# **Principled Programming**

Introduction to Coding in Any Imperative Language

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## **Cellular Automata**

We illustrate two-dimensional arrays, and enumerations over them, using the examples of Cellular Automata and the Game of Life.

Cellular Automata model the Universe as a rectangular grid of *cells*, each in a given *state*. Time progresses in discrete steps. On each clock tick, each cell simultaneously decides what state to enter based on its current state and the current states of its neighbors. Each cell makes its decision independently, but all cells follow the same rules.

The Game of Life is a particular Cellular Automaton that models birth and death.

Systematic top-down development of an entire program is illustrated. Deeply-nested **for**statements in the code arise naturally as a consequence of stepwise refinement, but are readily understood. Class Sim models the notion of a Cellular Automaton, and its simulation.

class Sim:

☞ Aggregate the definitions of related variables and methods in a class.

class Sim:

@classmethod
def main(cls) -> None:

☞ Aggregate the definitions of related variables and methods in a class.



- Decorator @classmethod
- First parameter **cls**

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@classmethod
def main(cls) -> None:

Aggregate the definitions of related variables and methods in a class.



- Decorator @classmethod
- First parameter **cls**

class Sim:

@classmethod

def main(cls) -> None:

The simulation will be run by invoking this class method using its qualified name, i.e., Sim.main(), which passes the class itself as an implicit first argument matching cls.

**Aggregate the definitions of related variables and methods in a class.** 

```
class Sim:
```

```
@classmethod
def main(cls) -> None:
    """Simulate a cellular automaton."""
```

A method header-comment specifies the effect of invoking it, and (if the method has non-None type) the value returned. If the method has parameters, the specification is written in terms of those parameters. For the implementation of main, adopt the pattern that first initializes, then computes.

class Sim:

```
@classmethod
def main(cls) -> None:
    """Simulate a cellular automaton."""
    #.Initialize.
    #.Compute.
```

Master stylized code patterns, and use them.

Instantiate placeholders *Initialize* and *Compute* for the problem at hand.

class Sim:

@classmethod
def main(cls) -> None:
 """Simulate a cellular automaton."""
 #.Create the initial Universe and display it.
 #.Simulate and display LAST\_GEN additional generations.

Master stylized code patterns, and use them.

Refine the specifications.

class Sim:

```
@classmethod
def main(cls) -> None:
    """Simulate a cellular automaton."""
    # Create the initial Universe and display it.
    Sim.initialize()
    Sim.display()
```

#.Simulate and display LAST\_GEN additional generations.

Program top-down, outside-in.

Refine the specifications.

class Sim:

```
@classmethod
def main(cls) -> None:
    """Simulate a cellular automaton."""
    # Create the initial Universe and display it.
    Sim.initialize()
    Sim.display()
    # Simulate and display LAST_GEN additional generations.
    for Sim.generation in range(1, Sim.LAST_GEN + 1):
        Sim.next_generation()
        Sim.display()
```

Program top-down, outside-in.

initialize, display, and next\_generation are other class methods to be defined. Class methods are always invoked using their qualified names, e.g., Sim.display(). Refine the specifications. @classmethod def main(cls) -> None:

```
"""Simulate a cellular/automaton."""
# Create the initial Universe and display it.
Sim.initialize()
Sim.display()
```

class Sim:

```
# Simulate and display LAST_GEN additional generations.
for Sim.generation in range(1, Sim.LAST_GEN + 1):
    Sim.next generation()
    Sim.display()
```

#### Many short procedures are better than large blocks of code. ß

Method specifications use a syntactic construct are known as a "docstring"

```
Refine the specifications.
```

```
class Sim:
```

```
@classmethod
def main(cls) -> None:
    """Simulate a cellular automaton."""
    # Create the initial Universe and display it.
    Sim.initialize()
    Sim.display()
# Simulate and display LAST GEN additional genen
```

```
# Simulate and display LAST_GEN additional generations.
for Sim.generation in range(1, Sim.LAST_GEN + 1):
    Sim.next_generation()
    Sim.display()
```

Create stubs for the methods that have been introduced, which you can do mindlessly.

class Sim:

```
@classmethod
def initialize(cls) -> None:
    """Create the initial Universes old. """
    pass
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    pass
@classmethod
def next_generation(cls) -> None:
    """Update Universe to be the next generation."""
    pass
```

**Defer challenging code for later; do the easy parts first.** 

The simulation of a cellular automaton models a finite M-by-N Universe of cells. The states of each generation of next cells is determined from the states of the old cells, where generations are numbered from 0 through LAST\_GEN. The state of each cell is modeled as an **int**.

```
Specify and declare the data representation.
```

```
class Sim:
```

. . .

```
M: int = 5
N: int = 20
old: list[list[int]] = []
next: list[list[int]] = []
LAST_GEN: int = 40
generation: int = 0
```

```
# Height of Universe.
# Width of Universe.
# old Universe.
# next Universe.
# Last generation.
# Generation number.
```

Aggregate the definitions of related variables and methods in a class.

The simulation of a cellular automaton models a finite M-by-N Universe of cells. The states of each generation of next cells is determined from the states of the old cells, where generations are numbered from 0 through LAST\_GEN. The state of each cell is modeled as an **int**.

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class Sim:
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. . .

```
M: int = 5
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old: list[list[int]] = []
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LAST_GEN: int = 40
generation: int = 0
```

```
N.B. The term "state" is overloaded.
Each cell of the Universe has a "state".
```

# Height of Universe. # Width of Universe. # old Universe. # next Universe. # Last generation. # Generation number.

Introduce program variables whose values describe "state".

The simulation of a cellular automaton models a finite M-by-N Universe of cells. The states of each generation of next cells is determined from the states of the old cells, where generations are numbered from 0 through LAST\_GEN. The state of each cell is modeled as an **int**.

```
Specify and declare the data representation.
```

```
class Sim:
```

. . .

```
M: int = 5
N: int = 20
old: list[list[int]] = []
next: list[list[int]] = []
LAST_GEN: int = 40
generation: int = 0
```

N.B. The term "state" is overloaded. Each cell of the Universe has a "state", and the simulation as a whole has a "state".

```
# Height of Universe.
# Width of Universe.
# old Universe.
# next Universe.
# Last generation.
# Generation number.
```

Introduce program variables whose values describe "state".

```
class Sim:
```

. . .

```
M: int = 5
N: int = 20
old: list[list[int]] = []
next: list[list[int]] = []
LAST_GEN: int = 40
generation: int = 0
```

# Height of Universe. # Width of Universe. # old Universe. # next Universe. # Last generation. # Generation number.

Names of variables intended to be constant throughout program execution are, by convention, all capital letters.

### Minimize use of literal numerals in code; define and use symbolic constants.

class Sim:

. . .

```
M: int = 5
N: int = 20
old: list[list[int]] = []
next: list[list[int]] = []
LAST_GEN: int = 40
generation: int = 0
```

# Height of Universe. # Width of Universe. # old Universe. # next Universe. # Last generation. # Generation number.

Variables <u>declared and initialized</u> at the top-level of a class are called *class variables*, and are shared among all methods of the class.

class Sim:

. . .

```
M: int = 5
N: int = 20
old: list[list[int]] = []
next: list[list[int]] = []
LAST_GEN: int = 40
generation: int = 0
```

# Height of Universe.
# Width of Universe.
# old Universe.
# next Universe.
# Last generation.
# Generation number.

The initial value of the 2-D array must be created in method initialize because the needed construct (list comprehension) is not permitted here.

class Sim:

. . .

```
M: int = 5
N: int = 20
old: list[list[int]] = []
next: list[list[int]] = []
LAST_GEN: int = 40
generation: int = 0
```

# Height of Universe. # Width of Universe. # old Universe. # next Universe. # Last generation. # Generation number.

The initial value of the 2-D array must be created in method initialize — because the needed construct (list comprehension) is not permitted here.

Technically, initialization to [] is a violation of the representation invariant because [] is not M-by-N.

```
class Sim:
```

• • •

```
@classmethod
def initialize(cls) -> None:
    """Create the initial Universe."""
    pass
```

```
class Sim:
```

• • •

• • •

```
@classmethod
def initialize(cls) -> None:
    """Create the initial Universe."""
    #.Initialize old and new Universes to M-by-N arrays of 0.
```

```
class Sim:
```

• • •

### Long lines can be split if between matched parentheses or brackets.

We now turn to implementation of the methods: initialize.

```
class Sim:
```

• • •

```
class Sim:
```

• • •

. . .

The temporary violation of the representation invariant has now been corrected. Turn to implementing method **display**.

```
class Sim:
```

• • •

```
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    pass
```

Turn to implementing method display.

```
class Sim:
```

• • •

```
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    print( "Generation: ", Sim.generation )
```

Use a standard row-major-order traversal.

```
class Sim:
...
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    print( "Generation: ", Sim.generation )
    for r in range(0, Sim.M):
        for c in range(0, Sim.N): print(Sim.old[r][c], end='')
        print()
```

Master stylized code patterns, and use them.

Use a standard row-major-order traversal, output cells.

```
class Sim:
...
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    print( "Generation: ", Sim.generation )
    for r in range(0, Sim.M):
        for c in range(0, Sim.N): print(Sim.old[r][c], end='')
        print()
```

Master stylized code patterns, and use them.

Use a standard row-major-order traversal, output cells, and output newlines at row ends.

```
class Sim:
```

. . .

. . .

```
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    print( "Generation: ", Sim.generation )
    for r in range(0, Sim.M):
        for c in range(0, Sim.N): print(Sim.old[r][c], end='')
        print()
```

Master stylized code patterns, and use them.

of the next line after printing.

Use a standard row-major-order traversal, output cells, and output newlines at row ends.

```
class Sim:
```

. . .

```
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    print( "Generation: ", Sim.generation )
    for r in range(0, Sim.M):
        for c in range(0, Sim.N): print(Sim.old[r][c], end='')
        print()
.... Means: Don't go to the beginning
```

Master stylized code patterns, and use them.

Use a standard row-major-order traversal, output cells, and output newlines at row ends.

```
class Sim:
```

. . .

. . .

Variables r and c are local variables of method display. Refer to local variables without qualification. Use a standard row-major-order traversal, output cells, and output newlines at row ends.

```
class Sim:
```

. . .

. . .

Variables generation, M, N, and old are class variables. Refer to class variables wth qualified names. A method stub may suffice for a program test.

• • •

```
class Sim:
...
@classmethod
def next_generation(cls) -> None:
   """Update Universe to be the next generation."""
   pass
```

**Write degenerate program stubs that allow partial programs to execute.** 

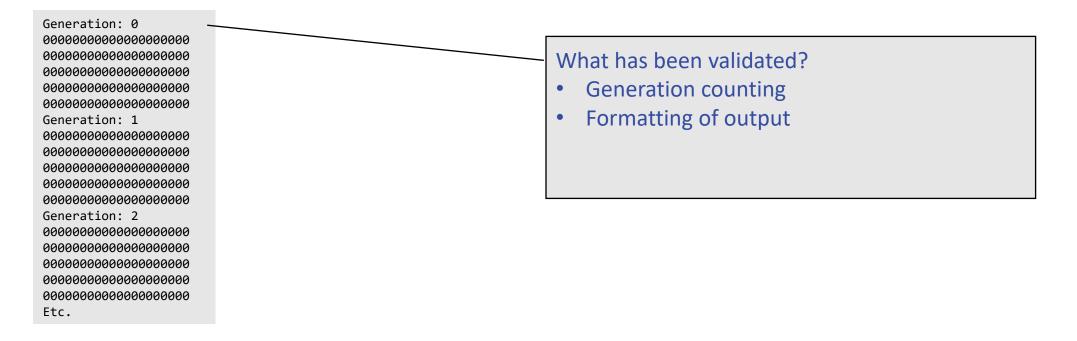
Test early and often.

To try it out, invoke Sim.main().

**Test programs incrementally.** 

Never be (very) lost. Don't stray far from a correct (albeit, partial) program.

#### Output:



### **Validate output thoroughly.**

We now turn to implementation of the method stub: next\_generation.

```
class Sim:
```

. . .

```
...
@classmethod
def next_generation(cls) -> None:
    """Update Universe to be the next generation."""
    pass
```

Master stylized code patterns, and use them.

Instance of the standard *compute-use* pattern.

```
class Sim:
```

```
• • •
```

@classmethod
def next\_generation(cls) -> None:
 """Update Universe to be the next generation."""
 #.Determine the states of next[][] as F(old[][] states).
 #.Swap old[][] and next[][] Universes.

Master stylized code patterns, and use them.

Standard row-major-order traversal for determining new states of each cell of next.

class Sim:

. . .

. . .

#.Swap old[][] and next[][] Universes.

Standard row-major-order traversal for determining new states of each cell of next.

class Sim:

. . .

. . .

```
@classmethod
def next_generation(cls) -> None:
    """Update Universe to be the next generation."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            # Sim.next[r][c] = F(Sim.old[r][c] and its neighbors)
```

#.Swap old[][] and next[][] Universes.

#### Standard code for swap

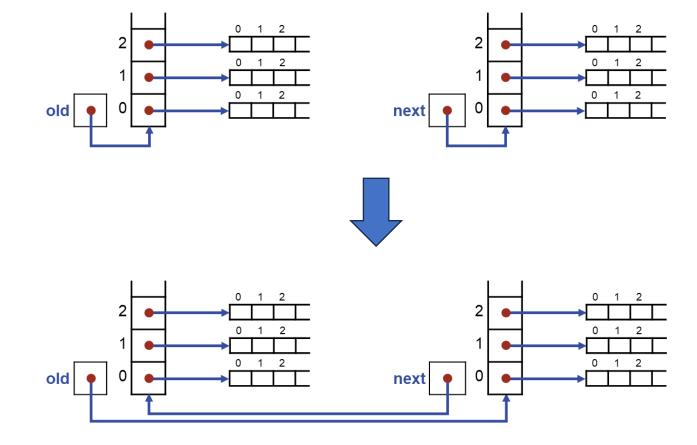
class Sim:

• • •

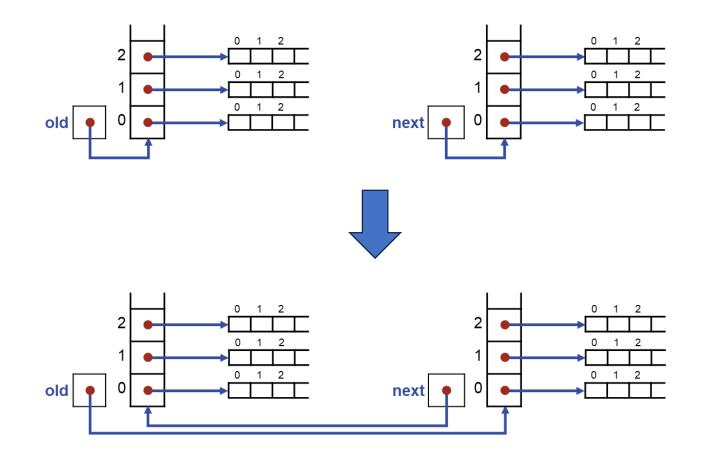
```
@classmethod
def next_generation(cls) -> None:
    """Update Universe to be the next generation."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            # Sim.next[r][c] = F(Sim.old[r][c] and its neighbors)
    # Swap old[][] and next[][] Universes.
    temp = Sim.old; Sim.old = Sim.next; Sim.next = temp
....
```

Master stylized code patterns, and use them.

Notice that swap is a constant-time operation, independent of the size of the Universes.



Notice that swap is a constant-time operation, independent of the size of the Universes.\*



\***C/C++** Constant-time swap is not available for C-style arrays in C/C++. Rather, this can be read as describing one of the alternatives to C-style arrays that are available in C++.

Completed next\_generation for a generic cellular automaton.

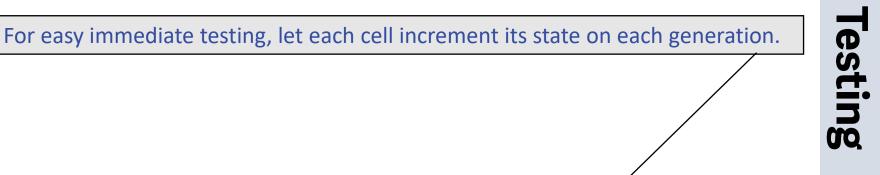
class Sim:

. . .

. . .

```
@classmethod
def next_generation(cls) -> None:
    """Update Universe to be the next generation."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            # Sim.next[r][c] = F(Sim.old[r][c] and its neighbors)
```

```
# Swap old[][] and next[][] Universes.
temp = Sim.old; Sim.old = Sim.next; Sim.next = temp
```



```
class Sim:
...
@classmethod
def next_generation(cls) -> None:
    """Update Universe to be the next generation."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            Sim.next[r][c] = Sim.old[r][c] + 1
```

# Swap old[][] and next[][] Universes. temp = Sim.old; Sim.old = Sim.next; Sim.next = temp

**Test programs incrementally.** 

. . .

Test early and often.

To try it out again, invoke Sim.main().

**Test programs incrementally.** 

Never be (very) lost. Don't stray far from a correct (albeit, partial) program.

#### Output:

#### What has been validated?

- Generation counting
- Formatting of output
- Creation of next Universe from old Universe
- Swapping of old and next Universes

**Validate output thoroughly.** 

Game of Life. A Cellular Automaton in which each cell is either dead or alive.

In each generation:

- Each live cell with 2 or 3 live neighbors lives on to the next generation (life) otherwise it dies (death).
- Each dead cell with 3 live neighbors comes alive in the next generation (birth) otherwise it remains dead.

Each cell is either dead or alive, so specialize the Universes as **Boolean 2-D** arrays.

class Sim:

```
M: int = 5
N: int = 20
old: list[list[bool]] = []
next: list[list[bool]] = []
LAST_GEN: int = 50
generation: int = 0
```

# Height of Universe. # Width of Universe. # old Universe. # next Universe. # Last generation. # Generation number.

Choose representations that by design don't have nonsensical configurations.

Revise method initialize similarly.

```
class Sim:
...
@classmethod
def initialize(cls) -> None:
    """Create the initial Universe."""
    # Initialize old and new Universes to M-by-N arrays of False.
    Sim.old = [[False for _ in range(0, Sim.N)]
        for _ in range(0, Sim.N)]
        Sim.next = [[False for _ in range(0, Sim.N)]
        for _ in range(0, Sim.N)]
        for _ in range(0, Sim.M)]
```

**Choose representations that by design don't have nonsensical configurations.** 

Revise method display to compactly render dead as "\_" and alive as "X".

```
class Sim:
...
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    print( "Generation: ", Sim.generation )
    for r in range(0, Sim.M):
        for c in range(0, Sim.N): print(Sim.old[r][c], end='')
        print()
```

• • •

Revise method **display** to compactly render dead as "\_" and alive as "X".

```
class Sim:
...
@classmethod
def display(cls) -> None:
    """Display the present Universe."""
    print( "Generation: ", Sim.generation )
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            if sim.old[r][c]: print("X", end='')
            else: print("_", end='')
            print()
```

. . .

While we are at it, revise the method header to be more informative.

```
class Sim:
...
@classmethod
def display(cls) -> None:
    """Display Universe old[][] as an M-by-N grid."""
    print( "Generation: ", Sim.generation )
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            if sim.old[r][c]: print("X", end='')
            else: print("_", end='')
            print()
```

**Repeatedly improve comments by relentless copy editing.** 

. . .

That way, for example, the tip provided in an IDE editor will be more helpful.

```
class Sim:
    . . .
    @classmethod
    def main(cls) -> None:
         """Simulate a cellular automaton."""
        # Create the initial Universe and display it.
         Sim.initialize()
         Sim.display()
                  @classmethod
        # Simula def display(cls) -> None
                                                  hing generations.
         for Sim.
                                                  ST_GEN + 1):
                  Display Universe old[][] as an M-by-N grid.
             Sim.
             Sim.display()
```

**Repeatedly improve comments by relentless copy editing.** 

Similarly, turn end-of-line comments of variable declarations into docstrings.

#### class Sim:

```
M: int = 5
N: int = 20
old: list[list[bool]] = []
next: list[list[bool]] = []
LAST_GEN: int = 50
generation: int = 0
```

# Height of Universe. # Width of Universe. # old Universe. # next Universe. # Last generation. # Generation number.

**Repeatedly improve comments by relentless copy editing.** 

Similarly, you can turn end-of-line comments of variable declarations into docstrings.

```
class Sim:
   M: int = 5
    """M is the height of the Universe."""
   N: int = 20
    """N is the width of the Universe."""
    old: list[list[bool]] = []
    """old is the present state of the Universe."""
    next: list[list[bool]] = []
    """next is the upcoming generation of the Universe, in preparation."""
    LAST GEN: int = 40
    """LAST GEN is last generation to be simulated."""
    generation: int = 0
    """generation is the number of the present generation."""
```

The docstring of a variable is the variable's representation invariant.

**Repeatedly improve comments by relentless copy editing.** 

. . .

The docstring of a variable is the variable's representation invariant.

```
class Sim:
    . . .
    @classmethod
    def display(cls) -> None:
         """Display Universe old[][] as an M-by-N grid."""
         print( "Generation: ", Sim.generation )
         for r in range(0, Sim.M):
             for c in range(0, Sim.N):
                 if Sim.old[r][c]: print("X", end='')
                 else: class attribute old of Sim
                        old: list[list[bool]] = []
             print()
    . . .
                         old is the present state of the Universe.
```

**Repeatedly improve comments by relentless copy editing.** 

Implement the Game of Life rules.

```
class Sim:
```

```
@classmethod
def next_generation(cls) -> None:
    """Update old[][] to be the next generation of the Universe."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            # Set next[r][c] = according to the Game of Life rules.
    # Swap old[][] and next[][] Universes.
    temp = Sim.old; Sim.old = Sim.next; Sim.next = temp
```

• • •

Refine the specification using the standard *compute-use* pattern.

```
@classmethod
def next_generation(cls) -> None:
    """Update old[][] to be the next generation of the Universe."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            #.Compute.
            #.Use.
```

```
# Swap old[][] and next[][] Universes.
temp = Sim.old; Sim.old = Sim.next; Sim.next = temp
```

## Master stylized code patterns, and use them.

Instantiate placeholders *Compute* and *Use* for the problem at hand.

```
@classmethod
def next_generation(cls) -> None:
    """Update old[][] to be the next generation of the Universe."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            #.Let live_neighbors be number of alive cells around old[r][c].
            #.Set next[r][c] according to the rules re. live_neighbors.
        # Swap old[][] and next[][] Universes.
```

```
temp = Sim.old; Sim.old = Sim.next; Sim.next = temp
```

#### Master stylized code patterns, and use them.

*Use* is a structured four-way case analysis.

```
@classmethod
def next_generation(cls) -> None:
    """Update old[][] to be the next generation of the Universe."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            #.Let live_neighbors be number of alive cells around old[r][c].
```

```
# Set next[r][c] according to the rules re. live_neighbors.
if Sim.old[r][c]: # Currently live.
    if (live_neighbors == 2) or (live_neighbors == 3):
        Sim.next[r][c] = True
    else: Sim.next[r][c] = False
else: # Currently dead.
    if live_neighbors == 3:
        Sim.next[r][c] = True
    else: Sim.next[r][c] = False
```

```
# Swap old[][] and next[][] Universes.
temp = Sim.old; Sim.old = Sim.next; Sim.next = temp
```

*Compute* is a 3x3 row-major-order traversal, counting **live\_neighbors** as appropriate.

```
@classmethod
def next generation(cls) -> None:
    """Update old[][] to be the next generation of the Universe."""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            # Let live_neighbors be number of alive cells around old[r][c].
                live_neighbors = 0
                for dr in range(-1, 2):
                    for dc in range(-1, 2):
                        if not((dr == 0) and (dc == 0)) and (
                             Sim.old[r + dr][c + dc]):
                                 live neighbors += 1
```

# Set next[r][c] following the rules re. live\_neighbors.

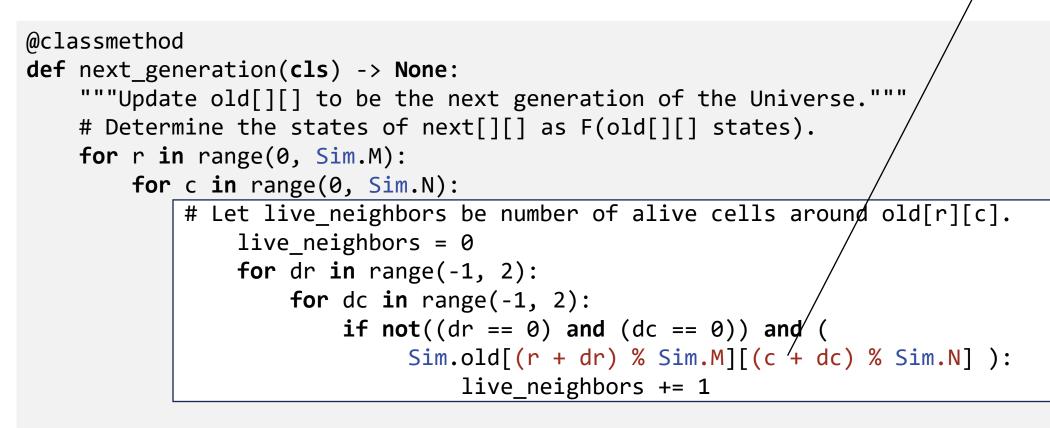
```
# Swap old[][] and next[][] Universes.
temp = Sim.old; Sim.old = Sim.next; Sim.next = temp;
```

#### To prevent the subscripts from going out of bounds

```
@classmethod
def next generation(cls) -> None:
    """Update old[][] to be the next generation of the Universe.""
    # Determine the states of next[][] as F(old[][] states).
    for r in range(0, Sim.M):
        for c in range(0, Sim.N):
            # Let live_neighbors be number of alive cells around old[r][c].
                live_neighbors = 0
                for dr in range(-1, 2):
                    for dc in range(-1, 2):
                        if not((dr == 0) and (dc \neq= 0)) and (
                             Sim.old[r + dr][c + dc] ):
                                  live neighbors += 1
            # Set next[r][c] following the rules re. live neighbors.
            . . .
```

```
# Swap old[][] and next[][] Universes.
temp = Sim.old; Sim.old = Sim.next; Sim.next = temp;
```

To prevent the subscripts from going out of bounds, simulate on a torus.



# Set next[r][c] following the rules re. live\_neighbors.

```
# Swap old[][] and next[][] Universes.
temp = Sim.old; Sim.old = Sim.next; Sim.next = temp;
```

Create some life, which will glide diagonally down and to the right.

```
@classmethod
def initialize(cls) -> None:
    """Create the initial Universe."""
    # Initialize old and new Universes to M-by-N arrays of False.
         . . .
                                                 old
    # Glider
                                                     0
                                                         1
                                                            2 ...
        Sim.old[0][1] = True
                                                   0
                                                         Т
                                                                . . .
        Sim.old[1][2] = True
                                                                . . .
        Sim.old[2][0] = True
                                                   2
                                                         T.
        Sim.old[2][1] = True
         Sim.old[2][2] = True
                                                               ...
                                                     ...
                                                         ...
                                                            ...
```

To let it rip, invoke Sim.main() yet again.

## And presto ...

Generation: 0 \_X\_\_\_\_\_ \_X\_\_\_\_\_ XXX\_\_\_\_\_

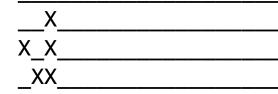
## And presto ...

### Generation: 1

X_X	 	
XX		
_X		

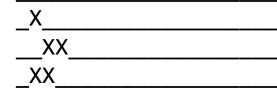
## And presto ...

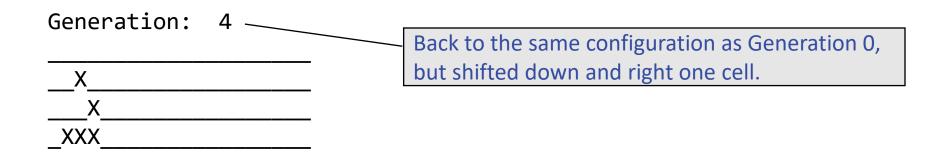
### Generation: 2



## And presto ...

### Generation: 3





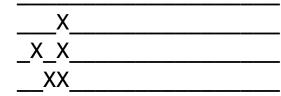
And presto ...

Generation: 5

\_X\_X\_\_\_\_\_\_ \_\_XX\_\_\_\_\_\_ \_\_X\_\_\_\_\_

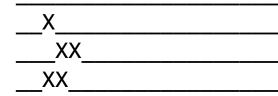
And presto ...

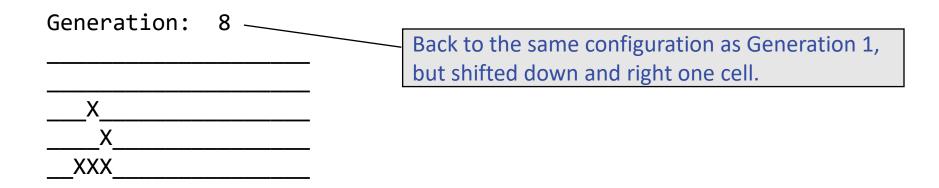
Generation: 6

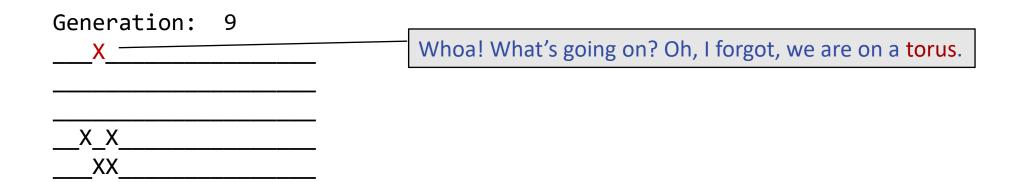


### And presto ...

Generation: 7







#### And presto ...

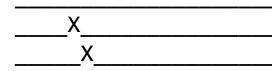
Generation: 10 \_\_\_\_\_X \_\_\_\_\_X \_\_\_\_\_X\_\_\_\_ Generation: 11

### And presto ...

Generation: 11 \_\_\_\_XX\_\_\_\_\_\_ \_\_\_\_X\_\_\_\_\_XX\_\_\_\_\_

### And presto ...

Generation: 12 \_\_\_\_XXX\_\_\_\_\_



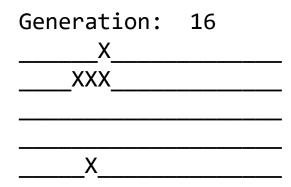
### And presto ...

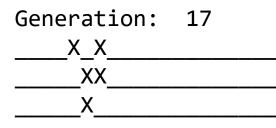
Generation: 13 \_\_\_\_X\_\_\_\_\_\_ \_\_\_X\_\_\_\_\_\_ \_\_\_X\_\_\_\_\_\_

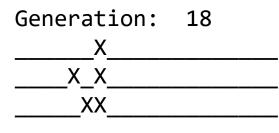
### And presto ...

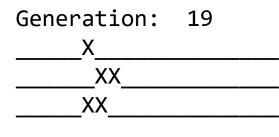
### And presto ...

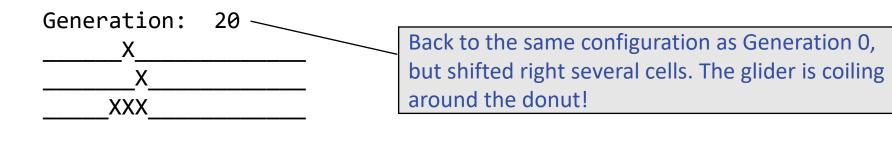
Generation: 15 \_\_\_\_XX\_\_\_\_\_ \_\_\_XX\_\_\_\_\_\_ \_\_\_\_X\_





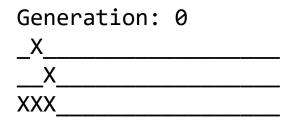






What are the boundary conditions for this problem, and did we forget them?

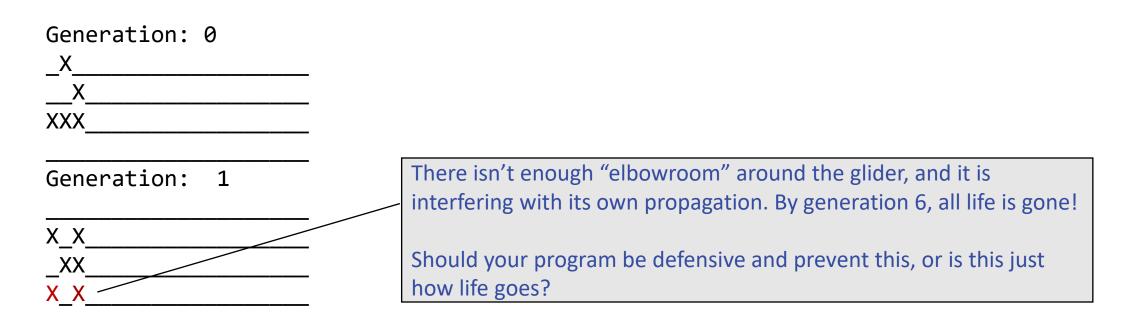
For example, what if the height of the Universe were only 4? To try it out, change N, and invoke Sim.main().



**Boundary conditions.** Dead last, but don't forget them.

What are the boundary conditions for this problem, and did we forget them?

For example, what if the height of the Universe were only 4? To try it out, change N, and invoke Sim.main().



**Boundary conditions.** Dead last, but don't forget them.

#### Summary:

The notion of a **class** has been introduced as a means for aggregating variables and methods.

Many standard precepts, patterns, and recommended coding techniques have been illustrated.

Representation invariants for data structures and their components have been emphasized, and their effective use in IDE's shown.

And the Game of Life itself is fascinating.