Cosmic Security

Benjamin Chan

Joint work with Cody Freitag & Rafael Pass

Cornell Tech

Reading Group, TAU. 12/16/2021

The Duality of Progress

To Know the Unknown

The desire to model the world, to build a *Theory of Everything*, so that we can know everything. A World of Uncertainty Acknowledgement of the Unknown, so that we can prepare for (and survive) the unknown tribulations of the cosmos.



Cryptography Embraces the Unknown

To Know the Unknown

The desire to model the world, to build a *Theory of Everything*, so that we can know everything.



A World of Uncertainty

We want to build protocols that are secure against *any real-life attacker*, no matter when, where, or how (assuming computational hardness.)



Reduction-Based Computational Security

Suppose A wins a security game C (e.g. breaking an encryption scheme):



Then R^A breaks C' (e.g. inverting a OWF)

$$\mathbf{C}^{\mathsf{f}(\mathsf{x})} \xrightarrow{f^{-1}(\mathsf{f}(\mathsf{x}))} \mathbf{R}^{\mathsf{A}} \xrightarrow{\mathsf{Classically, R can rewind}} or restart the attacker.}$$

Contradiction! (assuming security of the OWF for R^A.)

Embracing Uncertainty has done us remarkably well.

From OWFs, we can build,

- Hardcore Bits [GL89]
- Pseudorandom Generators [HILL99]
- Private-Key Encryption [GGM85+86]
- Commitment Schemes [Nao91]
- Digital Signatures [Lam79, Rom90, Mer90]
- Zero-Knowledge Proofs for all of NP [GMW86]



No-Cloning Theorem: can't copy qubits without collapsing superposition.

No-Cloning Theorem: can't copy qubits without collapsing superposition.



No-Cloning Theorem: can't copy qubits without collapsing superposition.



Can no longer reason about **Nature** as classical algorithm: It is bizarrely stateful (Behavior changes on repeated invocations).

No-Cloning Theorem: can't copy qubits without collapsing superposition.



Can no longer reason about **Nature** as classical algorithm: It is bizarrely stateful (Behavior changes on repeated invocations).



Classically, R can just "rerun" A, where A is not stateful across runs

No-Cloning Theorem: can't copy qubits without collapsing superposition.



Can no longer reason about **Nature** as classical algorithm: It is bizarrely stateful (Behavior changes on repeated invocations).

It isn't a stretch to imagine a fully stateful real world attacker...



Questioning Our Faith

Our world view is built on two implicit assumptions:

Extended Church Turing Hypothesis: Real-life attackers are stateless, and (nu)PPT.

Quantum Extended Church Turing Hypothesis: Real-life attackers are stateless, and (nu)QPT.



Questioning Our Faith

Our world view is built on two implicit assumptions:

Extended Church Turing Hypothesis: Real-life attackers are stateless, and (nu)PPT.

Quantum Extended Church Turing Hypothesis: Real-life attackers are stateless, and (nu)QPT.

OKAY, HOLD STILL. AND REMEMBER, IF YOU REALLY BELIEVE IN THE LAWS OF PHYSICS, YOU WON'T FLINCH.

These are religious, not scientific assumptions.

Popper's Falsifiability Test [Pop05]

"It must be possible for a scientific theory to be refuted by experience."

"It is easy to obtain confirmations for nearly every theory — if we look for confirmations."

"ALL MEN ARE MORTAL"

"Real Life Attackers Are Efficient Algorithms" Thus, a scientific theory must be systematically falsifiable.

"ALL MEN ARE IMMORTAL"

The Extended Church Turing Hypothesis doesn't pass the test!

If Our Faith is Wrong... If Nature is Stateful...



It took 1400 years for humankind to develop the tools necessary to falsify geocentrism.

This talk:

Can we build a reduction-based theory of cryptography without making a *religious* assumption on the properties of real-world attackers?

(acknowledging that unknown, that the world is vaster than we previously thought it to be, and not just modeled by stateless algorithms?)

This talk:

Can we build a reduction-based theory of cryptography in the face of *newfound uncertainty* about the Nature of real-world attackers?

(What if real-world attackers can change the way they play security games over time?)

Our work: Cosmic Security (Informal)

Suppose PPT A uses Nat to 'break' C (e.g. breaking an encryption scheme):

$$C \xrightarrow[m]{Enc(m)} A \longrightarrow Nat Can be stateful Can be unbounded$$

Then **∃** PPT **A'** that uses **Nat** to 'break' **C'** (e.g. inverting a OWF)

$$C' \xrightarrow{f(x)} A' \xrightarrow{f^{-1}(f(x))} A' \xrightarrow{f^{-1}(f(x))}$$

Contradiction! (assuming security of the OWF vs PPT+Nat)

Roadmap

1. Motivation (10min)

- 2. Defining Cosmic Security (15min)
- 3. Properties of Cosmic Security: a Sanity Check
 - a. Composition, Black-box reductions (5min)
- 4. Summary of Key Results
 - a. Feasibilities and Impossibilities (20min)
- 5. Other Notions of Cosmic Security (10min)
- 6. Conclusion (5min)

Roadmap

1. Motivation (10min)

2. Defining Cosmic Security (15min)

- 3. Properties of Cosmic Security: a Sanity Check
 - a. Composition, Black-box reductions (5min)
- 4. Summary of Key Results
 - a. Feasibilities and Impossibilities (20min)
- 5. Other Notions of Cosmic Security (10min)
- 6. Conclusion (5min)

Defining Cosmic Security

A "Attacker" uniform PPT "Nature" nonuniform any choice of runtime Stateful! "Cosmic Adversary" (A, Nat)

Cosmic Adversaries



Nat \longleftrightarrow

"Attacker" uniform **PPT** "Nature" nonuniform any choice of runtime Stateful

"Cosmic Adversary" (A, Nat)

to capture our uncertainty regarding the "Power of Nature"

A classic attacker that uses Nature to break some scheme

Cosmic Security Game

$C \leftrightarrow A \leftrightarrow Nat$

"Challenger" uniform **PPT** outputs "win"/"lose"

"Attacker" "Nature" uniform PPT nonunifo

"Nature" nonuniform any choice of runtime Stateful

"Cosmic Adversary" (A, Nat)

Observe: the attacker can alter the state of Nature during the interaction. This is intentional and a key property of our definition. Nature can be invoked many times, "statefully"

> 412023436986659543855531365332575948 WIN. $C \leftrightarrow A \leftrightarrow Nat$ 79828454556264338764455652484261986 57624033946373269391

7926142024718886949256093177637503 150944909106910269861031862704114 36536588674337317208131041051908

2397

3282601393

Nat refuses to play more than once.

time

LOSE. $\mathbf{C} \leftrightarrow \mathbf{A} \leftrightarrow$

Maybe Nat measured a qubit.

The cat was let out of the box.

Nat simply isn't useful!

Useful adversaries win repeatedly.

No point in inverting a OWF only once.

WIN. $\mathbf{C} \leftrightarrow \mathbf{A}$

time

Classically, we always have the option of re-running A from scratch, with independent coins. We consider only (**A**, **Nat**) that "wins repeatedly" over time.

time

WIN. $\mathbf{C} \leftrightarrow \mathbf{A} \leftrightarrow$ WIN. $\mathbf{C} \leftrightarrow \mathbf{A} \leftrightarrow$ WIN. $C \leftrightarrow A \leftrightarrow$ WIN. $C \leftrightarrow A \leftrightarrow Nat$

It doesn't matter that Nat is "stateful" if it only wins once.





robust winning



Interaction prefix p:

a transcript of messages previously sent to Nat before the beginning of execution, including coins flipped by Nat.

(A, Nat) wins for C regardless of any interactions that Nat had in the past!

robust winning



Interaction prefix p:

the past

a transcript of messages previously sent to Nat before the beginning of execution, including coins flipped by Nat.

Definition: (A, Nat) has **robust advantage** a(.) for C, if \forall *interaction prefixes* ρ , $\forall \lambda$: Pr[(A, Nat(ρ)) wins C] $\geq a(\lambda)$

robust winning



Definition:(A, Nat) has robust advantageA weaka(.) for C, if \forall interaction prefixes ρ , $\forall \lambda$:Can winPr[(A, Nat(ρ)) wins C] $\geq a(\lambda)$ for each

A weak notion! Can win in a different way for each prefix.

Interaction prefix p:

the past

a transcript of messages previously sent to Nat before the beginning of execution, including coins flipped by Nat.

 \exists an ϵ -cosmic reduction from C to C' if \forall PPT A, \exists PPT A' s.t. \forall Nat: Suppose (A, Nat) has robust advantage a(·) for C

$$C \xrightarrow[m]{Enc(m)} A \longrightarrow Nat$$

 \exists an ϵ -cosmic reduction from C to C' if \forall PPT A, \exists PPT A' s.t. \forall Nat: Suppose (A, Nat) has robust advantage a(·) for C

$$C \xrightarrow{Enc(m)} A \longrightarrow Nat$$

Then (A', Nat) has robust advantage ε(·,a(·)) for C'.

C'
$$\xrightarrow{f(x)}$$
 A' \longrightarrow Nat

 \exists an ϵ -cosmic reduction from C to C' if \forall PPT A, \exists PPT A' s.t. \forall Nat:

Suppose (A, Nat) has robust advantage a(·) for C



 \exists an ϵ -cosmic reduction from C to C' if \forall PPT A, \exists PPT A' s.t. \forall Nat:

Suppose (A, Nat) has robust advantage a(•) for C



 \exists an ϵ -cosmic reduction from C to C' if \forall PPT A, \exists PPT A' s.t. \forall Nat:

Suppose (A, Nat) has robust advantage a(•) for C



Roadmap

- 1. Motivation (10min)
- 2. Defining Cosmic Security (15min)

3. Properties of Cosmic Security: a Sanity Check

- a. Composition, Black-box reductions (5min)
- 4. Summary of Key Results
 - a. Feasibilities and Impossibilities (20min)
- 5. Other Notions of Cosmic Security (10min)
- 6. Conclusion (5min)

Two Nice Properties

What makes us think that cosmic security is a good definition?

 <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".


What makes us think that cosmic security is a good definition?

 <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".



Follows nicely from definition!

What makes us think that cosmic security is a good definition?

- <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".
- 2. <u>Dummy Lemma</u>: Regular cosmic reductions are *equivalent* to black-box cosmic reductions.



What makes us think that cosmic security is a good definition?

- <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".
- 2. <u>Dummy Lemma</u>: Regular cosmic reductions are *equivalent* to black-box cosmic reductions.

Let's quickly intuit this to understand cosmic security better



What makes us think that cosmic security is a good definition?

- <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".
- 2. <u>Dummy Lemma</u>: Regular cosmic reductions are *equivalent* to black-box cosmic reductions.

Let's quickly intuit this to understand cosmic security better



What makes us think that cosmic security is a good definition?

- <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".
- 2. <u>Dummy Lemma</u>: Regular cosmic reductions are *equivalent* to black-box cosmic reductions.

Let's quickly intuit this to understand cosmic security better





 $(\mathbf{R}^{\mathsf{A}}, \mathbf{Nat})$ has robust adv $\varepsilon(.,a(.))$ for **C**'.





(**R**^A, **Nat**) has *robust adv* ε(.,a(.)) for **C**'.









Recap So Far: A Necessary, Natural Definition

Why do we like Cosmic Security?

1. <u>Acknowledges the Unknown</u>: allows Nature to be *stateful* in a way previously not considered, but that we are now compelled to acknowledge.

Moreover, the definition is Well-Behaved.

- 2. <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".
- 3. <u>Dummy Lemma</u>: Regular cosmic reductions are *equivalent* to black-box cosmic reductions.



Recap So Far: A Necessary, Natural Definition

Why do we like Cosmic Security?

1. <u>Acknowledges the Unknown</u>: allows Nature to be *stateful* in a way previously not considered, but that we are now compelled to acknowledge.

Moreover, the definition is Well-Behaved.

- 2. <u>Composability</u>: Cosmic reductions are composable; that is, if C reduces to C' and C' reduces to C", then C reduces to C".
- 3. <u>Dummy Lemma</u>: Regular cosmic reductions are *equivalent* to black-box cosmic reductions.





Roadmap

- 1. Motivation (10min)
- 2. Defining Cosmic Security (15min)
- **3.** Properties of Cosmic Security: a Sanity Check
 - a. Composition, Black box reductions (5min)
- 4. Summary of Key Results
 - a. Feasibilities and Impossibilities (20min)
- 5. Other Notions of Cosmic Security (10min)
- 6. Conclusion (5min)

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

New 1-shot straight-line black-box proof for WI! (See Paper)

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

<u>Corollaries</u>: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

What about reductions that use the attacker multiple times?

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

<u>Corollaries</u>: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>Thm 2</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>Thm 3</u> (*Impossibility*): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible.

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

<u>Corollaries</u>: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>Thm 2</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>Thm 3</u> (*Impossibility*): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated inputs.

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

<u>Corollaries</u>: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>Thm 2</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>Thm 3</u> (*Impossibility*): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated inputs.

Claim: we need new techniques to build advanced cosmic cryptography!

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>**Thm 2**</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>**Thm 3**</u> (*Impossibility*): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated inputs.

<u>Thm 4</u> (*Feasibility*): Hardness amplification is possible for "re-randomizable" OWFs.

Thm 1 (Feasibility): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

"A First Stab" at feasibility in

cess to

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

Thm 2 (Impossibility): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>**Thm 3**</u> (*Impossibility*): A Goldreich Levin Theorem, where the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated the cosmic setting

<u>Thm 4</u> (*Feasibility*): Hardness amplification is possible for "re-randomizable" OWFs.

Claim: a non-trivial setting (for feasibility), since we rule out some classical approaches.

Claim: a non-trivial setting (for feasibility), since we rule out some classical approaches.

Later: "Relaxed" Cosmic Security

Suppose Nature isn't fully stateful; the only state it keeps is the "time".

Claim: a non-trivial setting (for feasibility), since we rule out some classical approaches.

Later: "Relaxed" Cosmic Security

Suppose Nature isn't fully stateful; the only state it keeps is the "time".

<u>Thm 5</u> (*Informal*): Non-adaptive straight-line black-box reductions give cosmic reductions for Natures that evolve over time (but are otherwise stateless).

Claim: a non-trivial setting (for feasibility), since we rule out some classical approaches.

Later: "Relaxed" Cosmic Security

Intermission

Next Up: Walkthrough of our results

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>Thm 2</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>**Thm 3**</u> (*Impossibility*): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated inputs.

<u>**Thm 4**</u> (*Feasibility*): Hardness amplification is possible for "re-randomizable" OWFs.

<u>Thm 1</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>**Thm 2</u>** (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.</u>

Thm 3 (Impossibility): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated inputs.

<u>**Thm 4**</u> (*Feasibility*): Hardness amplification is possible for "re-randomizable" OWFs.

Thm 1 (Feasibility): classical 1-shot straight-line black-box reductions imply cosmic reductions.

1-shot straightline black-box ${\bf R}$



<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.



Thm 1 (Feasibility): classical 1-shot straight-line black-box reductions imply cosmic reductions.



Thm 1 (Feasibility): classical 1-shot straight-line black-box reductions imply cosmic reductions.



Thm 1 (Feasibility): classical 1-shot straight-line black-box reductions imply cosmic reductions.



Thm 1 (Feasibility): classical 1-shot straight-line black-box reductions imply cosmic reductions.



Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>Thm 2</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>**Thm 3**</u> (*Impossibility*): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated inputs.

<u>**Thm 4**</u> (*Feasibility*): Hardness amplification is possible for "re-randomizable" OWFs.
Key Results: Feasibilities and Impossibilities

<u>Thm 2</u> (Impossibility of Hardness Amplification):

Suppose there is an ε -cosmic black reduction from the OWF security of $g^n(x_1...x_n) = (g(x_1), ..., g(x_n))$ to the OWF security of g(x) that uses only black-box access to g, and that works for any function g. Then, there exists a negligible function μ such that $\varepsilon(\lambda, a) \le a + \mu(\lambda)$.

Recall: Classical Hardness Amplification



Recall: Classical Hardness Amplification











Thus, Black-Box Cosmic Hardness Amplification is *Impossible*.

Classical reductions that use an attacker repeatedly on correlated inputs may fail, if the attacker notices the correlation and halts.

We may need to hide the correlation.

Key Results: Feasibilities and Impossibilities

<u>**Thm 1**</u> (*Feasibility*): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>**Thm 2**</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>Thm 3</u> (*Impossibility*): A Goldreich Levin Theorem, where the reduction has only black-box access to the OWF, is impossible. Same High Level Idea!

> Teaser: the cosmic adversary can notice when it is sent correlated inputs.

<u>**Thm 4**</u> (*Feasibility*): Hardness amplification is possible for "re-randomizable" OWFs.

Key Results: Feasibilities and Impossibilities

<u>Thm 3</u> (Impossibility of a Goldreich Levin Theorem):

Suppose there is an ε -cosmic black-box reduction from the security of the hardcore predicate $h(x,r) = \langle x,r \rangle$ w.r.t. f(x, r) = (g(x), r) to the OWF security of g that uses only black-box access to g and that works for any function g. Then, there is a negligible function μ such that $\varepsilon(\lambda, a) \leq \mu(\lambda)$ for all a.

Recall: Goldreich Levin Theorem



Thus, a Goldreich Levin Theorem is *Impossible*.

Takeaway: classical techniques fail.

To get around it, need techniques that are non-black-box in the OWF.

Key Results: Feasibilities and Impossibilities

Thm 1 (Feasibility): classical 1-shot straight-line black-box reductions imply cosmic reductions.

Corollaries: PRFs/SKE/Commitments/Witness Indistinguishability/PRG Length Extension

"A First Stab" at feasibility in

cess to

> Not surprising, since it doesn't matter that Nature is stateful: a 1-shot reduction only uses Nature once.

<u>**Thm 2**</u> (*Impossibility*): Hardness amplification of arbitrary weak OWFs via direct product, using only black-box access to the OWF, is impossible.

<u>**Thm 3**</u> (*Impossibility*): A Goldreich Levin Theorem, where the OWF, is impossible.

> Teaser: the cosmic adversary can notice when it is sent correlated the cosmic setting

<u>Thm 4</u> (*Feasibility*): Hardness amplification is possible for "re-randomizable" OWFs.

Feasibility: Hardness Amplification

Suppose g(x) is <u>re-randomizable</u>.

Def: A one-way function g is **re-randomizable** if \exists PPT *rand*(.), *recover*(.) s.t. $\forall x, \{r \leftarrow \{0,1\}^{\lambda} : rand(g(x), r)\} \equiv \{x' \leftarrow \{0,1\}^{\lambda} : g(x')\}$ $\forall x, r, let y \leftarrow rand(g(x), r), x' \leftarrow recover(g^{-1}(y), r), then g(x) = g(x').$

Denote $g(x) = rand(g(x), U_{\lambda})$ and x' = recover(x', r), where r is the previous seed used.

Feasibility: Hardness Amplification

Suppose g(x) is <u>re-randomizable</u>.



Feasibility: Hardness Amplification

Suppose g(x) is <u>re-randomizable</u>.



We Can Go Beyond Single-Shot Straight-Line Reductions!

Key Point: We need to hide any correlation between queries in the view of the adversary. For example, by <u>re-randomizing</u> g(x).

How Far Can We Go?

Open Problem: Even though a Goldreich-Levin Theorem (that is black-box in the OWF) is impossible, can we build cosmic PRGs from OWFs?

For now... ...let's climb a different mountain.



Roadmap

- 1. Motivation (10min)
- 2. Defining Cosmic Security (15min)
- **3.** Properties of Cosmic Security: a Sanity Check
 - a. Composition, Black box reductions (5min)
- 4. Summary of Key Results

a. Feasibilities and Impossibilities (20min)

- 5. Other Notions of Cosmic Security (10min)
- 6. Conclusion (5min)

Recall: Why did this fail?

Because Nat can adjust future behavior based on prior game outcomes.



Small Games, Large World

It may be presumptuous to think that **C** or **A** can *influence* the future behavior of **Nat**. Nat

Small Games, Large World

It may be presumptuous to think that **C** or **A** can *influence* the future behavior of **Nat**. Nat

What if Nat plays every game independently, the same way? "Restartable", and *classic*.

maybe **Nat** evolves over time...









maybe **Nat** evolves over time...



...the same way regardless of any interaction we have with it.

Let's formalize it.

<u>Def</u>: (A, Nat) is μ -weakly restartable if \exists Sim s.t. $\forall \lambda$, $\forall C$, \forall interaction prefixes ρ ,



In other words, the behavior of (A, Nat) in the view of any C is pre-programmed and depends only on $|\rho|$.

Regular Cosmic Adversaries



Weakly-Restartable Cosmic Adversaries

(A, Nat) is now a sequence of attackers Sim(0), Sim(1), Sim(2), ...



Weakly-Restartable Cosmic Adversaries

(A, Nat) is now a sequence of attackers $A_1 A_2 A_3 ...$



Weakly-Restartable Cosmic Adversaries

(A, Nat) is now a sequence of attackers $A_1 A_2 A_3 \dots$ (informal)



<u>Theorem:</u> Suppose there is a non-adaptive (straight-line
black-box) reduction R_{classic} from C to C '.
Then there is a cosmic reduction from C to C ', assuming
(A , Nat) is weakly restartable.
Corollaries: Hardcore Bits from OWFs, Hardness
Amplification.












<u>Theorem:</u> Suppose there is a non-adaptive (straight-line black-box) reduction R_{classic} from C to C'. Then there is a cosmic reduction from C to C', assuming (A, Nat) is weakly restartable.



<u>Theorem:</u> Suppose there is a non-adaptive (straight-line black-box) reduction R_{classic} from C to C'. Then there is a cosmic reduction from C to C', assuming (A, Nat) is weakly restartable.



<u>Theorem:</u> Suppose there is a non-adaptive (straight-line black-box) reduction R_{classic} from C to C'. Then there is a cosmic reduction from C to C', assuming (A, Nat) is weakly restartable.



<u>Theorem</u>: Suppose there is a non-adaptive (straight-line black-box) reduction $R_{classic}$ from C to C'. Then there is a cosmic reduction from C to C', assuming (A, Nat) is weakly restartable.



<u>Theorem:</u> Suppose there is a non-adaptive (straight-line black-box) reduction $R_{classic}$ from C to C'. Then there is a cosmic reduction from C to C', assuming (A, Nat) is weakly restartable.



Roadmap

- 1. Motivation (10min)
- 2. Defining Cosmic Security (15min)
- **3.** Properties of Cosmic Security: a Sanity Check
 - a. Composition, Black box reductions (5min)
- 4. Summary of Key Results
 - a. Feasibilities and Impossibilities (20min)
- 5. Other Notions of Cosmic Security (10min)
- 6. Conclusion (5min)

Let's wrap it up.

In Sum

It is perhaps surprising that classical non-adaptive reductions "work" for the cosmic weakly-restartable " $A_1 A_2 A_3$ " model.

Implication: we can treat slightly stateful Natures (that keep only time as state) as stateless! (if we don't make adaptive queries)

In Greater Sum

For fully stateful Natures,

- Several black-box classical reductions that run the adversary repeatedly on correlated inputs cannot have cosmic equivalents.
- So far, feasibility for cosmic reductions is limited to either re-randomizing correlated queries, or sticking to one-shot reductions.

In Greater, Greater Sum



Biggest Takeaway: we should consider **stateful** adversaries that may behave differently each time its run.

What's Next? An Unexplored Universe.

- PRGs from OWFs?
- MPC?
- New techniques to deal with a stateful Cosmos?

What's Next? An Unexplored Universe.

- PRGs from OWFs?
- MPC?
- New techniques to deal with a stateful Cosmos?



Image Credits: NASA/ESA/Hubble

Extra Slides