Principled Programming

Introduction to Coding in Any Imperative Language

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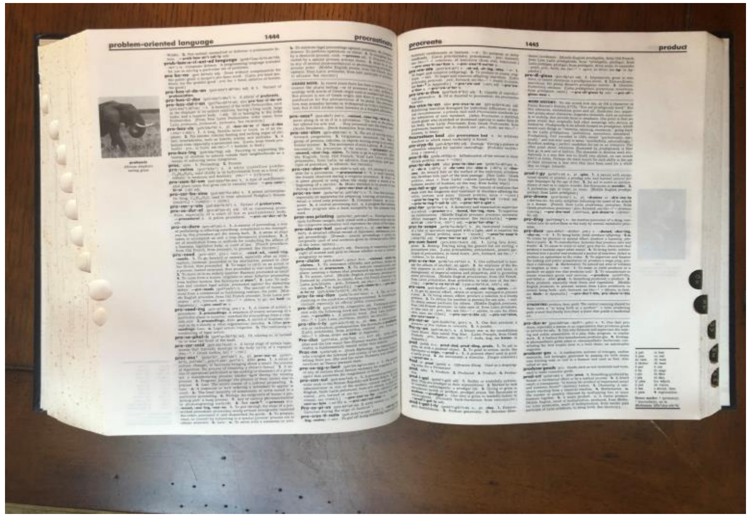
Binary Search

If you want to find the definition of the word **proboscis** in a 512-page dictionary, you wouldn't use Sequential Search starting on the first page, say, with **aardvark**. Rather, you would start roughly in the middle. From there, you would:

- Repeatedly halve the portion of the dictionary that remains under consideration, doing so by looking at the middle page of the region in hand, and discarding whichever half is revealed thereby to not contain **proboscis**.
- Once the search has been narrowed to a single page, you would look on that page to see if **proboscis** is there.
- If it is, you found its definition; otherwise, it isn't in the dictionary.



The method is called Binary Search, and is an example of a Divide and Conquer algorithm. Binary Search is astoundingly fast.

Application: Search for a value v in an unordered array `A[0..n-1]`.

```
/* Given array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
```

☞ **A statement-comment says exactly what code must accomplish, not how it does so.**

Binary Search

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
```

☞ **A statement-comment says exactly what code must accomplish, not how it does so.**

Binary Search

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int k = 0;
   while ( A[k]!=v && k<n ) k++;
```

☞ **Master stylized code patterns, and use them.**

Sequential search works, but ignores the order. We can do better.

Binary Search

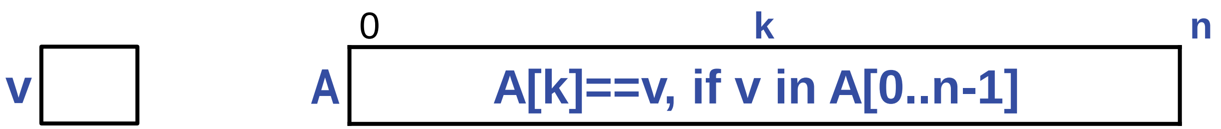
Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   __________
   while ( ______ ) ___________
   __________
```

Coding order
(1) body
(2) termination
(3) initialization
(4) finalization
(5) boundary conditions

☞ **If you "smell a loop", write it down.**

Binary Search



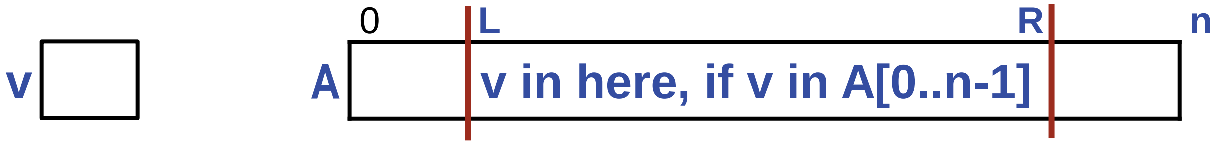
POST

Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   __________
   while ( ______ ) ___________
   __________
```

☞ **Invent (or learn) diagrammatic ways to express concepts.**

Binary Search



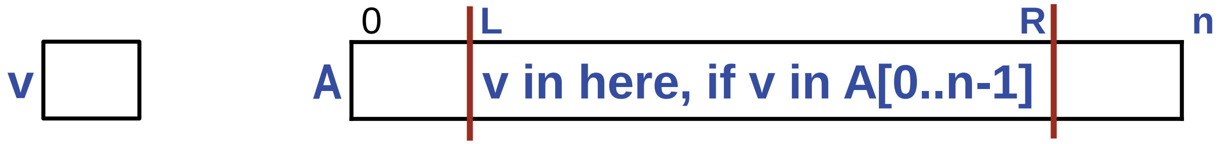
INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   __________
   while ( ______ ) ___________
   __________
```

☞ **To get to POST iteratively, choose a weakened POST as INVARIANT.**

Binary Search



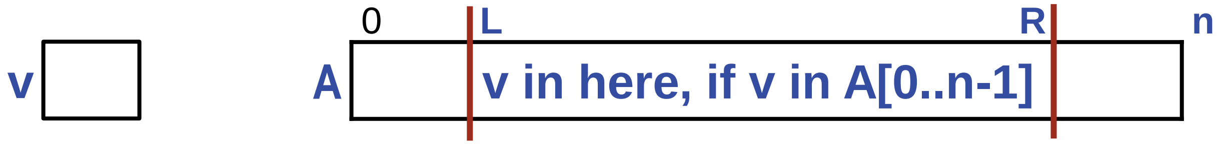
INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int L = ______; int R = ______;
   while ( ______ ) ___________
   __________
```

☞ **Introduce program variables whose values describe “state”.**

Binary Search



VARIANT: R-L

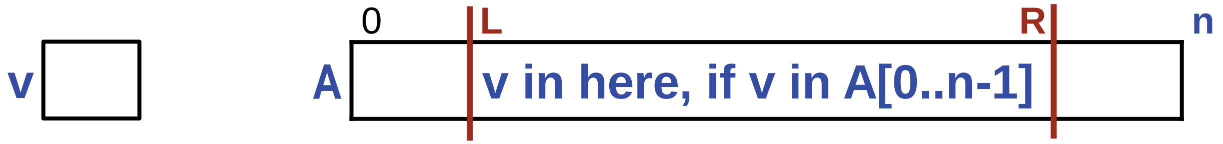
INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int L = ______; int R = ______;
   while ( ______ )
      if ( ________ ) ___________ else ___________
   __________
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

Binary Search



VARIANT: R-L

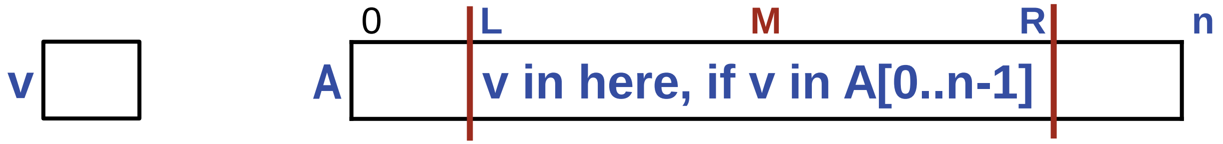
INVARIANT

Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int L = ______; int R = ______;
   while ( ______ )
      if ( ________ )
         R = ______;      // Select left “half”.
      else L = ______;    // Select right “half”.
   __________
```

☞ **A Case Analysis in the loop body is often needed for characterizing different ways in which to decrease the loop variant while maintaining the loop invariant.**

Binary Search



v

A

0

L

M

R

n

v in here, if v in A[0..n-1]

VARIANT: R-L

INVARIANT

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int L = ______; int R = ______;
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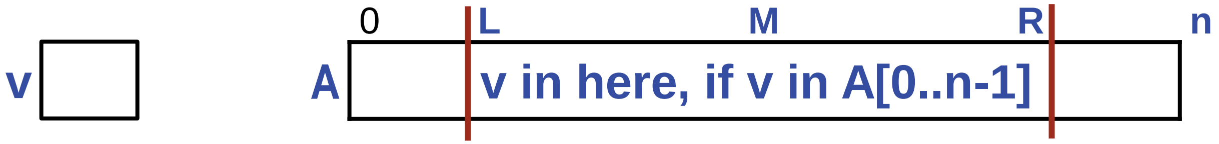
Binary Search



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Application: Search for a value v in an ordered array `A[0..n-1]`.

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      if ( ________ )
         R = ______;        // Select left “half”.
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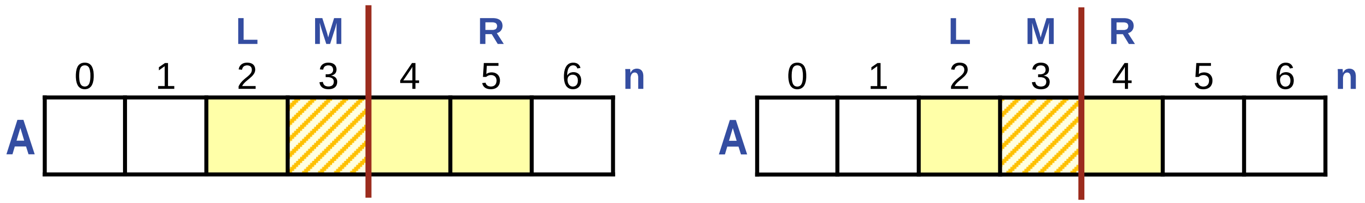
   __________
```

If you object to `A[L..R]` straddling the midpoint of `A[0..n-1]`, understand that in “schematic diagrams”, the exact locations of boundaries are immaterial.

Binary Search



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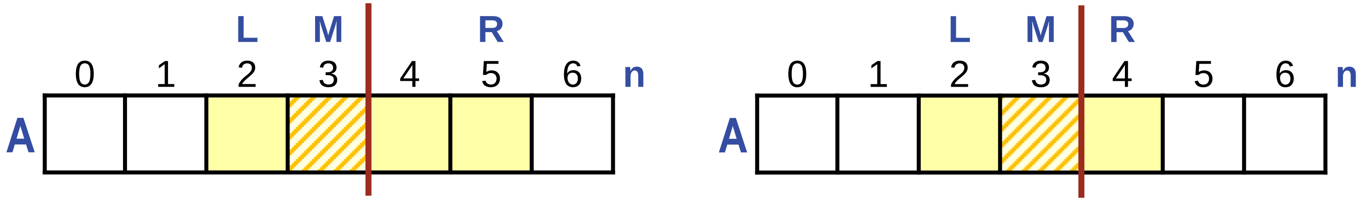
☞ **Be alert to high-risk coding steps associated with binary choices.**

Recognize that regions of even and odd lengths may need **distinct** treatments.

Binary Search



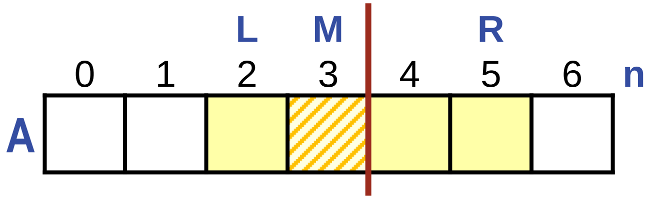
M is index of rightmost element of left "half".

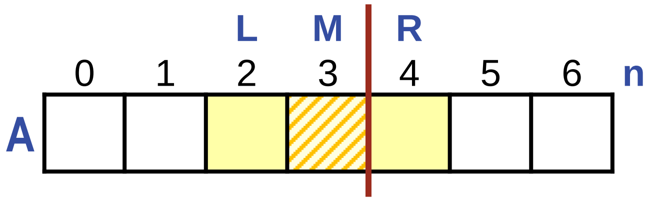
Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int L = ______; int R = ______;
   M = (L+R)/2;             // Compute "midpoint".
   while ( ______ )
      if ( ________ )
         R = ______;        // Select left "half".
      else L = ______;      // Select right "half".
   __________
```

☞ **Be alert to high-risk coding steps associated with binary choices.**

Recognize that regions of even and odd lengths may need **distinct** treatments, but hope for a **uniform** treatment.





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   int L = ______; int R = ______;
   M = (L+R)/2;              // Compute “midpoint”.
   while ( ______ )
      if ( ________ )
         R = M;              // Select left “half”.
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   __________
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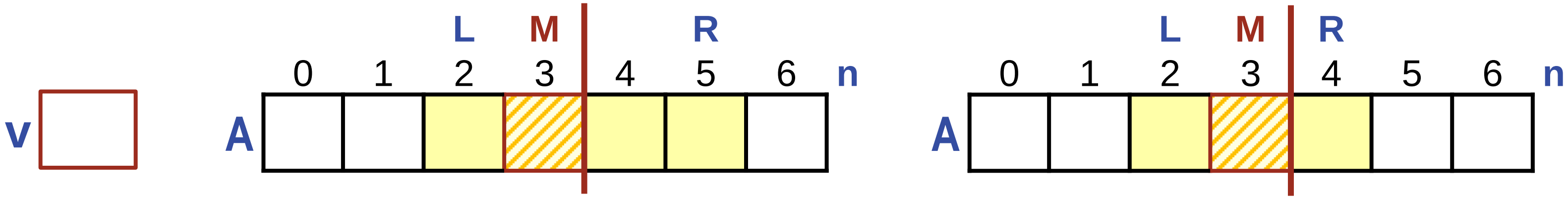
Binary Search



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         R = M;             // Select left "half".
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   __________
```

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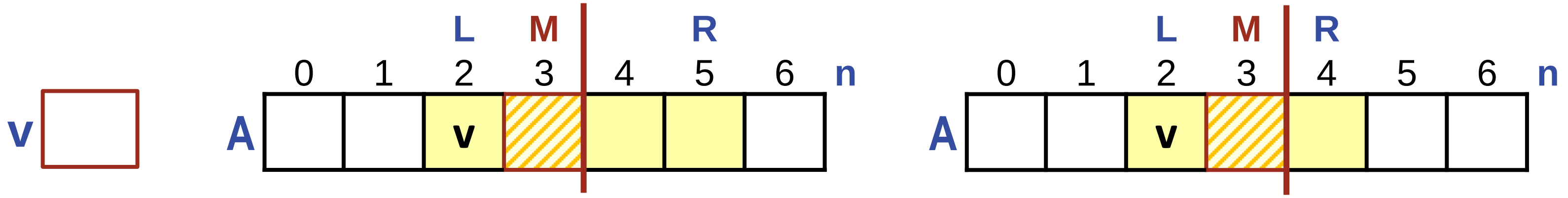
Binary Search



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   while ( ______ )
      if ( v <= A[M] )
         R = M;                // Select left "half".
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   __________
```

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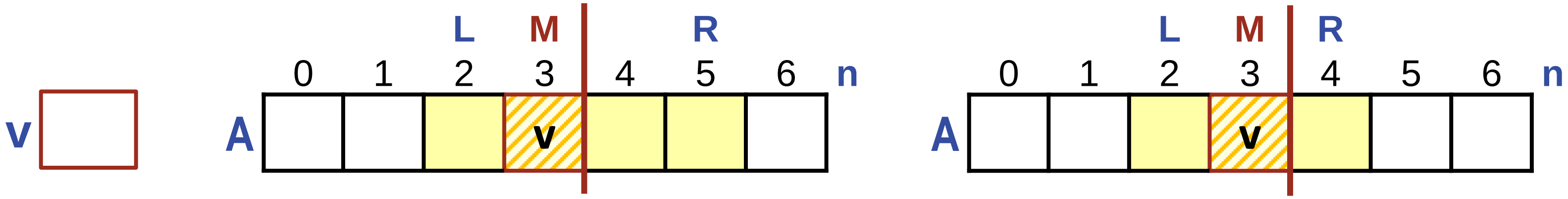
Binary Search



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   __________
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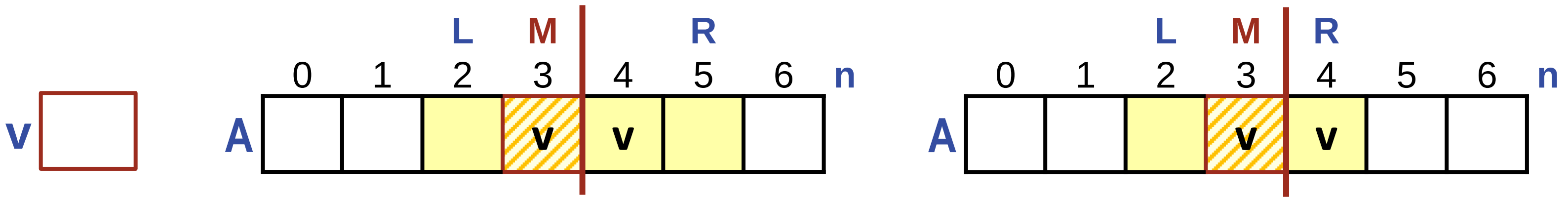
Binary Search



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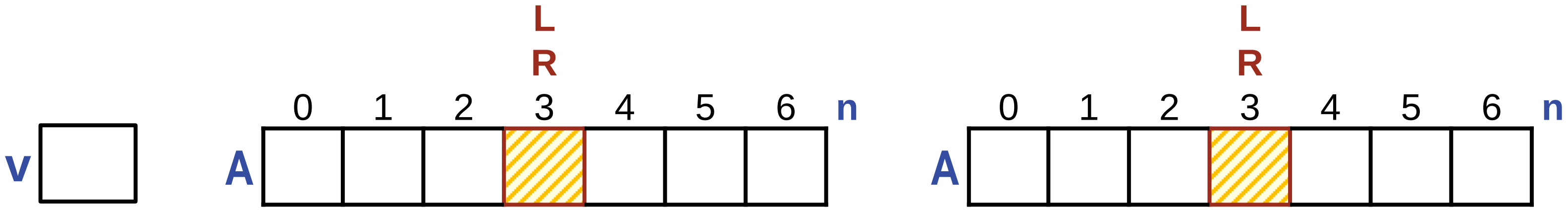
         R = M;              // Select left “half”.
      else L = M+1;          // Select right “half”.

   __________
```

☞ **Be alert to high-risk coding steps associated with binary choices.**

Duplicate instances of v in `A[L..R]` may escape, but not all of them.



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   M = (L+R)/2;                // Compute "midpoint".
   while ( L != R )
      if ( v <= A[M] )
         R = M;                // Select left "half".
      else L = M+1;            // Select right "half".
   __________
```

Coding order
(1) body
(2) termination
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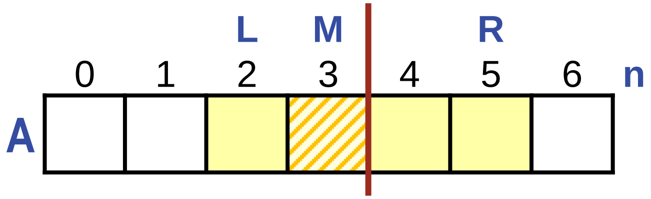
Binary Search



VARIANT:

Before			After (left)			After (right)		
L	R	R-L	L	R	R-L	L	R	R-L
2	5	3	2	3	1	4	5	1

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int L = ______; int R = ______;
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   while ( L != R )
      if ( v <= A[M] )
         R = M;              // Select left “half”.
      else L = M+1;          // Select right “half”.
   __________
```

Coding order
(1) body
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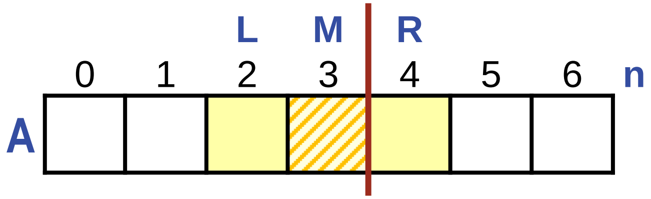
Confirm that the **VARIANT** decreases on every iteration.

Binary Search



VARIANT:

Before			After (left)			After (right)		
L	R	R-L	L	R	R-L	L	R	R-L
2	4	2	2	3	1	4	4	0

Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   int L = ______; int R = ______;
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   while ( L != R )
      if ( v <= A[M] )
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   __________
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Coding order
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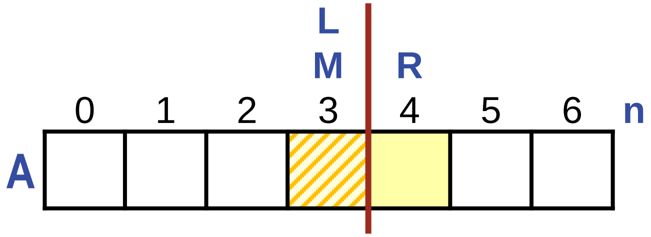
Confirm that the **VARIANT** decreases on every iteration.

Binary Search



VARIANT:

Before			After (left)			After (right)		
L	R	R-L	L	R	R-L	L	R	R-L
3	4	1	3	3	0	4	4	0

Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
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   __________
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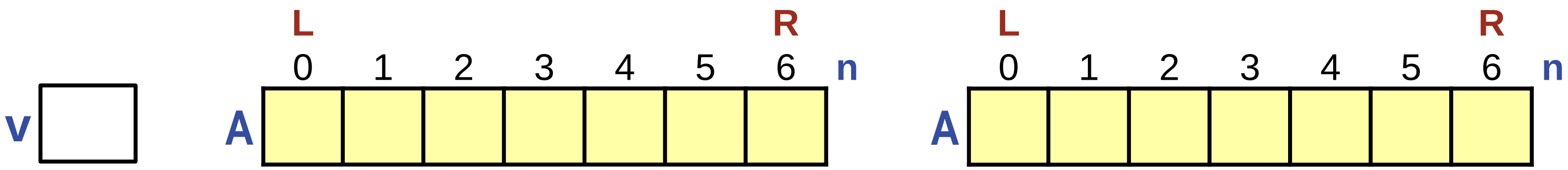

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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   int L = 0; int R = n-1;
   M = (L+R)/2;             // Compute “midpoint”.
   while ( L != R )
      if ( v <= A[M] )
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   __________
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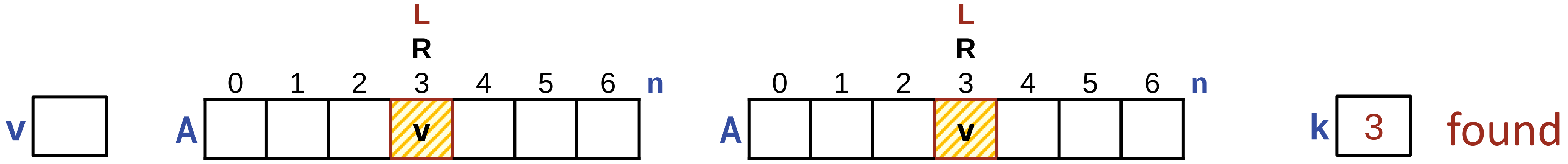

Application: Search for a value v in an ordered array A[0..n-1].

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/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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   int L = 0; int R = n-1;
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   while ( L != R )
      if ( v <= A[M] )
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   if ( A[L]==v ) k = L; else k = n;
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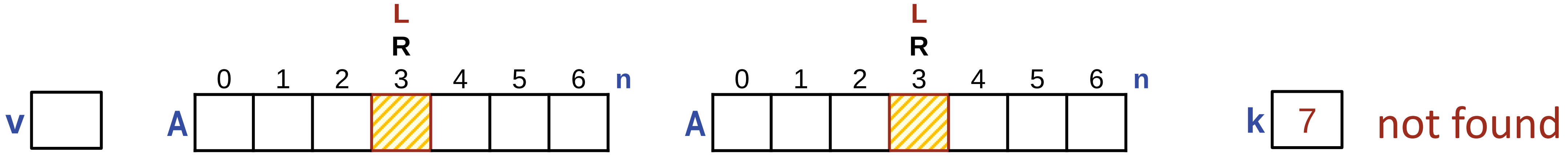

Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
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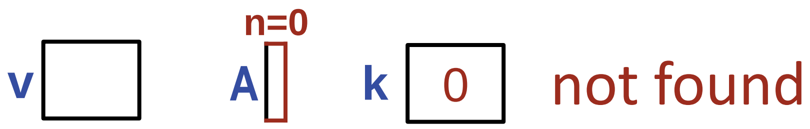

Application: Search for a value v in an ordered array A[0..n-1].

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   if ( n==0 ) k = 0;
   else {
      int L = 0; int R = n-1;
      M = (L+R)/2;               // Compute “midpoint”.
      while ( L != R )
         if ( v <= A[M] )
            R = M;               // Select left “half”.
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      if ( A[L]==v ) k = L; else k = n;
      }
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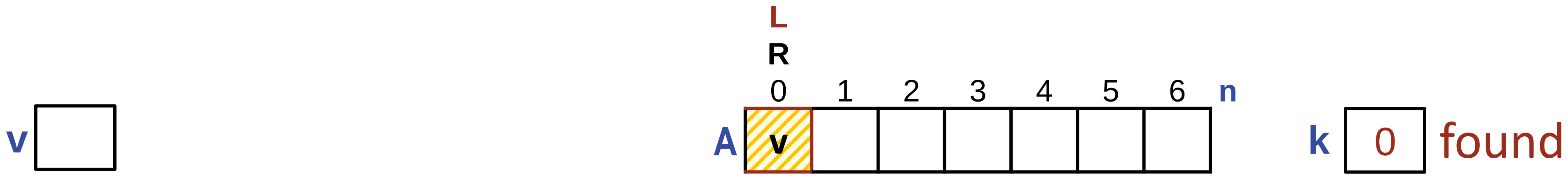

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      while ( L != R )
         if ( v <= A[M] )
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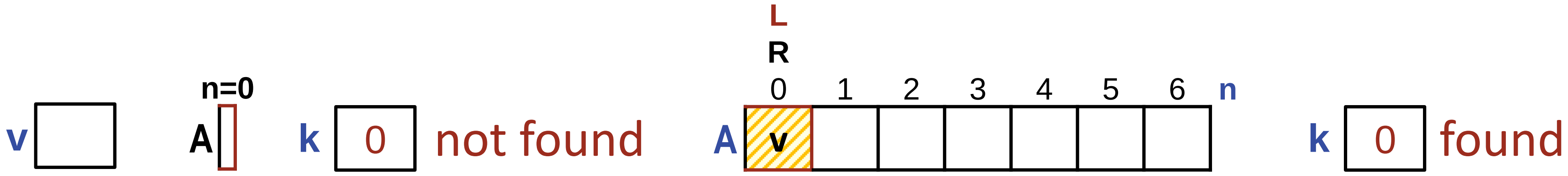
         else L = M+1;       // Select right “half”.
      if ( A[L]==v ) k = L; else k = n;
      }
```

Coding order
(1) body
(2) termination
(3) initialization
(4) finalization
(5) boundary conditions

Binary Search



Application: Search for a value v in an ordered array `A[0..n-1]`.

```
/* Given ordered array A[0..n-1], n≥0, and value v, let k be an index of A
   where A[k]==v, or n if there is no v in A. */
   if ( n==0 ) k = 0;
   else {
      int L = 0; int R = n-1;
      M = (L+R)/2;              // Compute “midpoint”.
      while ( L != R )
         if ( v <= A[M] )
            R = M;              // Select left “half”.
         else L = M+1;          // Select right “half”.
      if ( A[L]==v ) k = L; else k = n;
      }
```

Coding order
(1) body
(2) termination
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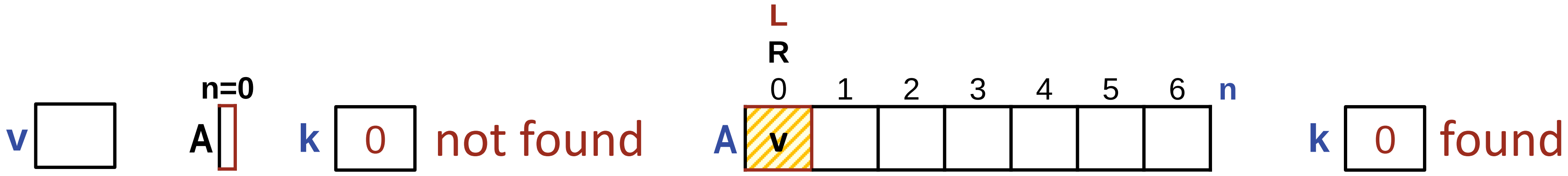
Is it a problem that k==0 represents both “not found” and “found in 0th element”?

Binary Search



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         else L = M+1;        // Select right “half”.
      if ( A[L]==v ) k = L; else k = n;
      }
if ( k<n ) /* Found. */ else /* Not found. */
```

Coding order
(1) body
(2) termination
(3) initialization
(4) finalization
(5) boundary conditions

No. What matters is whether k<n, not whether k==0.

Binary Search

Binary Search is astoundingly fast. If n==512, just 9 iterations to termination!

Iteration #	VARIANT
0	512
1	256
2	128
3	64
4	32
5	16
6	8
7	4
8	2
9	1

Running time is logarithmic in n,

and independent of whether v is in A or not.