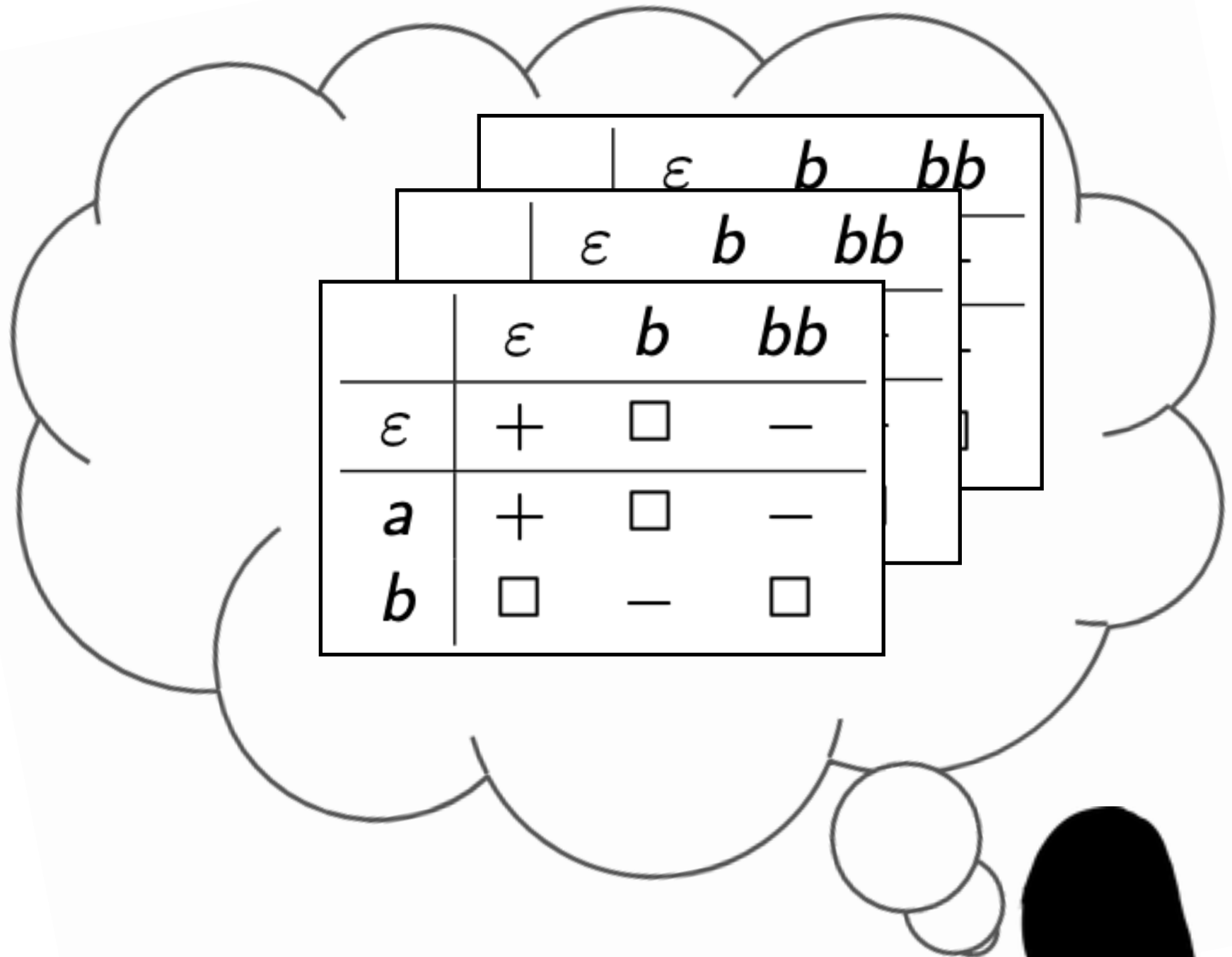
$L_{+}, L_{-}$

	$\varepsilon$	$b$	$bb$
$\varepsilon$	+	□	-
$a$	+	□	-
$b$	□	-	□

# Our solution: $L_{\square}^{\star}$

- Uses a worklist of *observation tables with blanks*
- Rely on a *solver* to fill blanks

$L_{+}, L_{-}$



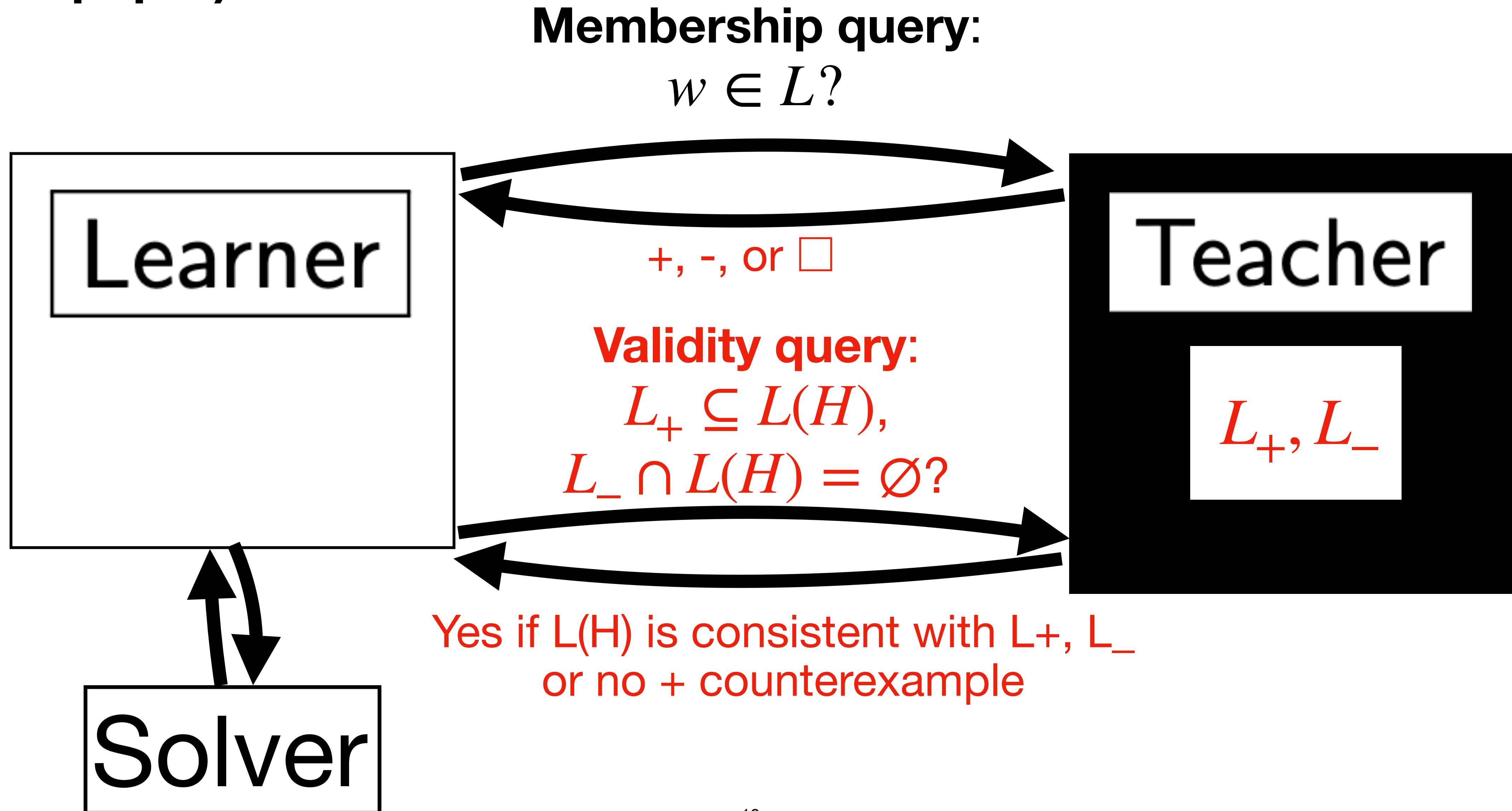
	$\epsilon$	$b$	$bb$
$\epsilon$	+	□	-
$a$	+	□	-
$b$	□	-	□

## Our solution: $L_{\square}^{\star}$

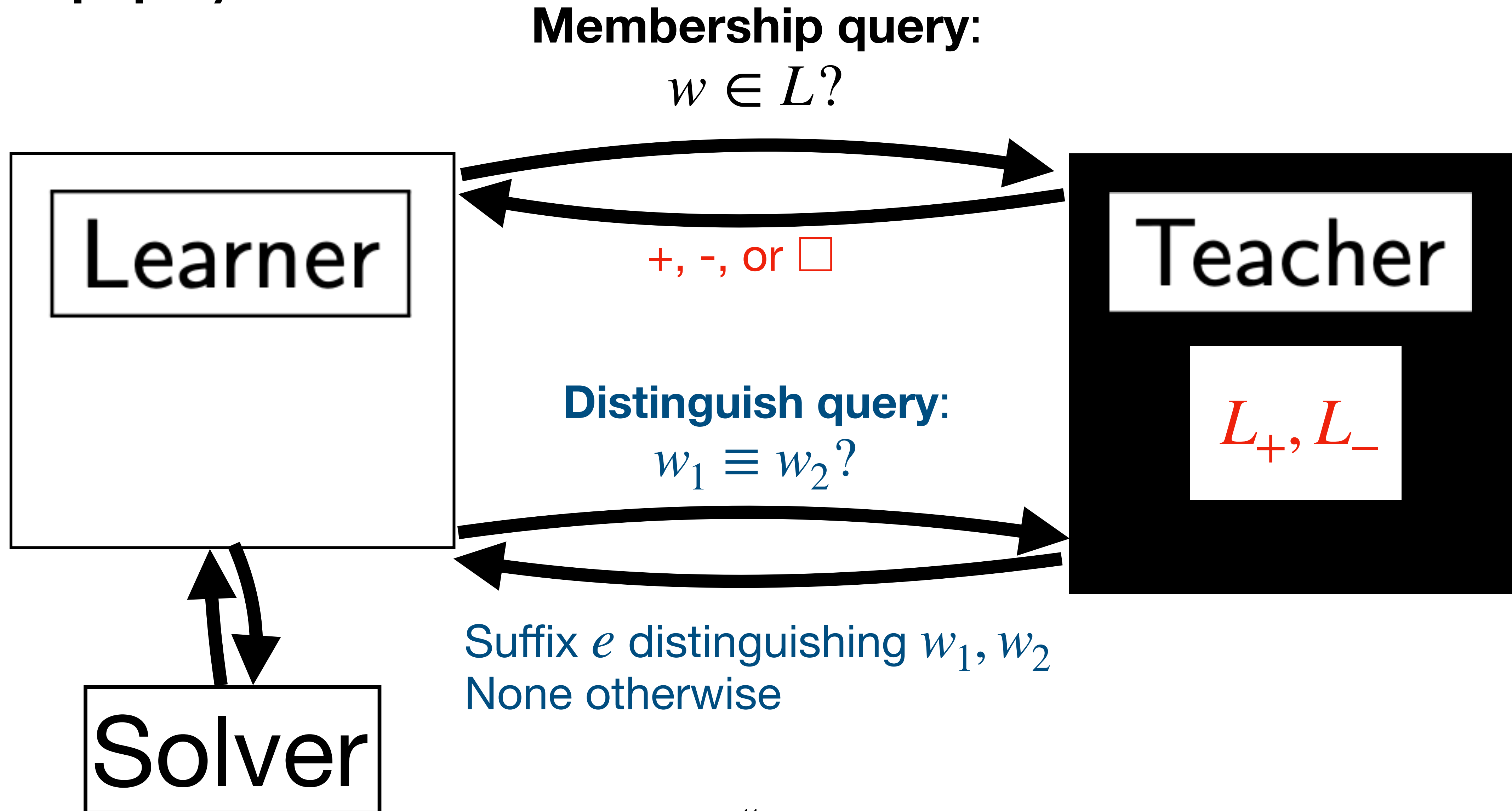
- Uses a worklist of *observation tables with blanks*
- Rely on a *solver* to fill blanks
- Guaranteed to learn *minimum-size* DFA accepting  $L_{+}$  and rejecting  $L_{-}$

$L_{+}, L_{-}$

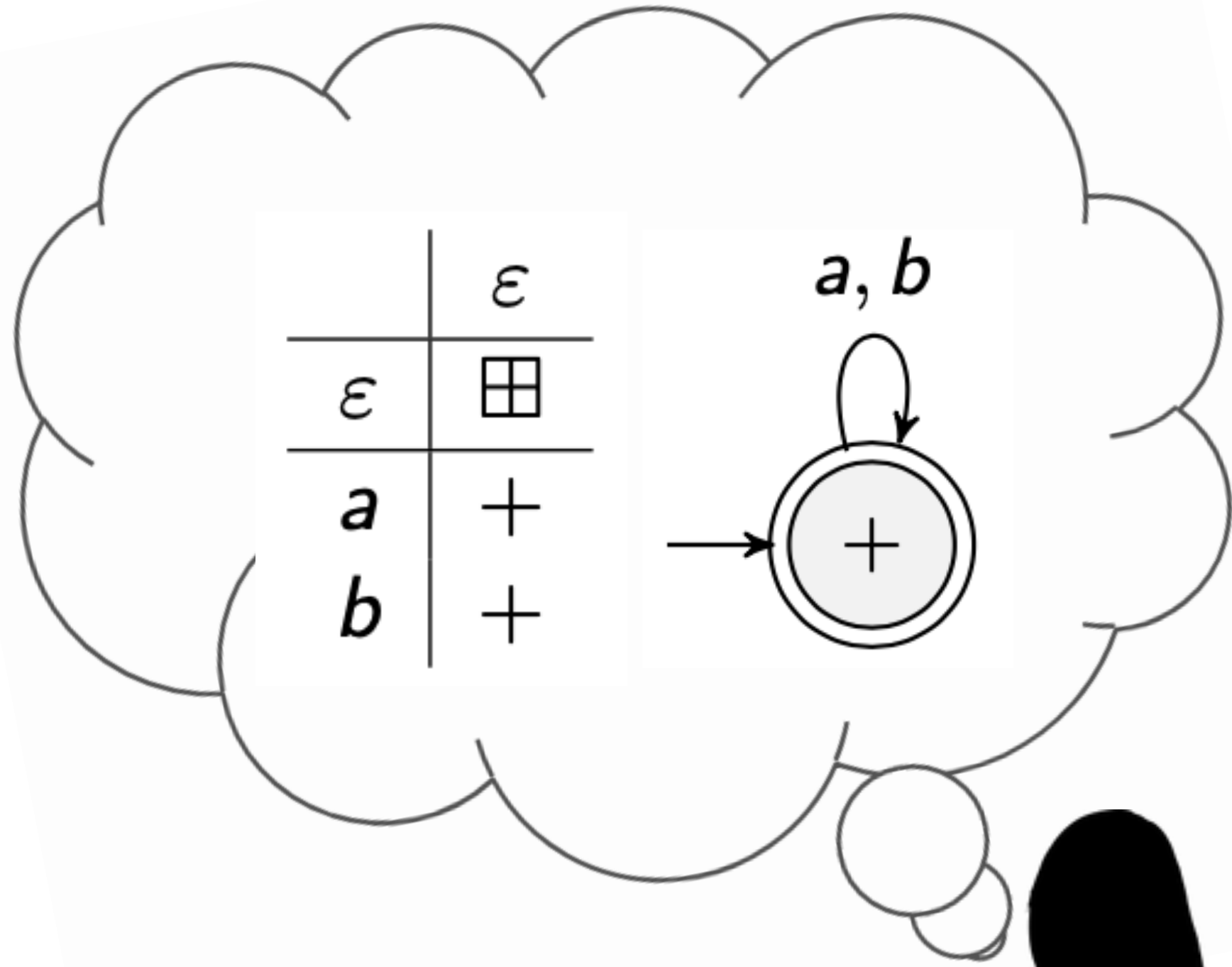
# Incomplete Minimally Adequate Teacher (iMAT) Framework (This paper)



# Incomplete Minimally Adequate Teacher (iMAT) Framework (This paper)



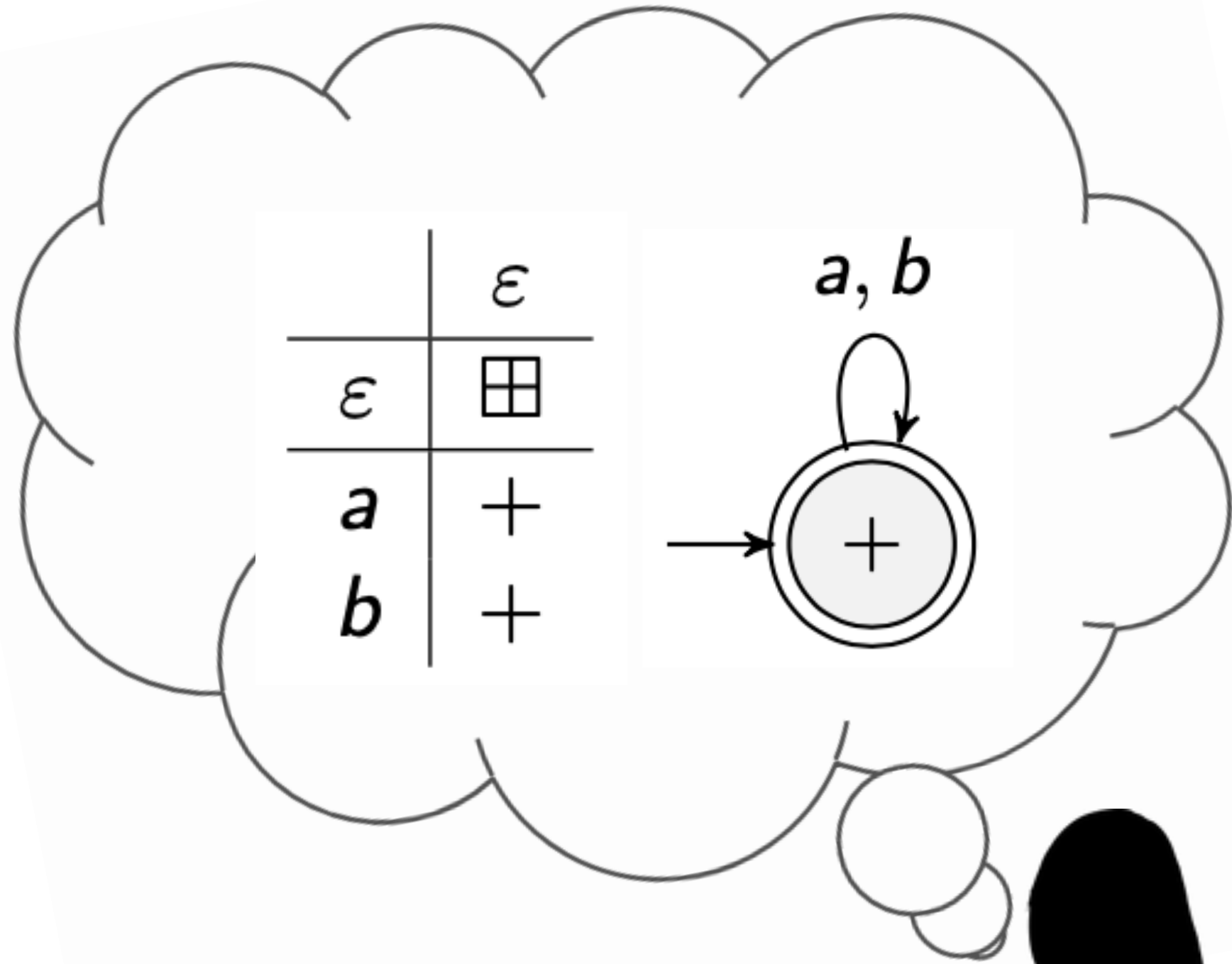
# Distinguish query



$L_+$	$L_-$
$a$	$aa$
$b$	



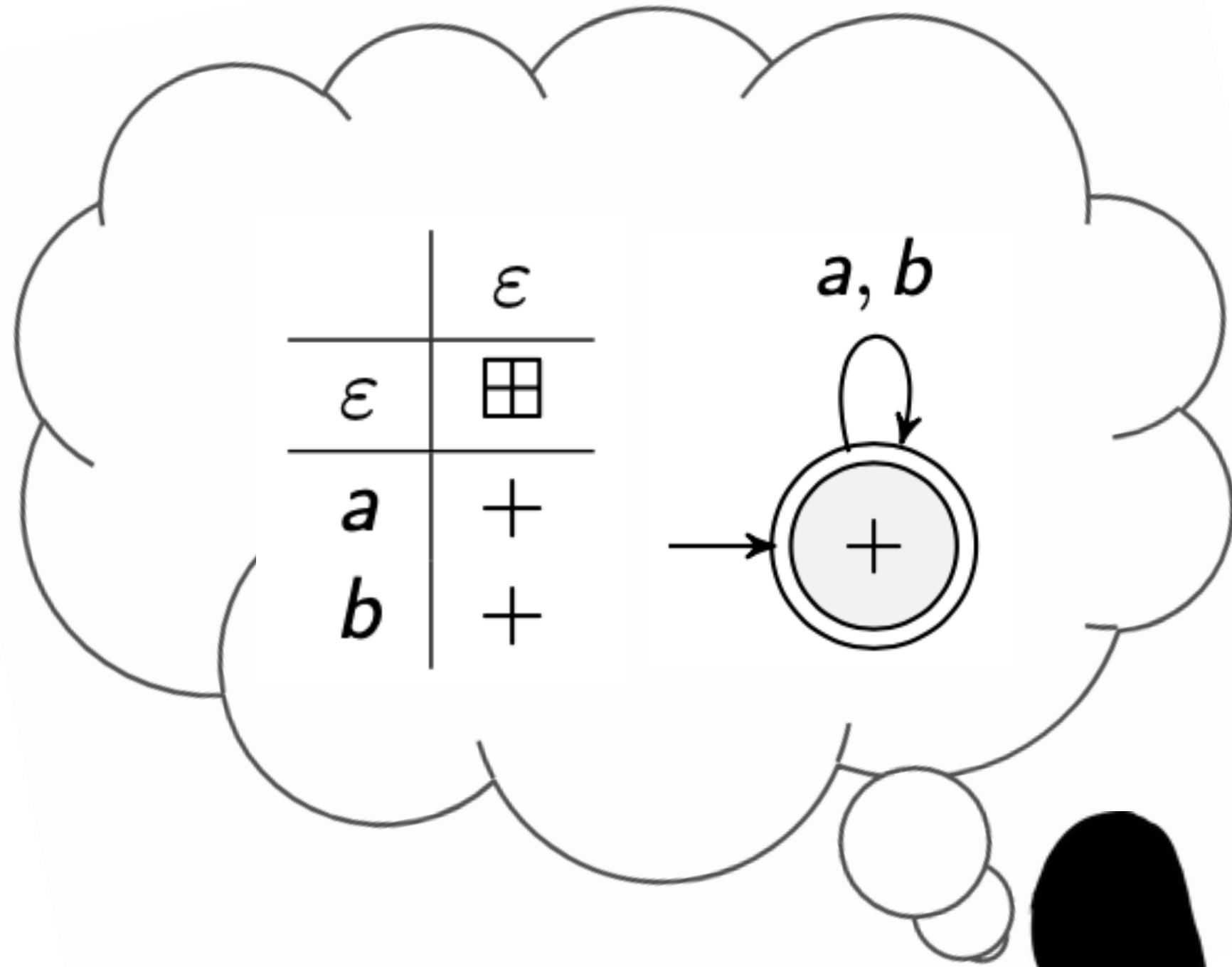
# Distinguish query



Distinguish  $\epsilon$ ,  $a$ ?

$L_+$	$L_-$
$a$	$aa$
$b$	

# Distinguish query



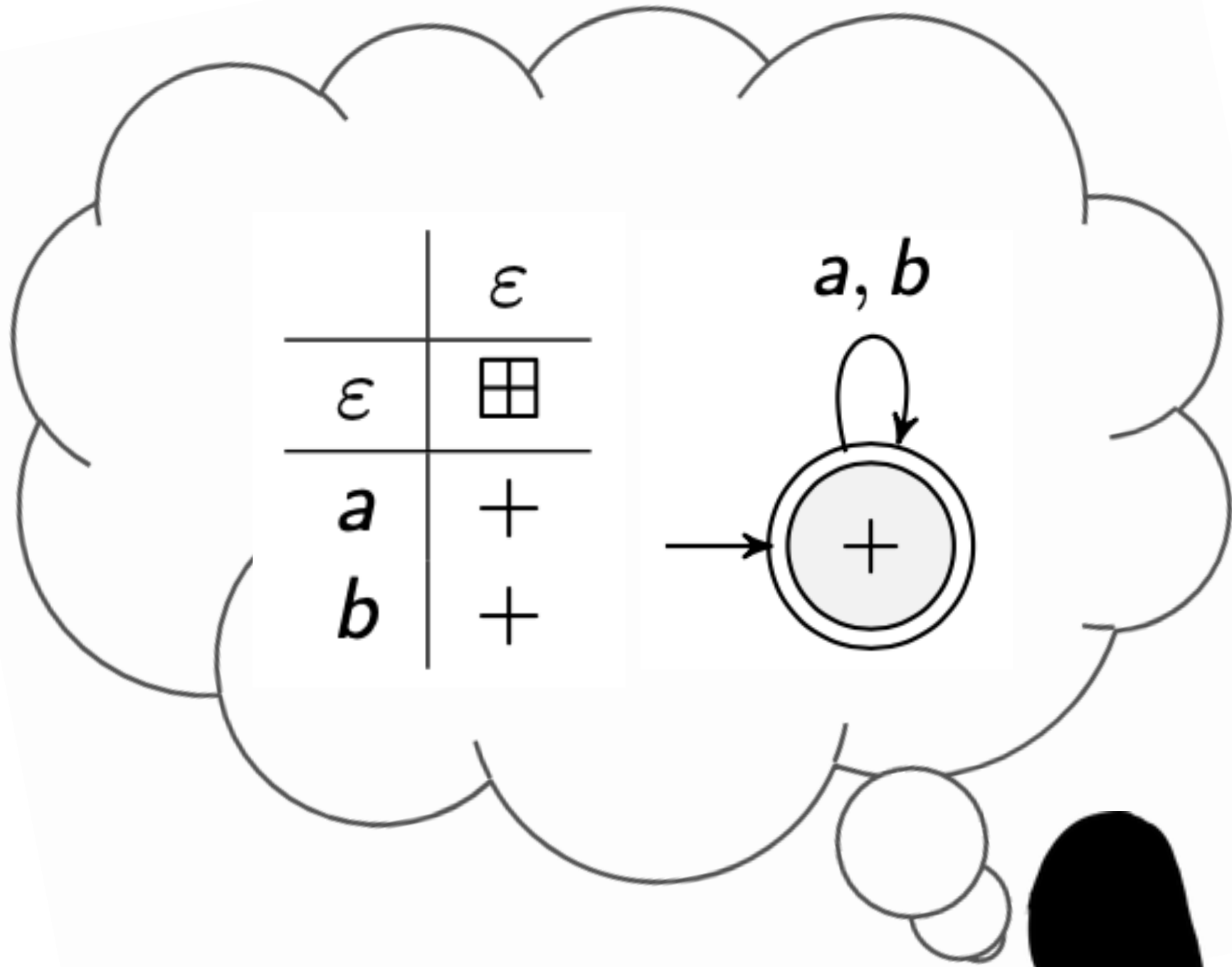
Distinguish  $\epsilon$ ,  $a$ ?

Yes:  $a$

$$\begin{aligned}\epsilon \cdot a &\in L_+ \\ a \cdot a &\in L_-\end{aligned}$$

$L_+$	$L_-$
$a$	$aa$
$b$	

# Distinguish query



Distinguish  $\varepsilon, a$ ?

Yes:  $a$

$$\begin{aligned} \varepsilon \cdot a &\in L_+ \\ a \cdot a &\in L_- \end{aligned}$$

$L_+$	$L_-$
$a$	$aa$
$b$	

$L_{\square}^{\star}$  modified to use distinguish is guaranteed to learn minimum size DFA for finite  $L_+, L_-$

# Implementation



GitHub link:



- $L_{\square}^{\star}$  is written in OCaml
  - Self contained automata library
  - Library for building automata learning algorithms
- The *solver* is implemented using the Z3 SMT solver
- Optimizations based on unsatisfiable cores and worklist heuristic prioritization



# Related Work

- **Leucker and Neider**, ISoLA 2012, “Learning Minimal Deterministic Automata from Inexperienced Teachers”
- **Grinchtein, Leucker, and Piterman**, IJCAR 2006, “Inferring Network Invariants Automatically”
- **Chen, Farzan, Clarke, Tsay, and Wang**, TACAS 2009, “Learning Minimal Separating DFA’s for Compositional Verification”

# Q & A

GitHub link:



# Q & A

GitHub link:

