## **Principled Programming**

Introduction to Coding in Any Imperative Language

#### Tim Teitelbaum

Emeritus Professor Department of Computer Science Cornell University

## **Online Algorithms**

We introduce the *online-computation* pattern for processing an unbounded file of input. We use it to:

- Process exam grades
- Compress the file
- Decompress a compressed file

We illustrate many important programming precepts.

Application: Process an input file of unbounded length.



**Offline-computation pattern:** calls for reading all values first.





# **Offline-computation pattern:** calls for reading all values first, then processing them.

/\* Input. \*/
/\* Compute. \*/
/\* Output. \*/



**Offline-computation pattern:** calls for reading all values first, then processing them, then outputting results.

<b></b>		٦
i /*	Input. */	i
/*	Compute. */	i
¦/*	Output. */	i
		-



**Offline-computation pattern:** A mismatch because the memory is finite, but the input is unbounded.

/\* Input. \*/
/\* Compute. \*/
/\* Output. \*/



**Offline-computation pattern:** A mismatch because the memory is finite, but the input is unbounded.\*

/\* Input. \*/ /\* Compute. \*/ /\* Output. \*/

\*Virtual memory is also effectively unbounded, so the real issue is paging time.



#### **Online-computation pattern:** An alternative is to process input values on the fly.

```
v = first-input-value;
/* Initialize. */
while ( v != stoppingValue ) {
    /* Process v. */
    v = next-input-value;
    }
/* Finalize. */
```

#### **Online-computation pattern:** A specialization of the general-iteration pattern.

v = first-input-value; /\* Initialize. \*/ while ( v != stoppingValue ) { /\* Process v. \*/ v = /\* next input value \*/; } /\* Finalize. \*/

#### **Online-computation pattern:** Not all problems amenable to online computation.

```
v = first-input-value;
/* Initialize. */
while ( v != stoppingValue ) {
    /* Process v. */
    v = /* next input value */;
    }
/* Finalize. */
```

Amenable if:

 Inputs are independent and can be fully processed on the fly.

#### **Online-computation pattern:** Not all problems amenable to online computation.

```
v = first-input-value;
/* Initialize. */
while ( v != stoppingValue ) {
    /* Process v. */
    v = /* next input value */;
    }
/* Finalize. */
```

Amenable if:

- Inputs are independent and can be fully processed on the fly, or
- Inputs can be summarized on the fly, and the final result computed from those summary values.

# **Online-computation pattern:** Assume inputs are nonnegative integers, followed by -1.

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
/* Initialize. */
while ( v != -1 ) {
    /* Process v. */
    v = in.nextInt();
    }
/* Finalize. */
```

#### **Online-computation pattern:** Parametric in $\alpha$ , $\beta$ , and $\gamma$ .

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
/* Initialize. (α) */
while ( v != -1 ) {
    /* Process v. (β) */
    v = in.nextInt();
    }
/* Finalize. (γ) */
```

Application: Process exam grades (in range 0-100).

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Application	α	β	γ
Print			
Count			
Average			
Highest			
Distribution			

Application: Process exam grades (in range 0-100).

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

ש
<b>D</b>
-
-
Ō
X
$\mathbf{O}$
Ω
Ö
<u>v</u>
Ę
$\mathbf{m}$

	Print
	Count
	Average
ge 0-100).	Highest

Application

Distribution

β

α

90 80 85 90 100 0 85	-1
----------------------	----

Application: Process exam grades (in range 0-100).

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

#### **There is no shame in reasoning with concrete examples.**

Application	α	β	γ
Print			
Count			
Average			
Highest			
Distribution			

Application: Process exam grades (in range 0-100).

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Code iterations in the following order: (1) body, (2) termination, (3) initialization,
 (4) finalization, (5) boundary conditions.

Application	α	β	γ
Print			
Count			
Average			
Highest			
Distribution			

Application: Process exam grades (in range 0-100).

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

**Data Processing** 

# Print grades

Application: Print grades.

Program top-down, outside-in.

Data Processing Print grades

Master stylized code patterns, and use them.

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

#### Master stylized code patterns, and use them.

int

(JO)

rades

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (a) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

int

<u>(</u>

rades

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (a) */
while ( grade != -1 ) {
    System.out.println(grade);
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

int

(JO)

rades

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
while ( grade != -1 ) {
   System.out.println(grade);
   grade = in.nextInt();
   }
/* Finalize. (γ) */
```

Coding order	
(1) body	β; grade=in.nextInt();
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

int

grades

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
while ( grade != -1 ) {
   System.out.println(grade);
   grade = in.nextInt();
   }
```

Coding order	
(1) body	β; grade=in.nextInt();
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

grades

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Seek algorithmic inspiration from experience. Hand-simulate an algorithm that is in your "wetware". Be introspective. Ask yourself: What am I doing?

Counting the input values.

ount

grades

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = ____; // count is the number of grades processed so far.
/* Initialize. (a) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Introduce program variables whose values describe "state".

A countercount value . Establish and maintain its representation invariant.

ount

grades

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = _____; // count is the number of grades processed so far.
/* Initialize. (a) */
while ( grade != -1 ) {
    count++;
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

ata

rocessing

Count

Maintain invariant.

(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

**Coding order** 

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = 0; // count is the number of grades processed so far.
while ( grade != -1 ) {
    count++;
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

Count

grade

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = 0; // count is the number of grades processed so far.
while ( grade != -1 ) {
    count++;
    grade = in.nextInt();
    }
System.out.println(count);
```

Coding order	
(1) body	β; grade=in.nextInt();
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

Count

<u>M</u>

rade

90	80	85	90	100	0	85	-1
-			-		-		

Application: Average grade.

œ

Seek algorithmic inspiration from experience. Hand-simulate an algorithm that is in your "wetware". Be introspective. Ask yourself: What am I doing?

Data Processin

(JO)

Averag

Ð

(na

rade

#### Application: Average grade.

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Seek algorithmic inspiration from experience. Hand-simulate an algorithm that is in your "wetware". Be introspective. Ask yourself: What am I doing?

Remember to do online, not offline, computation.

#### Application: Average grade.

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

#### Introduce program variables whose values describe "state".

werag

Ð

(na

rade
```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = _____; // count is the number of grades processed so far.
int sum = _____; // sum is the sum of the grades processed so far.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Introduce program variables whose values describe "state".

A counter count value and a running sum sum value

verag

D

gra

**D**e

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = _____; // count is the number of grades processed so far.
int sum = _____; // sum is the sum of the grades processed so far.
/* Initialize. (a) */
while ( grade != -1 ) {
    count++; sum = sum+count;
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Maintain invariants.

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

verag

Ð

gra

Qe

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = 0; // count is the number of grades processed so far.
int sum = 0; // sum is the sum of the grades processed so far.
while ( grade != -1 ) {
    count++; sum = sum+count;
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

Averag

Ð

grad

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = 0; // count is the number of grades processed so far.
int sum = 0; // sum is the sum of the grades processed so far.
while ( grade != -1 ) {
    count++; sum = sum+count;
    grade = in.nextInt();
    }
System.out.println(sum/count);
```

Coding order	
(1) body	β; grade=in.nextInt();
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

verag

Ð

grad

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int count = 0; // count is the number of grades processed so far.
int sum = 0; // sum is the sum of the grades processed so far.
while ( grade != -1 ) {
    count++; sum = sum+count;
    grade = in.nextInt();
    }
if (count==0) System.out.println("no grades"):
else System.out.println(sum/count); Coding order
```

verag

Ð

grad

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Seek algorithmic inspiration from experience. Hand-simulate an algorithm that is in your "wetware". Be introspective. Ask yourself: What am I doing?

Keeping track of highest.

ghest

grad

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int highest = ____; // highest is max of the grades processed so far.
/* Initialize. (a) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Introduce program variables whose values describe "state".



N

nest

grad

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int highest = _____; // highest is max of the grades processed so far.
/* Initialize. (a) */
while ( grade != -1 ) {
    if ( grade > highest ) highest = grade;
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Maintain invariant.

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

ighest

grad

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int highest = _____; // highest is max of the grades processed so far.
/* Initialize. (a) */
while ( grade != -1 ) {
    highest = Math.max(highest,grade);
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Maintain invariant.

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

ghest

grad

# Wrong! Need to distinguish between no grades and everyone got a 0; **Application:** Highest grade. int grade = in.nextInt(); // grade is the next grade to be processed, or -1. int highest = 0; // highest is max of the grades processed so far. while ( grade != -1 ) { highest = Math.max(highest,grade); grade = in.nextInt(); /\* Finalize. $(\gamma)$ \*/

Establish invariants.

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

# )ata Processing Highest grade

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int highest = -1; // highest is max of grades processed so far, or -1
while ( grade != -1 ) {
    highest = Math.max(highest,grade);
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

ighest

grad

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int highest = -1; // highest is max of grades processed so far, or -1
while ( grade != -1 ) {
    highest = Math.max(highest,grade);
    grade = in.nextInt();
    }
if (highest==-1) System.out.println("no grades");
else System.out.println(sum/count); Coding order
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

ghest

50

rad

ന

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

90	80	85	90	100	0	85	-1
					-		

```
Application: Distribution of grades.
```

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (α) */
while ( grade != -1 ) {
    /* Process v. (β) */
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Seek algorithmic inspiration from experience. Hand-simulate an algorithm that is in your "wetware". Be introspective. Ask yourself: What am I doing?

90	80	85	90	100	0	85	-1
50	00	05	50	TOO	0	05	

```
Application: Distribution of grades.
```

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
/* Initialize. (\alpha) */
while ( grade != -1 ) {
   /* Process v. (\beta) */
   grade = in.nextInt();
/* Finalize. (\gamma) */
```

Seek algorithmic inspiration from experience. Hand-simulate an algorithm ß that is in your "wetware". Be introspective. Ask yourself: What am I doing?

Counting the number of occurrences of each grade.

ata

roc

**B**S

Sin

```
Application: Distribution of grades.
```

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int freq[] = new int[101]; // For each k, freq[k] is # of grades of k so far.
/* Initialize. (a) */
while (grade != -1) {
   /* Process v. (β) */
   grade = in.nextInt();
/* Finalize. (\gamma) */
```

Introduce program variables whose values describe "state". ß

	-	0	1	2	•••	99	100
Need 101 counters.	freq						

ata

roc

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int freq[] = new int[101]; // For each k, freq[k] is # of grades of k so far.
/* Initialize. (a) */
while ( grade != -1 ) {
    freq[grade]++;
    grade = in.nextInt();
    }
/* Finalize. (y) */
```

Maintain invariant.

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

istributio

3

0

<u>m</u>

rad

**e**s

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int freq[] = new int[101] // For each k, freq[k] is # of grades of k so far.
while ( grade != -1 ) {
    freq[grade]++;
    grade = in.nextInt();
    }
/* Finalize. (γ) */
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

istribution

0

**M** 

rad

**e**s

```
int grade = in.nextInt(); // grade is the next grade to be processed, or -1.
int freq[] = new int[101] // For each k, freq[k] is # of grades of k so far.
while ( grade != -1 ) {
    freq[grade]++;
    grade = in.nextInt();
    }
for(int g=0; g<101; g++)
    System.out.println(g + " " + freq[g]); Coding order</pre>
```

Coding order	
(1) body	<pre>β; grade=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

istribution

0

<u>(</u>

rad

Φ

S

#### **Application:** Compressing a file of integers.

10 10 10 10 10 10 1 1 1 1 1 1 1 1 7 7 7 8 9 10 10 10 -1

A sequence of equal values is called a run. Each run of *n* instances of *r* can be encoded as a pair of integers,  $\langle r, n \rangle$ .

10 6 1 8 7 3 8 1 9 1 10 3 -1 -1

A run-encoded file will be shorter if there aren't too many runs of length one.

**Application:** Write a program to run encode an input file.

- Program top-down, outside-in.
- Master stylized code patterns, and use them.

Use the online-computation pattern.

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
/* Initialize. (α) */
while ( v != -1 ) {
    /* Process v. (β) */
    v = in.nextInt();
    }
/* Finalize. (γ) */
```

Program top-down, outside-in.

Master stylized code patterns, and use them.



- Program top-down, outside-in.
- Master stylized code patterns, and use them.

Follow the standard coding order.

```
int v = in.nextInt();  // v is the next integer to be processed, or -1.
a
while ( v != -1 ) {
    β
    v = in.nextInt();
    }
γ
```

Coding order	
(1) body	<pre>β; v=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

```
10 10 10 10 10 10 1 1 1 1 1 1 1 1 1 7 7 8 9 10 10 10 -1
```

Stop the music at an arbitrary, but well-chosen, place marked by the bar.

```
int v = in.nextInt();  // v is the next integer to be processed, or -1.
a
while ( v != -1 ) {
    β
    v = in.nextInt();
    }
γ
```

Body. Do 1st. Play "musical chairs" and "stop the music".

The state: The next value to be processed is v.

The state: The next value to be processed is v, and we are in a run of r values.



The state: The next value to be processed is v, and we are in a run of r values of length n.



**The state:** The next value to be processed is v, and we are in a run of r values of length n. All completed runs seen have been output.

**INVARIANT:** The next value to be processed is v, and we are in a run of r values of length n. All completed runs seen have been output.

Body. Do 1st. Play "musical chairs" and "stop the music". Characterize the "program state" when the music stops, i.e., at the instant the loop-body is about to execute yet again. If you had stopped one iteration later, what would have looked the same (the "loop invariant"), and what would have changed (the "loop variant")?

**VARIANT:** The number of input values remaining to be processed.

Body. Do 1st. Play "musical chairs" and "stop the music". Characterize the "program state" when the music stops, i.e., at the instant the loop-body is about to execute yet again. If you had stopped one iteration later, what would have looked the same (the "loop invariant"), and what would have changed (the "loop variant")?

**VARIANT:** The number of input values remaining to be processed (which the online-computation pattern reduces by 1).

Body. Do 1st. Play "musical chairs" and "stop the music". Characterize the "program state" when the music stops, i.e., at the instant the loop-body is about to execute yet again. If you had stopped one iteration later, what would have looked the same (the "loop invariant"), and what would have changed (the "loop variant")?

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
α
while ( v != -1 ) {
   β
   v = in.nextInt();
γ
```

n

A Case Analysis in the loop body is often needed for characterizing different ß ways in which to decrease the loop variant while maintaining the loop invariant.

$$\begin{bmatrix} n & v \\ r & v \\ 10 & 10 & 10 & 10 \end{bmatrix} \begin{bmatrix} n & v \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 7 & 7 & 8 & 9 & 10 & 10 & -1 \end{bmatrix}$$
  
INVARIANT: The next value to be processed is v, and we are in a run of r values of length n. All completed runs seen have been output.

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
α
while ( v != -1 ) {
                                               First case: Still in the middle of a run.
   if ( v==r ) n++;
   else
   v = in.nextInt();
γ
```

n

10 10 10 10 10 10 1 1 1 1 1 1 1 1 1 7 7 7 8

values of length n. All completed runs seen have been

A Case Analysis in the loop body is often needed for characterizing different œ ways in which to decrease the loop variant while maintaining the loop invariant.



**INVARIANT:** The next value to be processed is v, and we are in a run of r values of length n. All completed runs seen have been output.

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.

while ( v != -1 ) {
    if ( v==r ) n++;
    else {
        System.out.print(r + " " + c + " ");
        Second case: Output the now-completed run.
        }
        v = in.nextInt();
     }
```

Maintain invariant.


```
int v = in.nextInt(); // v is the next integer to be processed, or -1.

a
while ( v != -1 ) {
    if ( v==r ) n++;
    else {
        System.out.print(r + " " + c + " ");
        r = v; n = 1;
        }
    v = in.nextInt();
    }

y
```

Maintain invariant.



```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
a
while ( v != -1 ) {
    if ( v==r ) n++;
    else {
        System.out.print(r + " " + c + " ");
        r = v; n = 1;
        }
    v = in.nextInt();
    }
v
```

Maintain invariant.



```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
int r = ___; int n = ___; // In a run of r values of length n.
while ( v != -1 ) {
    if ( v==r ) n++;
    else {
        System.out.print(r + " " + c + " ");
        r = v; n = 1;
        }
    v = in.nextInt();
    }
    V
```

Establish invariant.

(1) body	<pre>β; v=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

ata

Q

ompression



```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
int r = v; int n = 0; // In a run of r values of length n.
while ( v != -1 ) {
    if ( v==r ) n++;
    else {
        System.out.print(r + " " + c + " ");
        r = v; n = 1;
        }
    v = in.nextInt();
    }
    v
```

Establish invariant.

at
ja j
Q
9
E
P
<b>e</b>
S

Coding order	
(1) body	<pre>β; v=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case



```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
int r = v; int n = 0; // In a run of r values of length n.
while ( v != -1 ) {
   if ( v==r ) n++;
   else {
      System.out.print(r + " " + c + " ");
      r = v; n = 1;
                                                        Coding order
   v = in.nextInt();
                                                                      β; v=in.nextInt();
                                                     (1) body
                                                     (2) termination
                                                                      -
γ
                                                     (3) initialization
                                                                      α
```

(4) finalization

(5) boundary conditions

γ

exceptions to the general case



```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
int r = v; int n = 0; // In a run of r values of length n.
while ( v != -1 ) {
   if ( v==r ) n++;
   else {
      System.out.print(r + " " + c + " ");
      r = v; n = 1;
                                                       Coding order
   v = in.nextInt();
                                                                     β; v=in.nextInt();
                                                    (1) body
                                                    (2) termination
                                                                     -
System.out.print(r + " " + c + " ");
                                                    (3) initialization
                                                                     α
System.out.println("-1 -1");
```

(4) finalization

(5) boundary conditions

γ

exceptions to the general case

n r v -1

**INVARIANT:** The next value to be processed is v, and we are in a run of r values of length n. All completed runs seen have been output.

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
int r = v; int n = 0; // In a run of r values of length n.
while ( v != -1 ) {
   if ( v==r ) n++;
   else {
      System.out.print(r + " " + c + " ");
      r = v; n = 1;
                                                       Coding order
   v = in.nextInt();
                                                                     β; v=in.nextInt();
                                                    (1) body
                                                    (2) termination
                                                                     -
System.out.print(r + " " + c + " ");
                                                    (3) initialization
System.out.println("-1 -1");
                                                                     α
```

(5) boundary conditions exceptions to the general case

γ

(4) finalization

n r v -1

**INVARIANT:** The next value to be processed is v, and we are in a run of r values of length n. All completed runs seen have been output.

```
int v = in.nextInt(); // v is the next integer to be processed, or -1.
int r = v; int n = 0; // In a run of r values of length n.
while ( v != -1 ) {
   if ( v==r ) n++;
   else {
      System.out.print(r + " " + c + " ");
      r = v; n = 1;
                                                        Coding order
   v = in.nextInt();
                                                                      β; v=in.nextInt();
                                                     (1) body
                                                     (2) termination
                                                                      -
if ( n!=0 )
                                                     (3) initialization
                                                                      α
   System.out.print(r + " " + c + " ");
System.out.println("-1 -1");
                                                     (4) finalization
                                                                      γ
```

(5) boundary conditions exceptions to the general case

**Application:** Write a program to decode a run-encoded file.

- Program top-down, outside-in.
- Master stylized code patterns, and use them.

Use online-computation pattern.

- Program top-down, outside-in.
- Master stylized code patterns, and use them.

Use online-computation pattern, generalized to read values two at a time.

# **INVARIANT:** Runs have been output for all (r,n) processed so far.

```
int r = in.nextInt(); int n = in.nextInt(); // Next ⟨r,n⟩ to process, or ⟨-1,-1⟩.
a
while ( r != -1 ) {
    β
    r = in.nextInt(); n = in.nextInt();
    }
γ
```

Coding order	
(1) body	<pre>β; r=in.nextInt(); n=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

# **INVARIANT:** Runs have been output for all (r,n) processed so far.

```
int r = in.nextInt(); int n = in.nextInt(); // Next (r,n) to process, or (-1,-1).
a
while ( r != -1 ) {
    for (int k=1; k<=n; k++) System.out.print(r + " ");
    r = in.nextInt(); n = in.nextInt();
    }
y</pre>
```

#### Maintain invariant.

Coding order	
(1) body	<pre>β; r=in.nextInt(); n=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

# **INVARIANT:** Runs have been output for all (r,n) processed so far.

```
int r = in.nextInt(); int n = in.nextInt(); // Next (r,n) to process, or (-1,-1).
while ( r != -1 ) {
    for (int k=1; k<=n; k++) System.out.print(r + " ");
    r = in.nextInt(); n = in.nextInt();
    }
y</pre>
```

### Establish invariant.

Coding order	
(1) body	<pre>β; r=in.nextInt(); n=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

```
int r = in.nextInt(); int n = in.nextInt(); // Next (r,n) to process, or (-1,-1).
while ( r != -1 ) {
    for (int k=1; k<=n; k++) System.out.print(r + " ");
    r = in.nextInt(); n = in.nextInt();
    }
System.out.println (-1);</pre>
```

Coding order	
(1) body	<pre>β; r=in.nextInt(); n=in.nextInt();</pre>
(2) termination	-
(3) initialization	α
(4) finalization	γ
(5) boundary conditions	exceptions to the general case

Precepts used without mention.

- Write the representation invariant of an individual variable as an end-of-line comment.
- Invent (or learn) vocabulary for concepts that arise in a problem.
- Invent (or learn) diagrammatic ways to express concepts.
- Alternate between using a concrete example to guide you in characterizing "program state", and an abstract version that refers to all possible examples.



Precepts used without mention.

- Initialization. Do 3rd. Initialize variables so that the loop invariant is established prior to the first iteration. Substitute those initial values into the invariant, and bench check the first iteration with respect to that initial instantiation of the invariant.
- Finalization. Do 4th, but don't forget. Leverage that the looping condition is false, the loop invariant remains true, and the loop variant is 0.
- Boundary conditions. Dead last, but don't forget them.
- Find boundary conditions at extrema, and at singularities, e.g., biggest, smallest, 0, edges, etc.