Principled Programming

Introduction to Coding in Any Imperative Language

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Running a Maze

Introduction

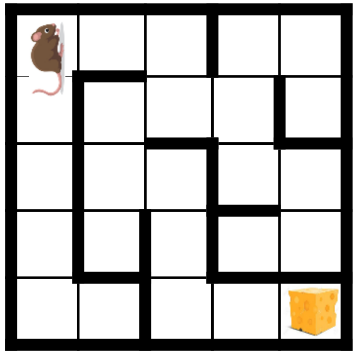
We present a systematic top-down development of an entire program to Run a Maze. We start from the beginning, but reference previous discussions from Chapters 1 and 4.

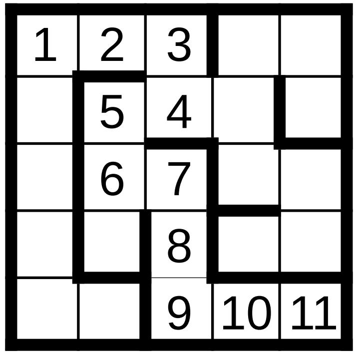
The main themes presented are:

- Use of a class to encapsulate a data representation.
- Consideration of alternative data representations.
- Structuring a program as two modules in a client/server relationship.
- The practice of information hiding.
- Incremental testing.
- Self-testing code.
- Exhaustive bounded testing of code.

Background. Define a maze to be a square two-dimensional grid of cells separated (or not) from adjacent cells by walls. One can move between adjacent cells if and only if no wall divides them. A solid wall surrounds the entire grid of cells, so there is no escape from the maze.

Problem Statement. Write a program that inputs a maze, and outputs a direct path from the upper-left cell to the lower-right cell if such a path exists, or outputs “Unreachable” otherwise. A path is direct if it never visits any cell more than once.





Top-level Code Structure

Establish a framework:

```
/* Run a rat through an arbitrary maze. */
class RunMaze {
   } /* RunMaze */
```

☞ **Program top-down, outside-in.**

Top-level Code Structure

Establish a framework:

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   /* Run maze. */
   public static void main() {
      } /* main */
   } /* RunMaze */
```

☞ **Program top-down, outside-in.**

Top-level Code Structure

Establish a framework:

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   /* Run maze. */
   public static void main() {
      /* Input. */
      /* Compute. */
      /* Output. */
      } /* main */
   } /* RunMaze */
```

☞ **Start by writing a top-level decomposition of the solution.**

Top-level Code Structure

Establish a framework:

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   /* Run a maze given as input, if possible. */
   public static void main() {
      /* Input a maze of arbitrary size, or output “malformed input”
         and stop if the input is improper. Input format: TBD.*/
      /* Compute a direct path through the maze, if one exists. */
      /* Output the direct path found, or “unreachable” if there is
         none. Output format: TBD. */
      } /* main */
   } /* RunMaze */
```

☞ **Repeatedly improve comments by relentless copy editing.**

Establish a framework:

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   /* Run a maze given as input, if possible. */
   public static void main() {
      /* Input a maze of arbitrary size, or output “malformed input”
         and stop if the input is improper. Input format: TBD.*/
         Input();
      /* Compute a direct path through the maze, if one exists. */
         Solve();
      /* Output the direct path found, or “unreachable” if there is
         none. Output format: TBD. */
         Output();
      } /* main */
   } /* RunMaze */
```

☞ **Many short procedures are better than large blocks of code.**

Top-level Code Structure

Stubs: Create stubs for the methods that have been introduced, which you can do mindlessly.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Input a maze of arbitrary size, or output “malformed input”
      and stop if the input is improper. Input format: TBD. */
   private static void Input() { } /* Input */
   /* Compute a direct path through the maze, if one exists. */
   private static void Solve() { } /* Solve */
   /* Output the direct path found, or “unreachable” if there is none.
      Output format: TBD. */
   private static void Output() { } /* Output */
   ...
 } /* RunMaze */
```

☞ **Don’t type if you can avoid it; clone. Cut and paste, then adapt.**

Private and internal to RunMaze. No other class needs to know about them.

Stubs: Create stubs for the methods that have been introduced, which you can do mindlessly.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Input a maze of arbitrary size, or output “malformed input”
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   private static void Input() { } /* Input */
   /* Compute a direct path through the maze, if one exists. */
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   /* Output the direct path found, or “unreachable” if there is none.
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   private static void Output() { } /* Output */
   ...
 } /* RunMaze */
```
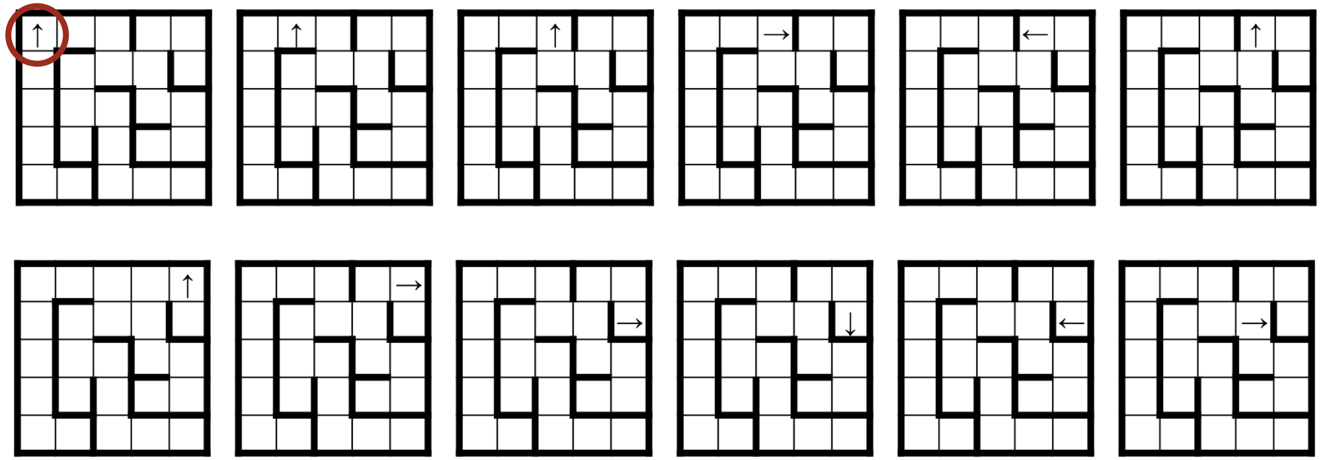
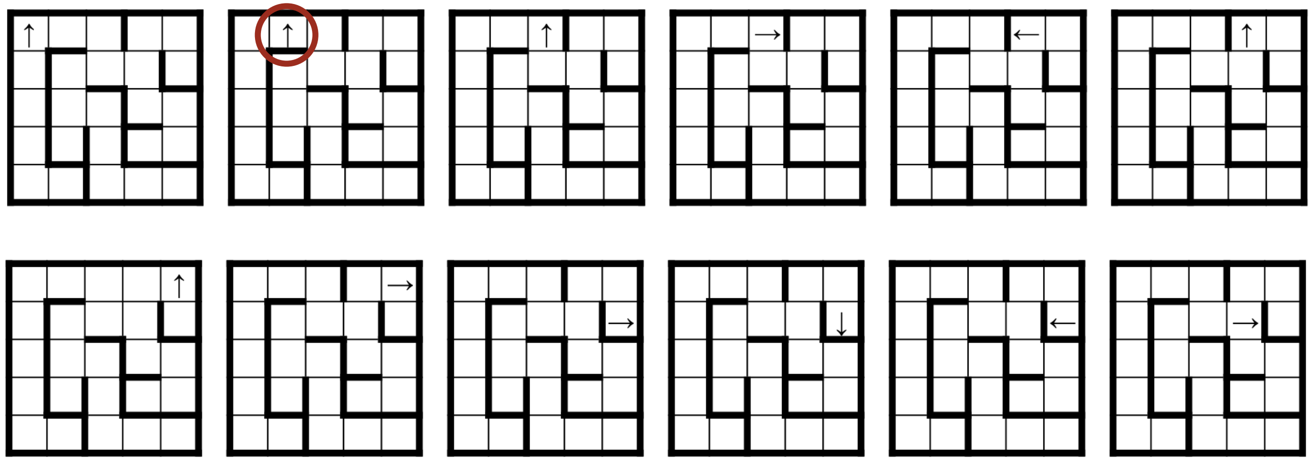

☞ **Practice information hiding.**

Algorithm (from Chapter 4):



☞ **Seek algorithmic inspiration from experience. Hand-simulate an algorithm that is in your "wetware". Be introspective. Ask yourself: What am I doing?**



Sidestep

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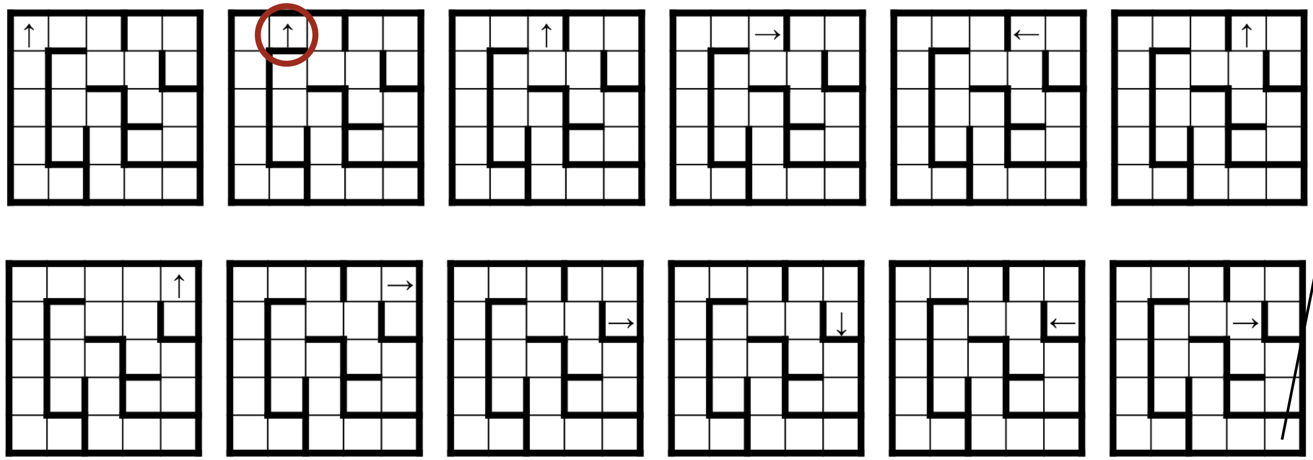
Algorithm

INVARIANT:
Left hand is on the interior surface of a peripheral wall.

VARIANT:
Get closer to goal.



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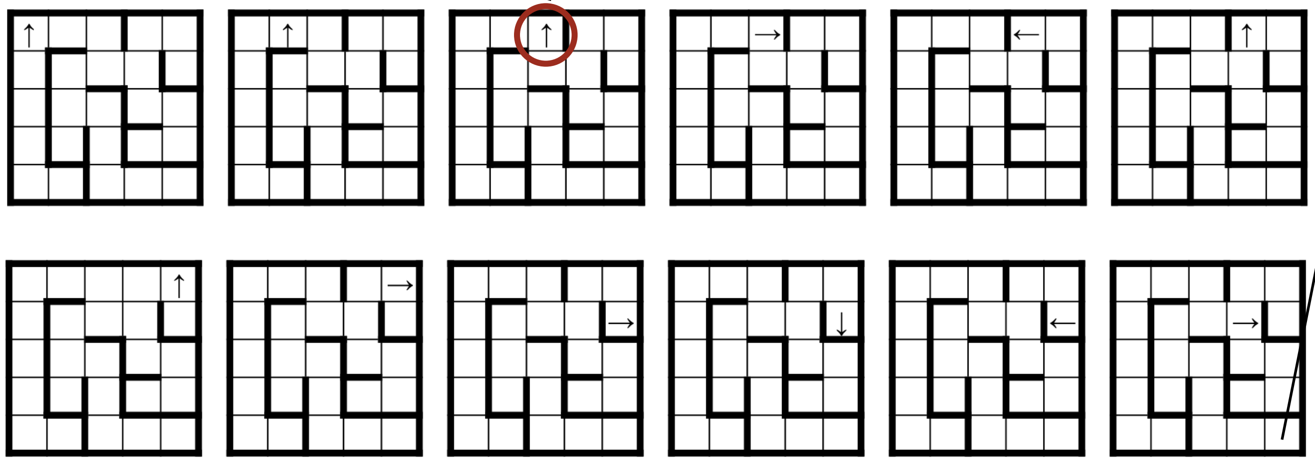
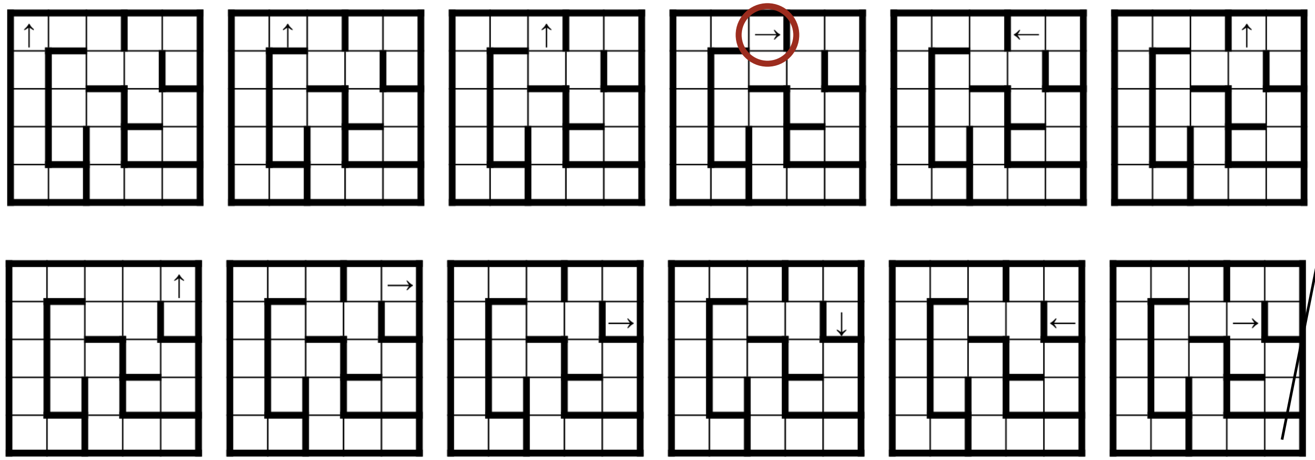


Sidestep

INVARIANT:
Left hand is on the interior surface of a peripheral wall.
"Peripheral" is not just "outer", but includes "attached" inner walls.

VARIANT:
Get closer to goal.



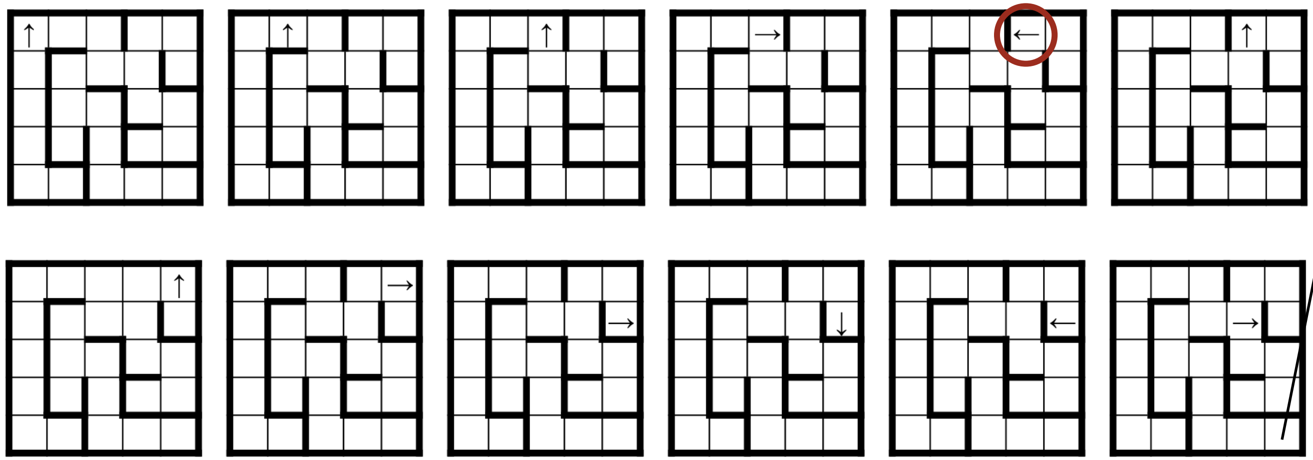
Turn convex corner

Algorithm

Algorithm

INVARIANT:
Left hand is on the interior surface of a peripheral wall.

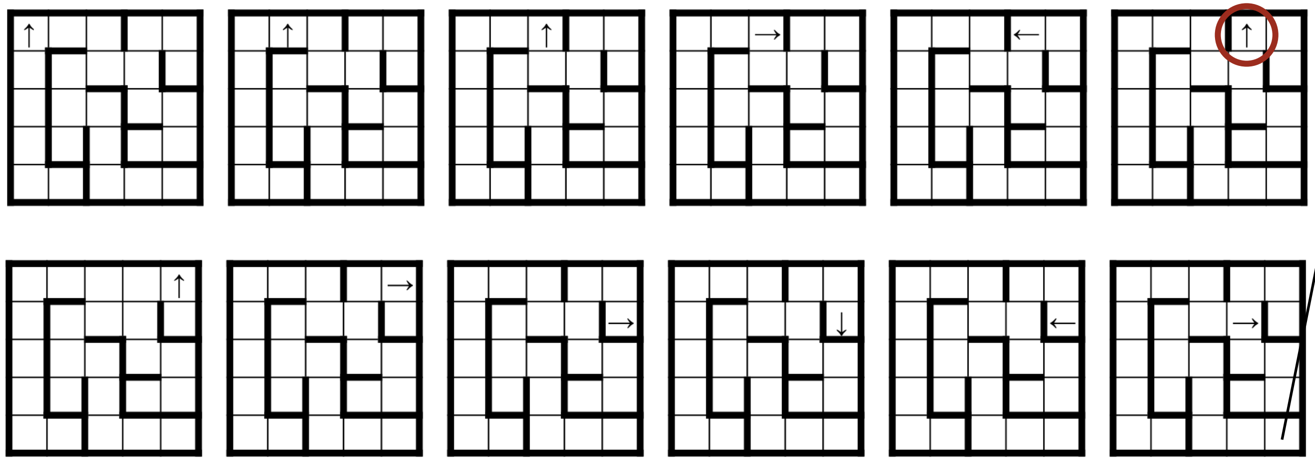
VARIANT:
Get closer to goal.



Pirouette to other side

INVARIANT:
Left hand is on the interior surface of a peripheral wall.

VARIANT:
Get closer to goal.

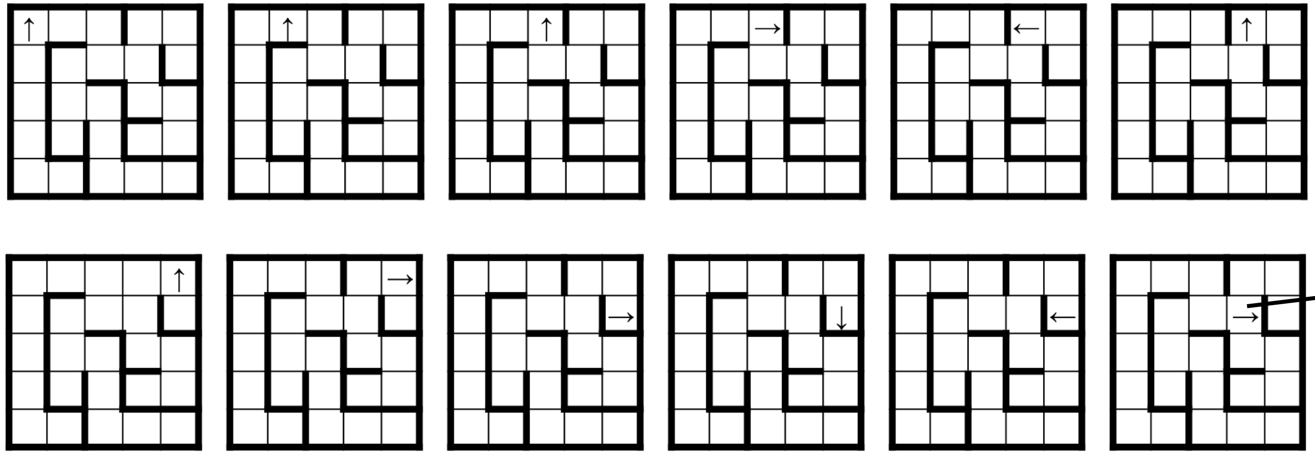


Turn convex corner

Algorithm

INVARIANT:
Left hand is on the interior surface of a peripheral wall.

VARIANT:
Get closer to goal.

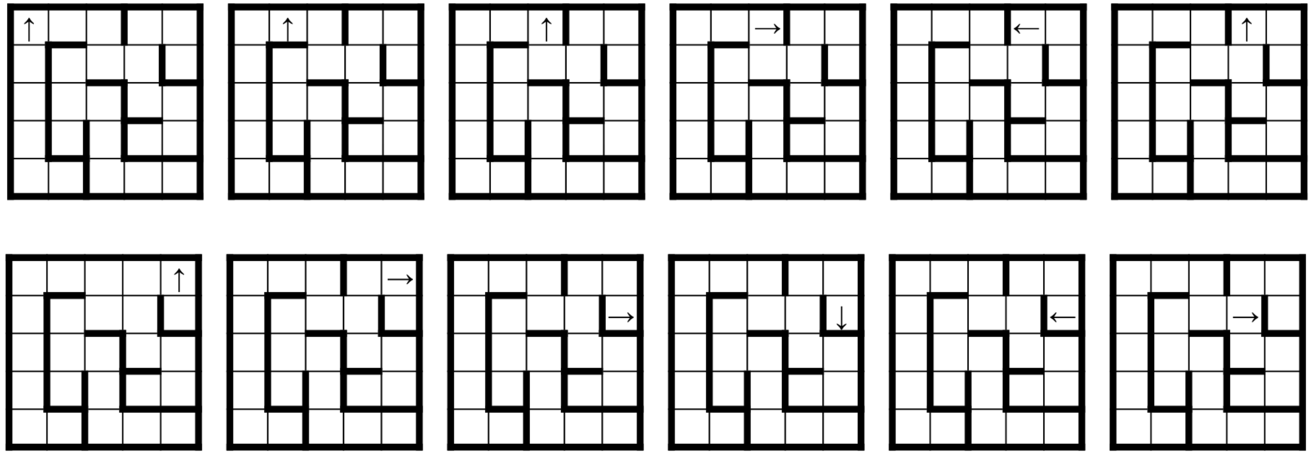


Actions:

- Sidestep
- Pirouette
- Turn convex corner
- (Turn concave corner)

INVARIANT:
Left hand is on the interior surface of a peripheral wall.

VARIANT:
Get closer to goal.

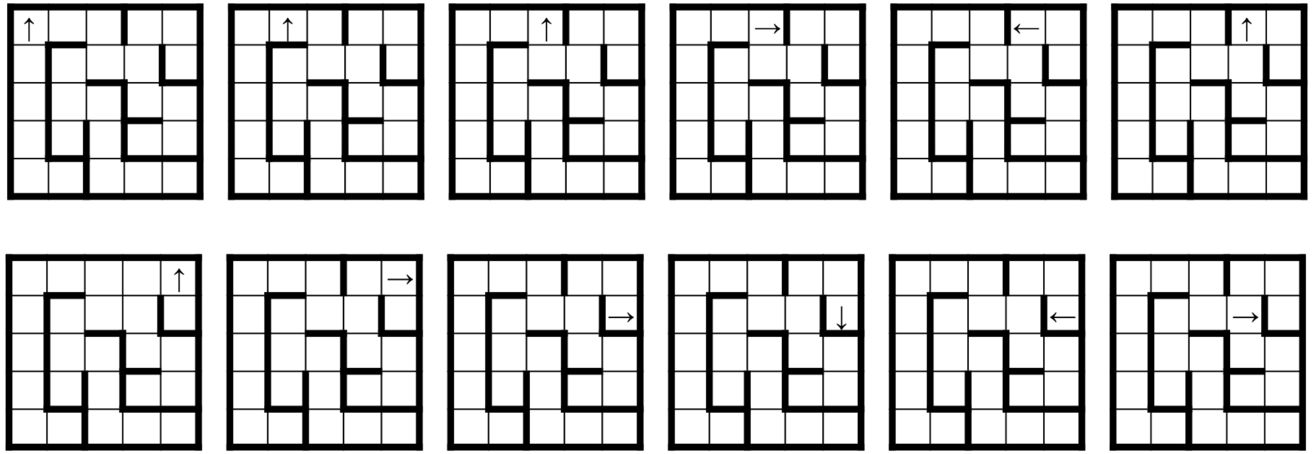


Actions:
- Sidestep
- Pirouette
- Turn convex corner
- (Turn concave corner)

Query:
- What action to perform?

INVARIANT:
Left hand is on the interior surface of a peripheral wall.

VARIANT:
Get closer to goal.

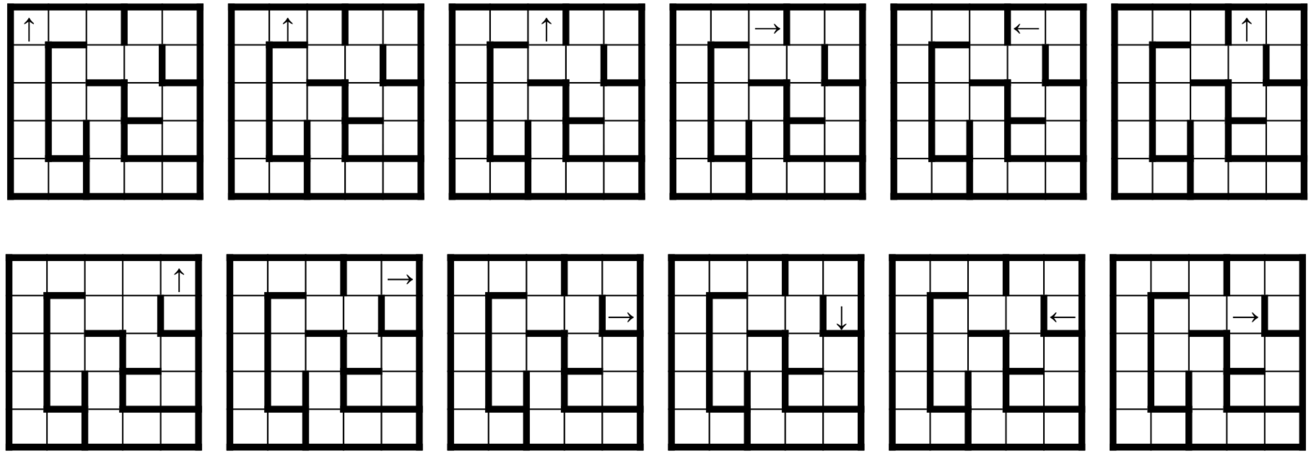


Actions:
- Sidestep
- Pirouette
- Turn convex corner
- (Turn concave corner)

Query:
- What action to perform?

Unit of progress:
- 1 wall-segment-surface

Algorithm



Physically, you don't need to distinguish cases, e.g., "just keep your hand on the wall and move to the right", but computationally, a case analysis must inspect the geometry, e.g.,

if (___________) Sidestep
else if (___________) Pirouette
else if (___________) Turn convex corner
else Turn concave corner

Alternative Formulation: From Chapter 4.

(allow left-hand off wall if it is at a **door**)

INVARIANT:

Left hand is on the interior surface of a peripheral wall, or at a door.

Actions:

- Turn clockwise 90°
- Turn counterclockwise 90°
- Step forward

Query:

- Facing a wall?

Unit of progress:

- 1 wall-segment-surface-or-door

Alternative Formulation: From Chapter 4.

(allow left-hand off wall if it is at a **door**)

INVARIANT:

Left hand is on the interior surface of a peripheral wall, or at a door.

Actions:

- Turn clockwise 90°
- Turn counterclockwise 90°
- Step forward

Finer-grained actions.

Query:

- Facing a wall?

Unit of progress:

- 1 wall-segment-surface-or-door

Alternative Formulation: From Chapter 4.

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INVARIANT:

Left hand is on the interior surface of a peripheral wall, or at a door.

Actions:

- Turn clockwise 90°
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- Step forward

Finer-grained actions.

Query:

- Facing a wall?

Local query.

Unit of progress:

- 1 wall-segment-surface-or-door

Alternative Formulation: From Chapter 4.

(allow left-hand off wall if it is at a **door**)

INVARIANT:

Left hand is on the interior surface of a peripheral wall, or at a door.

Actions:

- Turn clockwise 90°
- Turn counterclockwise 90°
- Step forward

Finer-grained actions.

Query:

- Facing a wall?

Local query.

Simpler to implement.

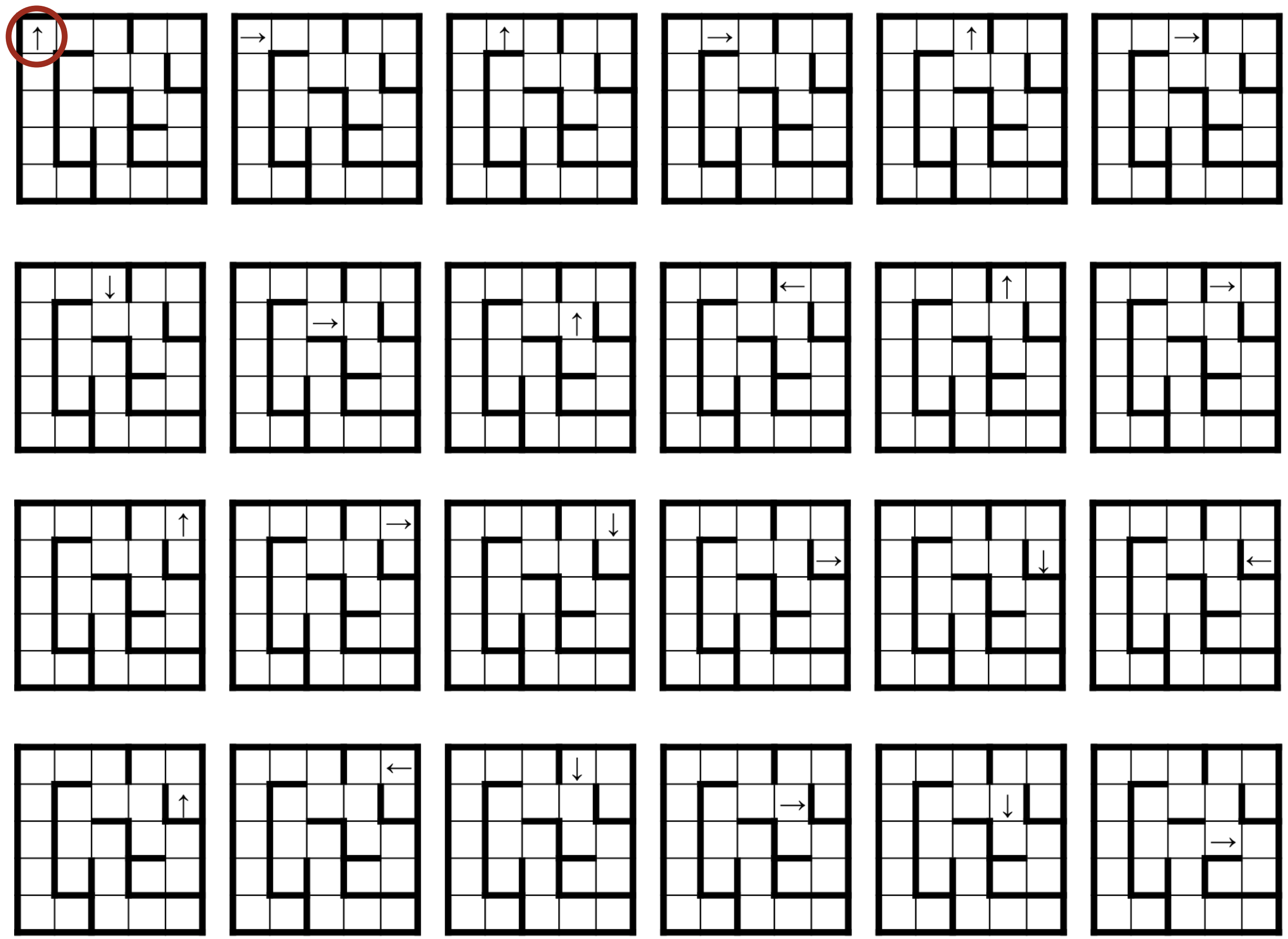
Unit of progress:

- 1 wall-segment-surface-or-door

Alternative Formulation: Pseudo-code, from Chapter 4.

```
/* Start in upper-left cell, facing up. */
while ( /* !in-lower-right && !in-upper-left-about-to-cycle */ )
   if ( /* facing-wall */ )
      /* Turn 90° clockwise. */
   else {
      /* Step forward. */
      /* Turn 90° counterclockwise. */
      }
```

INVARIANT:
Left hand is on the interior surface of a peripheral wall, or at a **door**.



```
while ( /* !in-lower-right && !in-upper-left-about-to-cycle */ )
   if ( /* facing-wall */ )
      /* Turn 90° clockwise. */
   else {
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      /* Turn 90° counterclockwise. */
      }
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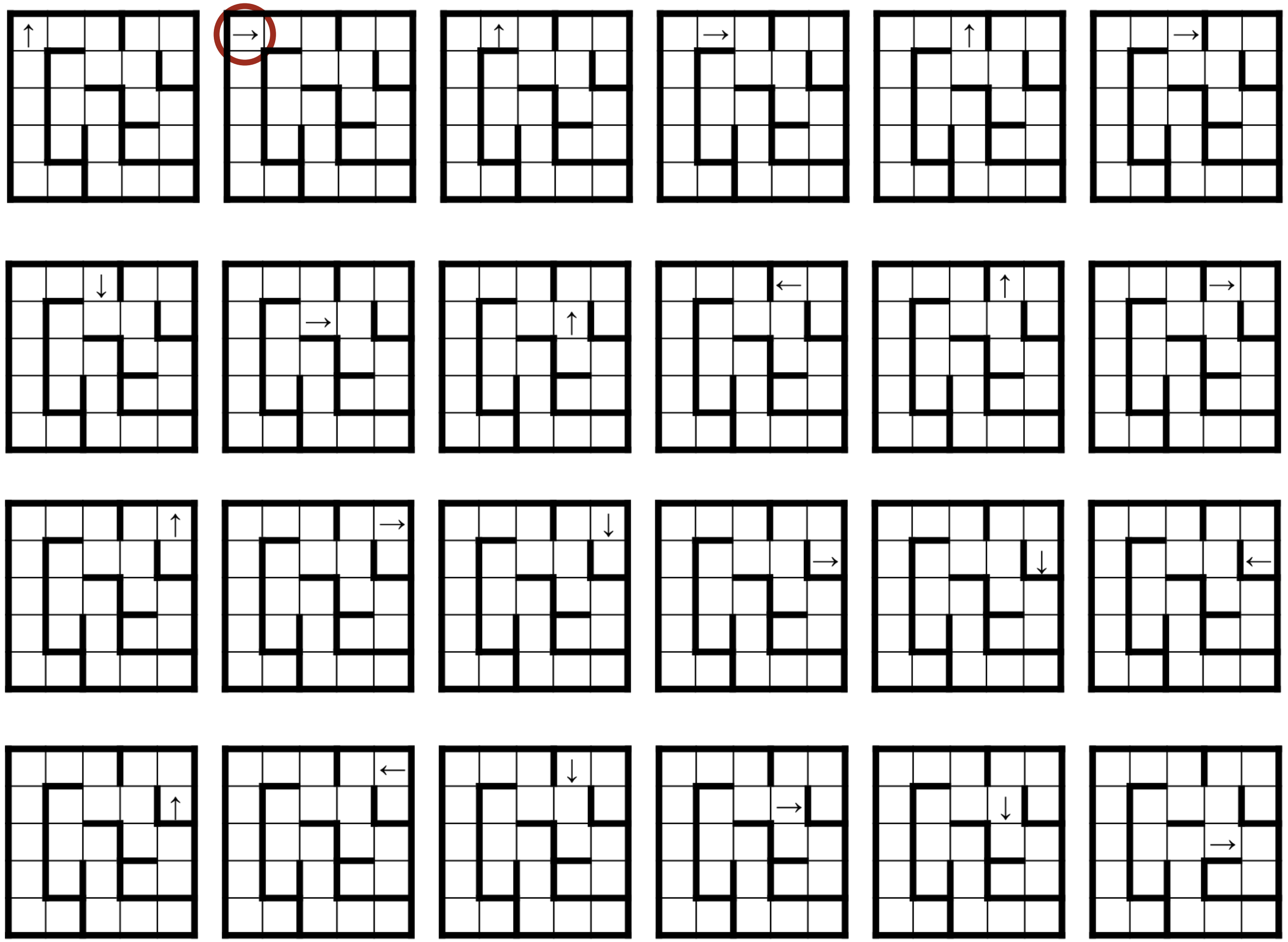

Algorithm

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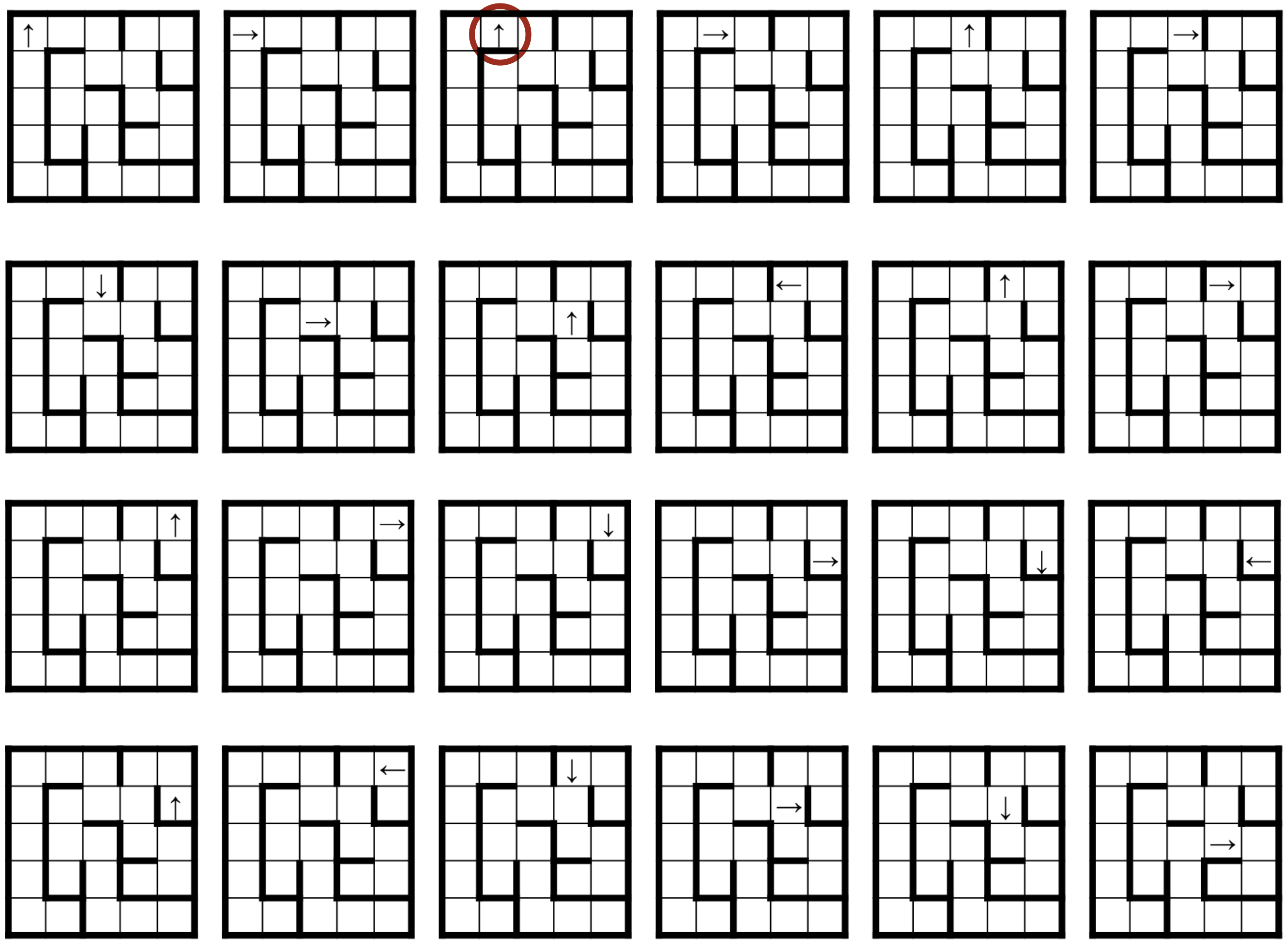

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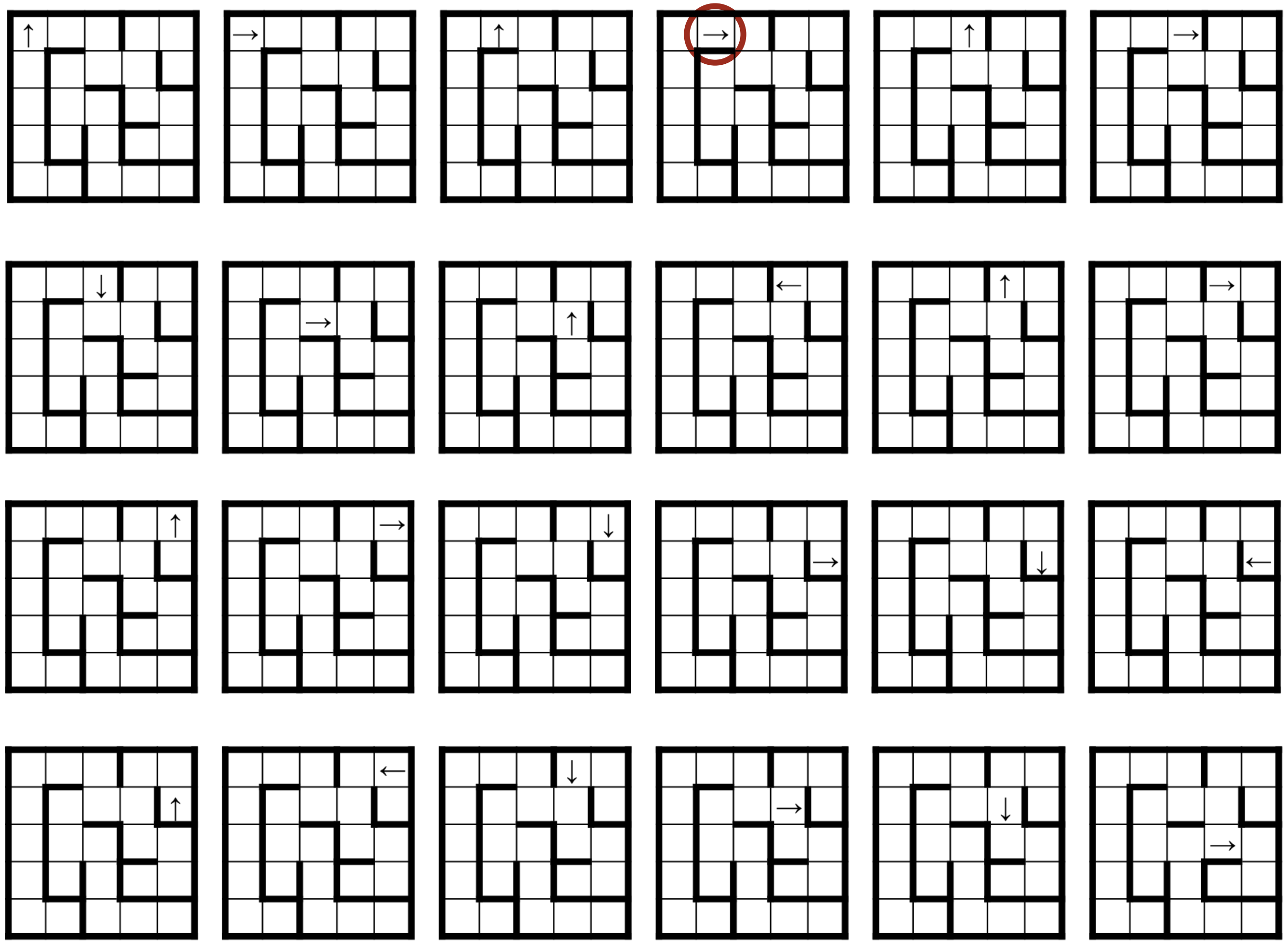

Algorithm

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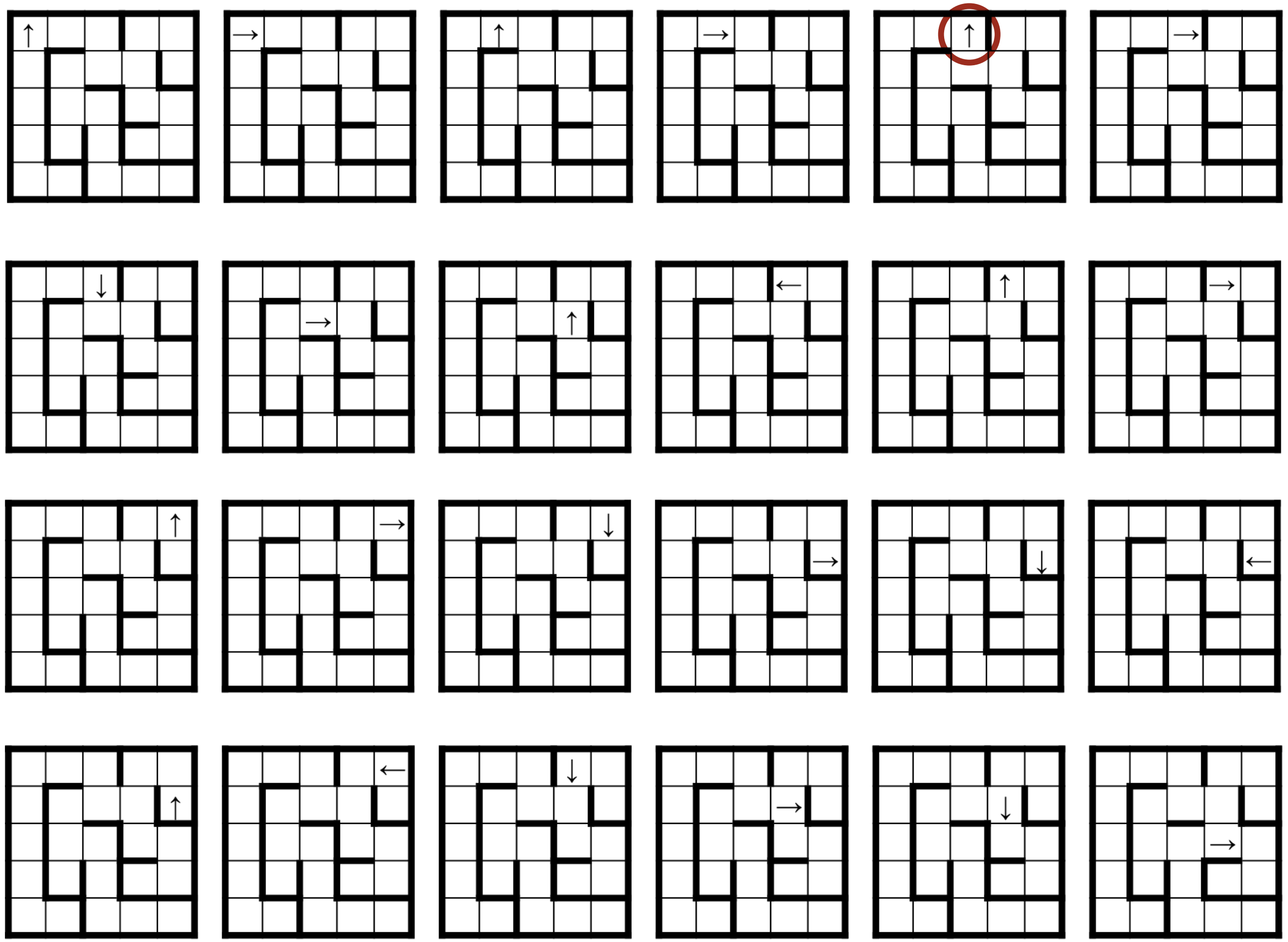

Algorithm

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while ( /* !in-lower-right && !in-upper-left-about-to-cycle */ )
   if ( /* facing-wall */ )
      /* Turn 90° clockwise. */
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      }
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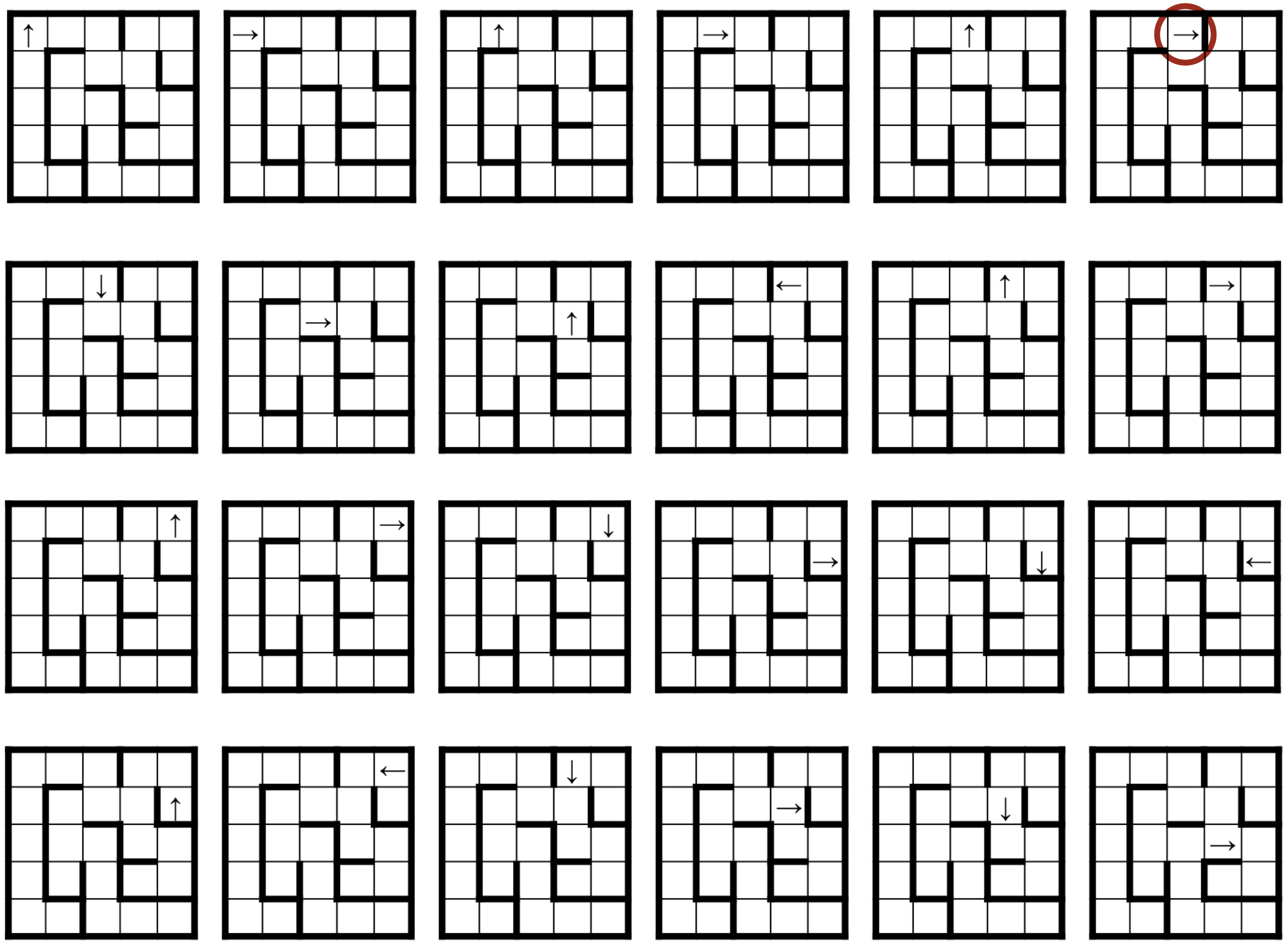

Algorithm

```
while ( /* !in-lower-right && !in-upper-left-about-to-cycle */ )
   if ( /* facing-wall */ )
      /* Turn 90° clockwise. */
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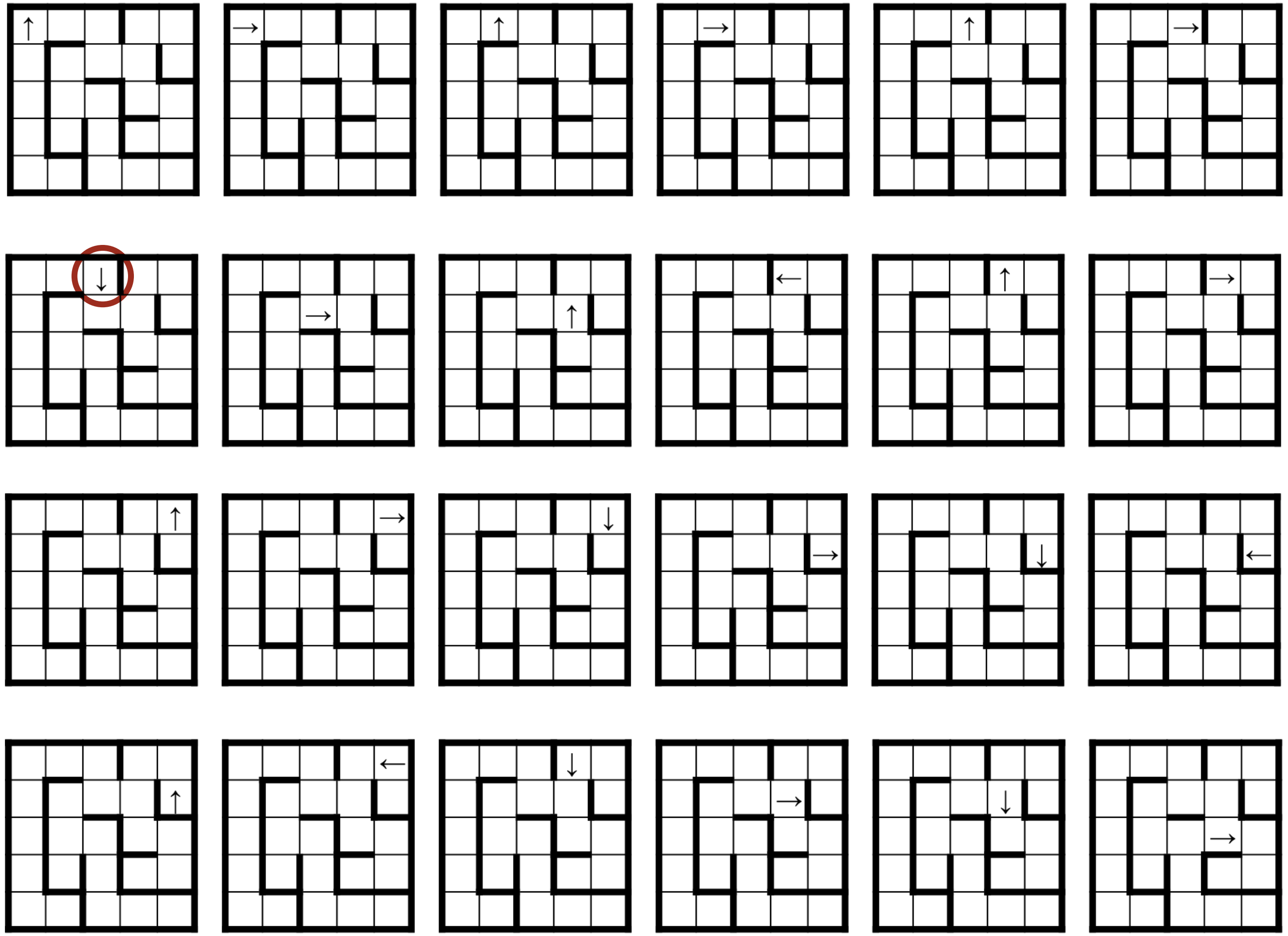

Algorithm

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while ( /* !in-lower-right && !in-upper-left-about-to-cycle */ )
   if ( /* facing-wall */ )
      /* Turn 90° clockwise. */
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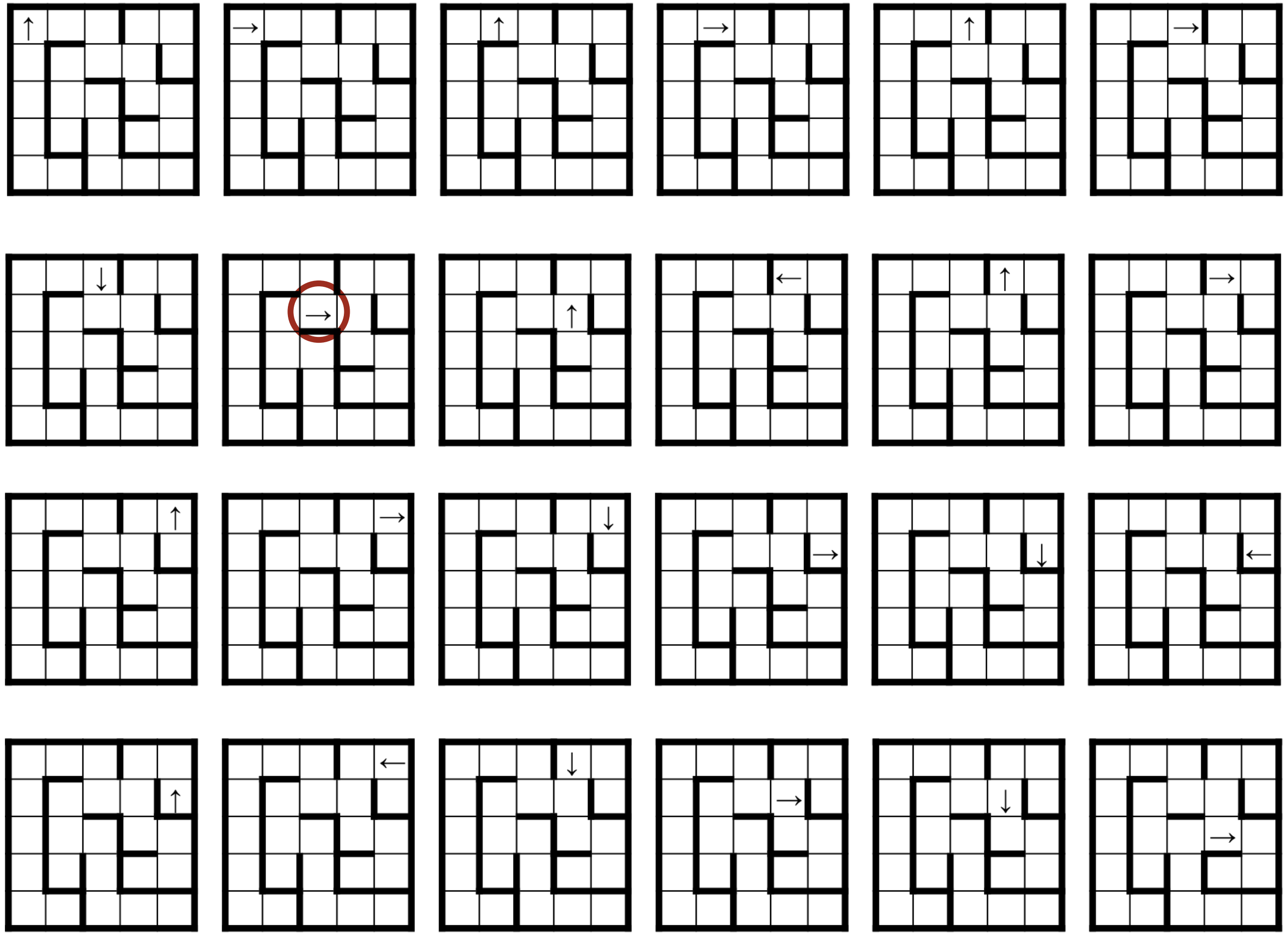

Algorithm

```
while ( /* !in-lower-right && !in-upper-left-about-to-cycle */ )
   if ( /* facing-wall */ )
      /* Turn 90° clockwise. */
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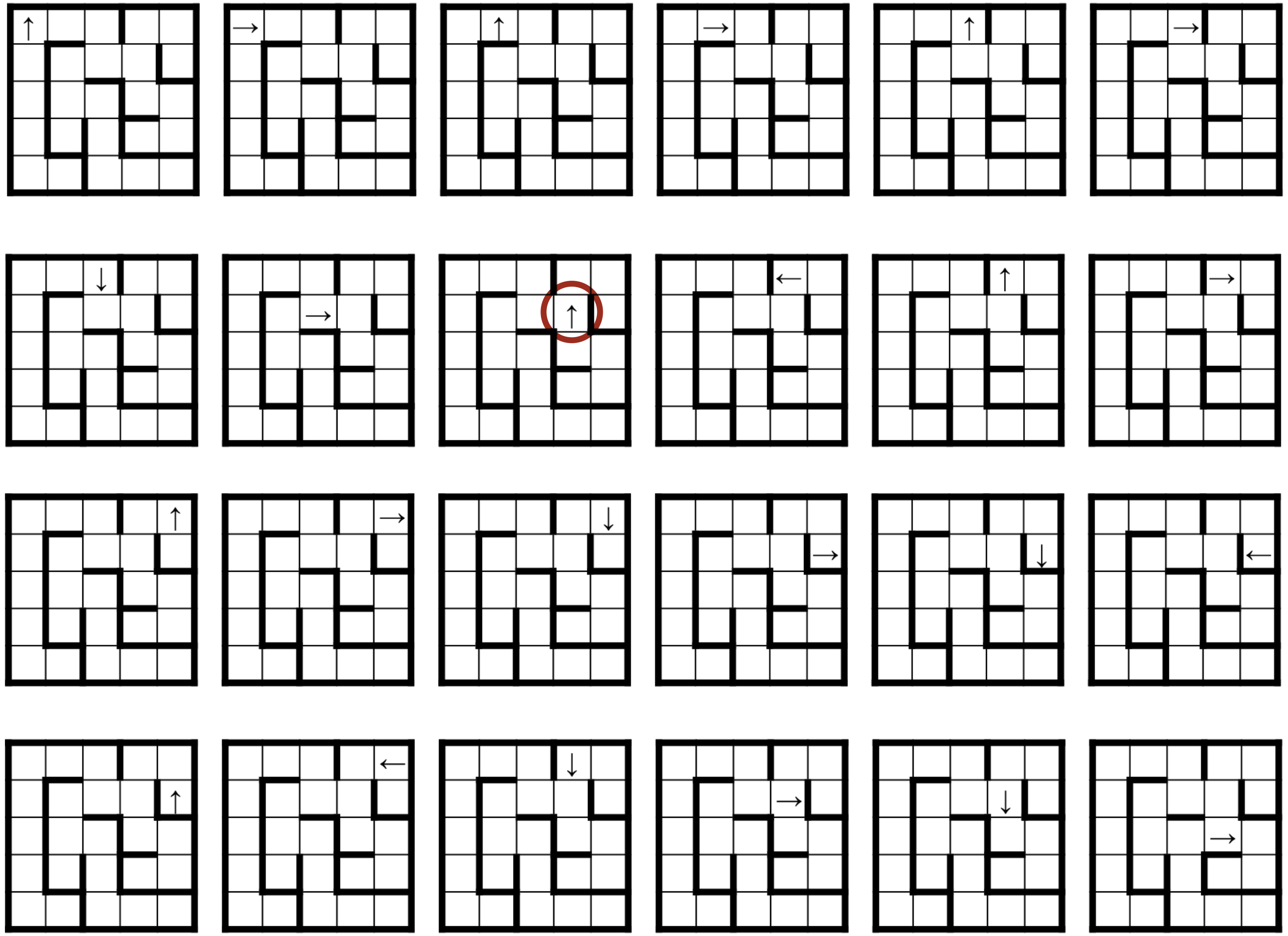

Algorithm

```
while ( /* !in-lower-right && !in-upper-left-about-to-cycle */ )
   if ( /* facing-wall */ )
      /* Turn 90° clockwise. */
   else {
      /* Step forward. */
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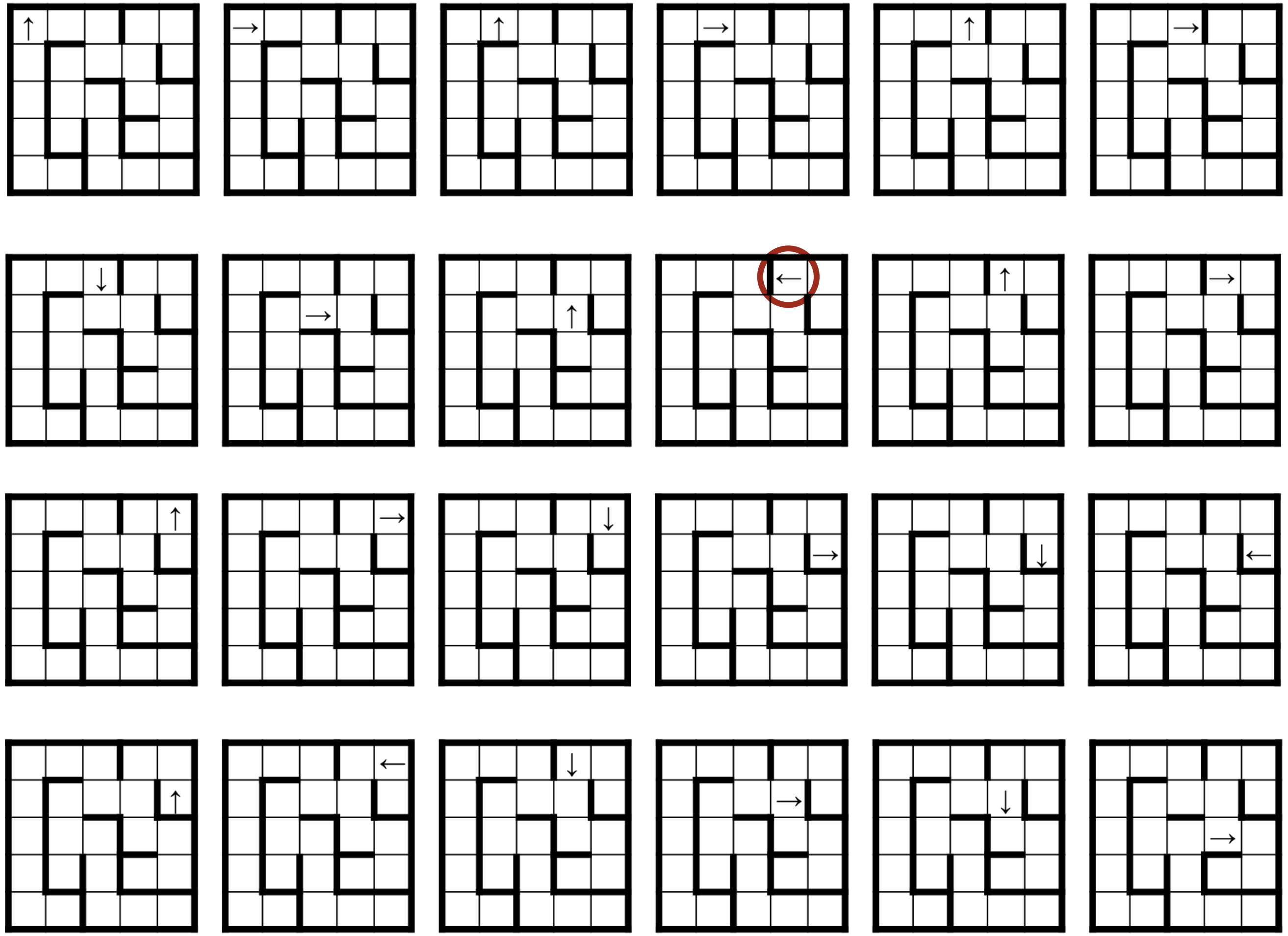
      }
```



INVARIANT:
Left hand is on the interior surface of a peripheral wall, or at a **door**.
Establish INVARIANT as part of initialization of state.

Algorithm: Drop code into RunMaze.

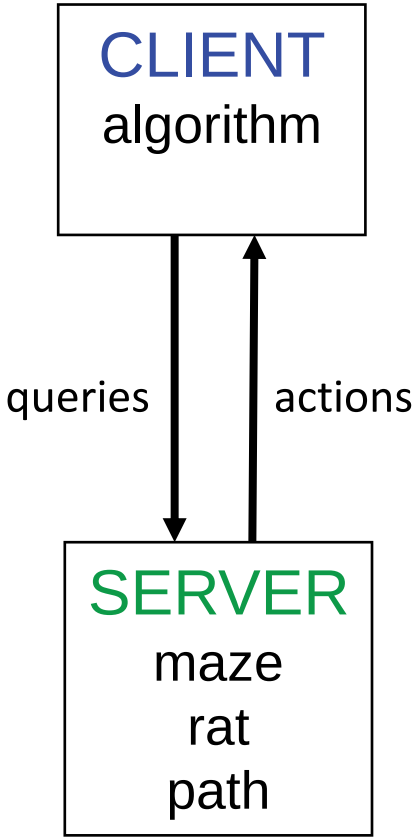
```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Input a maze of arbitrary size, or output "malformed input"
      and stop if the input is improper. Input format: TBD. */
   private static void Input() {
      ⟨Obtain maze from input.⟩
      ⟨Start in upper-left cell, facing up.⟩
      } /* Input */
   ...
   } /* RunMaze */
```

INVARIANT:
Left hand is on the interior surface of a peripheral wall, or at a **door**.
Maintain INVARIANT and make progress in Solve.

Algorithm: Drop code into RunMaze, with pseudo-operations turned into method calls.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Compute a ~~direct~~ path through the maze, if one exists. */
   private static void Solve() {
      while ( !isAtCheese() && !isAboutToRepeat() )
         if ( isFacingWall() ) TurnClockwise();
         else {
            StepForward();
            TurnCounterClockwise();
            }
      } /* Solve */
   ...
   } /* RunMaze */
```

Modular program structure: Separation of concerns.




```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Run a maze given as input, if possible. */
   public static void main() {
      ...
      } /* main *.
} /* RunMaze */

/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
    } /* MRP */
```

The algorithm is a client of services provided by class MRP.

Algorithm (from Chapter 4): Qualify names of methods of another class.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Compute a direct path through the maze, if one exists. */
   private static void Solve() {
      while ( !MRP.isAtCheese() && !MRP.isAboutToRepeat() )
         if ( MRP.isFacingWall() ) MRP.TurnClockwise();
         else {
            MRP.StepForward();
            MRP.TurnCounterClockwise();
            }
      } /* Solve */
   ...
   } /* RunMaze */
```

Procedure stubs for the services.

Operations:

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   public static void TurnClockwise() { }
   public static void TurnCounterClockwise() { }
   public static void StepForward() { }
   public static boolean isFacingWall() { return ____; }
   public static boolean isAtCheese() { return ____; }
   public static boolean isAboutToRepeat() { return ____; }
   } /* MRP */
```

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Stubs provide *signatures*, i.e., names, types for return values, types for parameters (none), and visibility.

Operations:

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   public static void TurnClockwise() { }
   public static void TurnCounterClockwise() { }
   public static void StepForward() { }
   public static boolean isFacingWall() { return ____; }
   public static boolean isAtCheese() { return ____; }
   public static boolean isAboutToRepeat() { return ____; }
   } /* MRP */
```

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Public to client classes of MRP, e.g., RunMaze.

Operations:

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   public static void TurnClockwise() { }
   public static void TurnCounterClockwise() { }
   public static void StepForward() { }
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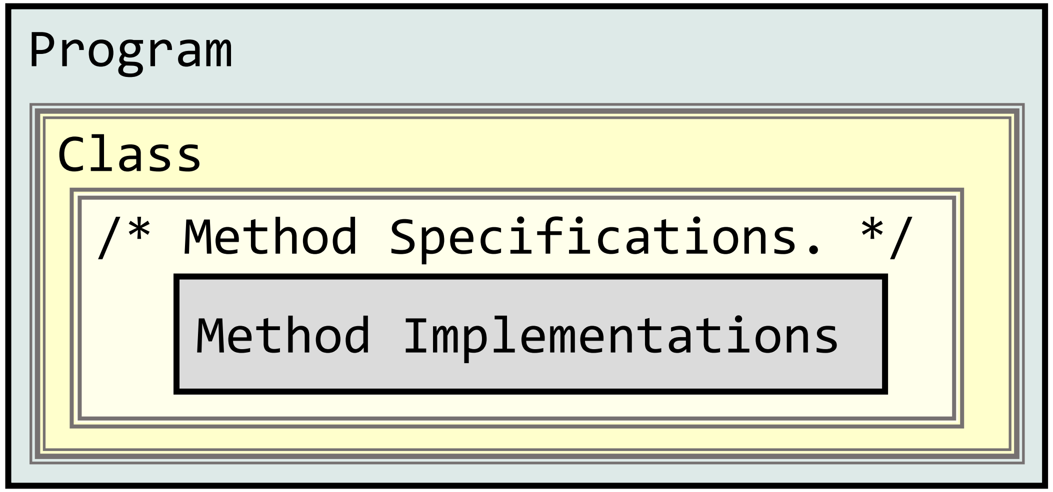
   } /* MRP */
```

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Data Representation

State: The Maze, Rat, and Path data representations.

We (the implementers of MRP) design the data representation to record the state, and code the query and action operations to update it.

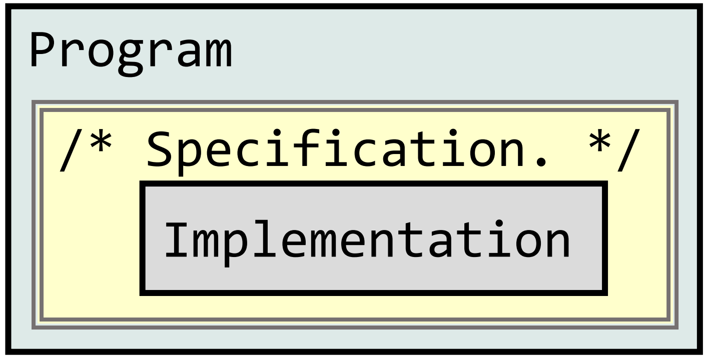


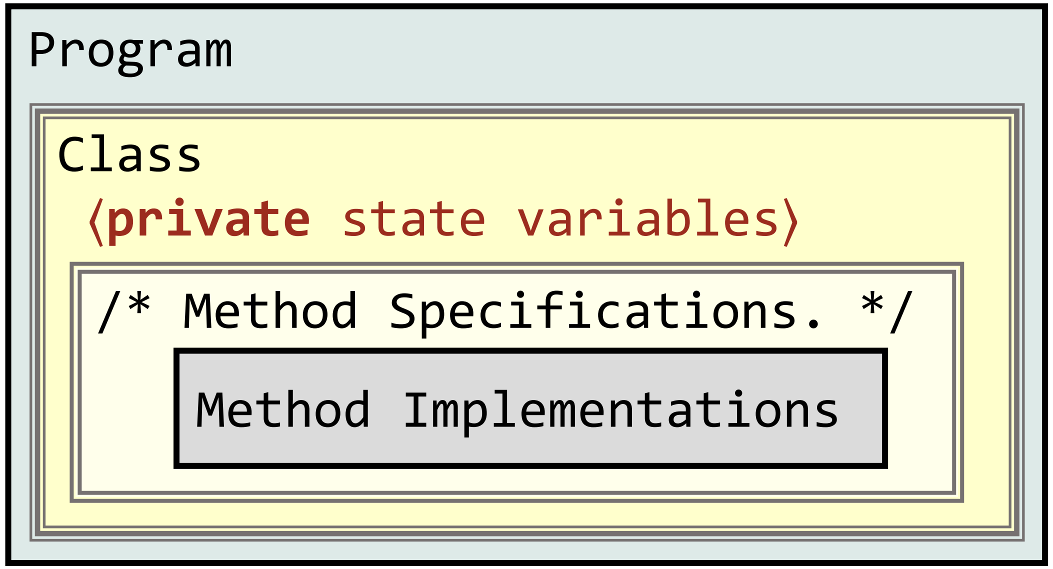
☞ **Practice information hiding.**

State: The Maze, Rat, and Path data representations.

We (the implementers of MRP) design the data representation to record the state, and code the operations to query and update it.

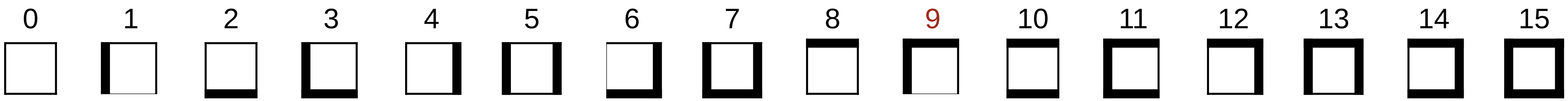
Clients of MRP will have no direct access to the state in MRP. Rather, they will only be able to interact with MRP via its operations, i.e., its interface. This is called an *abstract data type*, and generalizes our prior use of specifications for information hiding.

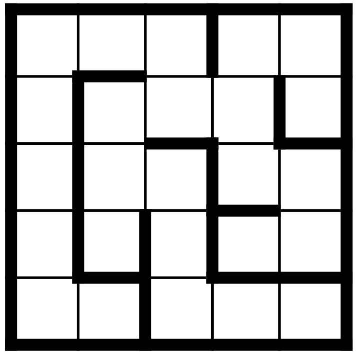


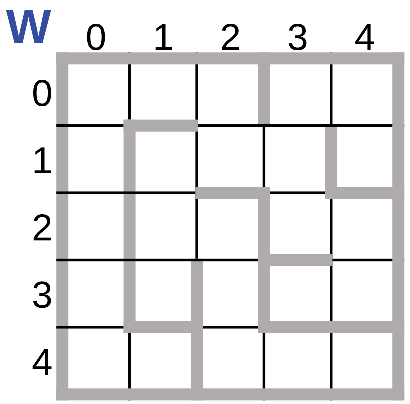


☞ **Practice information hiding.**

Maze Representation 1: N-by-N array `W` whose elements encode cell walls:



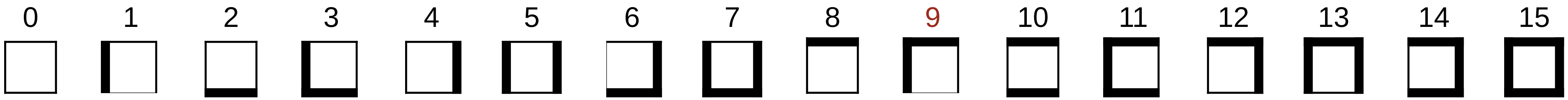


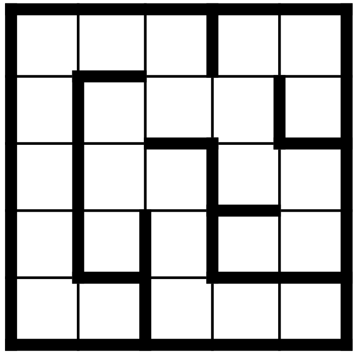


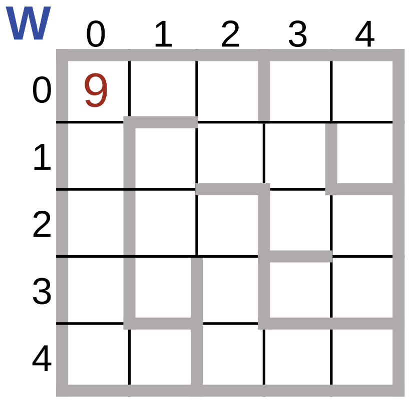
☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Data Representation

Maze Representation 1: N-by-N array `W` whose elements encode cell walls:

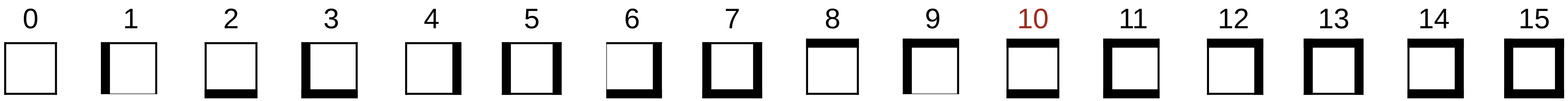


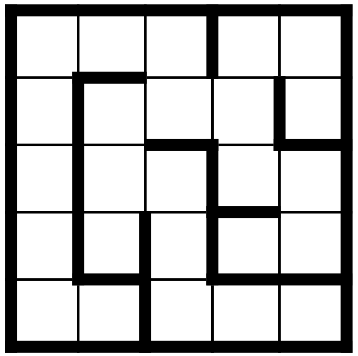


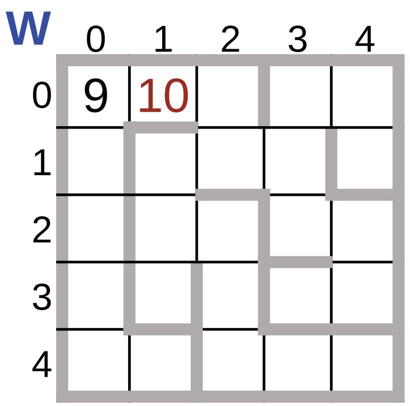


☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Maze Representation 1: N-by-N array `W` whose elements encode cell walls:







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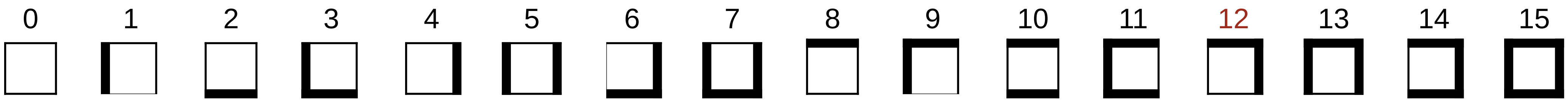

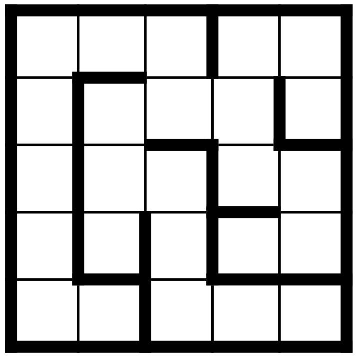

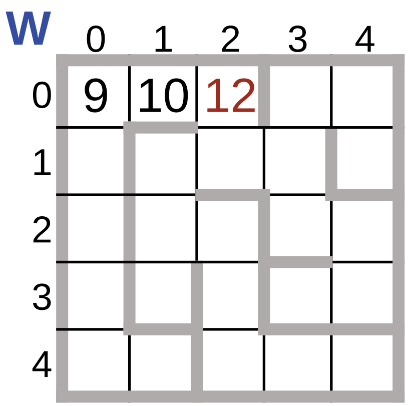

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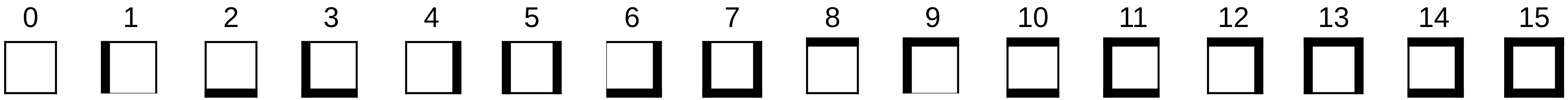
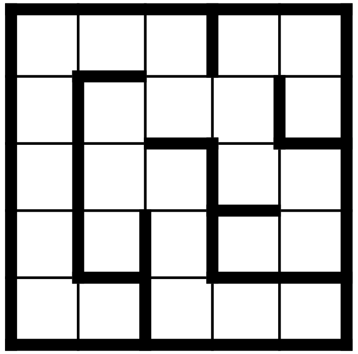
Maze Representation 1: N-by-N array `W` whose elements encode cell walls:



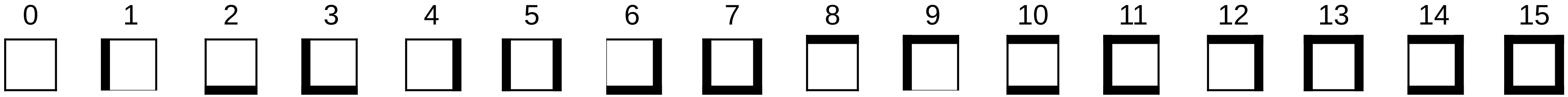


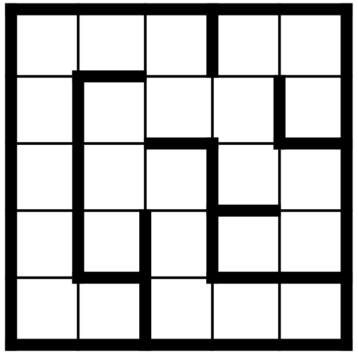
W

	0	1	2	3	4
0	9	10	12	9	12
1	5	9	2	4	7
2	5	1	12	3	12
3	5	7	5	11	6
4	3	14	3	10	14

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Maze Representation 1: N-by-N array `W` whose elements encode cell walls:





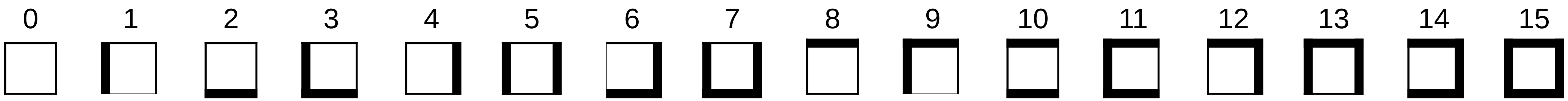
W	0	1	2	3	4
0	9	10	12	9	12
1	5	9	2	4	7
2	5	1	12	3	12
3	5	7	5	11	6
4	3	14	3	10	14

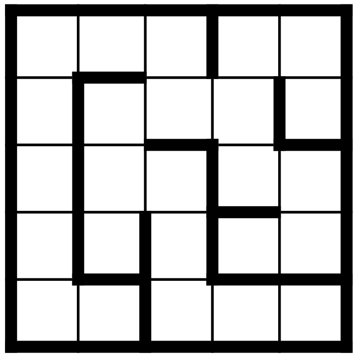
Anticipate

- Direction `d`, ⟨0,1,2,3⟩ = ⟨`up`,`right`,`down`,`left`⟩
- Decoder `isWall(r,c,d)`, **true** iff wall in direction d

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Maze Representation 1: N-by-N array `W` whose elements encode cell walls:





W

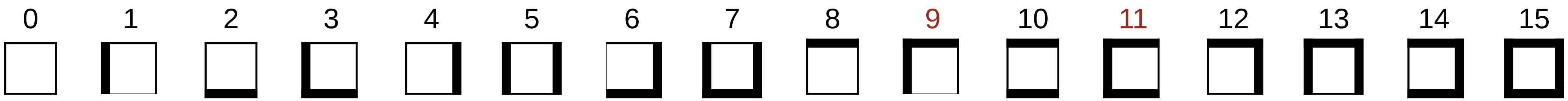
	0	1	2	3	4
0	9	10	12	9	12
1	5	9	2	4	7
2	5	1	12	3	12
3	5	7	5	11	6
4	3	14	3	10	14

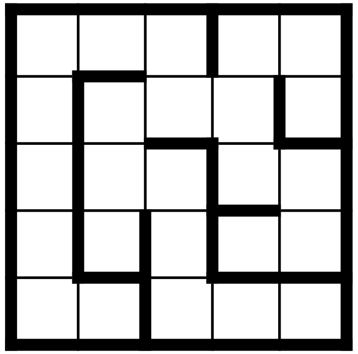
Positive

- Direct correspondence between physical maze and 2-D array **W**.

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Maze Representation 1: N-by-N array `W` whose elements encode cell walls:





W

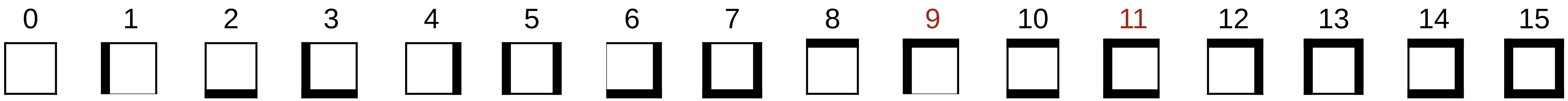
	0	1	2	3	4
0	9	11	12	9	12
1	5	9	2	4	7
2	5	1	12	3	12
3	5	7	5	11	6
4	3	14	3	10	14

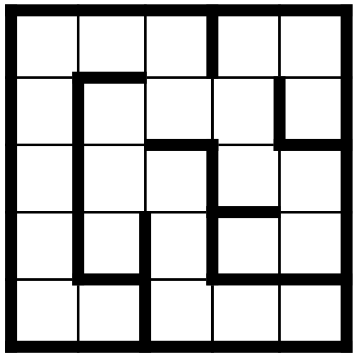
Negative

- Representation admits nonsensical data, e.g., 9 claims "there is no wall to the right", but 11 claims "there is a wall to the left".

☞ **Choose representations that by design do not have nonsensical configurations.**

Maze Representation 1: N-by-N array `W` whose elements encode cell walls:





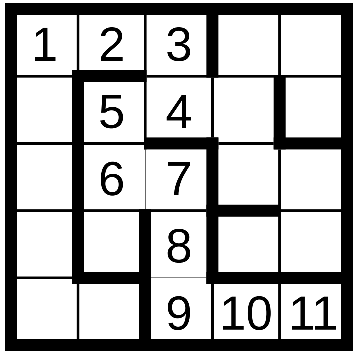
W

	0	1	2	3	4
0	9	11	12	9	12
1	5	9	2	4	7
2	5	1	12	3	12
3	5	7	5	11	6
4	3	14	3	10	14

Negatives

- Representation admits nonsensical data, e.g., 9 claims “there is no wall to the right”, but 11 claims “there is a wall to the left”.
- Decoder `isWall(r,c,d)` and corresponding encoder are somewhat fussy.

Path Representation 1: N-by-N array `P` whose elements are visit numbers or 0 (`Unvisited`).



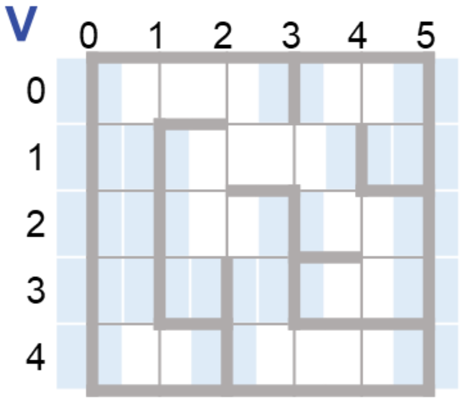
P	0	1	2	3	4
0	1	2	3	0	0
1	0	5	4	0	0
2	0	6	7	0	0
3	0	0	8	0	0
4	0	0	9	10	11

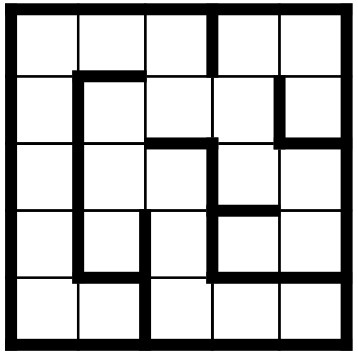
Positive

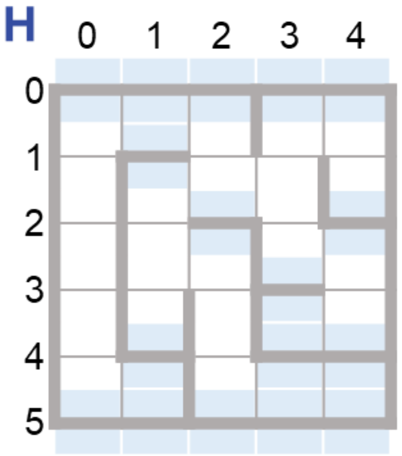
- Direct correspondence between physical maze and 2-D array **P**.

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Maze Representation 2: Separate **boolean** arrays, V and H, for **v**ertical and **h**orizontal walls.





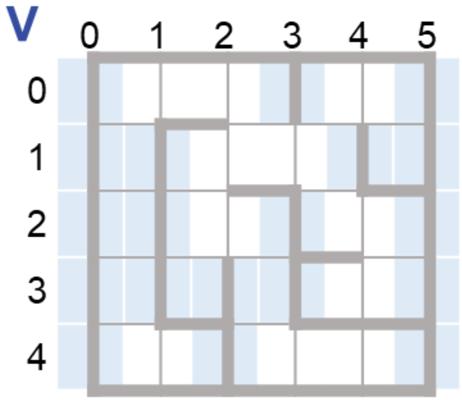


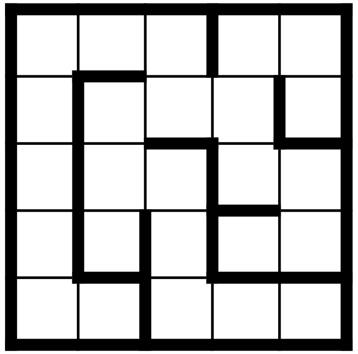
Eliminating Negatives of Representation 1

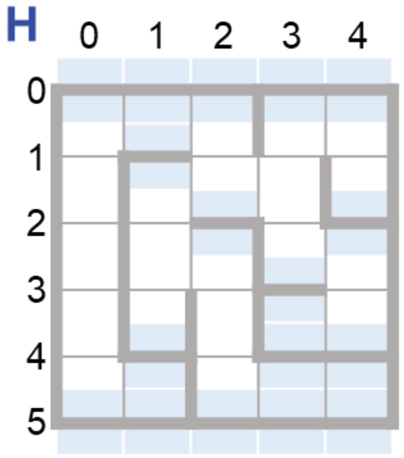
- Unique representation of each (possible) wall.
- Decoder and corresponding encoder are more straightforward.

☞ **Choose representations that by design do not have nonsensical configurations.**

Maze Representation 2: Separate **boolean** arrays, V and H, for **v**ertical and **h**orizontal walls.





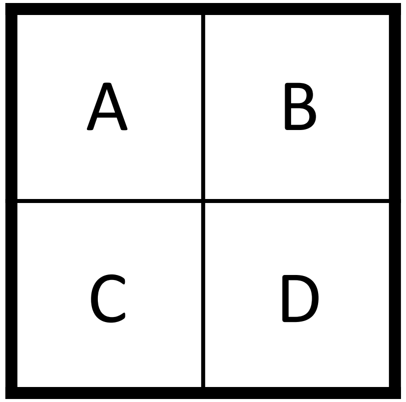


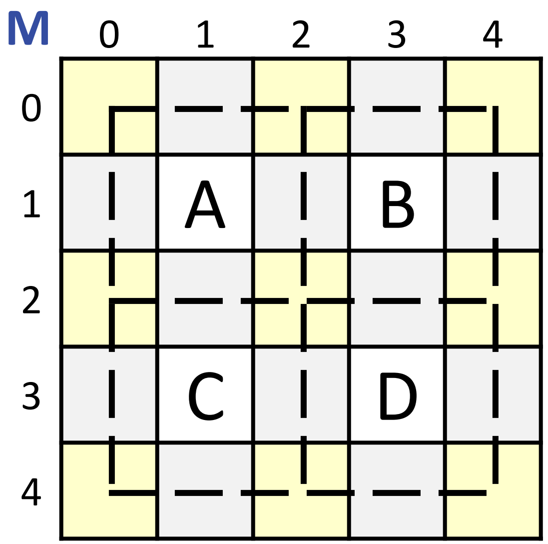
Negative of Representation 2

- Non-uniformity. Two arrays rather than one.

☞ **Choose data representations that are uniform, if possible.**

Maze Representation 3: (2·N+1)-by-(2·N+1) array `M` of walls and path visit numbers.



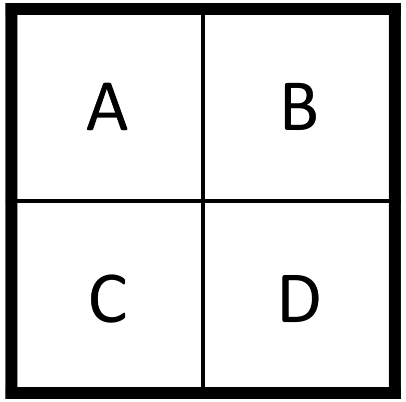


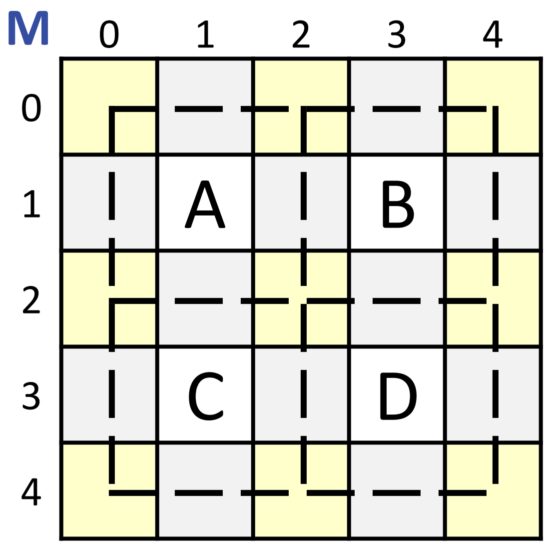
Positives

- Single 2-D array `M` for both walls and path.
- Unique array cell (gray) to represent each (possible) wall.
- Unique array cell (letters) for visit numbers.

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Maze Representation 3: (2·N+1)-by-(2·N+1) array `M` of walls and path visit numbers.



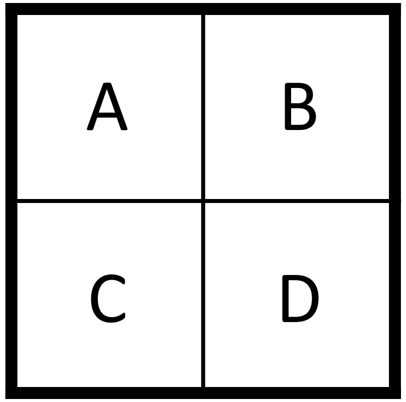


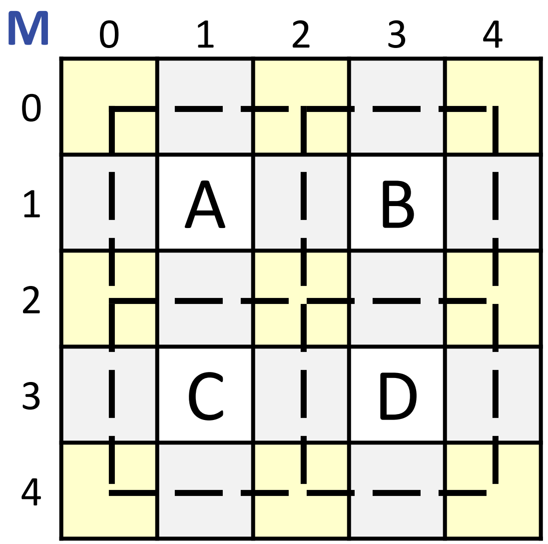
Negatives

- About ¼ of storage is wasted (yellow).
- Direct correspondence between maze coordinate system and 2-D array. indices lost.

☞ **The touchstone of a data representation is its utility in performing the needed operations.**

Maze Representation 3: Adopt it.





☞ **Don't let the "perfect" be the enemy of the "good". Be prepared to compromise because there may be no perfect representation. Don't freeze.**

Data Representation Invariant:

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   /* Maze. Cells of an N-by-N maze are represented by elements of a
      (2*N+1)-by-(2*N+1) array M. Maze cell ⟨r,c⟩ is represented by array
      element M[2*r+1][2*c+1]. The possible walls ⟨top, right, bottom,
      left⟩ of the maze cell corresponding to ⟨r,c⟩ are represented by
      Wall or NoWall in ⟨M[r-1][c], M[r][c+1], M[r+1][c], M[r][c-1]⟩.
      The remaining elements of M are unused. lo is 1, and hi is 2*N-1. */
      private static int N;                  // N is size of maze.
      private static int M[][];              // M is N-by-N maze, walls, and path.
      private static final int Wall = -1;    // WALL encodes presence of a wall.
      private static final int NoWall = 0;  // NO_WALL encodes absence of a wall.
      private static int lo;                 // lo is left/top maze indices.
      private static int hi;                 // hi is right/bottom maze indices.
   ...
   } /* MRP */
```

☞ **A representation invariant describes the value(s) of one or more program variables, and their relationships to one another as the program runs. The invariant is typically written as a comment associated with the declaration(s) of the relevant variable(s).**

Private and internal to MRP. No other class needs to know about them.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   /* Maze. Cells of an N-by-N maze are represented by elements of a
      (2*N+1)-by-(2*N+1) array M. Maze cell ⟨r,c⟩ is represented by array
      element M[2*r+1][2*c+1]. The possible walls ⟨top, right, bottom,
      left⟩ of the maze cell corresponding to ⟨r,c⟩ are represented by
      Wall or NoWall in ⟨M[r-1][c], M[r][c+1], M[r+1][c], M[r][c-1]⟩.
      The remaining elements of M are unused. lo is 1, and hi is 2*N-1. */
      private static int N;                       // N is size of maze.
      private static int M[][];                   // M is N-by-N maze, walls, and path.
      private static final int Wall = -1;         // WALL encodes presence of a wall.
      private static final int NoWall = 0;        // NO_WALL encodes absence of a wall.
      private static int lo;                      // lo is left/top maze indices.
      private static int hi;                      // hi is right/bottom maze indices.
   ...
   } /* MRP */
```

☞ **Practice information hiding.**

Data Representation

Names that are declared **final** are constant throughout program execution.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   /* Maze. Cells of an N-by-N maze are represented by elements of a
      (2*N+1)-by-(2*N+1) array M. Maze cell ⟨r,c⟩ is represented by array
      element M[2*r+1][2*c+1]. The possible walls ⟨top, right, bottom,
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      Wall or NoWall in ⟨M[r-1][c], M[r][c+1], M[r+1][c], M[r][c-1]⟩.
      The remaining elements of M are unused. lo is 1, and hi is 2*N-1. */
      private static int N;                  // N is size of maze.
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      private static final int Wall = -1;    // WALL encodes presence of a wall.
      private static final int NoWall = 0;   // NO_WALL encodes absence of a wall.
      private static int lo;                 // lo is left/top maze indices.
      private static int hi;                 // hi is right/bottom maze indices.
   ...
   } /* MRP */
```

☞ **Minimize use of literal numerals in code; define and use symbolic constants.**

Data Representation Invariant:

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Rat. The rat is located in cell M[r][c] facing direction d, where
      d=⟨0,1,2,3⟩ represents the orientation ⟨up,right,down,left⟩,
      respectively. */
      private static int r, c, d;
   ...
   } /* MRP */
```

☞ **A representation invariant describes the value(s) of one or more program variables, and their relationships to one another as the program runs. The invariant is typically written as a comment associated with the declaration(s) of the relevant variable(s).**

Data Representation Invariant:

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Path. When the rat has traveled to cell ⟨r,c⟩ via a given path
      through cells of the maze, the elements of M that correspond to
      those cells will be 1, 2, 3, etc., and all other elements of M
      that correspond to cells of the maze will be Unvisited. The
      number of the last step in the path is move. */
      private static final int Unvisited = 0;
      private static int move;
   ...
 } /* MRP */
```

☞ **A representation invariant describes the value(s) of one or more program variables, and their relationships to one another as the program runs. The invariant is typically written as a comment associated with the declaration(s) of the relevant variable(s).**

Variables declared at the top-level of a class are called *class variables*, and are shared among all of the methods of the class.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   /* Maze. Cells of an N-by-N maze are represented by elements of a
      (2*N+1)-by-(2*N+1) array M. Maze cell ⟨r,c⟩ is represented by array
      element M[2*r+1][2*c+1]. The possible walls ⟨top, right, bottom,
      left⟩ of the maze cell corresponding to ⟨r,c⟩ are represented by
      Wall or NoWall in ⟨M[r-1][c], M[r][c+1], M[r+1][c], M[r][c-1]⟩.
      The remaining elements of M are unused. lo is 1, and hi is 2*N-1. */
      private static int N;       // Size of maze. */
      private static int M[][];   // Maze, walls, and path.
      private static final int Wall = -1;
      private static final int NoWall = 0;
      private static int lo, hi; // Left/top and right/bottom maze indices.
   ...
   } /* MRP */
```

☞ **A representation invariant describes the value(s) of one or more program variables, and their relationships to one another as the program runs. The invariant is typically written as a comment associated with the declaration(s) of the relevant variable(s).**

Auxiliary Data:

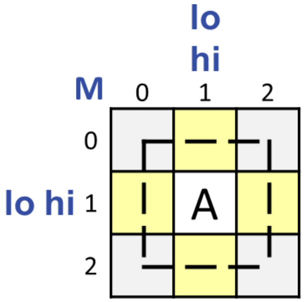
```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   // Unit vectors in direction d =          0,     1,    2,    3
   //                                       up, right, down, left
      private static final int deltaR[] = { -1,     0,    1,    0 };
      private static final int deltaC[] = {  0,     1,    0,   -1 };
   ...
   } /* MRP */
```

Operations: Complete the implementation

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Public Interface. */
   public static void TurnClockwise()
      { d = (d+1)%4; }
   public static void TurnCounterClockwise()
      { d = (d+3)%4; }
   public static voidStepForward()
      { r = r+2*deltaR[d]; c = c+2*deltaC[d]; move++; M[r][c] = move; }
   public static boolean isFacingWall()
      { return M[r+deltaR[d]][c+deltaC[d]]==Wall; }
   public static boolean isAtCheese()
      { return (r==hi)&&(c==hi); }
   public static boolean isAboutToRepeat()
      { return (r==lo)&&(c==lo)&&(d==3); }
   ...
   } /* MRP */
```

Interface includes I/O: Only MRP knows the data representation, so it must do the I/O.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Input N-by-N maze. */
   public static void Input() {
      } /* Input */
   /* Output N-by-N maze, with walls and path. */
   public static void PrintMaze() {
      } /* PrintMaze */
   } /* MRP */
```



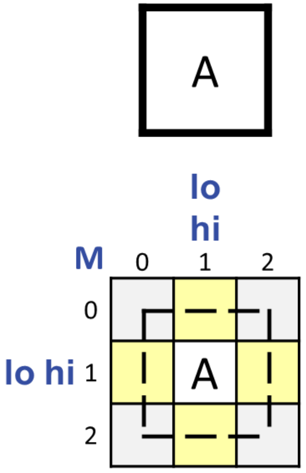
Input: Hard code a trivial initial example.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Input N-by-N maze. */
   public static void Input() {
      /* Maze. As per representation invariant. */
         N = 1;                     // Size of maze.
         lo = 1; hi = 2*N-1;        // First and last edges of maze.
         M = new int[2*N+1][2*N+1]; // Maze, walls, and path.
         M[0][1] = M[1][0] = M[1][2] = M[2][1] = Wall;
      /* Rat. Place rat in upper-left cell facing up. */
         r = lo; c = lo; d = 0;
      /* Path. Establish the rat in the upper-left cell. */
         move = 1; M[r][c] = move;
      } /* Input */
   } /* MRP */
```

Use the representation invariants as a guide in helping to establish correct values. Don't worry about trying to avoid needless assignments; it's better to be complete than to risk missing something.

Data Representation

Slight language extension: Multiple lefthand sides for assignment statement.



Input: Hard code a trivial initial example.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Input N-by-N maze. */
   public static void Input() {
      /* Maze. As per representation invariant. */
         N = 1;                         // Size of maze.
         lo = 1; hi = 2*N-1;            // First and last edges of maze.
         M = new int[2*N+1][2*N+1]; // Maze, walls, and path.
         M[0][1] = M[1][0] = M[1][2] = M[2][1] = Wall;
      /* Rat. Place rat in upper-left cell facing up. */
         r = lo; c = lo; d = 0;
      /* Path. Establish the rat in the upper-left cell. */
         move = 1; M[r][c] = move;
      } /* Input */
   } /* MRP */
```

Input: Invoke from the client.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Input a maze, or reject the input as malformed. */
   private static void Input() { MRP.Input(); } /* Input */
   ...
   } /* RunMaze */
```

Output: Straightforward, so knock it off, in general.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Output N-by-N maze, with walls and path. */
   public static void PrintMaze() {
      for (int r = lo-1; r<=hi+1; r++) {
         for (int c = lo-1; c<=hi+1; c++) {
            String s;
            if (M[r][c]==Wall) s = "#";
            else if (M[r][c]==NoWall || M[r][c]==Unvisited) s = " ";
            else s = M[r][c]+"";
            System.out.print((s+" ").substring(0,3));
            }
         System.out.println();
         }
      } /* PrintMaze */
   } /* MRP */
```

Output: Invoke from the client.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Output the direct path found, or “unreachable” if there is none. */
   private static void Output() {
      if (!MRP.isAtCheese()) System.out.println("Unreachable");
      else MRP.PrintMaze();
      } /* Output */
   ...
   } /* RunMaze */
```

Commentary : Design rules for abstract data types.

- Prefer fine-grained micro-operations over coarse-grained macro-operations.
  - E.g., `TurnClockwise` rather than `Pirouette`.
- It is better to support operations that are defined relative to the state than it is to reveal portions of the state itself. Avoid leaking details of any particular data representation.
  - E.g., `isAtCheese` rather than `getRow` and `getColumn`.
  - E.g., `TurnClockwise` rather than `getDirection` and `SetDirection`.
- Avoid macro-operations that embody algorithmic details that belong in the client.
  - E.g., `RunMaze.Solve` rather than `MRP.Solve` .

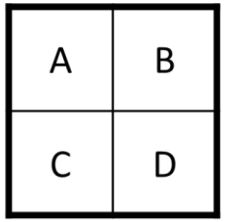
Controlled Testing: At first, use an empty stub for `Solve`.

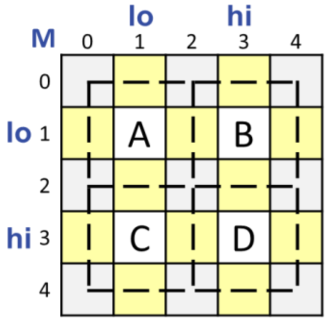
Test 1: Check for syntax errors, and check input/output framework.

	# # 1 # #
input	**output**

Correct output.

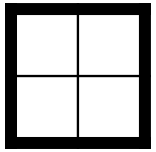
☞ **Test programs incrementally.**





Controlled Testing: Change `input` to hard-code a 2-by-2 maze, but still use an empty stub for `solve`.

Test 2: Check `output`.

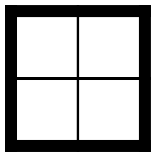
	Unreachable
input	**output**

Expected output since `solve` is just a stub. Validation of code for message.

☞ **Test programs incrementally.**

Controlled Testing: Now use real code for `Solve`.

Test 3: Further check of `Output`, and check of `Solve` for an empty 2-by-2 maze.



```
   #   #
#  1   2  #

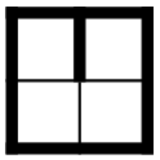
#      3  #
   #   #
```

input	output

Correct solution.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with an obstacle.

Test 4: Further check of `Solve`.



input

```
  #   #
# 1     #

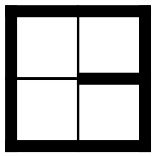
# 2   3 #
  #   #
```

output

Correct solution. Appears to be going counter-clockwise, but this is an illusion: It is making its way around the obstacle clockwise when it stumbles into the cheese.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

Test 5: Further check of `Solve`.



```
   #   #
#  3   2  #

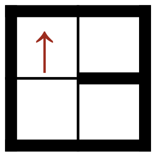
#  4   5  #
   #   #
```

input	output

Anticipated incorrect solution. We are doing a complete exploration, and don't bother to detect the cul-de-sac. As a result, we overwrite the path, and leave a mess.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

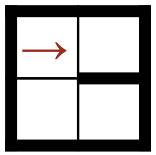
Test 5: Further check of `Solve`.

	# # # 1 # # # # #
input	**output**

Replay.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

Test 5: Further check of `Solve`.

<table>
<tr><td></td><td> # #
1

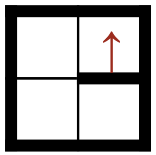
#   #
  # #</td></tr>
<tr><td>input</td><td>output</td></tr>
</table>

Replay.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

Test 5: Further check of `Solve`.

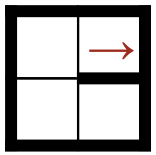
input	output

	`# #`
	 `# 1 2 #`
	 `# #`
	 `# #`

Replay.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

Test 5: Further check of `Solve`.

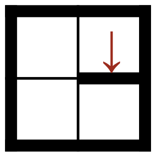
	`  #   #  `
	`# 1   2 #`
	
	`#       #`
	`  #   #  `

input	**output**

Replay.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

Test 5: Further check of `Solve`.



```
  #  #
# 1  2 #

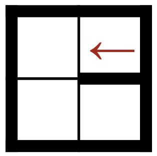
#      #
  #  #
```

input **output**

Replay.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

Test 5: Further check of `Solve`.



```
  #  #
# 1  2 #

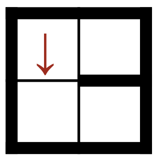
#      #
  #  #
```

input	**output**

Replay. This is the moment when we need to detect the imminent re-entry to a cell that is currently on the path.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

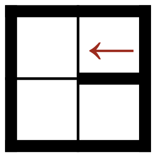
Test 5: Further check of `Solve`.

<table>
<tr><td></td><td>#   #
3   2
#   #
#</td></tr>
<tr><td>input</td><td>output</td></tr>
</table>

We ignored the issue, and overwrote the 1 with a 3.

Controlled Testing: Change `Input` to hard-code a 2-by-2, with a cul-de-sac.

Test 5: Further check of `Solve`.

<table>
<tr><td></td><td><pre>
  #   #
1   2

#   #
  #   #
</pre></td></tr>
<tr><td>input</td><td>output</td></tr>
</table>

Backing up, we need to prevent this.

Algorithm (Revised)

Algorithm: Proceed only if about to enter a cell that is not on the current path.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Compute a direct path through the maze, if one exists. */
   private static void Solve() {
      while ( !MRP.isAtCheese() && !MRP.isAboutToRepeat() )
         if ( MRP.isFacingWall() ) MRP.TurnClockwise();
         else if ( MRP.isFacingUnvisited() ) {
            MRP.StepForward();
            MRP.TurnCounterClockwise();
            }
         else Retract();
      } /* Solve */
   ...
   } /* RunMaze */
```

Add the check ...

... and introduce Retract to handle the cul-de-sac case.

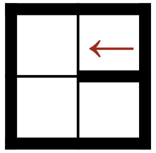
Algorithm (Revised)

Extend MRP: Add `isFacingUnvisited` to interface.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   public static boolean isFacingUnvisited()
      { return M[r+2*deltaR[d]][c+2*deltaC[d]] == Unvisited; }
   ...
 } /* MRP */
```

Algorithm (Revised)

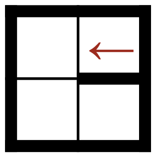
Retract:

	# # # 1 2 # # # # #
input	**output**

The next step from here needed to detect the imminent re-entry to a cell that is currently on the path, but didn't bother.

Algorithm (Revised)

Retract:

input	output
	see grid below

```
   #   #
#  1   2  #

#         #
   #   #
```

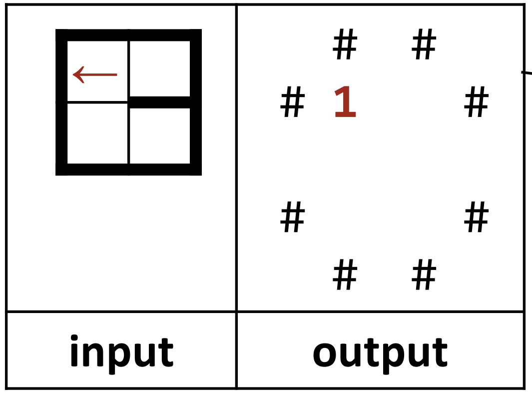
The next step from here needed to detect the imminent re-entry to a cell that is currently on the path, but didn't bother.

Need to undo the `StepForward` that took us into the cul-de-sac.

```
public static StepForward()
  { r = r+2*deltaR[d]; c = c+2*deltaC[d]; move++; M[r][c] = move; }
```

Algorithm (Revised)

Retract:



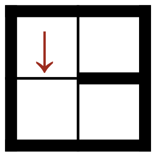
The next step from here needed to detect the imminent re-entry to a cell that is currently on the path, but didn't bother.

Need to undo the `StepForward` that took us into the cul-de-sac.

```
public static StepForward()
  { r = r+2*deltaR[d]; c = c+2*deltaC[d]; move++; M[r][c] = move; }
public static void StepBackward()
  { M[r][c] = Unvisited; move--; r = r+2*deltaR[d]; c = c+2*deltaC[d]; }
```

Algorithm (Revised)

Retract:



```
   #  #
#  1     #


#        #
   #  #
```

input	output

The next step from here needed to detect the imminent re-entry to a cell that is currently on the path, but didn't bother.

Need to undo the `StepForward` that took us into the cul-de-sac, and turn as if it had been skipped.

```
public static StepForward()
  { r = r+2*deltaR[d]; c = c+2*deltaC[d]; move++; M[r][c] = move; }
public static void StepBackward()
  { M[r][c] = Unvisited; move--; r = r+2*deltaR[d]; c = c+2*deltaC[d]; }
```

Algorithm (Revised)

Retract: Implemented as follows.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   private static void Retract () {
      MRP.StepBackward();
      MRP.TurnCounterClockwise();
      } /* Retract */
   ...
   } /* RunMaze */
```

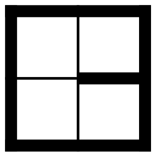
Retract: Implemented as follows.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   private static void Retract () {
      MRP.StepBackward();
      MRP.TurnCounterClockwise();
      } /* Retract */
   ...
   } /* RunMaze */
```

Marker: You have just been deliberately led astray, but we will keep going.

Algorithm (Revised)

Test 6: Redo Test 5.



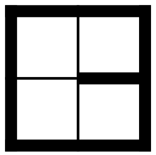
```
   #   #
#  1       #

#  2   3   #
   #   #
```

input	**output**

Correct solution. We backed out of the cul-de-sac, and proceeded to the lower-right cell.

Test 6: Redo Test 5.

<table>
<tr><td></td><td>   #   #
1

2   3
   #   #</td></tr>
<tr><td>input</td><td>output</td></tr>
</table>

Correct solution. We backed out of the cul-de-sac, and proceeded to the lower-right cell.

Could we be done? Perhaps, but we will need to test on bigger mazes. It's time to code the general-purpose `Input` method.

Input: Start with the hardcoded initial example.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Input N-by-N maze. */
   public static void Input() {
      /* Maze. As per representation invariant. */
         N = 1;                          // Size of maze.
         lo= 1; hi = 2*N-1;              // First and last indices of maze.
         M = new int[2*N+1][2*N+1]; // Maze, walls, and path.
         M[0][1] = M[1][0] = M[1][2] = M[2][1] = Wall;
      /* Rat. Place rat in upper-left cell facing up. */
         r = lo; c = lo; d = 0;
      /* Path. Establish the rat in the upper-left cell. */
         move = 1; M[r][c] = move;
      } /* Input */
   } /* MRP */
```

Input: Identify places to generalize.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Input N, and (2N+1)-by-(2N+1) values; non-blanks are walls. */
   public static void Input() {
      /* Maze. */
         Scanner in = new Scanner(System.in);
         N = ⟨value for N⟩;
         M = new int[2*N+1][2*N+1]; // Maze, walls, and path.
         ⟨Define each element of M⟩
      /* Rat. */
         r = lo; c = lo; d = 0;
      /* Path. */
         move = 1; M[r][c] = move;
      } /* Input */
   } /* MRP */
```

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Input N, and (2N+1)-by-(2N+1) values; non-blanks are walls. */
   public static void Input() {
      /* Maze. */
☞        Scanner in = new Scanner(System.in);
         N = in.nextInt(); in.nextLine();
         lo = 1; hi = 2*N-1;         // Left and right edges of maze.
         M = new int[2*N+1][2*N+1]; // Maze, walls, and path.
☞        for (int r=lo-1; r<=hi+1; r++) {
            String line = in.nextLine();
            for (int c=lo-1; c<=hi+1; c++)
               if ((r%2==1) && (c%2==1)) M[r][c] = Unvisited;
               else if (line.substring(c,c+1).equals(" "))
                  M[r][c] = NoWall;
               else M[r][c] = Wall;
            }
      /* Rat. */
         r = lo; c = lo; d = 0;
      /* Path. */
         move = 1; M[r][c] = move;
      } /* Input */
   } /* MRP */
```

Fussy detail that we shall skip over.

Input

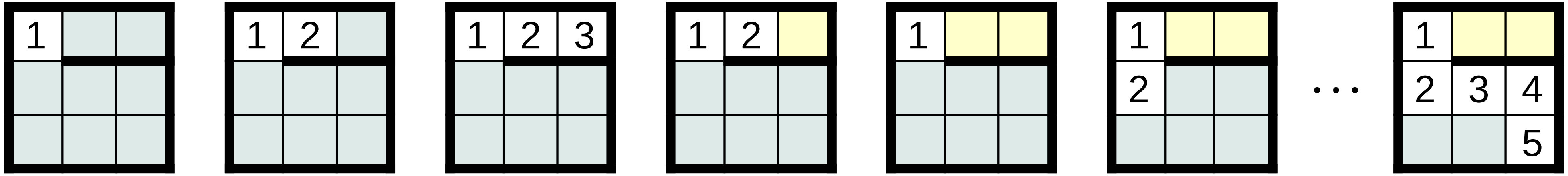
Input

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Input N, and (2N+1)-by-(2N+1) values; non-blanks are walls. */
   public static void Input() {
      /* Maze. */
         Scanner in = new Scanner("2\nxxxxx\nx x\nx x\nx x\nxxxxx\n");
         N = in.nextInt(); in.nextLine();
         lo = 1; hi = 2*N-1;          // Left and right edges of maze.
         M = new int[2*N+1][2*N+1]; // Maze, walls, and path.
         for (int r=lo-1; r<=hi+1; r++) {
            String line = in.nextLine();
            for (int c=lo-1; c<=hi+1; c++)
               if ((r%2==1) && (c%2==1)) M[r][c] = Unvisited;
               else if (line.substring(c,c+1).equals(" "))
                  M[r][c] = NoWall;
               else M[r][c] = Wall;
            }
      /* Rat. */
         r = lo; c = lo; d = 0;
      /* Path. */
         move = 1; M[r][c] = move;
      } /* Input */
   } /* MRP */
```

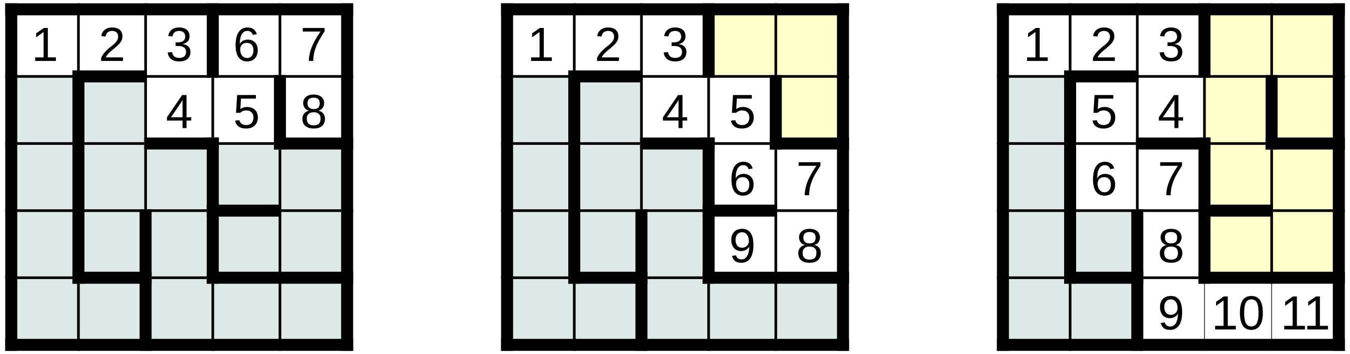
Hard code input as a string constant for easy retesting.

Controlled Testing: Try every sort of maze you can think of.

Deeper cul-de-sacs



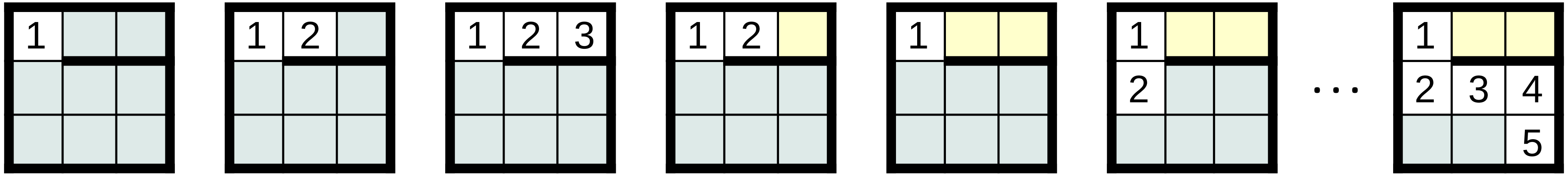
Higgledy-piggledy cul-de-sacs



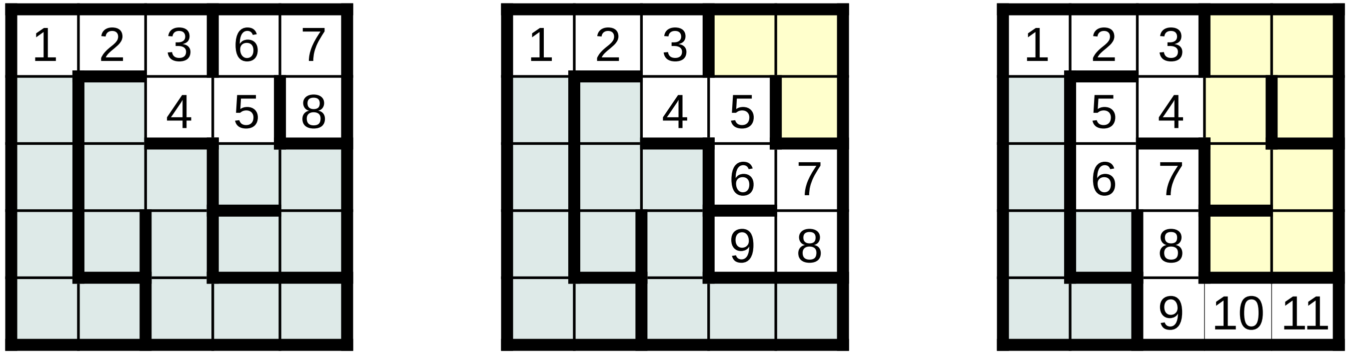
☞ **Test programs thoroughly.**

Controlled Testing: But how can you know when you are done?

Deeper cul-de-sacs



Higgledy-piggledy cul-de-sacs



☞ **Beware of premature self-satisfaction.**

Testing (Revisited)

Controlled Testing: But how can you know when you are done?

Review Code:

- You were supposed to be very systematic, but did you consider every case?

Review Test data:

- You were supposed to be very systematic, but did you consider every case?

☞ **Test programs thoroughly.**

Testing (Revisited)

Controlled Testing: But how can you know when you are done?

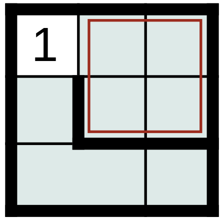
Review Code:

- You were supposed to be very systematic, but did you consider every case?

Review Test data:

- You were supposed to be very systematic, but did you consider every case?

Do you have to just keep trying until you think of a room-shaped cul-de-sac?



. . .

☞ **Test programs thoroughly.**

Controlled Testing: But how can you know when you are done?

Review Code:

- You were supposed to be very systematic, but did you consider every case?

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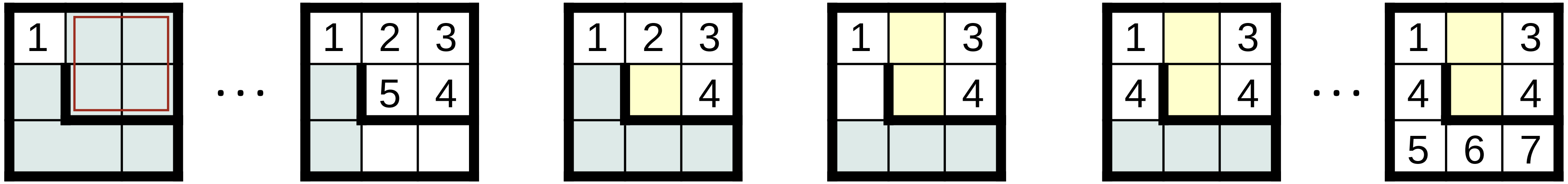
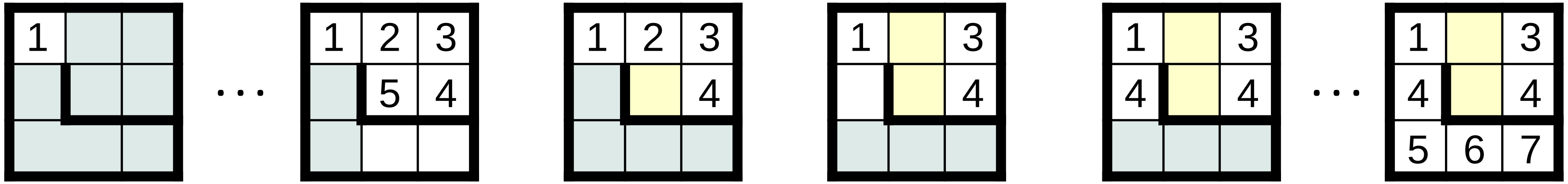
Do you have to just keep trying until you think of a room-shaped cul-de-sac?



Aargh! We only considered corridor-shaped cul-de-sacs.

☞ **Test programs thoroughly.**

Algorithm (Revised)



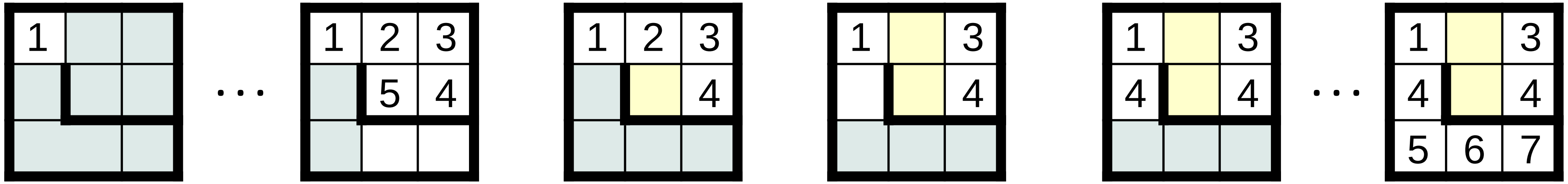
Retract: Recall that the implementation was as follows.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   private static void Retract () {
      MRP.StepBackward();
      MRP.TurnCounterClockwise();
      } /* Retract */
   ...
   } /* RunMaze */
```

Recall: We deliberately led you astray, but we kept going.

This didn't *unwind* the traversal of the cul-de-sac; it only undid the first step *into* the cul-de-sac. This worked fine even for deep corridor-shaped cul-de-sacs (which could be backed out of one "first-step" at a time).

Algorithm (Revised)

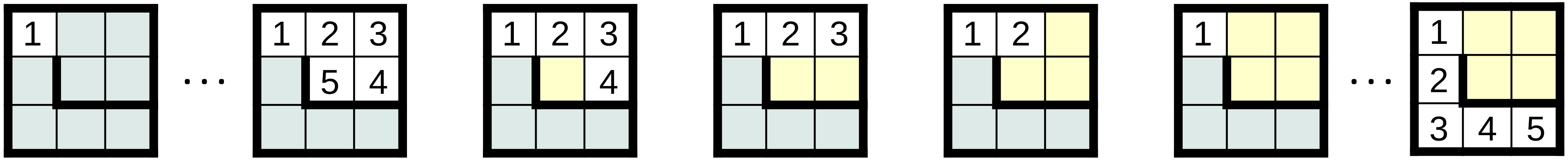


Retract: To be implemented now as follows.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   private static void Retract () {
      while ( /* not unwound */ ) {
         MRP.FacePrevious();
         MRP.StepBackward();
         }
      TurnCounterClockwise();
      } /* Retract */
   } /* RunMaze */
```

Picking up the “bread crumbs”.

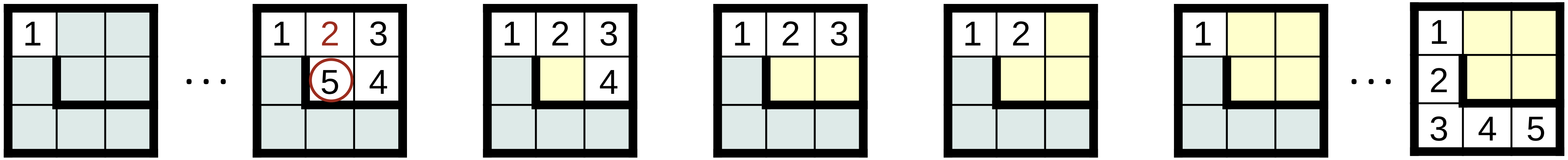
Correction: Now we will truly unwind the path.



Algorithm (Revised)

Retract is coded in class MRP in order to have direct access to the data representation.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]]
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```



Algorithm (Revised)

```
/* Maze, Rat, and Path (MRP) Representat
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]]
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         FacePrevious();
         StepBackward();
         }
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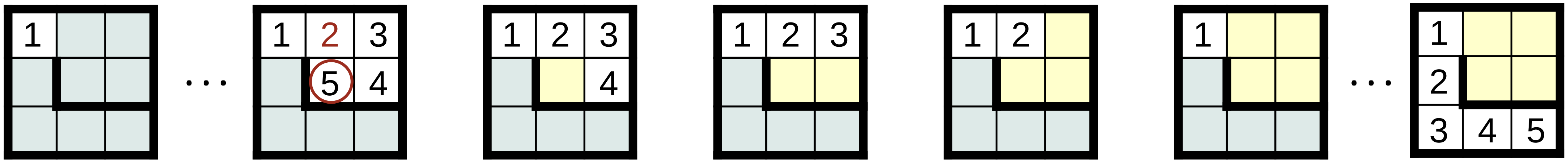
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```

We record the identity of the about-to-be-revisited neighbor



Algorithm (Revised)

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class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
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      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```
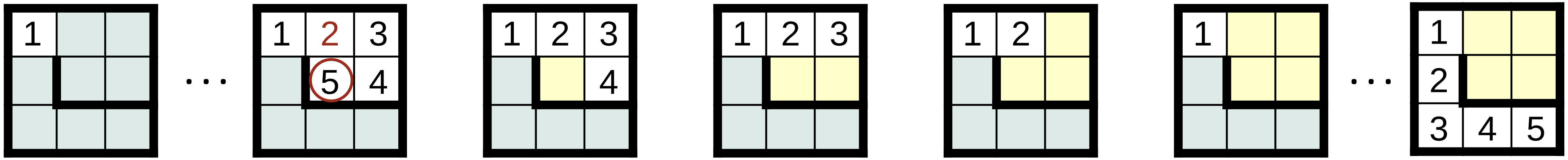

We record the identity of the about-to-be-revisited neighbor, and the direction we were facing when we detected it.



Algorithm (Revised)

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class MRP {
   ...
   /* Unwind abortive exploration. */
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      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
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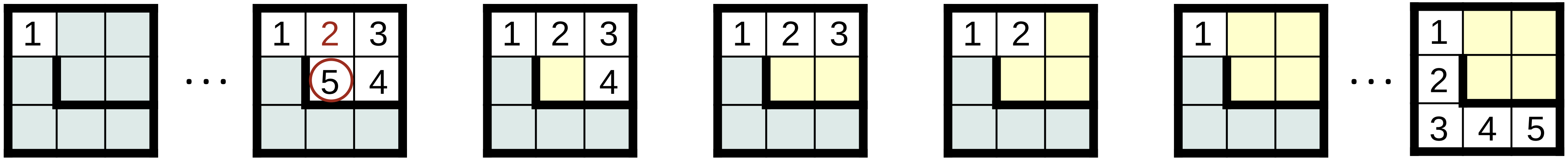
   } /* MRP */
```

We record the identity of the about-to-be-revisited neighbor, and the direction we were facing when we detected it. We stop unwinding when we get to it



Algorithm (Revised)

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         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```
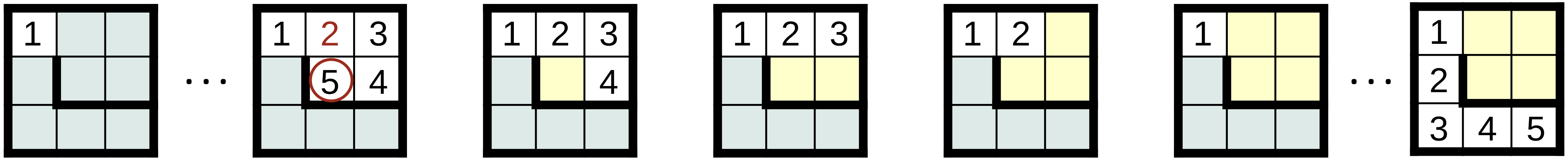

We record the identity of the about-to-be-revisited neighbor, and the direction we were facing when we detected it. We stop unwinding when we get to it, and restore the direction in which we were facing.



Algorithm (Revised)

Trace: There are actually two separate cul-de-sacs: one detected from 5 (facing 2), and the other from 2 (facing 1).

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]]
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
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         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```

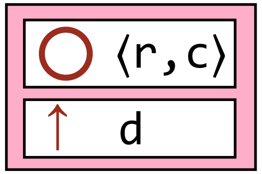

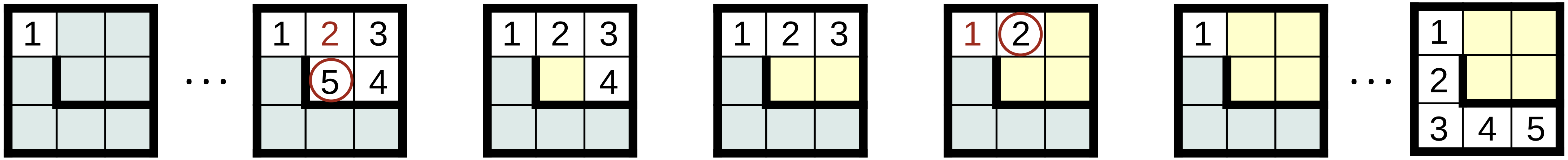

```
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   ...
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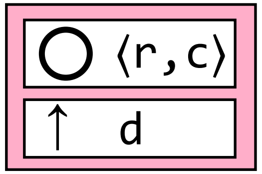

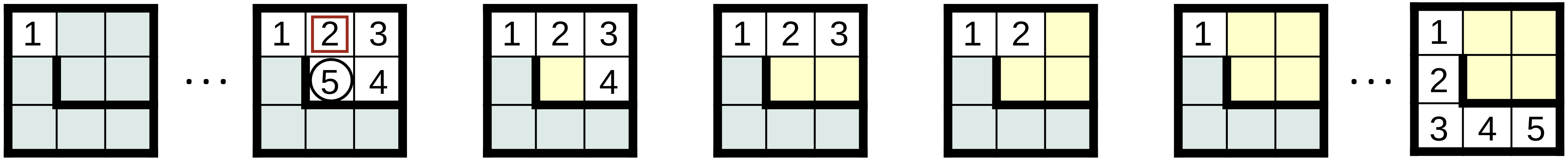

2

Algorithm (Revised)

```
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class MRP {
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         FacePrevious();
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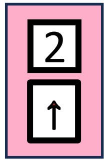
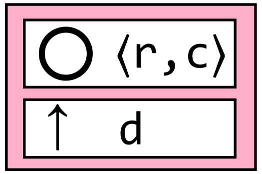
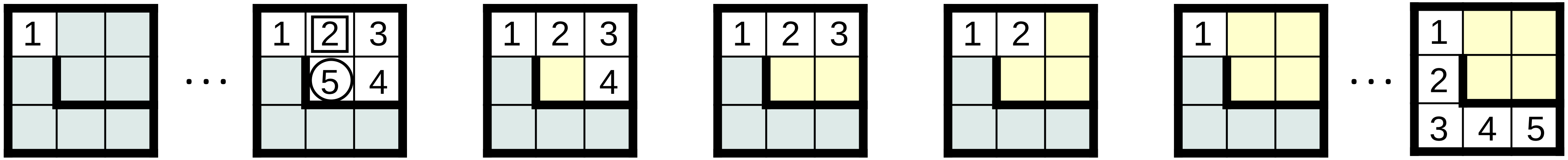
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```







Algorithm (Revised)

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class MRP {
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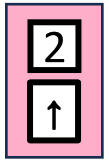

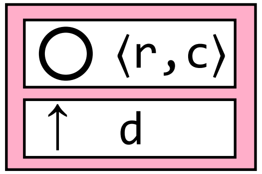

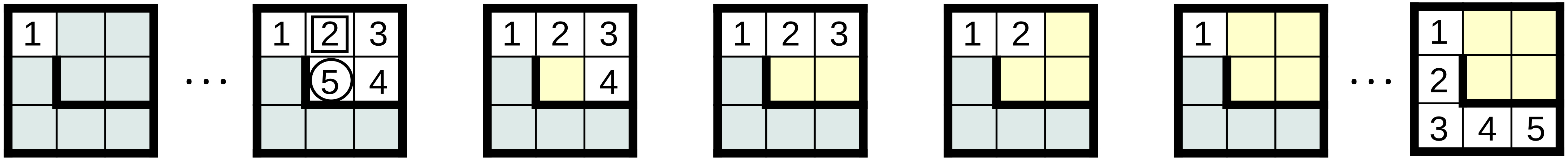

Algorithm (Revised)

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☞        FacePrevious();
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   ...
   } /* MRP */
```
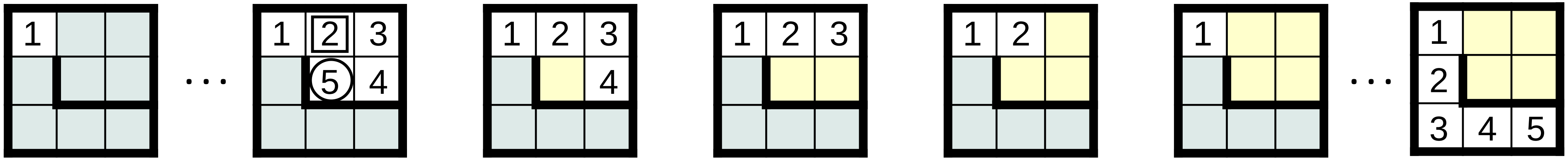

2
↑

○ ⟨r,c⟩
→ d



Algorithm (Revised)

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   ...
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```

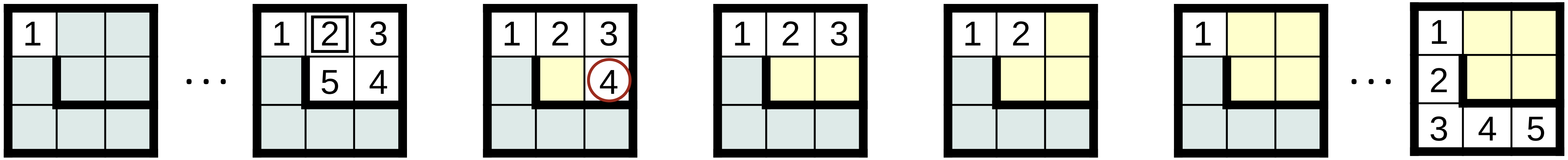
⟨r,c⟩

→ d

2

↑

time →



Algorithm (Revised)

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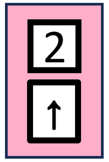

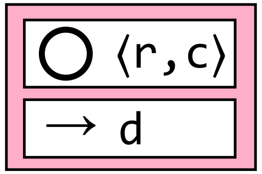

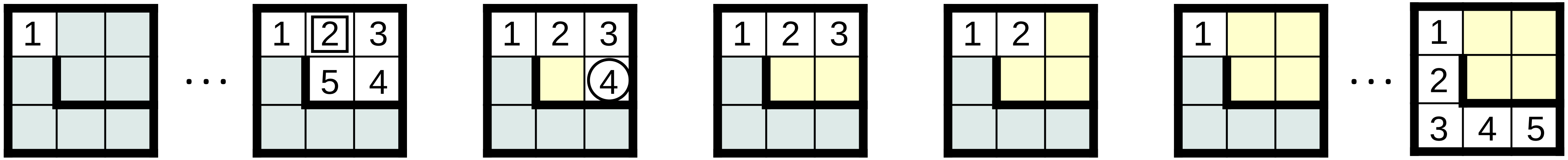

Algorithm (Revised)

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/* Maze, Rat, and Path (MRP) Representations. */
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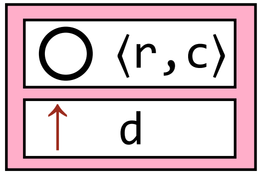

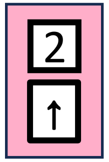

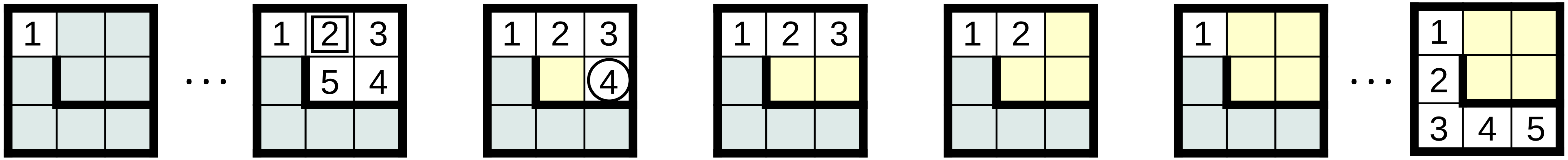

Algorithm (Revised)

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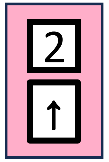

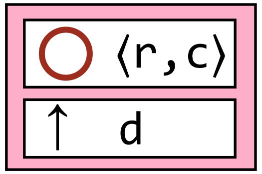

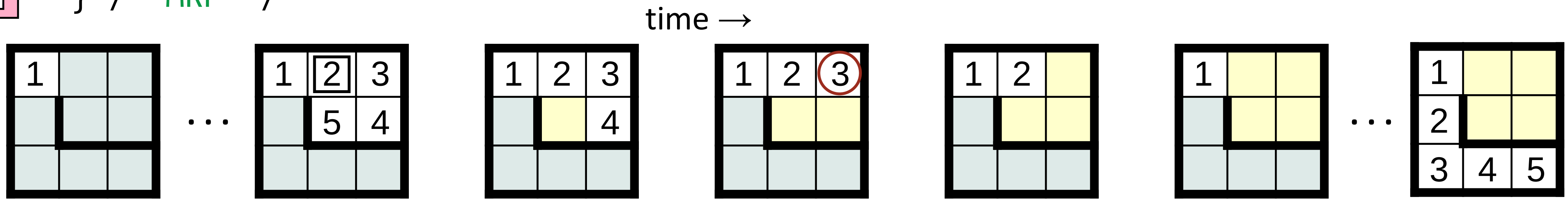

Algorithm (Revised)

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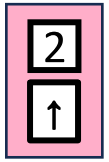

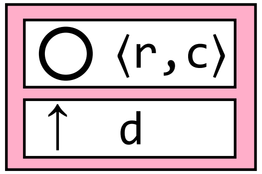

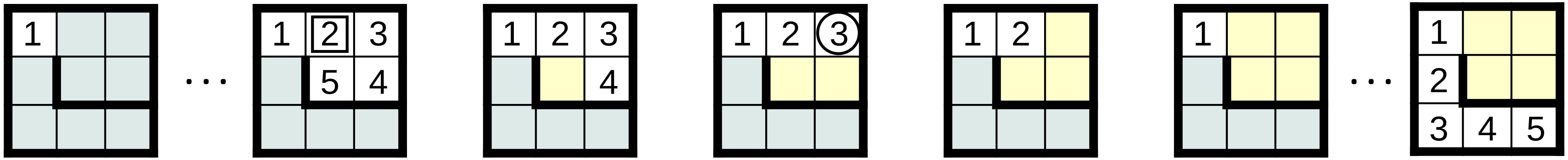

Algorithm (Revised)

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```

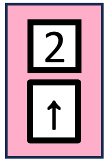

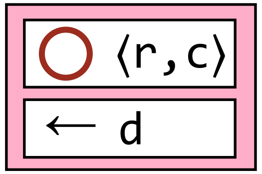

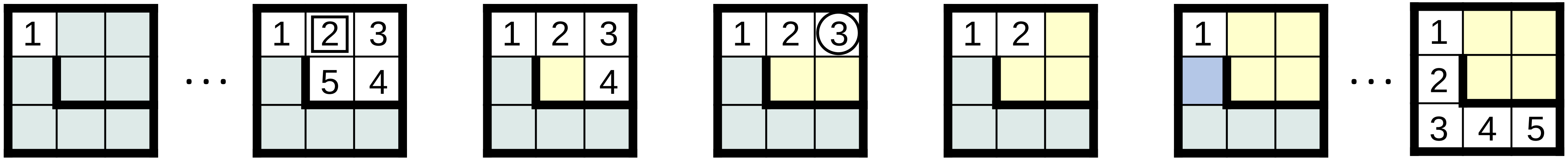

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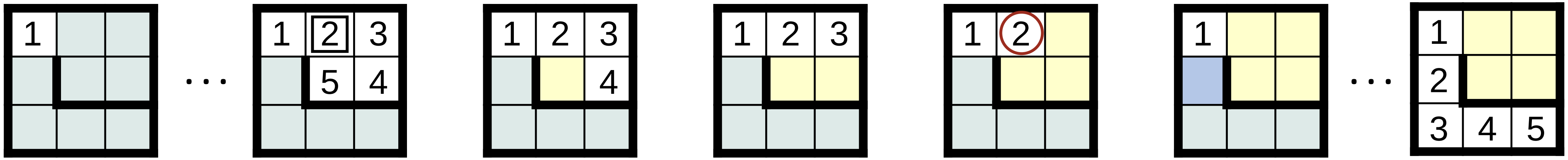
   ...
   } /* MRP */
```

2
↑

○ ⟨r,c⟩
← d

time →



Algorithm (Revised)

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      } /* Retract */
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```
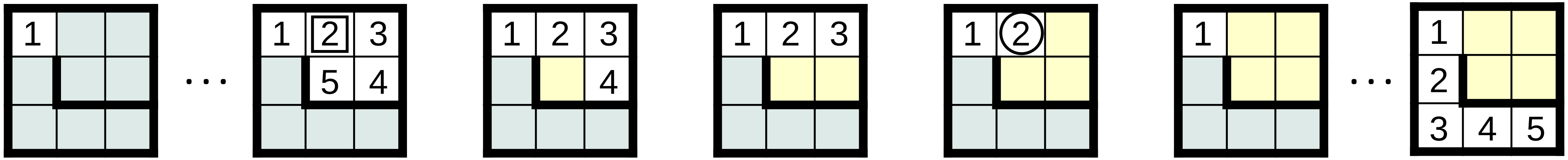

2

↑

○ ⟨r,c⟩

← d



Algorithm (Revised)

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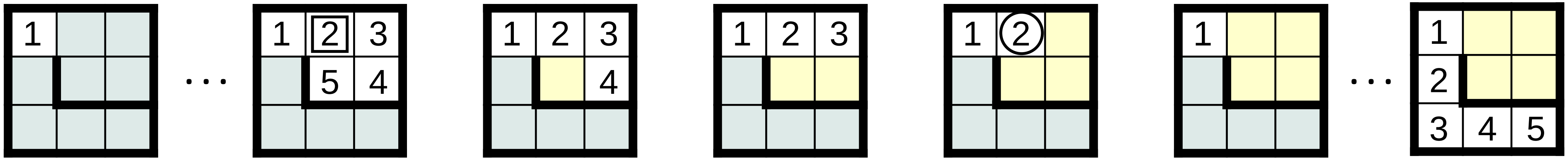
```

○ ⟨r,c⟩

↑ d

2

↑



Algorithm (Revised)

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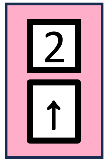

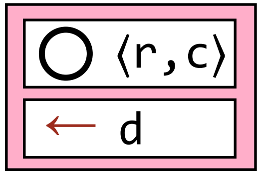

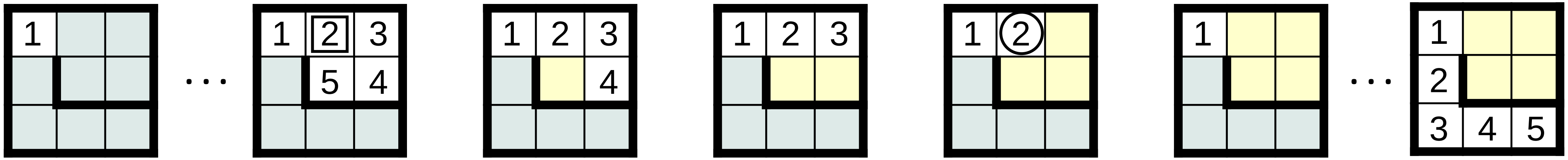

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      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```
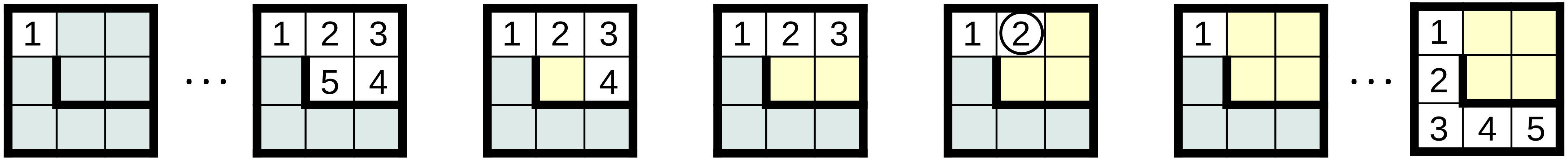

○ ⟨r,c⟩

← d



Second call to retract.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]]
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
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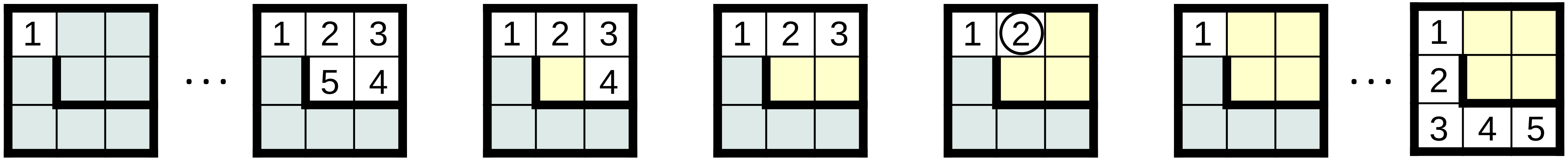
⟨r,c⟩

← d



Algorithm (Revised)

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]];
      int neighborDirection = d;  // Save direction.
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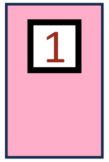

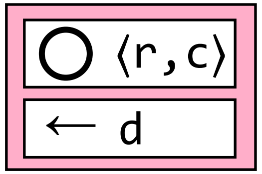

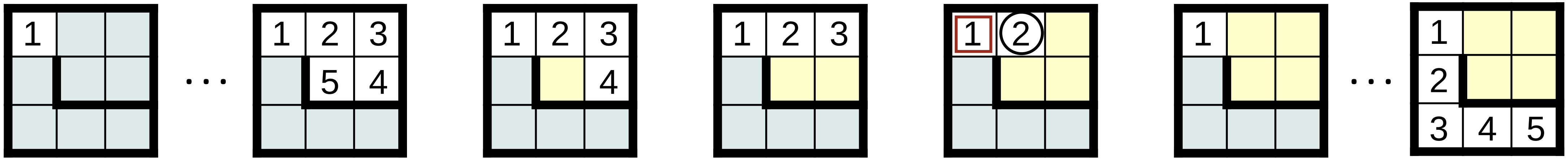

Algorithm (Revised)

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/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
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      TurnCounterClockwise();
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   ...
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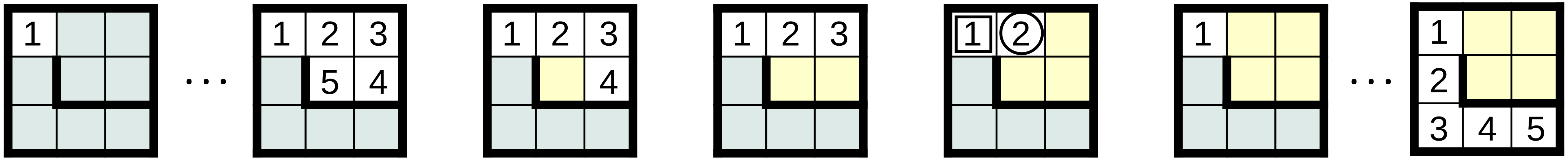
○ ⟨r,c⟩

← d



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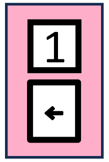
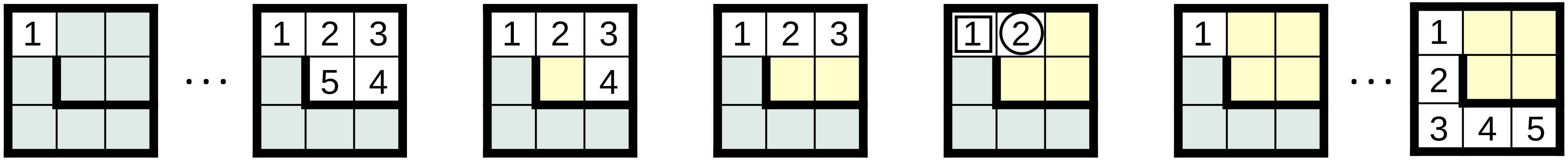


○ ⟨r,c⟩

← d



Algorithm (Revised)

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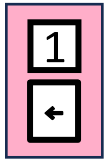

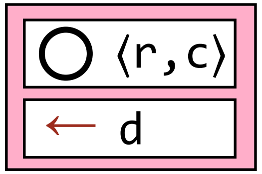

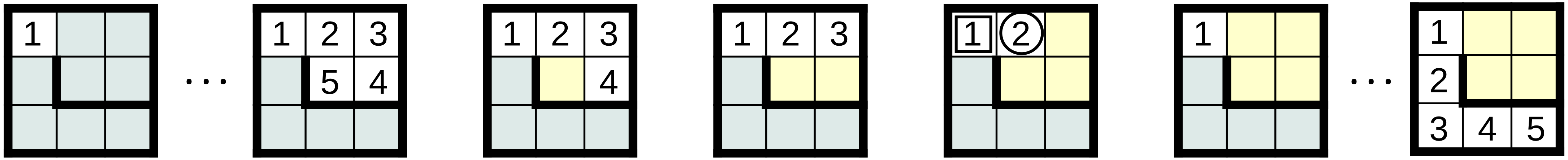

Algorithm (Revised)

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         }
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      TurnCounterClockwise();
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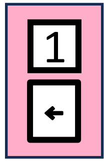

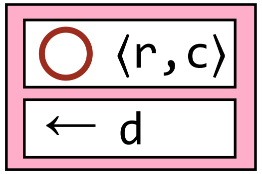

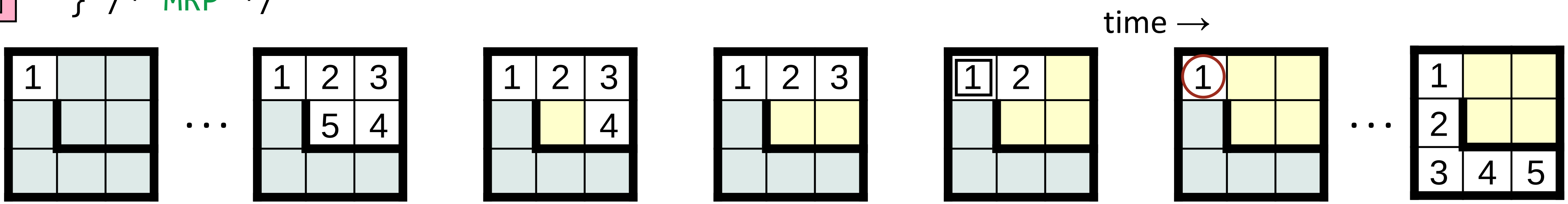

Algorithm (Revised)

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         }
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      TurnCounterClockwise();
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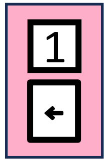

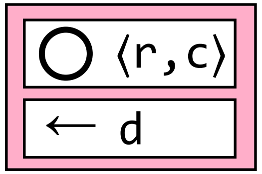

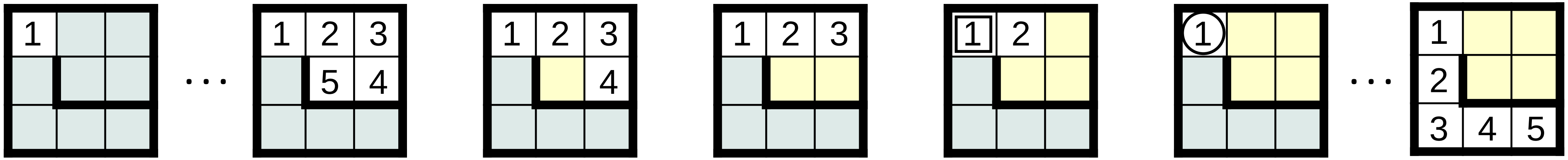

Algorithm (Revised)

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
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      TurnCounterClockwise();
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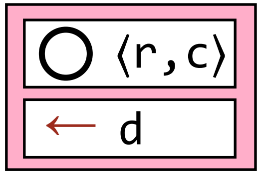
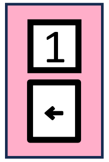
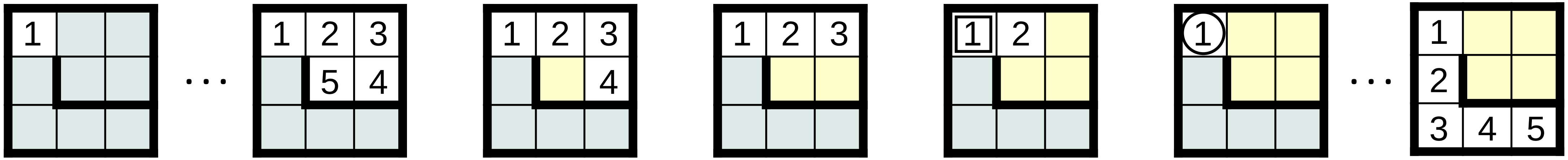
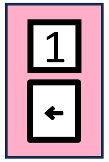


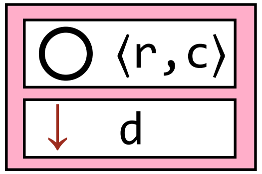


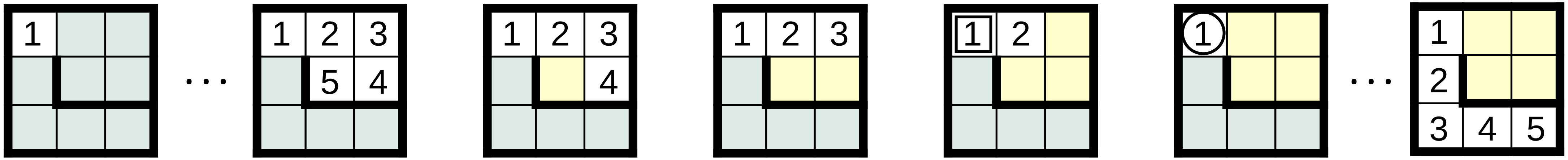


Algorithm (Revised)

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]];
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```

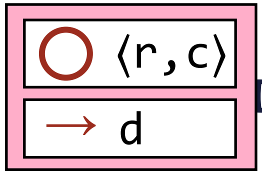




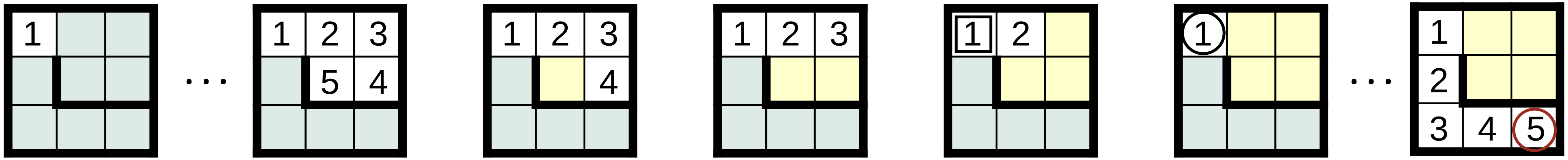


Algorithm (Revised)

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]];
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```



time →



Recall that Retract was coded in class MRP in order to have direct access to the data representation.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]]
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* MRP */
```

Algorithm (Revised)

Algorithm (Revised)

Recall that Retract was coded in class MRP in order to have direct access to the data representation. But it really is too algorithmic for MRP, and more properly belongs in RunMaze.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]]
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* RunMaze */
```

Recall that Retract was coded in class MRP in order to have direct access to the data representation.
But it really is too algorithmic for MRP, and more properly belongs in RunMaze.
But then it wouldn't have access to the data representation, which is private to MRP.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      int neighborNumber    = M[r+2*deltaR[d]][c+2*deltaC[d]]
      int neighborDirection = d;  // Save direction.
      while ( M[r][c] != neighborNumber ) {
         FacePrevious();
         StepBackward();
         }
      d = neighborDirection;     // Restore direction.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* RunMaze */
```

Algorithm (Revised)

Algorithm (Revised)

Recall that `Retract` was coded in class MRP in order to have direct access to the data representation.
But it really is too algorithmic for MRP, and more properly belongs in RunMaze.
But then it wouldn't have access to the data representation, which is *protected* in MRP.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      MRP.RecordNeighborAndDirection();
      while ( !MRP.isAtNeighbor() ) {
         MRP.FacePrevious();
         MRP.StepBackward();
         }
      MRP.RestoreDirection();.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* RunMaze */
```

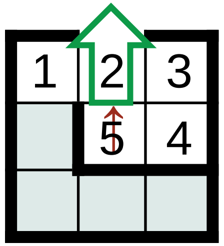
The solution is for MRP to encapsulate the needed code as an extension of its services:

- `RecordNeighborAndDirection`
- `isAtNeighbor`
- `RestoreDirection`

Algorithm (Revised)

First call to retract.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
   public static void Retract() {
      MRP.RecordNeighborAndDirection();
      while ( !MRP.isAtNeighbor() ) {
         MRP.FacePrevious();
         MRP.StepBackward();
         }
      MRP.RestoreDirection();.
      TurnCounterClockwise();
      } /* Retract */
   ...
   } /* RunMaze */
```



The solution is for MRP to encapsulate the needed code as an extension of its services:

- `RecordNeighborAndDirection`
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The MRP operations (colloquially) are:

- "Toss an arrow into a neighbor"

Algorithm (Revised)

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
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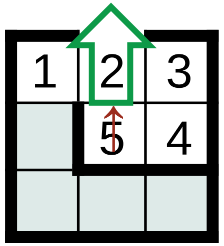

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- RestoreDirection

The MRP operations (colloquially) are:

- "Toss an arrow into a neighbor"
- "Detect being in that neighbor"

Algorithm (Revised)

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
   ...
   /* Unwind abortive exploration. */
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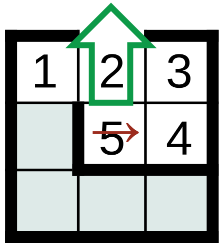

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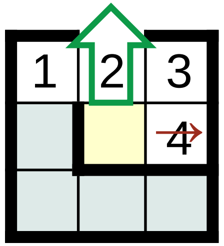

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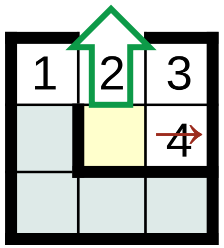

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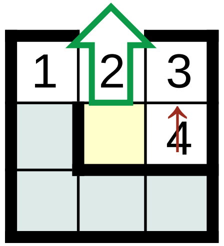

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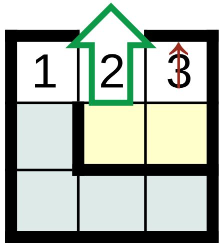

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- `isAtNeighbor`
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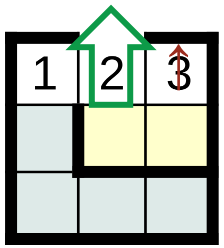

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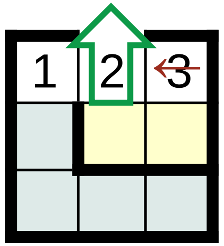

The solution is for MRP to encapsulate the needed code as an extension of its services:

- RecordNeighborAndDirection
- isAtNeighbor
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The MRP operations (colloquially) are:

- "Toss an arrow into a neighbor"
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Algorithm (Revised)

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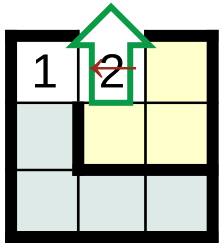

Algorithm (Revised)

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   ...
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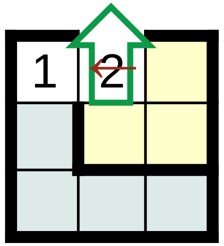

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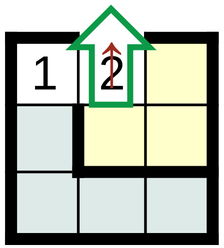

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Algorithm (Revised)

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/* Run a rat through an arbitrary maze.  */
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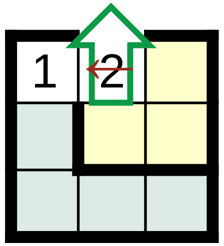

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Algorithm (Revised)

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
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   ...
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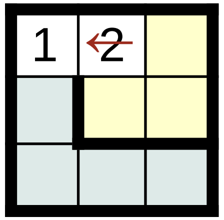

Algorithm (Revised)

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   ...
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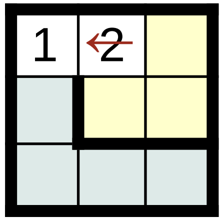

Algorithm (Revised)

```
/* Run a rat through an arbitrary maze.  */
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   ...
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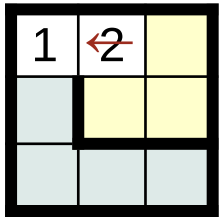

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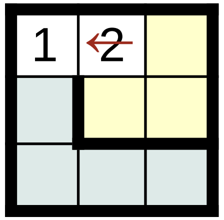

Algorithm (Revised)

Second call to retract.

```
/* Run a rat through an arbitrary maze.   */
class RunMaze {
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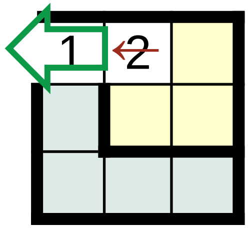
```

The MRP operations (colloquially) are:

- “Toss an arrow into a neighbor”



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The MRP operations (colloquially) are:

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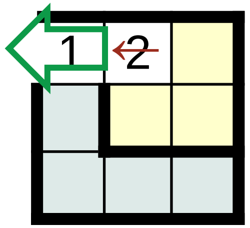

Algorithm (Revised)

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```
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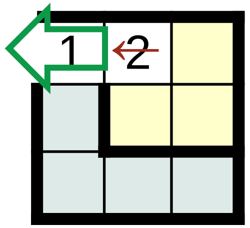

No change

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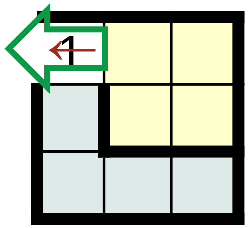

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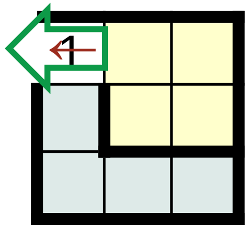

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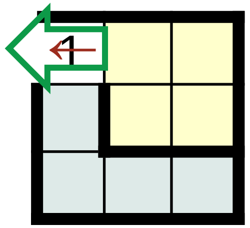

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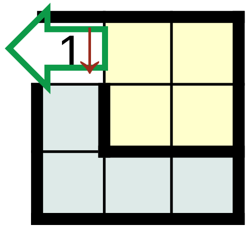

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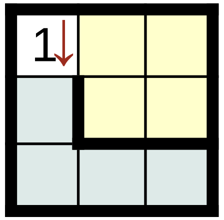

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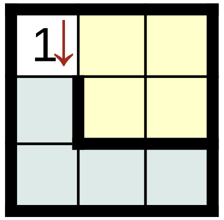

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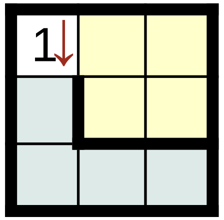

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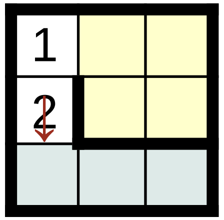

We’re on our way.

Algorithm (Revised)

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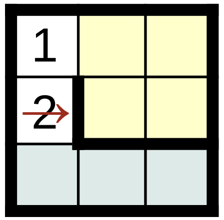


We’re on our way.

Algorithm (Revised)

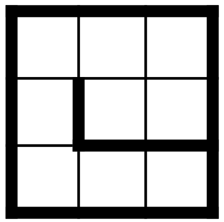
State variables of MRP supporting the notion of an “arrow in a cell”.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Recorded state. */
   private static int neighborNumber;    // Visit number of cell into which
                                         // the arrow was tossed.
   private static int neighborDirection; // Direction when the arrow was tossed.

   /* Toss an arrow into the neighboring cell in the direction faced. */
   public static void RecordNeighborAndDirection ()
     { neighborNumber = M[r+2*deltaR[d]][c+2*deltaC[d]]; neighborDirection = d; }

   /* Detect being in the same cell as the arrow. */
   public static boolean isAtNeighbor()
      { return M[r][c]==neighborNumber; }

   /* Align direction with the arrow. */
   public static void RestoreDirection()
      { d = neighborDirection; }
   ...
   } /* MRP */
```

Remaining Implementation: FacePrevious, just a Sequential Search.

```
/* Run a rat through an arbitrary maze.  */
class MRP {
   ...
   public static void FacePrevious() {
      int d = 0;
      while ( isFacingWall() || M[r][c]-1 != M[r+2*deltaR[d]][c+2*deltaC[d]] )
         d++;
      } /* FacePrevious */
   ...
   } /* MRP */
```

Test 7:

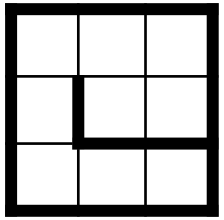


```
# # # # # # #
# 1         #
#   #       #
# 2 #       #
#   # # # # #
# 3   4   5 #
# # # # # # #
```

input	output

Testing (Revisited)

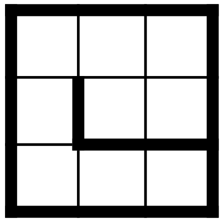
Test 7:



```
#  #  #  #  #  #  #
#  1              #
#     #           #
#  2  #           #
#     #  #  #  #  #
#  3     4     5  #
#  #  #  #  #  #  #
```

input	output

Test 7:

<table>
<tr><td></td><td><pre>
#   #   #   #   #   #   #
1
#   #   #
2   #
#   #   #   #   #   #
3   4   5
#   #   #   #   #   #   #
</pre></td></tr>
<tr><td>input</td><td>output</td></tr>
</table>

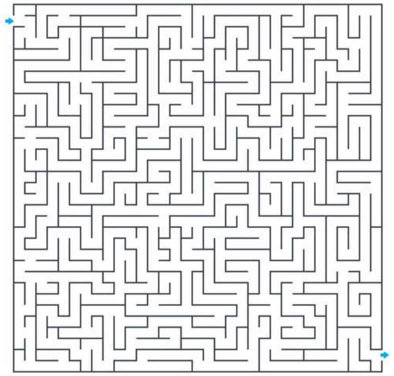
But how can we know there isn’t yet another lingering bug?

“Program testing can be used to show the presence of bugs, but never to show their absence!”

— Edsger W. Dijkstra

Self-checking: The setting.

It is often easier to automatically check the correctness of a problem solution than it is to find the solution in the first place.



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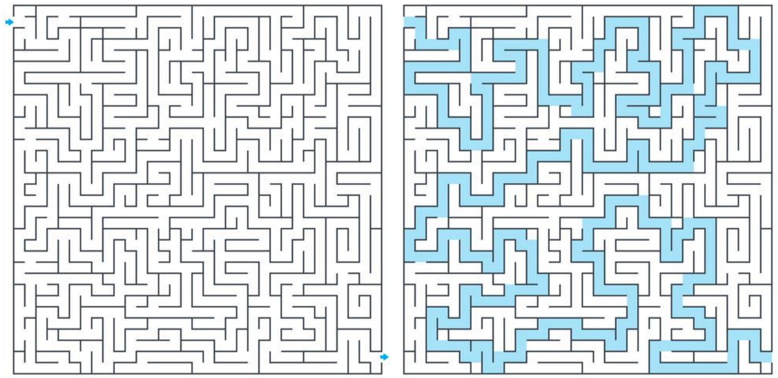

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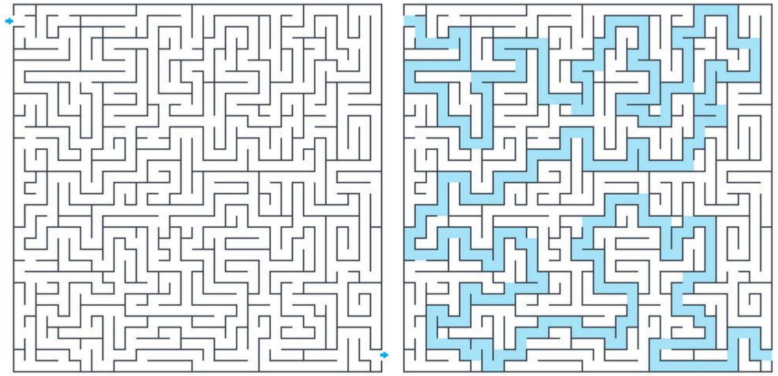
It is often easier to automatically check the correctness of a problem solution than it is to find the solution in the first place.

Running a Maze can be viewed as a search problem that either succeeds (by finding a path), or that announces “unreachable”.

Checking the answer “unreachable” is no easier than the original problem because it involves discovering a path that contradicts the unreachability claim.



Self-checking: The setting.

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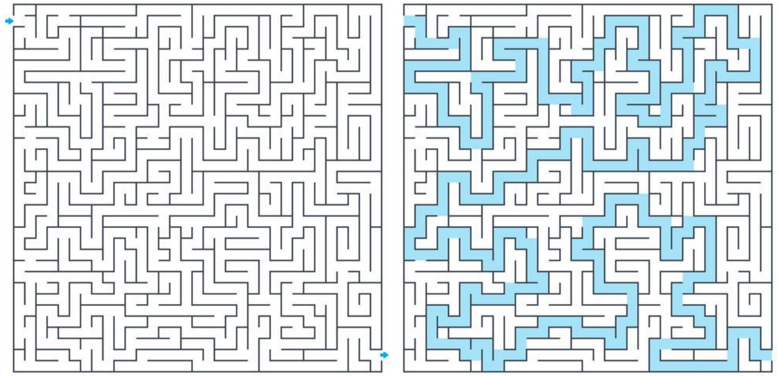
Running a Maze can be viewed as a search problem that either succeeds (by finding a path), or that announces "unreachable".

Checking the answer "unreachable" is no easier than the original problem because it involves discovering a path that contradicts the unreachability claim.

But if the program claims a path, it can be checked for correctness.



Self-checking: The checking code.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Return false iff rat reached cell ⟨r,c⟩ via an invalid path. */
   private static boolean isValidPath(int r, int c) {
      if ( M[r][c]==Unvisited ) return true; // No claim if Unvisited.
      else
         while ( !((r==lo)&&(c==lo)) ) {
            /* Go to any valid predecessor; return false if there is none. */
               int d = 0;
               while ( d<4 && (M[r+deltaR[d]][c+deltaC[d]] == Wall ||
                       M[r+2*deltaR[d]][c+2*deltaC[d]] != M[r][c]-1 ) ) d++;
               if (d==4) return false;
               r = r+2*deltaR[d]; c = c+2*deltaC[d];
            }
      return true; // Reached upper-left cell.
      } /* isValidPath */
   ...
   } /* MRP */
```

Self-checking: The checking code.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   ...
   /* Return false iff rat reached lower-right cell via an invalid path. */
   public static boolean isSolution() {
      return isValidPath(hi,hi);
      } /* isSolution */
   ...
   } /* MRP */
```

Self-Checking Code

Self-checking:. Make the assertion the last step in `RunMaze.main`.

```
/* Stop execution if path found is not a solution. */
   assert MRP.isSolution(): "internal program error";
```

N.B. No warning from the assert “confirms” that the solution is correct, provided, of course, that `MRP.isSolution()` does not contain its own bug. We should test that it does actually return False for (some) bad paths, by wantonly buggering paths (say) in `MRP.PrintMaze()`.

N.B. The code in `MRP.isValidPath()` is missing a check for the absence of noise off the path.

Testing (Revisited)

Exhaustive Bounded Testing:

There are an infinite number of mazes, so exhaustive testing is not possible.

For given N, there are a finite number of N-by-N mazes, so exhaustive testing of up to size N is feasible, in principle. How many are there?

Answer: 2^w, where *w* is the number of places where a wall can either exist or not exist:

- Outer walls must exist.
- Each of N rows of cells has N-1 interior vertical-wall positions.
- Each of N columns of cells has N-1 inerior horizontal-wall positions.

So w = 2*N*(N-1).

Feasible up through N=4.

N	$2^{2 \cdot N \cdot (N-1)}$
1	$2^0 = 1$
2	$2^4 = 16$
3	$2^{12} = 4{,}096$
4	$2^{24} = 16{,}777{,}216$
5	2^{90}

Exhaustive Bounded Testing: Maze generation.

```
/* Maze, Rat, and Path (MRP) Representations. */
class MRP {
   /* Create an N-by-N maze with walls given by the bits of w. */
   public static void GenerateInput(int N, int w) {
      /* Maze. */
         M = new int[2*N+1][2*N+1];
         lo = 1; hi = 2*N-1;
      /* Set boundary walls. */
         for (int i=0; i<=hi+1; i++)
            M[lo-1][i] = M[hi+1][i] = M[i][lo-1] = M[i][hi+1] = Wall;
      /* Set 2*n*(n-1) interior walls to the corresponding bits of w. */
         for (int r=lo; r<=hi; r++)
            for (int c=lo; c<=hi; c++)
               if ( (r%2==0 && c%2==1)||(r%2==1 && c%2==0) ) {
                  if ( w%2==1 ) M[r][c] = Wall; else M[r][c] = NoWall;
                  w = w/2;
                  }
      /* Rat. */
         r = lo; c = lo; d = 0;
      /* Path. */
         move = 1; M[r][c] = move;
      } /* GenerateInput */
   } /* MRP */
```

Exhaustive Bounded Testing: Iterating through mazes.

```
/* Run a rat through an arbitrary maze.  */
class RunMaze {
    ...
   /* Generate/solve all mazes of sizes up through 4, and validate paths found. */
    public static void exhaustiveTest() {
        for (int N = 1; N<=4; N++)
            for (int i=0; i<Math.pow(2,2*N*(N-1)); i++) {
                MRP.GenerateInput(N,i);
                Solve();
                assert MRP.isValidPath(): "internal program error";
                }
        System.out.println( "passed" );
        } /* exhaustiveTest */
    ...
    } /* RunMaze */
```

Random Testing: For larger mazes.

Can't test them all, but can generate and test random mazes of a given size (for as long as you want), and validate solutions. This is called *fuzz testing.*

N.B. Given the way wall configurations are expressed above, you will have to use Java's **`long`** or BigInteger integers for big values of N.