Principled Programming

Introduction to Coding in Any Imperative Language

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Creative Representations

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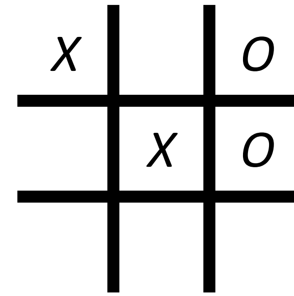
Choice of representations. Straightforward data representations are often effective and sufficient. However, creative representations can result in significant improvements. We present four examples of innovative data representations:

- Tic-Tac-Toe
- Checkers
- The Eight Queens Problem
- The Ricocheting Bee-Bee Problem

Tic-Tac-Toe

Background: Tic-Tac-Toe is played on a 3-by-3 board. Two players take turns marking cells with “X” or “O”. The objective is obtain 3 of your marks in a row, column, or diagonal.

Problem Statement: Write a program that plays Tic-Tac-Toe.



Tic-Tac-Toe

X		O
	X	O

```
/* Data Representation. */
```

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
```

```
/* Test for a win by “X”. */
```

Tic-Tac-Toe

T	0	1	2
0	X		O
1		X	O
2			

```
/* Data Representation. */
   char T[][] = new char[3][3]; // ⟨' ','X','O'⟩ for ⟨blank,“X”,“O”⟩.
```

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 'X';         // Mark T with 'X' for “X”.
```

IN PROGRESS

```
/* Test for a win by “X”. */
   if ( T[0][0]=='X' && T[0][1]=='X' && T[0][2]=='X' ||
        T[1][0]=='X' && T[1][1]=='X' && T[1][2]=='X' ||
        ...
        T[0][0]=='X' && T[1][1]=='X' && T[2][2]=='X' ||
        T[0][2]=='X' && T[1][1]=='X' && T[2][0]=='X' ) // “X” wins.
```

Tic-Tac-Toe

T	0	1	2
0	1	0	-1
1	0	1	-1
2	0	0	0

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
```

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;          // Mark T with 1 for “X”.
```

IN PROGRESS

```
/* Test for a win by “X”. */
   if ( T[0][0]+T[0][1]+T[0][1]==3 ||
        T[1][0]+T[1][1]+T[1][2]==3 ||
        ...
        T[0][0]+T[1][1]+T[2][2]==3 ||
        T[0][2]+T[1][1]+T[2][0]==3 ) // “X” wins.
```

Tic-Tac-Toe

T	0	1	2
0	1	0	-1
1	0	1	-1
2	0	0	0

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
```

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;          // Mark T with 1 for “X”.
```

```
/* Test for a win by “X”. */
   if ( T[0][0]+T[0][1]+T[0][1]==3 ||
        T[1][0]+T[1][1]+T[1][2]==3 ||
        ...
        T[0][0]+T[1][1]+T[2][2]==3 ||
        T[0][2]+T[1][1]+T[2][0]==3 ) // “X” wins.
```

Tic-Tac-Toe

T

	0	1	2
0	1	0	-1
1	0	1	-1
2	0	0	0

movesX: 2

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
   int movesX = 0;             // Number of moves made by “X”.
```

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;          // Mark T with 1 for “X”.
   movesX++;             // Increment count of “X” marks.
```

```
/* Test for a win by “X”. */
   if ( movesX<3 ) // Not a win for “X”.
   else // Win for “X” is still possible, but only in 3, 4, or 5 moves.
```

IN PROGRESS

Tic-Tac-Toe

T	0	1	2
0	1	0	-1
1	0	1	-1
2	0	0	1

movesX: 3   sumX: 15

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
   int movesX = 0;             // Number of moves made by “X”.
   int M[][]  = { {8,1,6},{3,5,7},{4,9,2} }; // Magic Square.
   int sumX   = 0; // Sum of magic values corresponding to “X”.
```

M	0	1	2
0	8	1	6
1	3	5	7
2	4	9	2

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;           // Mark T with 1 for “X”.
   movesX++;              // Increment count of “X” marks.
   sumX = sumX+M[r][c];   // Add magic value corresponding to ⟨r,c⟩.
```

```
/* Test for a win by “X”. */
   if ( movesX<3 ) // Not a win for “X”.
   else if ( movesX==3 && sumX==15 ) // “X” wins.
   else // Win for “X” is still possible, but only in 4 or 5 moves.
```

Tic-Tac-Toe

T

	0	1	2
0	1	1	-1
1	0	1	-1
2	0	0	0

movesX 3   sumX 14

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
   int movesX = 0;             // Number of moves made by “X”.
   int M[][]  = { {8,1,6},{3,5,7},{4,9,2} }; // Magic Square.
   int sumX   = 0; // Sum of magic values corresponding to “X”.
```

M

	0	1	2
0	8	1	6
1	3	5	7
2	4	9	2

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;           // Mark T with 1 for “X”.
   movesX++;              // Increment count of “X” marks.
   sumX = sumX+M[r][c];   // Add magic value corresponding to ⟨r,c⟩.
```

IN PROGRESS

```
/* Test for a win by “X”. */
   if ( movesX<3 ) // Not a win for “X”.
   else if ( movesX==3 && sumX==15 ) // “X” wins.
   else // Win for “X” is still possible, but only in 4 or 5 moves.
```

Tic-Tac-Toe

T

	0	1	2
0	1	1	-1
1	0	1	-1
2	0	-1	0

movesX **3**   sumX **14**

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
   int movesX = 0;             // Number of moves made by “X”.
   int M[][]  = { {8,1,6},{3,5,7},{4,9,2} }; // Magic Square.
   int sumX   = 0; // Sum of magic values corresponding to “X”.
```

M

	0	1	2
0	8	1	6
1	3	5	7
2	4	9	2

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;          // Mark T with 1 for “X”.
   movesX++;             // Increment count of “X” marks.
   sumX = sumX+M[r][c]; // Add magic value corresponding to ⟨r,c⟩.
```

IN PROGRESS

```
/* Test for a win by “X”. */
   if ( movesX<3 ) // Not a win for “X”.
   else if ( movesX==3 && sumX==15 ) // “X” wins.
   else // Win for “X” is still possible, but only in 4 or 5 moves.
```

Tic-Tac-Toe

T	0	1	2
0	1	1	-1
1	0	1	-1
2	0	-1	1

movesX: 4 sumX: 16

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
   int movesX = 0;             // Number of moves made by “X”.
   int M[][]  = { {8,1,6},{3,5,7},{4,9,2} }; // Magic Square.
   int sumX   = 0; // Sum of magic values corresponding to “X”.
   int TT[]   = new int[10];   // TT[m] is 1 iff “X” in T[r][c]
                               // and M[r][c]==m.
```

M	0	1	2
0	8	1	6
1	3	5	7
2	4	9	2

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;           // Mark T with 1 for “X”.
   movesX++;              // Increment count of “X” marks.
   sumX = sumX+M[r][c];   // Add magic value corresponding to ⟨r,c⟩.
   TT[M[r][c]] = 1;       // Set TT[m] to 1 iff square with magic
                          // value m is “X”.
```

	0	1	2	3	4	5	6	7	8	9
TT	0	1	1	0	0	1	0	0	1	0

```
/* Test for a win by “X”. */
   if ( movesX<3 ) // Not a win for “X”.
   else if ( movesX==3 && sumX==15 ) // “X” wins.
   else if ( movesX==4 && 9<=sumX && sumX<=24 && TT[sumX-15]==1 ) // “X” wins.
   else // Win for “X” is still possible, but only in 5 moves.
```

Tic-Tac-Toe

T	0	1	2
0	1	1	-1
1	0	1	-1
2	1	-1	0

movesX: 4   sumX: 18

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
   int movesX = 0;             // Number of moves made by “X”.
   int M[][]  = { {8,1,6},{3,5,7},{4,9,2} }; // Magic Square.
   int sumX   = 0; // Sum of magic values corresponding to “X”.
   int TT[]   = new int[10];   // TT[m] is 1 iff “X” in T[r][c]
                               // and M[r][c]==m.
```

M	0	1	2
0	8	1	6
1	3	5	7
2	4	9	2

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;           // Mark T with 1 for “X”.
   movesX++;              // Increment count of “X” marks.
   sumX = sumX+M[r][c];   // Add magic value corresponding to ⟨r,c⟩.
   TT[M[r][c]] = 1;       // Set TT[m] to 1 iff square with magic
                          // value m is “X”.
```

	0	1	2	3	4	5	6	7	8	9
TT	0	1	0	0	1	1	0	0	1	0

IN PROGRESS

```
/* Test for a win by “X”. */
   if ( movesX<3 ) // Not a win for “X”.
   else if ( movesX==3 && sumX==15 ) // “X” wins.
   else if ( movesX==4 && 9<=sumX && sumX<=24 && TT[sumX-15]==1 ) // “X” wins.
   else // Win for “X” is still possible, but only in 5 moves.
```

Tic-Tac-Toe

T

	0	1	2
0	1	0	-1
1	0	-1	1
2	-1	1	1

movesX 4 **sumX** 26

```
/* Data Representation. */
   int T[][]  = new int[3][3]; // ⟨0,1,-1⟩ for ⟨blank,“X”,“O”⟩.
   int movesX = 0;             // Number of moves made by “X”.
   int M[][]  = { {8,1,6},{3,5,7},{4,9,2} }; // Magic Square.
   int sumX   = 0; // Sum of magic values corresponding to “X”.
   int TT[]   = new int[10];   // TT[m] is 1 iff “X” in T[r][c]
                               // and M[r][c]==m.
```

M

	0	1	2
0	8	1	6
1	3	5	7
2	4	9	2

```
/* Update the data representation on a move by “X” at ⟨r,c⟩. */
   T[r][c] = 1;          // Mark T with 1 for “X”.
   movesX++;             // Increment count of “X” marks.
   sumX = sumX+M[r][c];  // Add magic value corresponding to ⟨r,c⟩.
   TT[M[r][c]] = 1;      // Set TT[m] to 1 iff square with magic
                         // value m is “X”.
```

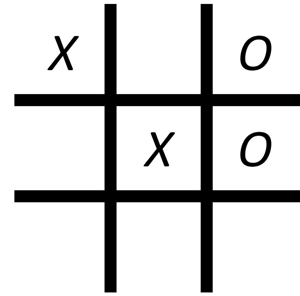
TT

0	1	2	3	4	5	6	7	8	9
0	0	1	0	0	0	0	1	1	1

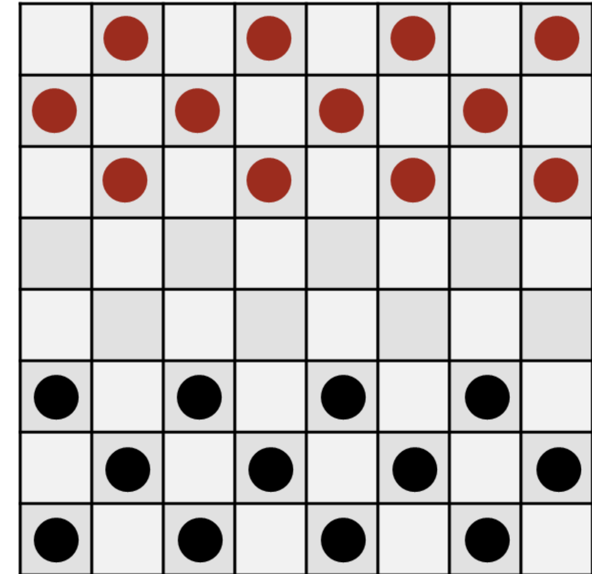
```
/* Test for a win by “X”. */
   if ( movesX<3 ) // Not a win for “X”.
   else if ( movesX==3 && sumX==15 ) // “X” wins.
   else if ( movesX==4 && 9<=sumX && sumX<=24 && TT[sumX-15]==1 ) // “X” wins.
   else // Win for “X” is still possible, but only in 5 moves.
```

IN PROGRESS

Reflection: Because Tic-Tac-Toe is so simple, the efficiency of the above code is not really needed. It is intended to stretch your imagination, and exhibit principles that can lead to significant code improvement.

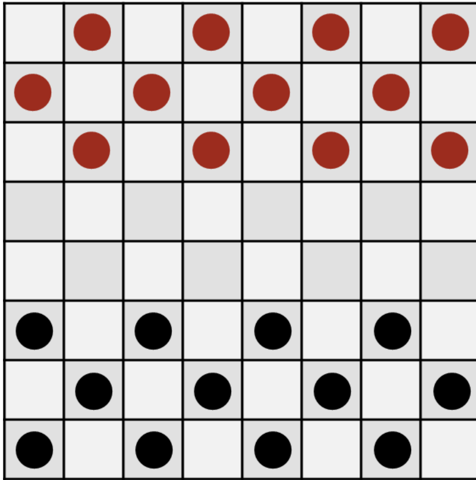


Checkers



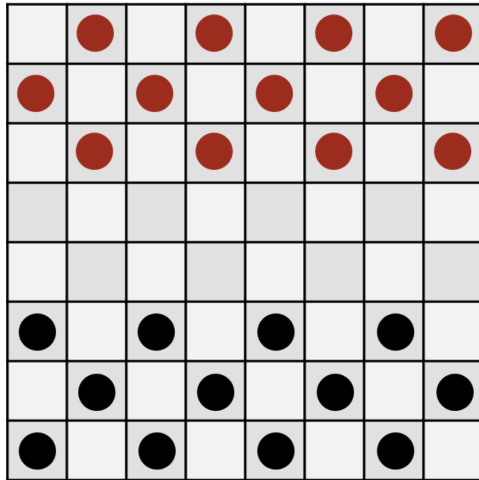
Background: Checkers is played on a 8-by-8 board, starting with the initial layout shown. The two players take turns making diagonal moves. In addition to the initial “men” shown, players can earn “queens”. Because all moves are diagonal, only half the squares are used. No other rules are relevant for the purpose of this presentation.

Problem Statement: Write a program that plays Checkers.



```
/* Data Representation. */
```

The touchstone of a data representation is its utility in performing the needed operations.

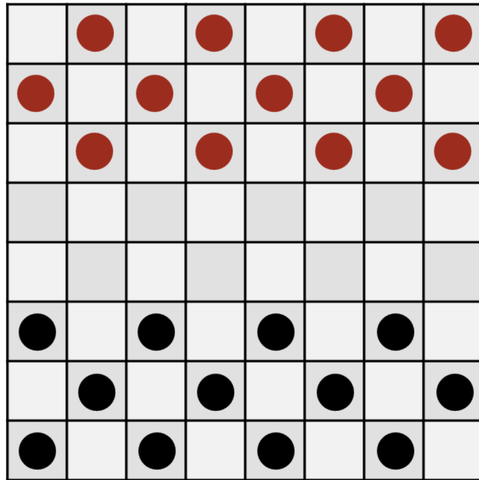


```
/* Data Representation. */
   /* ⟨Blank, BlackMan, BlackQueen, RedMan, RedQueen⟩
      represented by ⟨0,1,2,-1,-2⟩. */
      int Board[][] = new int[8][8];
```

0	-1	0	-1	0	-1	0	-1
-1	0	-1	0	-1	0	-1	0
0	-1	0	-1	0	-1	0	-1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
+1	0	+1	0	+1	0	+1	0
0	+1	0	+1	0	+1	0	+1
+1	0	+1	0	+1	0	+1	0

This representation requires 64*32 bits.

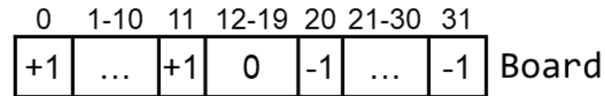
☞ **The touchstone of a data representation is its utility in performing the needed operations.**



```
/* Data Representation. */
   /* ⟨Blank, BlackMan, BlackQueen, RedMan, RedQueen⟩
      represented by ⟨0,1,2,-1,-2⟩. */
      int Board[] = new int[32];
```

	28		29		30		31
24		25		26		27	
	20		21		22		23
16		17		18		19	
	12		13		14		15
8		9		10		11	
	4		5		6		7
0		1		2		3	

Number the used squares 0..31 and adopt a 1-dimensional representation indexed by those numbers. This requires only half the space, i.e., 32*32 bits.



☞ **The touchstone of a data representation is its utility in performing the needed operations.**

	28		29		30		31
24		25		26		27	
	20		21		22		23
16		17		18		19	
	12		13		14		15
8		9		10		11	
	4		5		6		7
0		1		2		3	

```
/* Data Representation. */
   /* ⟨Blanks, BlackMen, BlackQueens, RedMen, RedQueens⟩
      represented by 5 separate Boolean arrays. */
      boolean Blanks[]      = new boolean[32];
      boolean BlackMen[]    = new boolean[32];
      boolean BlackQueens[] = new boolean[32];
      boolean RedMen[]      = new boolean[32];
      boolean RedQueens[]   = new boolean[32];
```

0	1-10	11	12-19	20	21-30	31	
F	...	F	T	F	...	F	Blanks

0	1-10	11	12-19	20	21-30	31	
T	...	T	F	F	...	F	BlackMen

0	1-10	11	12-19	20	21-30	31	
F	...	F	F	T	...	T	RedMen

Because there are only 5 things to represent, the same information can be stored in 5 **boolean** arrays. Thus, only 5*32 bits are required, i.e., 5/64 of the original.

N.B. The implementation of a programming language does not necessarily implement **boolean** as 1 bit, or **int** as 32 bits, but in principle it could.

The touchstone of a data representation is its utility in performing the needed operations.

Checkers

	28		29		30		31
24		25		26		27	
	20		21		22		23
16		17		18		19	
	12		13		14		15
8		9		10		11	
	4		5		6		7
0		1		2		3	

+4 +5 +4 +5 +4 +5 +4 (row increments, top to bottom)

0	1-10	11	12-19	20	21-30	31	
F	…	F	T	F	…	F	Blanks

0	1-10	11	12-19	20	21-30	31	
T	…	T	F	F	…	F	BlackMen

0	1-10	11	12-19	20	21-30	31	
F	…	F	F	T	…	T	RedMen

```
/* Data Representation. */
   /* ⟨Blanks, BlackMen, BlackQueens, RedMen, RedQueens⟩
      represented by 5 separate Boolean arrays. */
      boolean Blanks[]      = new boolean[32];
      boolean BlackMen[]    = new boolean[32];
      boolean BlackQueens[] = new boolean[32];
      boolean RedMen[]      = new boolean[32];
      boolean RedQueens[]   = new boolean[32];

         /* Compute targets of all black’s forward-right
            diagonal moves. */
```

Forward-right diagonal moves go to a square numbered 4 (or 5) higher, an irregularity that prevents uniformly shifting the 1-dimensional arrays right 4 (or 5) places to compute reachable squares.

Checkers

	32		33		34		35	
27		28		29		30		31
	23		24		25		26	
18		19		20		21		22
	14		15		16		17	
9		10		11		12		13
	5		6		7		8	
0		1		2		3		4

+5 +5 +5 +5 +5 +5 +5 +5

```
/* Data Representation. */
   /* ⟨Blanks, BlackMen, BlackQueens, RedMen, RedQueens⟩
      represented by 5 separate Boolean arrays. */
      boolean Blanks[]      = new boolean[36];
      boolean BlackMen[]    = new boolean[36];
      boolean BlackQueens[] = new boolean[36];
      boolean RedMen[]      = new boolean[36];
      boolean RedQueens[]   = new boolean[36];
```

```
/* Compute targets of all black's forward-right
   diagonal moves. */
```

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	…	35
BlackMen	T	T	T	T	F	T	T	T	T	T	T	T	T	F	F	F	F	F	…	F
BlackMen right shifted 5	F	F	F	F	F	T	T	T	T	F	T	T	T	T	T	T	T	T	…	F
Blanks	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F
all targets of forward-right moves	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F

We introduce 4 phantom squares, and renumber.
All forward-right diagonal moves are now to a square that is **uniformly** numbered **5** higher.

The touchstone of a data representation is its utility in performing the needed operations.

		32		33		34		35	
+5	27		28		29		30		31
+5		23		24		25		26	
+5	18		19		20		21		22
+5		14		15		16		17	
+5	9		10		11		12		13
+5		5		6		7		8	
+5	0		1		2		3		4

```
/* Data Representation. */
   /* ⟨Blanks, BlackMen, BlackQueens, RedMen, RedQueens⟩
      represented by 5 separate Boolean arrays. */
      boolean Blanks[]      = new boolean[36];
      boolean BlackMen[]    = new boolean[36];
      boolean BlackQueens[] = new boolean[36];
      boolean RedMen[]      = new boolean[36];
      boolean RedQueens[]   = new boolean[36];
```

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	…	35
BlackMen	T	T	T	T	F	T	T	T	T	T	T	T	T	F	F	F	F	F	…	F
BlackMen right shifted 5	F	F	F	F	F	T	T	T	T	F	T	T	T	T	T	T	T	T	…	F
Blanks	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F
all targets of forward-right moves	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F

```
/* Compute targets of all black's forward-right
   diagonal moves. */
```

As a result, we can compute all black's forward-right diagonal moves in a single Right-Shift-5 operation...

The touchstone of a data representation is its utility in performing the needed operations.

	32		33		34		35	
27		28		29		30		31
	23		24		25		26	
18		19		20		21		22
	14		15		16		17	
9		10		11		12		13
	5		6		7		8	
0		1		2		3		4

+5 +5 +5 +5 +5 +5 +5 +5

```
/* Data Representation. */
   /* ⟨Blanks, BlackMen, BlackQueens, RedMen, RedQueens⟩
      represented by 5 separate Boolean arrays. */
      boolean Blanks[]      = new boolean[36];
      boolean BlackMen[]    = new boolean[36];
      boolean BlackQueens[] = new boolean[36];
      boolean RedMen[]      = new boolean[36];
      boolean RedQueens[]   = new boolean[36];
```

```
/* Compute targets of all black's forward-right
   diagonal moves. */
```

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	…	35
BlackMen	T	T	T	T	F	T	T	T	T	T	T	T	T	F	F	F	F	F	…	F
BlackMen right shifted 5	F	F	F	F	F	T	T	T	T	F	T	T	T	T	T	T	T	T	…	F
Blanks	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F
all targets of forward-right moves	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F

perform a bitwise-logical-**and** operation with the array of non-blank non-phantom squares…

The touchstone of a data representation is its utility in performing the needed operations.

	32		33		34		35	
27		28		29		30		31
	23		24		25		26	
18		19		20		21		22
	14		15		16		17	
9		10		11		12		13
	5		6		7		8	
0		1		2		3		4

+5 +5 +5 +5 +5 +5 +5

```
/* Data Representation. */
   /* ⟨Blanks, BlackMen, BlackQueens, RedMen, RedQueens⟩
      represented by 5 separate Boolean arrays. */
      boolean Blanks[]      = new boolean[36];
      boolean BlackMen[]    = new boolean[36];
      boolean BlackQueens[] = new boolean[36];
      boolean RedMen[]      = new boolean[36];
      boolean RedQueens[]   = new boolean[36];
```

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	…	35
BlackMen	T	T	T	T	F	T	T	T	T	T	T	T	T	F	F	F	F	F	…	F
BlackMen right shifted 5	F	F	F	F	F	T	T	T	T	F	T	T	T	T	T	T	T	T	…	F
Blanks	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F
all targets of forward-right moves	F	F	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	…	F

```
/* Compute targets of all black's forward-right
   diagonal moves. */
```

and obtain the positions of all blank squares reachable by BlackMen in a forward-right diagonal move.

The touchstone of a data representation is its utility in performing the needed operations.

History and Serendipity: Arthur Samuels, a founder of the field of Machine Learning, wrote a Checkers playing program in the 1950s, for which he invented the above representation.

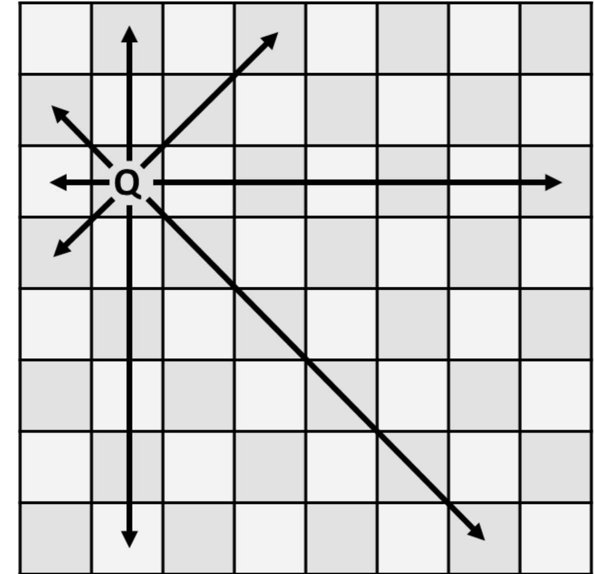
At that time, the fundamental word length of computers was **36** bits. An operation like Shift-Right-5 could be performed in a single machine instruction, and so too could an operation like bitwise-logical-**and**. Thus, the target of all forward-right moves by BlackMen to a vacant square could be performed in just 2 machine instructions!

Shortly thereafter, IBM changed to 32-bit words, which eliminated the possibility of representing phantom squares in words, and the efficiency of uniform shifts of them to compute possible moves in parallel.

Words are now 64-bits (or more), and so the representation is viable once again.

The touchstone of a data representation is its utility in performing the needed operations.

Eight Queens Problem



Background: Chess is played on a 8-by-8 board. A Queen can move or capture at any distance vertically, horizontally, or diagonally. It is possible to place eight Queens on the board so that no two are on the same row, column, or diagonal..

Problem Statement: Write a program that finds a layout that solves the problem.

Eight Queens Problem

```
/* Data Representation. */
   boolean Board[][] = new boolean[8][8];
```

	0	1	2	3	4	5	6	7
0	Q							
1					Q			Q
2		Q						
3						Q		
4			Q					
5							Q	
6								
7				Q				

But why choose a representation (with 2^64 possible states) that allows two-in-the-same row to be expressed?

Not a solution because there are two Queens in same row.

Choose representations that by design do not have nonsensical configurations.

Eight Queens Problem

```
/* Data Representation. */
   int R[] = new int[8]; // R[c] is r if Queen in ⟨r,c⟩.
```

	0	1	2	3	4	5	6	7
0	Q							
1					Q			Q
2		Q						
3						Q		
4			Q					
5							Q	
6								
7				Q				

	0	1	2	3	4	5	6	7
R	0	2	4	7	1	3	4	1

Why not choose a representation (with 8^8 states) in which two-in-the-same-row cannot even be expressed.

Not a solution because there are two Queens in same row.

Choose representations that by design do not have nonsensical configurations.

Eight Queens Problem

```
/* Data Representation. */
   int R[] = new int[8]; // R[c] is r if Queen in ⟨r,c⟩.
```

	0	1	2	3	4	5	6	7
0	Q							
1					Q			Q
2		Q						
3						Q		
4			Q					
5							Q	
6								
7				Q				

	0	1	2	3	4	5	6	7
R	0	2	4	7	1	3	4	1

But why choose a representation (with 8^8 states) that allows two-in-the-same column to be expressed?

Require R to be a permutation of 1,2,3,4,5,6,7,8.

Choose representations that by design do not have nonsensical configurations.

Eight Queens Problem

```
/* Data Representation. */
   int R[] = new int[8]; // R[c] is r if Queen in ⟨r,c⟩.
```

	0	1	2	3	4	5	6	7
0	Q							
1					Q			Q
2		Q						
3						Q		
4			Q					
5							Q	
6								
7				Q				

	0	1	2	3	4	5	6	7
R	permutation of 1,2,3,4,5,6,7,8							

But not choose a representation (with 8! states) in which two-in-the-same-column cannot even be expressed.

The failing configuration (above) cannot be expressed as a permutation of 1,2,3,4,5,6,7,8.

Choose representations that by design do not have nonsensical configurations.

Eight Queens Problem

	0	1	2	3	4	5	6	7
R	permutation of 1,2,3,4,5,6,7,8							

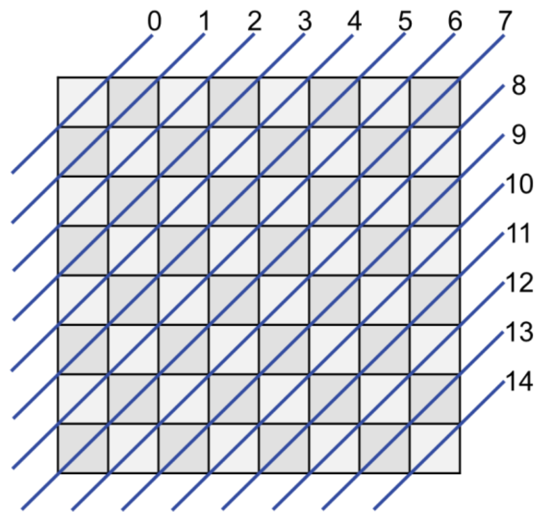
```
/* Solve the Eight Queens problem. */
static void main() {
   /* R[c] is r if Queen in ⟨r,c⟩.*/
      int R[] = { 0, 1, 2, 3, 4, 5, 6, 7 };
   /* Consider each permutation of R until one is found that represents a
      solution. (At least one such permutation is known to exist.) */
      while ( condition ) NextPermutation(R);
   /* Output solution R. */
      ...
   } /* main */
```

0 1 2 3 4 5 6 7

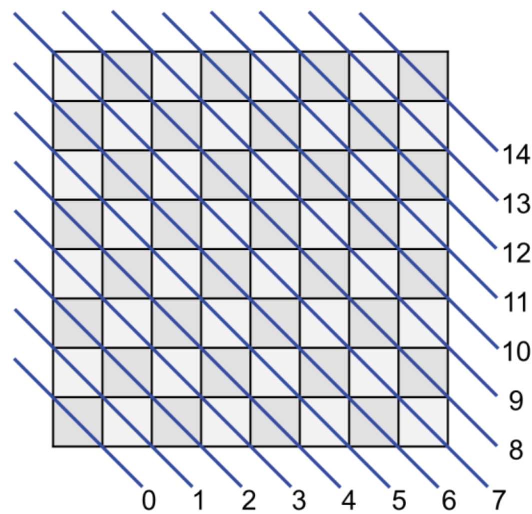
R | permutation of 1,2,3,4,5,6,7,8 |

```
/* Solve the Eight Queens problem. */
static void main() {
   /* R[c] is r if Queen in ⟨r,c⟩.*/
      int R[] = { 0, 1, 2, 3, 4, 5, 6, 7 };
   /* Consider each permutation of R until one is found that represents a
      solution. (At least one such permutation is known to exist.) */
      while ( hasSameDiagonal(R) ) NextPermutation(R);
   /* Output solution R. */
      ...
   } /* main */
```

Eight Queens Problem



Positive diagonal index: row+column



Negative diagonal index: column-row+7

```
/* Return true iff R has two Queens on same diagonal. */
static boolean hasSameDiagonal( int R[] ) {
   /* PosDiag[k] (resp., NegDiag[k]) true iff a Queen in R[0..c] occurs on the
      positive (resp., negative) diagonal with index k.
      boolean PosDiag[] = new boolean[15]; // Initially false, by default.
      boolean NegDiag[] = new boolean[15]; // Initially false, by default.
   int c = 0;
   while ( c<8 && !PosDiag[R[c]+c] && !NegDiag[c-R[c]+7] ) {
      PosDiag[R[c]+c] = true; NegDiag[c-R[c]+7] = true;
      c++;
      }
   return c!=8;
   } /* hasSameDiagonal */
```

Iterate through the 8 queens until the first that occurs on an already-occupied diagonal. If no such queen is found, then the permutation represents a solution.

Commentary. Who would have guessed that the Eight Queens Problem reduces to:

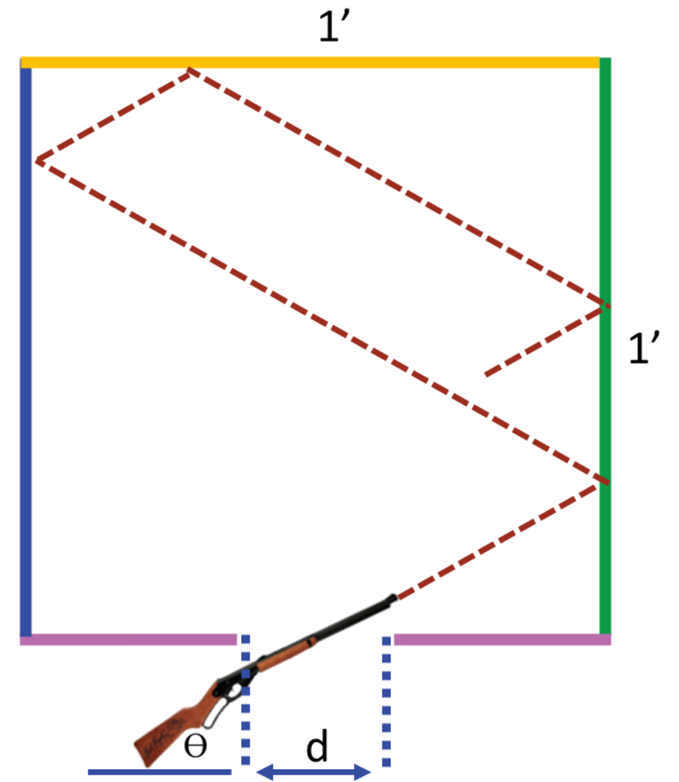
```
/* Update R[0..7] to be the next permutation of 1,2,3,4,5,6,7,8
   in a cycle of all 8! Such permutations. */
static void NextPermutation( int R[] ) {
   ...
   }
```

which is left as a (not so easy) exercise.

Ricocheting Bee-Bee

Background. A square tin box measuring one foot on each side has a slit of size d centered on one side. Insert a bee-bee gun at the center of the slit at angle Θ, and shoot. The bee-bee ricochets off sides, one after another. On each ricochet, the angle of reflection is equal to the angle of incidence.

Problem Statement. Write a program that inputs d and Θ, and outputs the total distance the bee-bee travels before it exits.



Ricocheting Bee-Bee

Background. A square tin box measuring one foot on each side has a slit of size d centered on one side. Insert a bee-bee gun at the center of the slit at angle Θ, and shoot. The bee-bee ricochets off sides, one after another. On each ricochet, the angle of reflection is equal to the angle of incidence.

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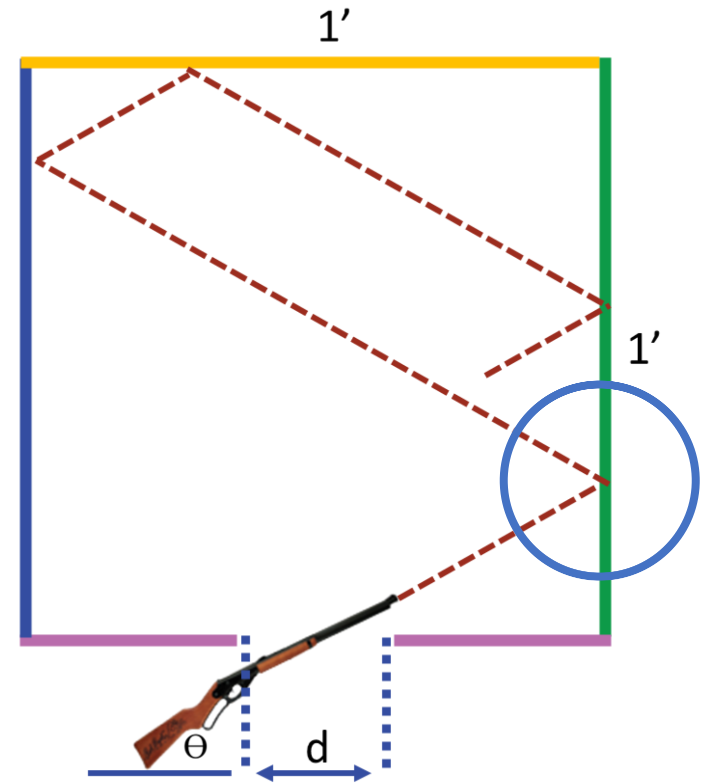
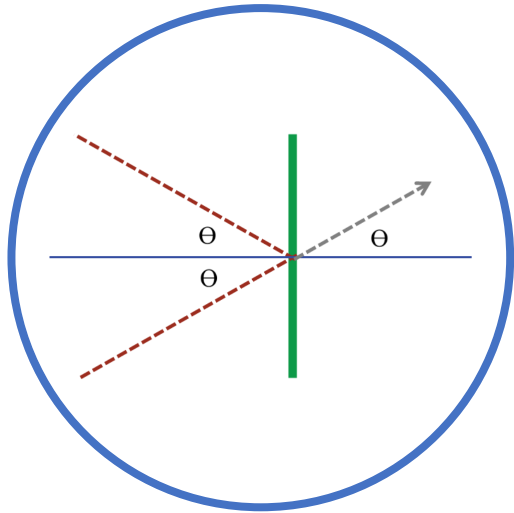
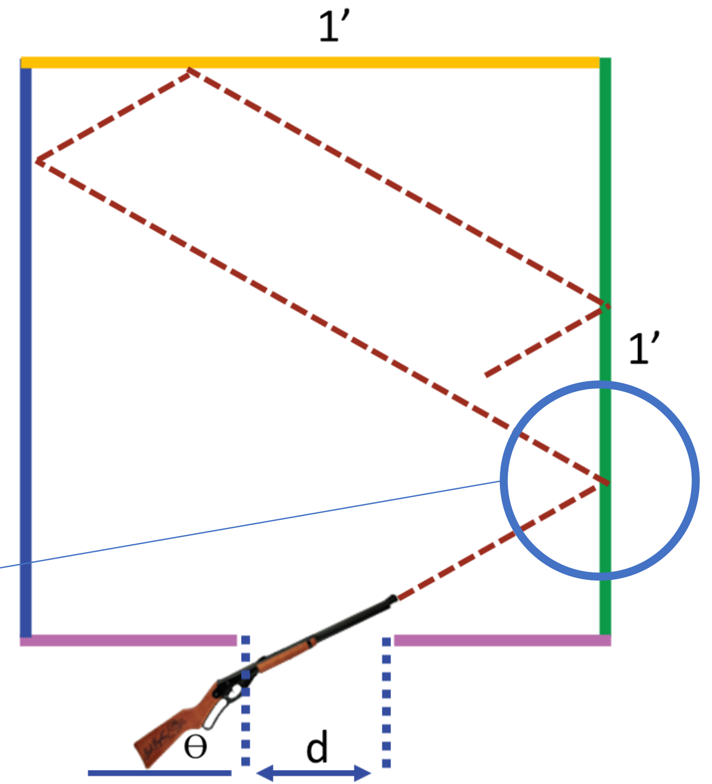


Ricocheting Bee-Bee

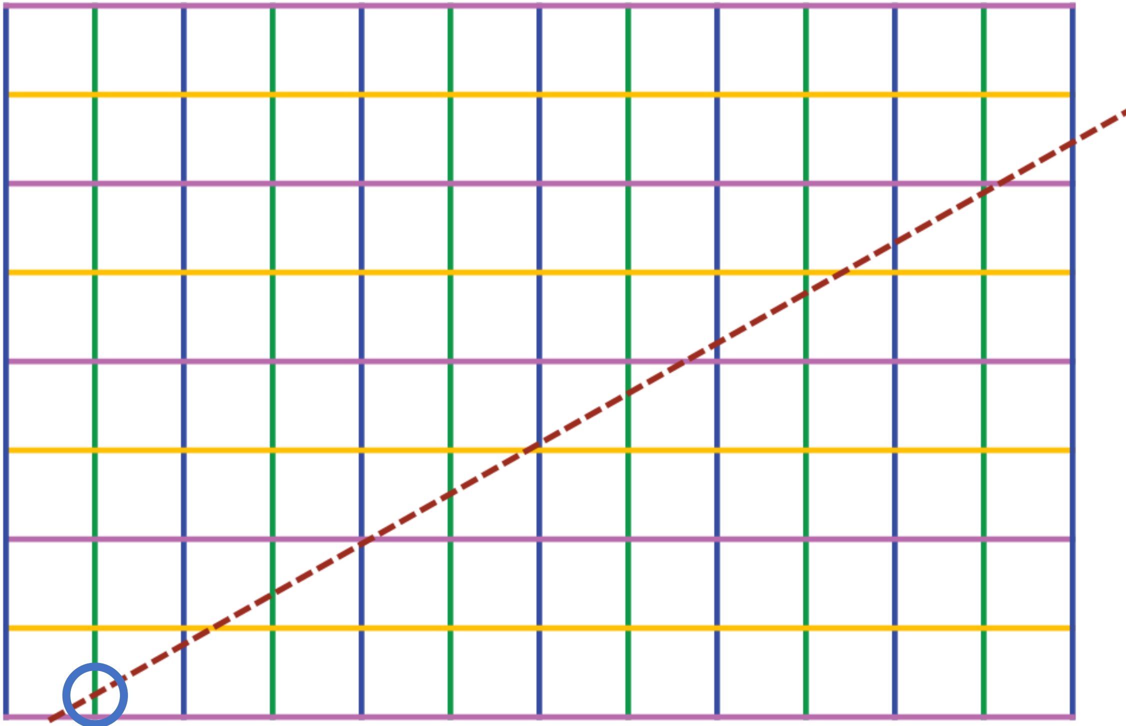
Problem Reduction. Allow the bee-bee to break through the wall of the box, and proceed in a straight line trajectory.

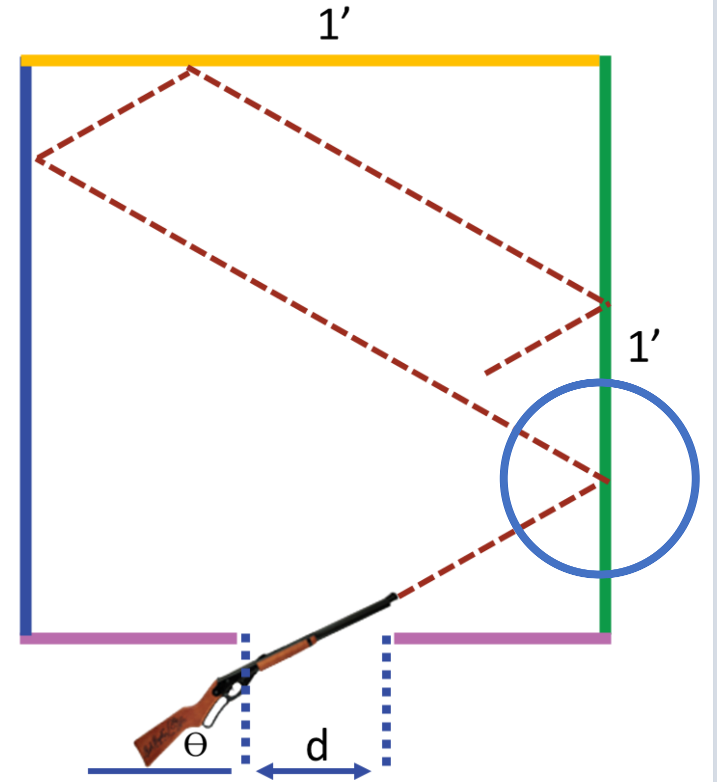




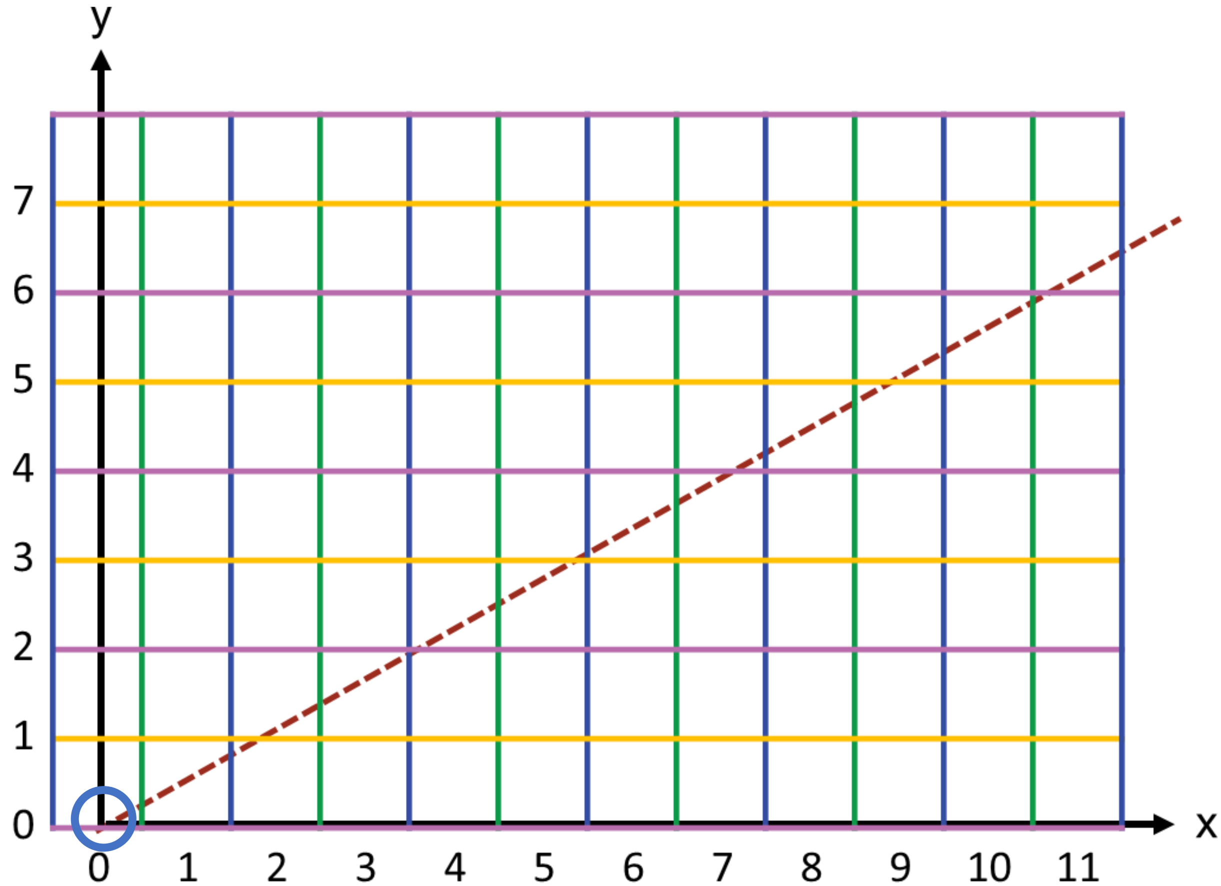
Ricocheting Bee-Bee

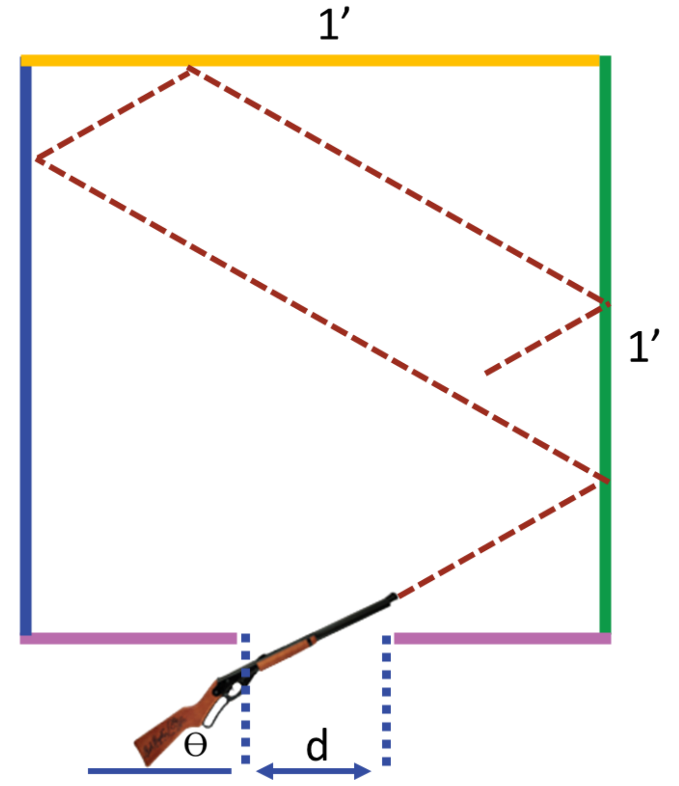
Problem Reduction. Consider the plane to be tiled with reflections of the box.





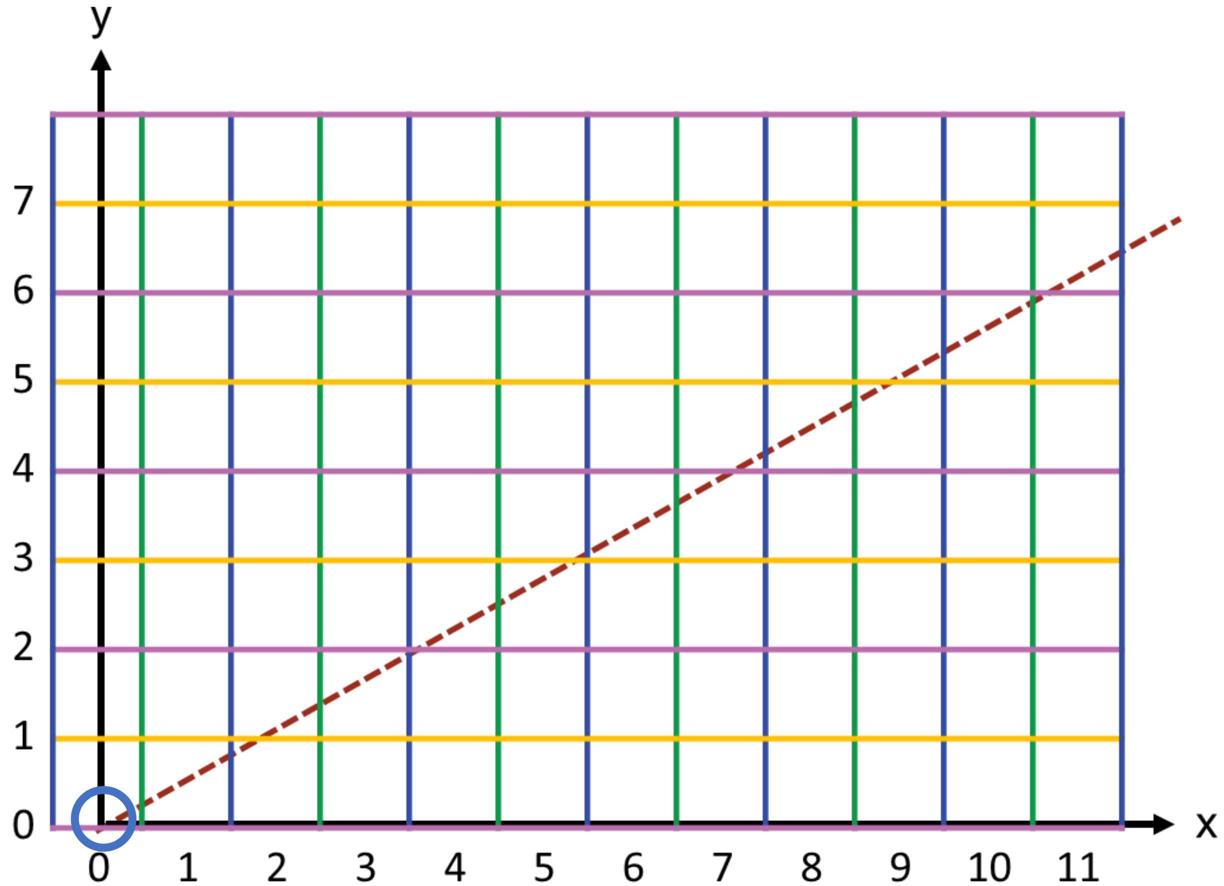
Problem Reduction. Add Cartesian coordinates with origin centered in the middle of the slit of the lower-leftmost box.

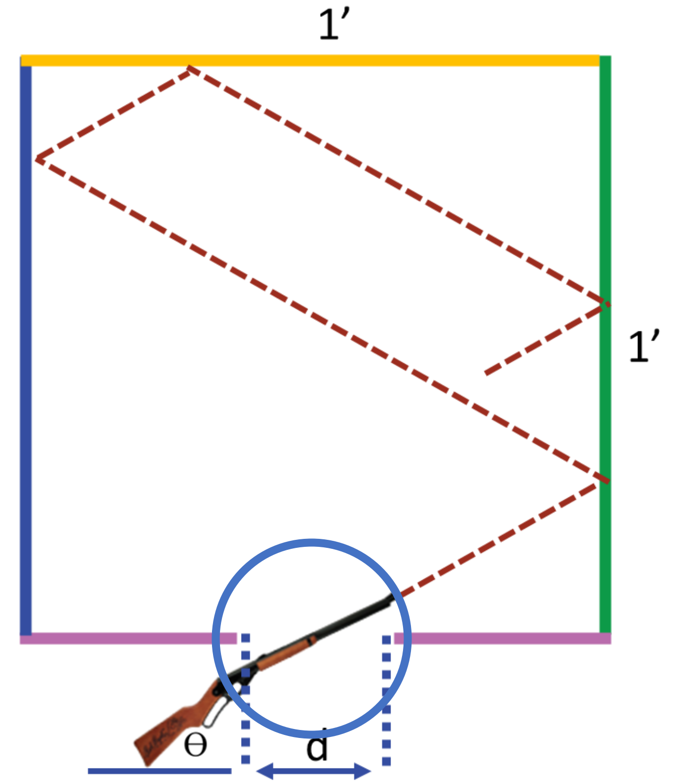




Ricocheting Bee-Bee

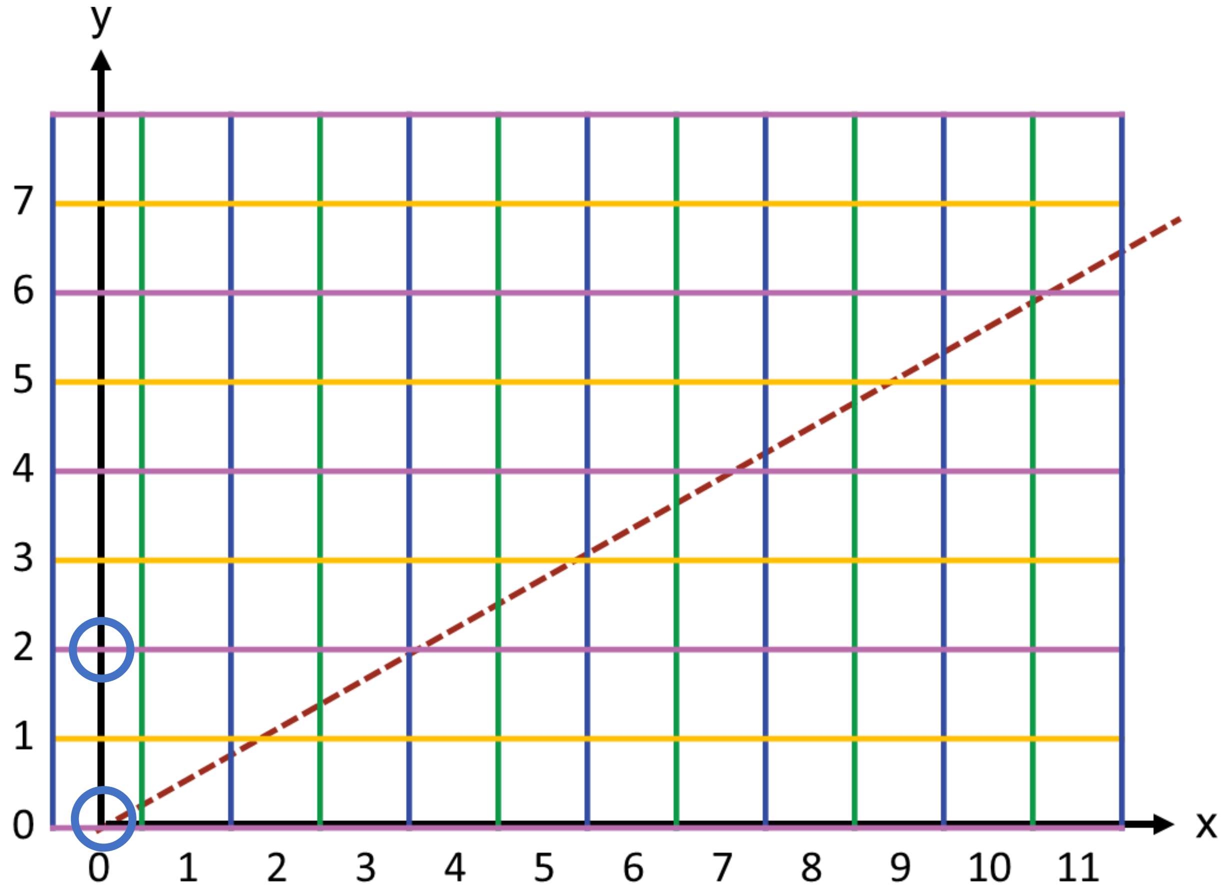
Problem Reduction. The slit replicas are on even values of y, centered on integer values of x.

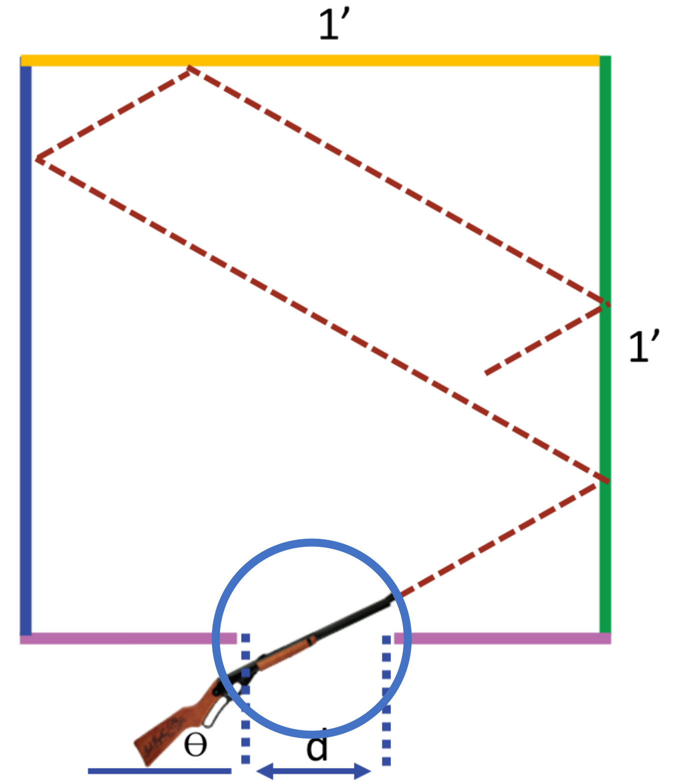




Ricocheting Bee-Bee

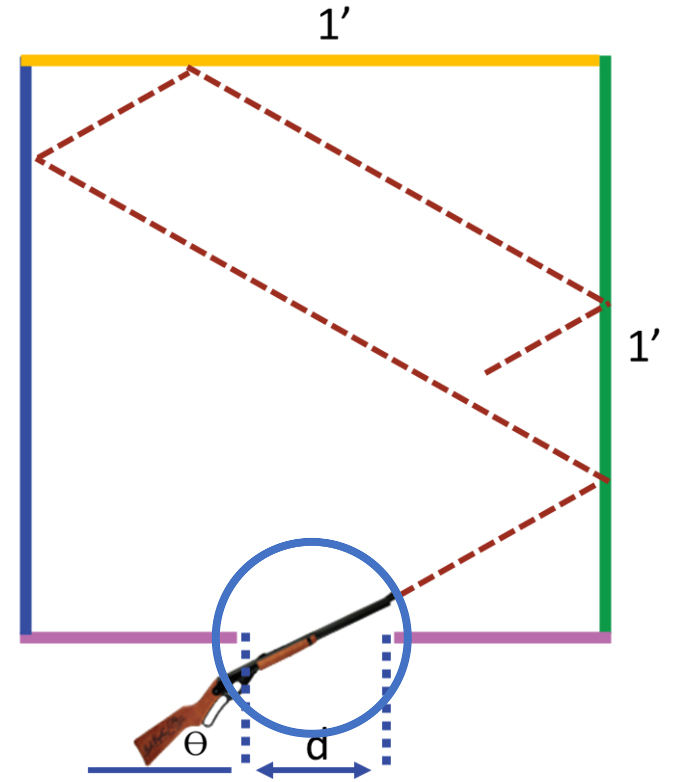
Problem Reduction. The slit replicas are on even values of y, centered on integer values of x.



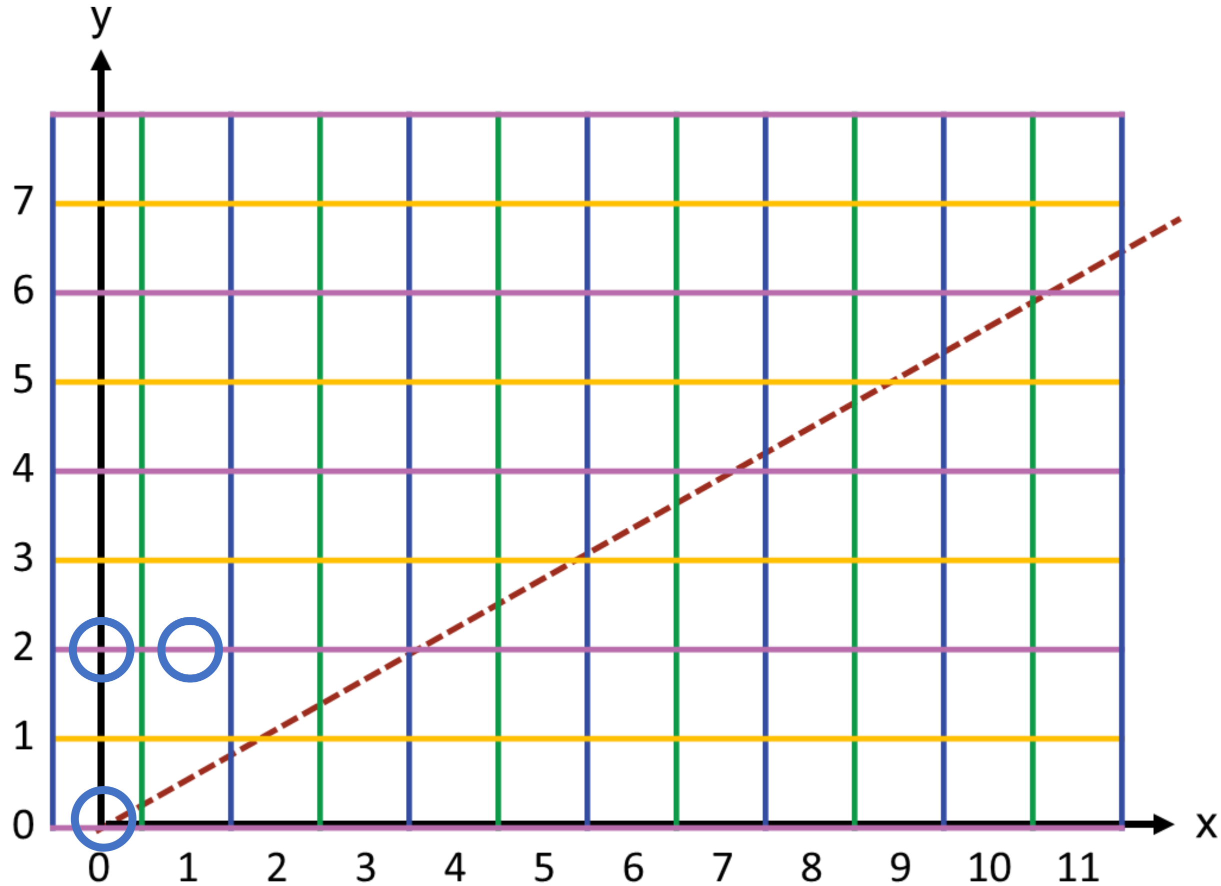


Ricocheting Bee-Bee

Ricocheting Bee-Bee

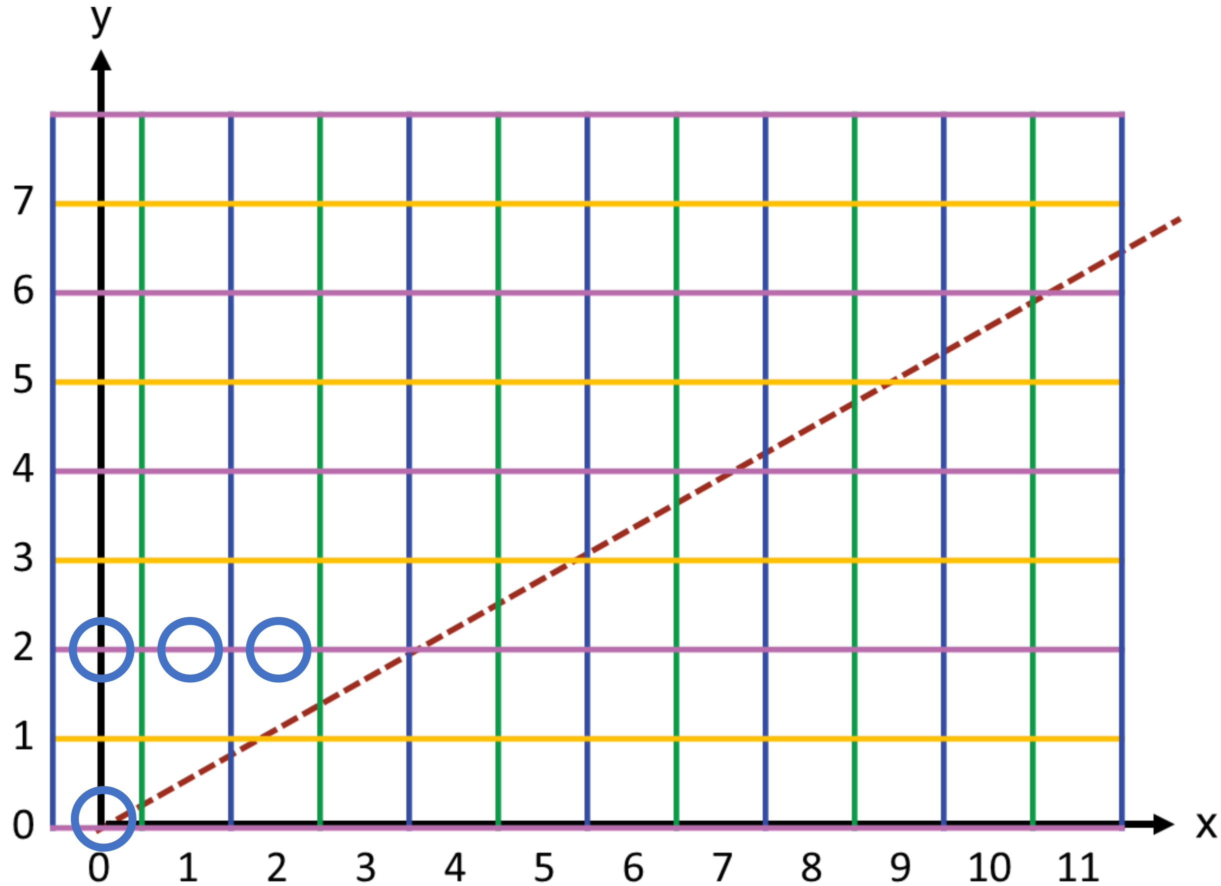


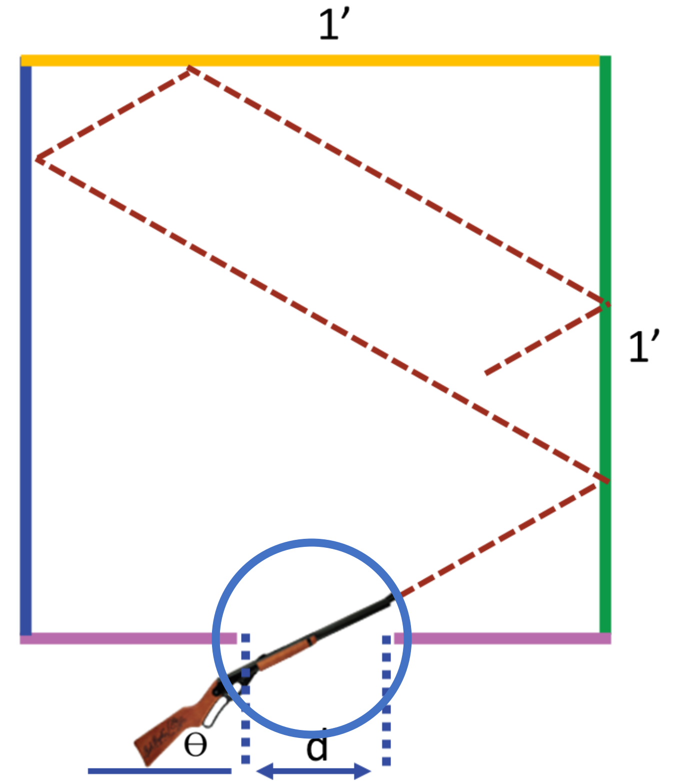
Problem Reduction. The slit replicas are on even values of y, centered on integer values of x.



Ricocheting Bee-Bee

Problem Reduction. The slit replicas are on even values of y, centered on integer values of x.





Ricocheting Bee-Bee

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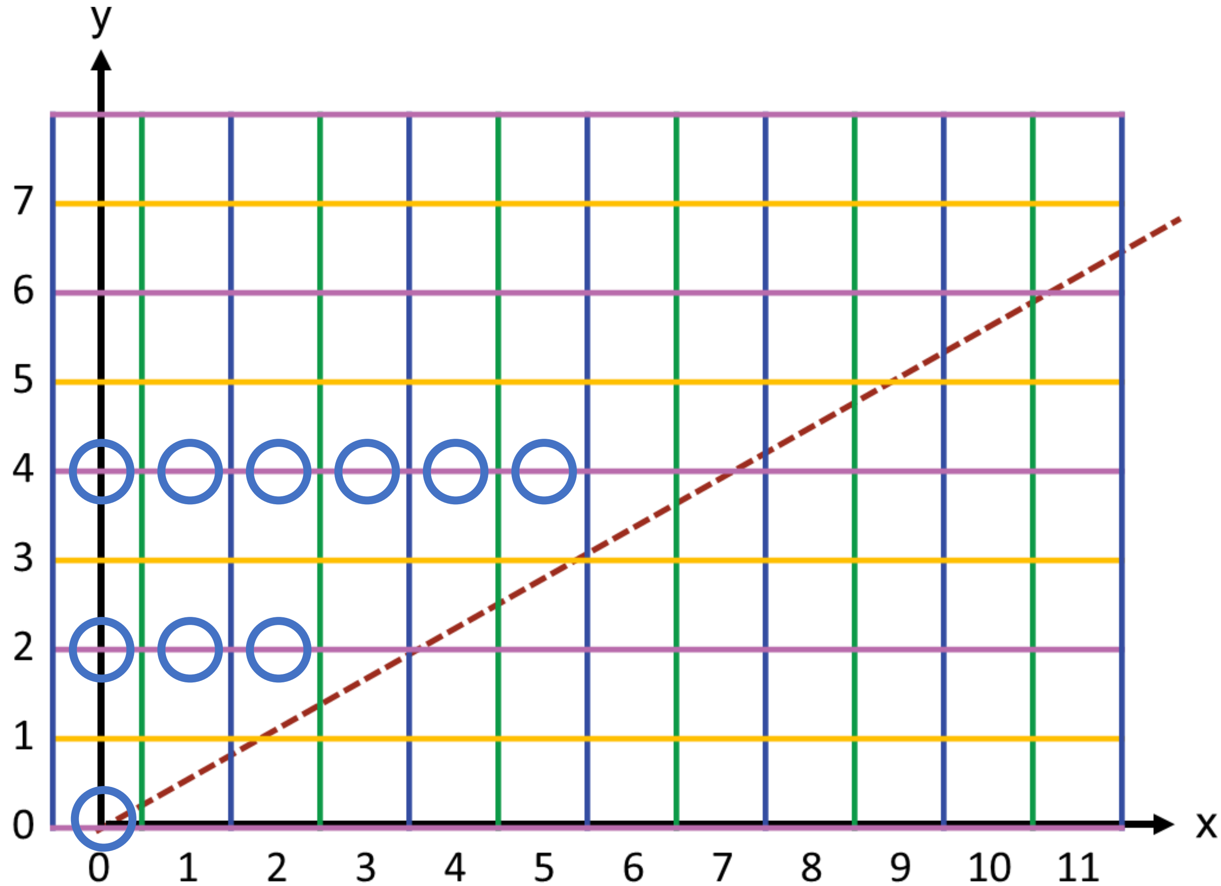
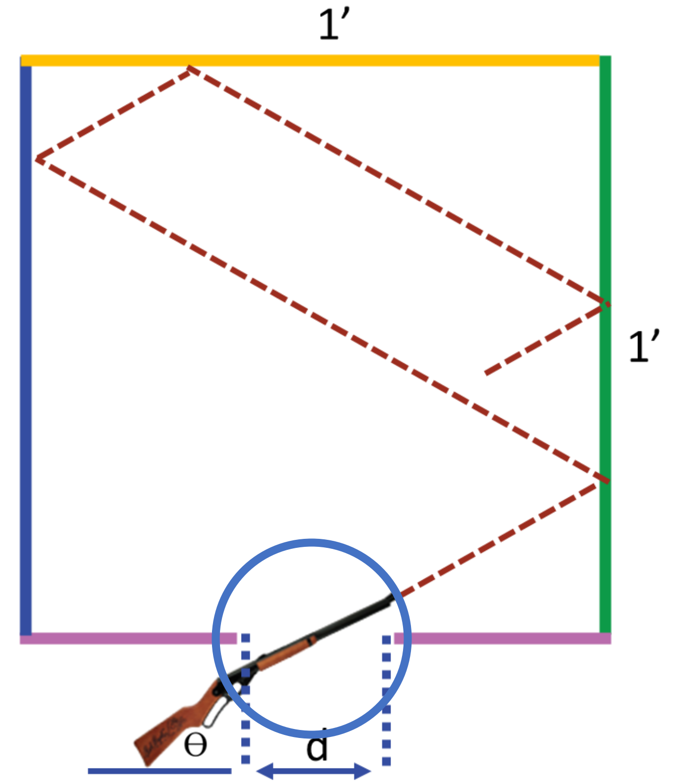
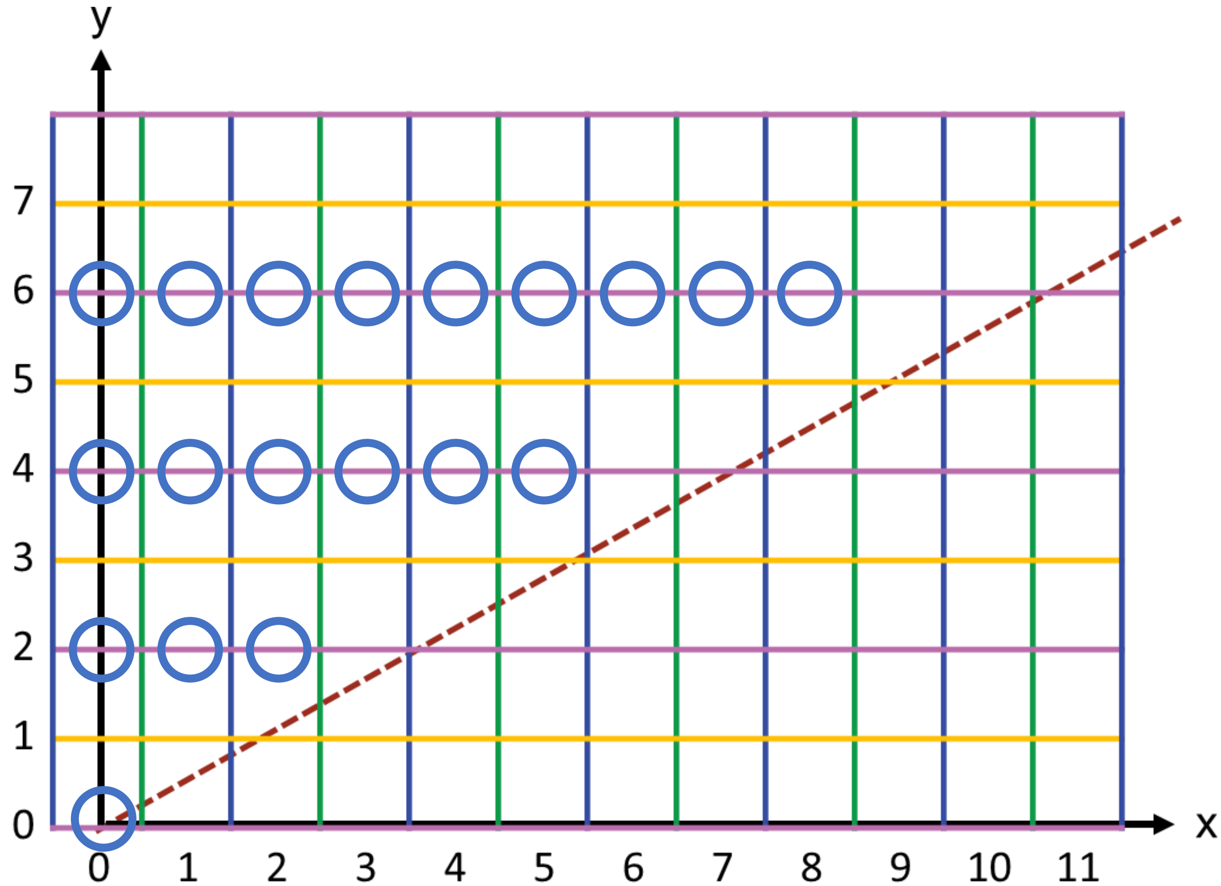
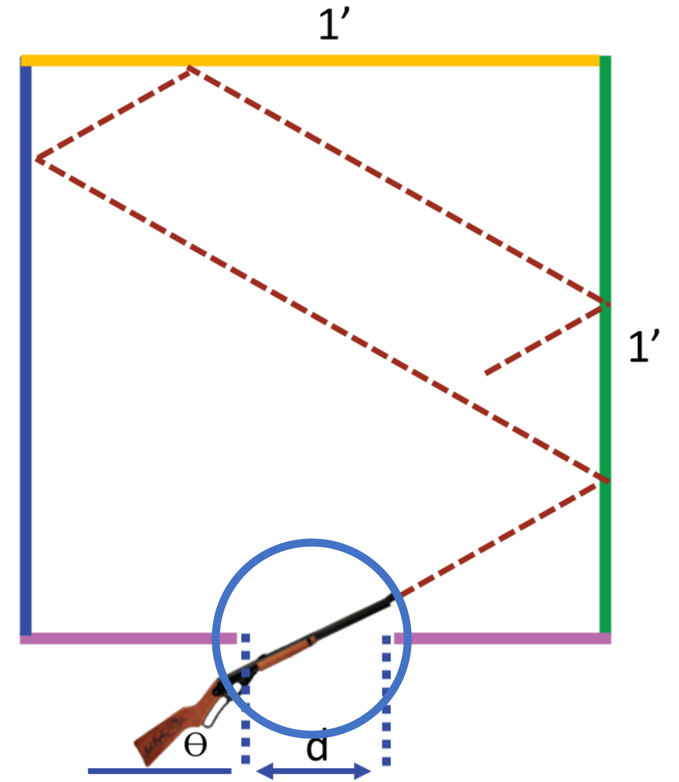




Ricocheting Bee-Bee

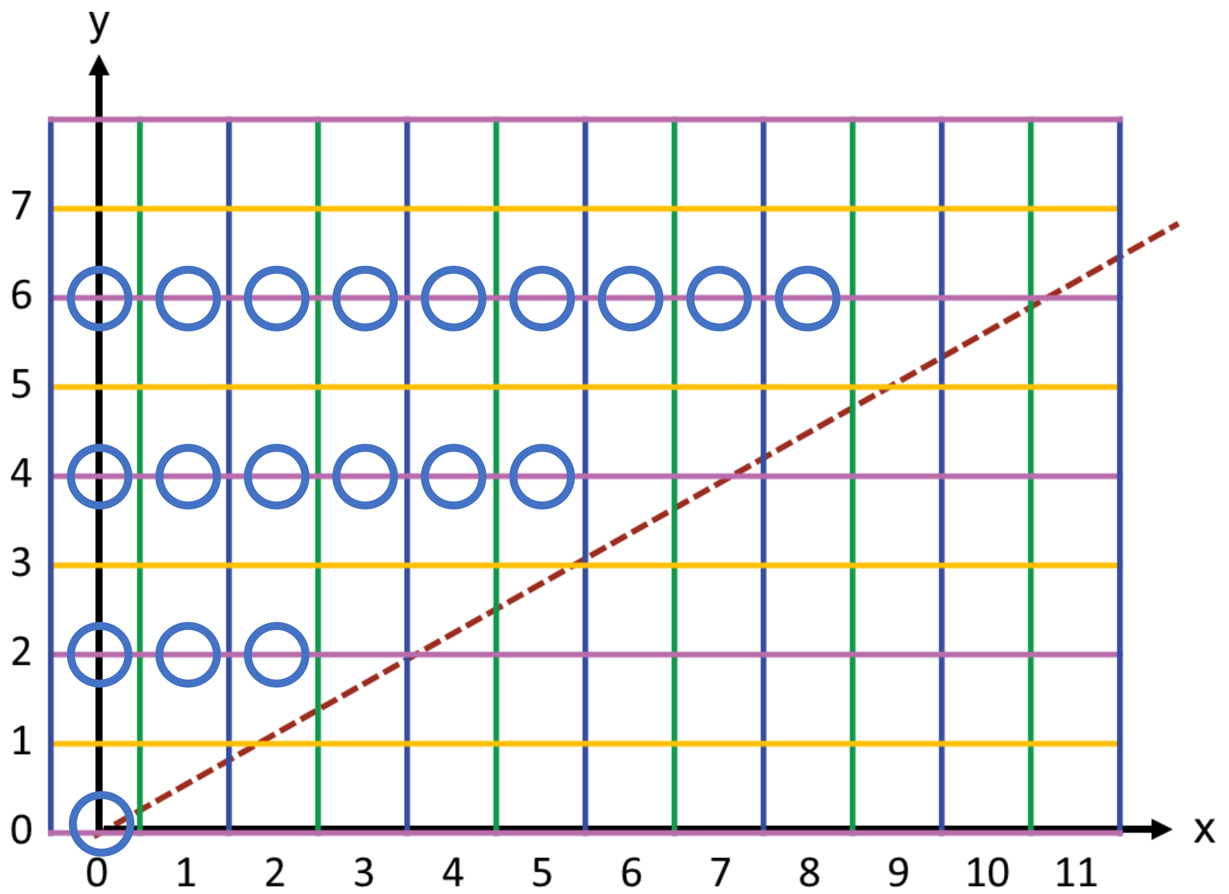
Problem Reduction. The slit replicas are on even values of y, centered on integer values of x, etc.

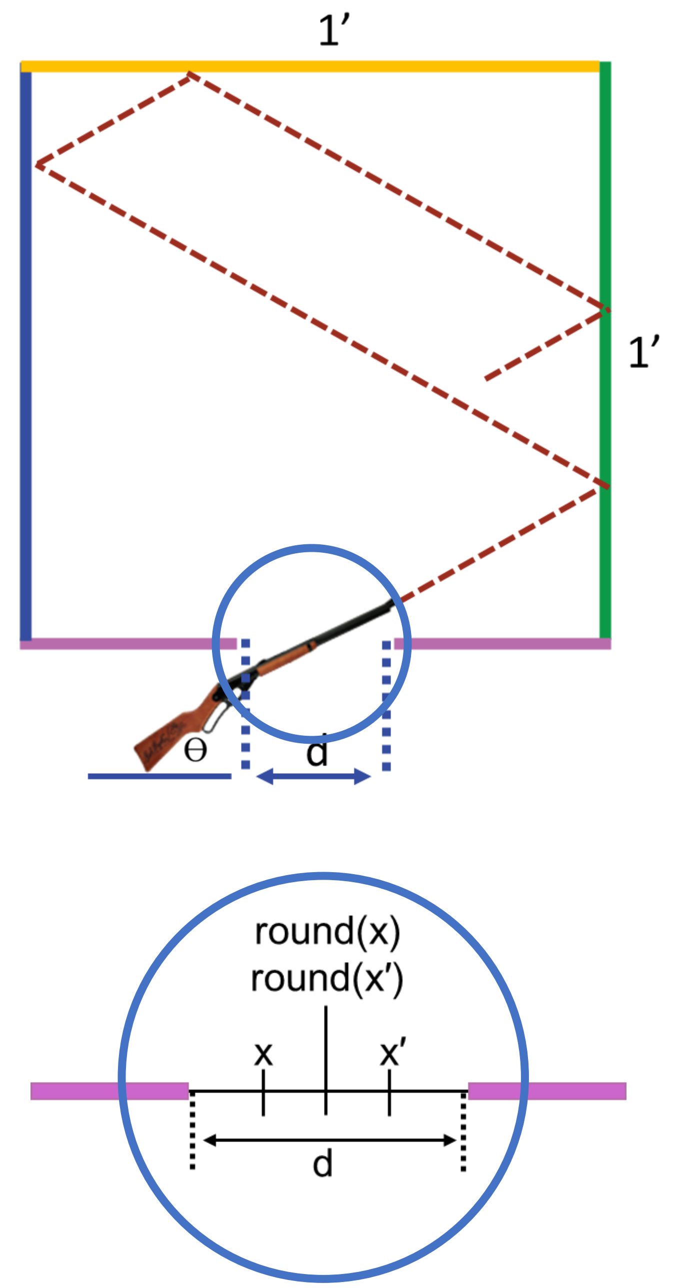




Ricocheting Bee-Bee

Problem Reduction. We seek the smallest (even) y such that the trajectory falls within d/2 of the center of a slit replica.





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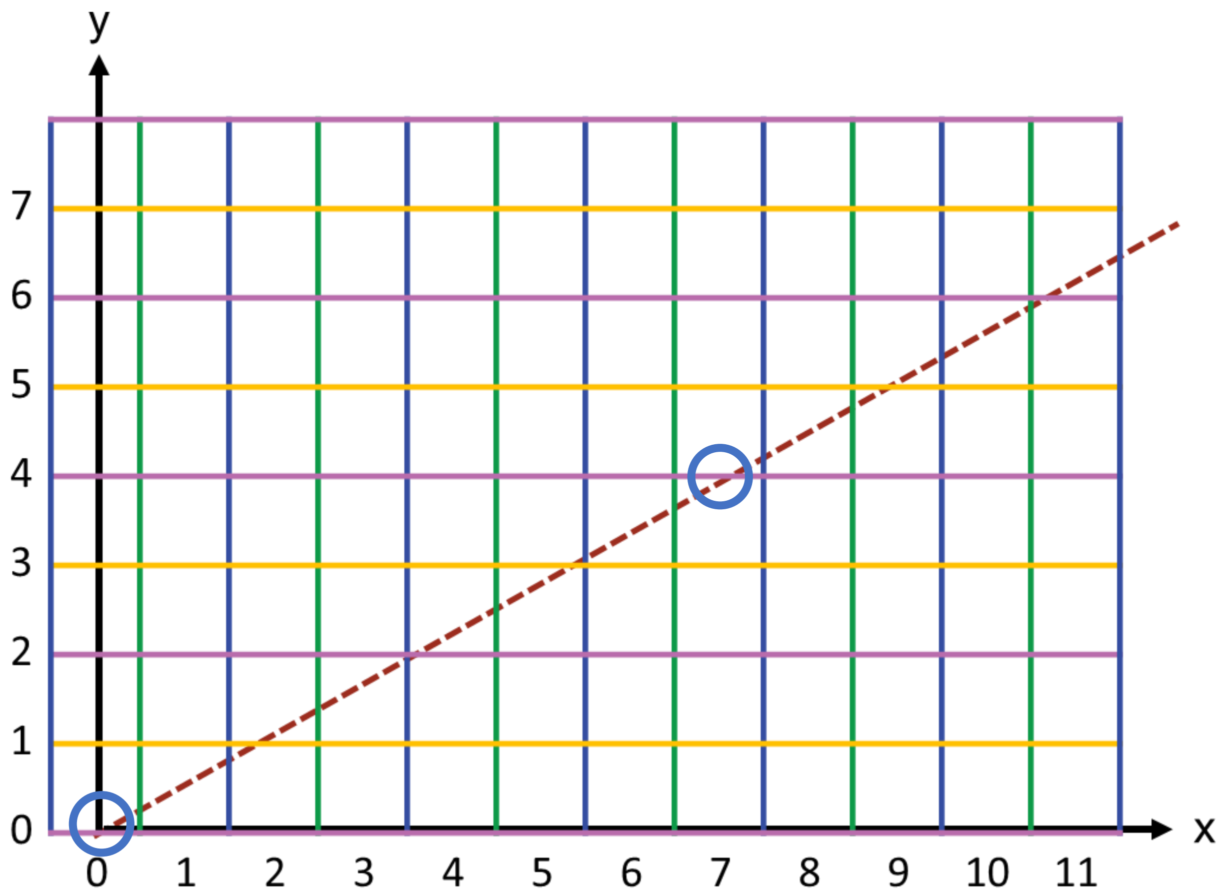
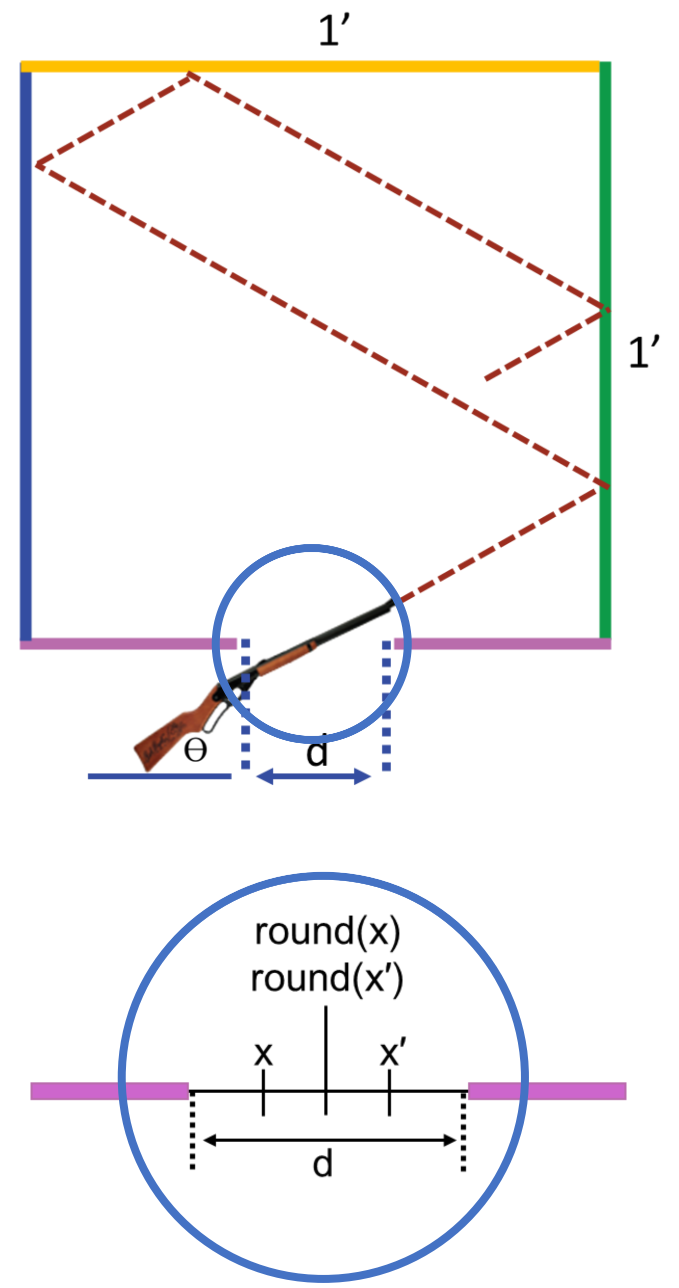
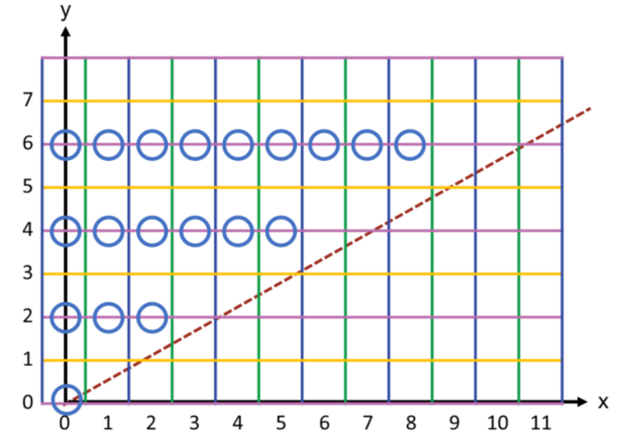
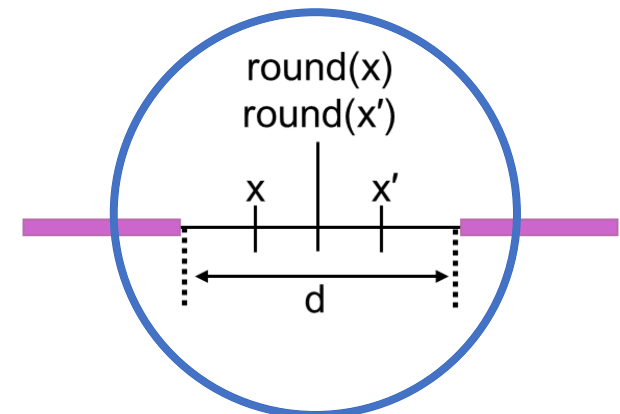




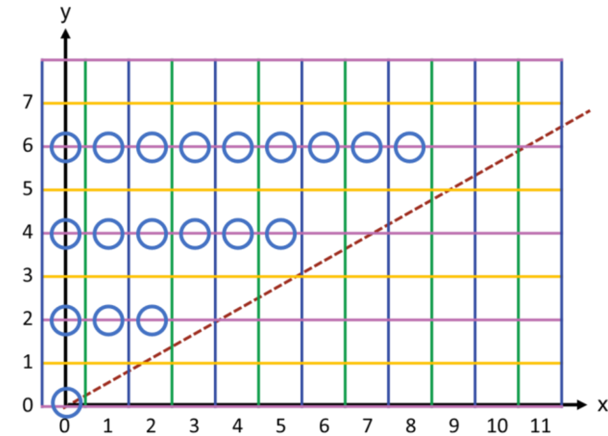


Problem Reduction. We seek the smallest (even) y such that the trajectory falls within d/2 of the center of a slit replica.

```
int y = 2;
while ( /* line does not pass through a slit at y */ ) y = y+2;
/* Output the length of the line between ⟨0,0⟩ and ⟨x,y⟩, where x is
   computed from y and theta. */
```



Ricocheting Bee-Bee

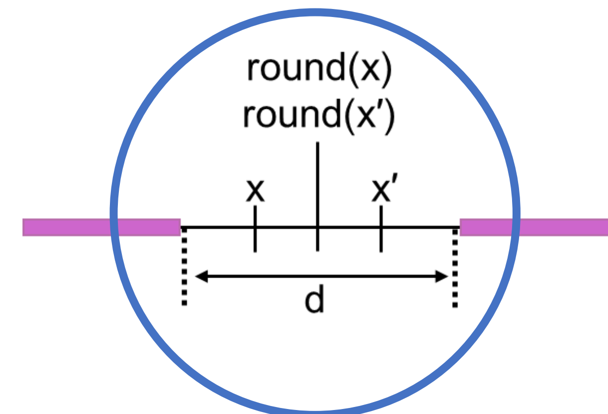


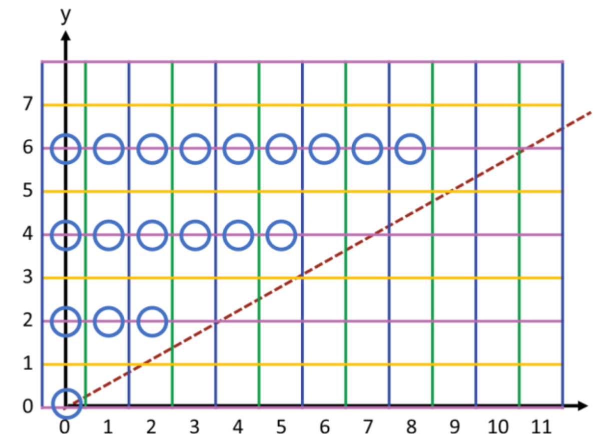
Problem Reduction. We seek the smallest (even) y such that the trajectory falls within d/2 of the center of a slit replica.

```
int y = 2;
while ( Math.abs(x(y,theta)-Math.round(x(y,theta)))>=d/2 ) y = y+2;
/* Output the length of the line between ⟨0,0⟩ and ⟨x,y⟩, where x is
   computed from y and theta. */

where

static double x(double y, double theta) {
   ⟨compute x from y and theta⟩
   } /* x */
```



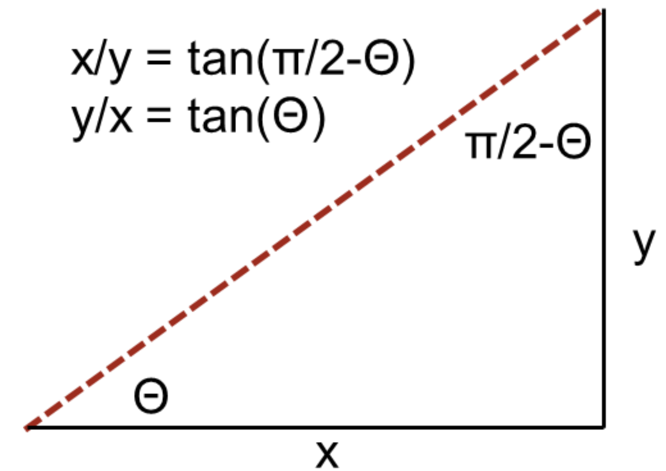


Problem Reduction. Output the length of the trajectory for that y and the corresponding x.

```
int y = 2;
while ( Math.abs(x(y,theta)-Math.round(x(y,theta)))>=d/2 ) y = y+2;
System.out.println( Hypotenuse( x(y,theta),y) );
```

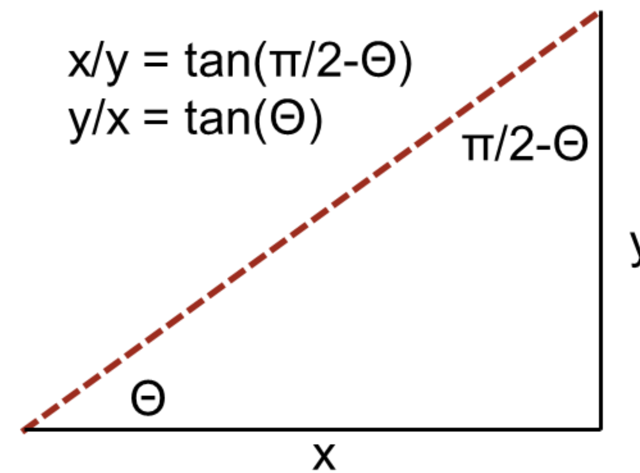
where

```
/* Return length of hypotenuse of triangle with sides x and y. */
static double Hypotenuse( double x, double y ) {
   return Math.sqrt( x*x + y+y );
   } /* Hypotenuse */
```



Final detail: Compute `x` from `y` and `theta` using a bit of trigonometry and care for numerical stability.

```
static double x(double y, double theta) {
  ⟨compute x from y and theta⟩
  } /* x */
```



Final detail: Compute x from y and `theta` using a bit of trigonometry and care for numerical stability.

```
static double x(double y, double theta) {
   if ( theta<Math.PI/4 || theta>3*Math.PI/4 )
      return y/Math.tan(theta);
   else return y*Math.tan(Math.PI/2-theta);
   } /* x */
```

Ricocheting Bee-Bee

Reflection. Analysis and problem reduction resulted in simple code.

Contrast it with the complexity of the code that would have been needed to simulate each leg the bee-bee's trajectory.

```
static void main() {
   double d = in.nextDouble();
   double theta = in.nextDouble();
   int y = 2;
   while math.abs(x(y,theta) - Math.round(x(y,theta))) >= d / 2:
       y = y + 2
   print( Hypotenuse(x(y, theta), y))
```