On Speeding Up Language Model Evaluation

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Abstract

Large language models (LLMs) currently dominate the field of natural language processing (NLP), representing the state-of-the-art across a diverse array of tasks. Developing a model of this nature, from training to inference, requires make numerous decisions which define a combinatorial search problem. For example, selecting the optimal pre-trained LLM, prompt, or hyperparameters to attain the best performance for a task often requires evaluating multiple candidates on an entire test set. This exhaustive evaluation can be time-consuming and costly, as both inference and metric computation with LLMs are resource-intensive. In this paper, we address the challenge of identifying the best method within a limited budget for evaluating methods on test examples. By leveraging the well-studied multi-armed bandit framework, which sequentially selects the next method-example pair to evaluate, our approach, combining multi-armed bandit algorithms with low-rank factorization, significantly reduces the required resources. Experiments show that our algorithms can identify the top-performing method using only 5-15% of the typically needed resources, resulting in an 85-95% reduction in cost.

1 Introduction

Large language models (LLMs) have demonstrated remarkable proficiency in diverse tasks such as question answering, machine translation and mathematical reasoning [5, 16, 43]. They are employed across numerous applications, ranging from automated customer support to content generation [45]. As the development of new LLMs continues, practitioners find themselves with a plethora of options, selecting the optimal model, prompt [40], or hyperparameters for their specific needs from hundreds available. For instance, initiatives like the Chatbot Arena [15] actively maintain nearly 100 models for benchmarking on user-specified open-ended queries. Similarly, the AlpacaEval [44] project benchmarks over 200 models against a diverse set of 805 questions.

Querying an LLM is resource-intensive, and therefore such extensive evaluations require significant investments in time, compute and financial resources. In Figure 1 Left, we show the estimated cost to fully evaluate (denoted as full evaluation) the 153 models officially included in AlpacaEval [44] as of May 20, 2024 to be almost 800 USD. In Figure 1 Right, we show that 78.2 Nvidia A6000 GPU hours are needed for evaluating 205 zero-shot prompts on 784 GSM8K [18] questions using Mistral-7B [32].

Despite the convention of exhaustive evaluations of all data points in a test set from a task, practitioners often only care about the overall performance rankings. Typically, the goal is to identify the top-performing methods or simply the best ones, discarding the lower-ranked alternatives. Therefore, while full-scale evaluation of each method on every data point is thorough, it may not be cost-effective when the goal is merely to identify the superior method.

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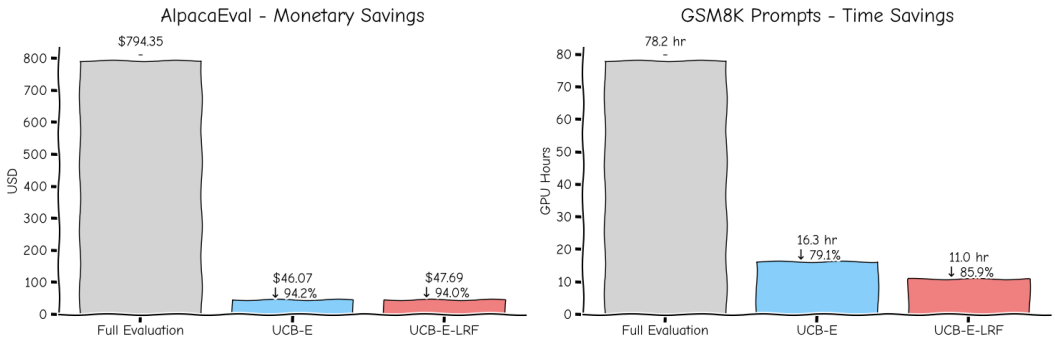

Figure 1: Comparison of cost for finding the best model or prompt between our proposed algorithms (UCB-E, UCB-E-LRF) and full evaluation on two datasets. **Left:** AlpacaEval [44] data as of May 20, 2024. The data includes 153 models evaluated on 805 instructions. The response on each instruction for each model is compared against a baseline response generated by GPT4-turbo, with GPT4-turbo being the LLM judge [66]. The monetary saving is calculated based on the inference cost of proprietary models among the 153 models and GPT4-turbo judging cost. **Right:** GSM8K [18] prompt engineering data. The data includes 205 zero-shot prompts for Mistral-7B generated by GPT4 evaluated on 784 questions (10%) of GSM8K training set. The time saving is calculated based on inference speed of Mistral-7B using the Huggingface library with one Nvidia A6000 GPU.

In this paper, we explore a limited-budget setting. The best method among *a set of methods* is the method that has the best average performance among *a set of test examples*. We want to identify this best method given a fixed budget for evaluating method-example pairs. A simple baseline is to evenly split the budget for each method. However, this can be very inefficient – as we observed in our experiments, for some datasets, we need to evaluate more than 90% of all method-example pairs to predict the best method correctly with high probability. Intuitively, it is easier to recognize that low-performance methods are unlikely to be the best. Therefore, a more effective budget allocation strategy is to spend less on low-performing methods and more on promising ones. Multi-armed bandit algorithms enable this dynamic allocation by actively selecting the next method-example pair to evaluate based on the results of previous evaluations.

We propose two active selection algorithms UCB-E and UCB-E-LRF. Our first algorithm is an extension of the classical UCB-E [2] to solve the multi-armed bandit problem. It estimates the upper confidence bound (UCB) to guide the selection of the method which is paired with a randomly chosen example for the next evaluation in order to efficiently estimate the best method. This algorithm enjoys a theoretical guarantee that the chance of selecting the best arm converges to 100% by an exponential decay of the number of evaluations. Our second algorithm, UCB-E-LRF, leverages the intrinsic low-rankness of the scoring matrices. We find that in practice, scoring matrices, where each row (column) is a method (an example) and the value is some metric score to measure how good the method is on the example, usually can be well-approximated by a low-rank matrix. Intuitively we can predict the remaining unobserved method-example pairs by the low-rank factorization (LRF) and prioritize evaluations of the pairs with large uncertainties in this prediction. By deploying this intuition in addition to UCB-E, UCE-E-LRF actively selects both the method and example to evaluate and it potentially improves the efficiency of the budget usage.

We study the performance of these algorithms as well as common baselines in a number of practical settings. Through the empirical analysis, we find that our two active selection algorithms are much better than non-active baselines. In addition, we find for all algorithms, the harder settings, where the method set is large or the gaps between the top method and other methods are small, generally need more budget for evaluating the method-example pairs. More interestingly, we observe that the UCB-E works best for the easier settings; conversely, UCB-E-LRF has superiority over all other algorithms when the settings are harder.

2 Preliminaries

2.1 Evaluation Workflow in Large Language Model Applications

Table 1: Selected benchmarks where LLMs are state-of-the-art methods. We group them based on the task type and whether additional LLM-based / human-based scoring is needed for evaluation.

Task Type	LLM Inference + Rule-based Scoring	LLM Inference + LLM-based Scoring
Natural Language Understanding	BOLD [22] GLUE [57] HellaSwag [64] SQuAD [52] TriviaQA [34] WinoGrande [1]	CNN/Dailymail [48] Newsroom [25] XSUM [49]
Open-ended QA	Google NQ [41] HotpotQA [62] QASPER [19]	AlpacaEval [44] Chatbot Arena [15] MT-Bench [66]
STEM and Reasoning	APPS [27] Arc [17] GPQA [53] GSM8K [18] MATH [29] MMLU [28] PIQA [3]	–

A typical evaluation workflow in large language model (LLM) applications involves three steps: inference, scoring, and performance aggregation, the first two of which LLMs can play important roles.

1. **Inference.** Given a dataset such as one shown in Table 1 and an LLM-based *method*, the output from this *method* is generated through an LLM. Each *method* can be a distinct LLM for benchmarking different LLM performance, the same LLM with different prompts for prompt engineering [60], or other different configurations such as temperature, decoding strategies, etc for hyperparameter tuning.
2. **Scoring.** The outputs from different methods are scored with a scoring function i.e. metric. The scoring function can either be rule-based (exact string match, BLEU [50], ROUGE [46]), LLM-based (BERTScore [65], LLM judge [66]), or human-based i.e. user study. Depending on the task and dataset format, researchers have employed different types of scoring functions. In Table 1, we classify a selected group of commonly used datasets according to the task type and scoring function applied to them. We group LLM-based and human-based scoring together since they are usually considered alternatives to each other and have been shown to have relatively high correlation [44].
3. **Performance aggregation.** The performance of each method is aggregated across the dataset, typically with a simple average over all examples in the dataset.

Notice that LLMs can play an important role in both inference and scoring. However, LLMs are becoming increasingly large and many state-of-the-art LLMs for LLM-based scoring functions are black-box APIs [42]. Both inference and scoring in the evaluation workflow start to become massively resource-intensive.

Despite the intensive resources needed, in many practical scenarios, we are only interested in identifying the best method among all methods. For example, for prompt engineering and hyperparameter tuning, knowing which method (prompt / configuration) is the best is usually sufficient for next round of iteration. Intuitively, evaluating all methods on all examples on a dataset is excessive for this purpose and therefore we are interested in studying the identification of the best method with as few evaluation pairs as possible.

2.2 Notations and Problem Formulation

With the motivation above, we now define our notation and provide a formal problem formulation. Suppose we have a set of methods $\mathcal{F} = \{f_1, \ldots, f_n\}$ and a set of examples $\mathcal{X} = \{x_1, \ldots, x_m\}$. Let $e : \mathcal{F} \times \mathcal{X} \to [0, 1]$ denote our scoring function and without loss of generality, we assume $e(f, x) > e(f', x)$ indicates f has better performance than f' on the example x. Suppose $E \in [0, 1]^{n \times m}$ is the underlying scoring matrix for a given problem $(\mathcal{F}, \mathcal{X}, e)$ by defining $E_{i,j} := e(f_i, x_j)$. The score of a given method f_i is defined as $\mu_i = \frac{1}{m} \sum_{j=1}^{m} E_{i,j}$ and want to find the best method $i^* = \arg\max_i \mu_i$.

As motivated above, we are interested in the scenario where we have a limited budget of T evaluations. The question is: how can we select these T method-example pairs to maximize the chance of finding the best method i^*? Formally, we want to study the evaluation algorithm $\mathcal{A}$. The input is the

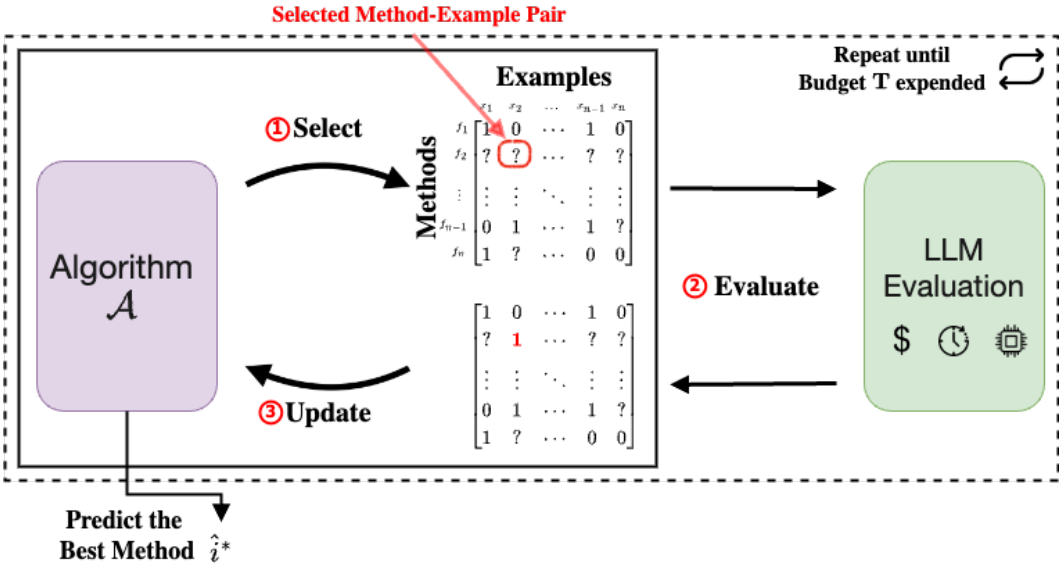

Figure 2: **Active method-example pair selection**: After LLM evaluated t method-example pairs, we then call Algorithm $\mathcal{A}$ to *select* the next method-example pair. Then we *query* LLM for evaluating this pair and *fill* the scoring received from LLM into the scoring matrix. Algorithm $\mathcal{A}$ then *updates* its internal status prepared for the next method-example pair selection. This process is repeated T times and, in the end, the algorithm $\mathcal{A}$ predicts the best method $f_{\hat{i}^*}$

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evaluation budget T, a set of methods $\mathcal{F}$ and a set of examples $\mathcal{X}$. In the evaluation algorithm $\mathcal{A}$, we can evaluate at most T method-example pairs (f_{i_t}, x_{j_t}) $(t = 1, \cdots T)$. The output is the predicted best method. Our goal is to maximize $\mathbb{P}_{\mathcal{A}}(\mathcal{A}(T, \mathcal{F}, \mathcal{X}) = i^*)$, the probability of returning the best method i^*.

3 Algorithms

One simple baseline for designing the evaluation algorithm $\mathcal{A}$ is: for each method $f_i \in \mathcal{F}$, we uniformly sample $\lfloor T/n \rfloor$ examples $\mathcal{X}_{i,T}$ from $\mathcal{X}$, estimate the mean $\hat{\mu}_i = \frac{1}{\lfloor T/n \rfloor} \sum_{x \in \mathcal{X}_{i,T}} e(f_i, x)$, and pick the method $f_{\hat{i}^*}$ with the highest estimated mean $\hat{\mu}_{\hat{i}^*}$ as the prediction for the best method. However, this algorithm can be very inefficient. We will see in the experiments that for some datasets, we need to evaluate at least 90% of all method-example pairs to predict the best method correctly with high probability. Different from this simple baseline, two algorithms proposed in this section actively decide the method-example pair to evaluate next and each decision is made based on previous evaluations; Figure 2 illustrates this high level idea.

3.1 Algorithm 1 – UCB-E

The algorithm. Notice the simple baseline evenly distributes its total budget across the different methods f_i. This allocation of resources is the main limitation of the simple baseline. In order to distinguish the best method f_{i^*} from other good methods f_i (i.e. $\mu_{i^*} - \mu_i$ is small), we may need more than $\lfloor T/n \rfloor$ examples, while $\leq \lfloor T/n \rfloor$ examples are sufficient to distinguish f_{i^*} from bad methods (i.e. $\mu_{i^*} - \mu_i$ is large).

We address the limitation above with our first algorithm UCB-E ($\mathcal{A}_{ucb-e}$; Algorithm 1), a simple extension of the classic multi-arm bandit algorithm UCB-E [2]. At every step t, we estimate the upper confidence bound of each method f_i from the examples evaluated with that method and pick the method f_{i_t} with the largest upper confidence bound. Then we uniformly sample one example x_{j_t}

Algorithm 1 UCB-E ($\mathcal{A}_{ucb-e}(T, \mathcal{F}, \mathcal{X}; \eta)$)

Input: The evaluation budget T, a set of methods $\mathcal{F}$ and a set of examples $\mathcal{X}$, the uncertainty scaling η.

Output: The prediction $\hat{i}^*$ for best method i^*.

1: $\forall f_i \in \mathcal{F}$, the upper confidence bound $B_i = +\infty$, the set of evaluated examples $S_i = \{\}$, the observed scoring matrix $E^{\text{obs}} := \{?\}^{n \times m}$.
2: **for** $t = 1, \cdots, T$ **do**
3: **Select:** Draw uniform randomly $i_t \in \arg\max_i B_i$; Draw uniform randomly $j_t \in [m] \backslash S_i$.
4: **Evaluate:** Run inference for the method-example pair (f_{i_t}, x_{j_t}), send the result to the annotator, and receive $e(f_{i_t}, x_{j_t})$; $E^{\text{obs}}_{i_t, j_t} \leftarrow e(f_{i_t}, x_{j_t})$.
5: **Update:** $S_{i_t} \leftarrow S_{i_t} \cup \{j_t\}$; $B_{i_t} \leftarrow \frac{\sum_{j \in S_{i_t}} E^{\text{obs}}_{i_t, j}}{|S_{i_t}|} + \sqrt{\frac{\eta}{|S_{i_t}|}}$.
6: **end for**

Return: $\hat{i}^* = \arg\max_i \frac{\sum_{j \in S_i} E^{\text{obs}}_{i,j}}{|S_i|}$

that has not been evaluated with the method f_{i_t}, run the inference procedure for (f_{i_t}, x_{j_t}) and send the result to the annotator for scoring.

Comparison with the simple baseline. The UCB-E algorithm addresses the limitation of the simple baseline. The upper confidence bounds of bad methods (i.e. $\mu_{i^*} - \mu_i$ is large) will never exceed the upper confidence bound of f_{i^*}, as long as they are evaluated with a sufficient number of examples, and hence UCB-E will never select this method again and pay more attentions on more promising methods.

Theoretical results. We state the theoretical guarantee of UCB-E in our context, which is a corollary of the theorem for UCB-E in the original multi-arm bandit setting.

Corollary 1 (Theoretical guarantee of UCB-E) *Define* $H_1 = \sum_{i=1, i \neq i^*}^{n} \frac{1}{(\mu_i - \mu_i^*)^2}$ *and suppose* $\eta = \frac{25}{36} \frac{T-n}{H_1}$, $\mathbb{P}_{\mathcal{A}_{ucb-e}}(\mathcal{A}_{ucb-e}(T, \mathcal{F}, \mathcal{X}; \eta) = i^*) \geq 1 - 2Tn \exp\left(-\frac{T-n}{18H_1}\right)$.

Comment. Given a problem set-up $(\mathcal{F}, \mathcal{X}, e)$, H_1 in the corollary indicates the hardness of the problem. The larger the gaps between the highest score and the remaining scores are, the higher the probability UCB-E will pick the best method given a fixed budget T. In our experiments, we will observe that the datasets with smaller H_1 tend to have a higher chance of predicting i^* correctly given the same evaluation budget T.

3.2 Algorithm 2 – UCB-E with Low-Rank Factorization (UCB-E-LRF)

Main idea/Inspiration/Motivation. We find that the scoring matrix E for real-world problem instances can often be approximated well by a low rank matrix where rank $r \ll n, m$. This observation means that it is possible to estimate the score of all method-example pairs from the scores of just a few. Consider if the matrix E were exact rank-1, then only $n + m - 1$ scores would be needed to exactly recover the full scoring matrix E, which is extremely efficient compared to exhaustively evaluating all $n \cdot m$ combinations. Suppose E^{obs} is a partially observed scoring matrix, where the non-zero elements are evaluated pairs with the ground truth score for that pair, and $\hat{E}$ is the estimated scoring matrix, where the non-zero elements are unevaluated pairs with the estimated score. Suppose $O \in \{0, 1\}^{n \times m}$ is the observation matrix where $O_{i,j}$ indicates whether the method-example pair (f_i, x_j) has been observed. When the scores of unevaluated method-example pairs are estimated accurately, the score estimation $\hat{\mu}_i = \frac{1}{m} \sum_{j=1}^{m} (O_{i,j} E^{\text{obs}}_{i,j} + (1 - O_{i,j}) \hat{E}_{i,j})$, which combines both evaluated pairs and the estimated pairs, has the potential to be more accurate than the estimation by only the evaluated pairs.

The algorithm. Leveraging the intuition of estimating the unevaluated method-example pairs, we adopt a low rank factorization (LRF) approach and propose our second algorithm UCB-E-LRF

Algorithm 2 UCB-E-LRF ($\mathcal{A}_{uel}(T, \mathcal{F}, \mathcal{X}; \mathcal{M}, r, K, T_0, b, \eta)$)

Input: The evaluation budget T, a set of methods $\mathcal{F}$ and a set of examples $\mathcal{X}$, the LRF oracle $\mathcal{M}$, the rank r, # solutions in ensemble K, the warm-up budget T_0, the batch size b, the uncertainty scaling η.
Output: The prediction $\hat{i}^*$ for best method i^*.

1: Uniformly sample T_0 method-example pairs from $[n] \times [m]$ and get the observed scoring matrix $E^{\text{obs}} \in ([0,1] \cup \{?\})^{n \times m}$ and the observation matrix $O \in \{0,1\}^{n \times m}$ w.r.t. these T_0 evaluations.
2: $\hat{E}, R \leftarrow \mathcal{M}(E^{\text{obs}}, O); \forall f_i \in \mathcal{F}, B_i \leftarrow \frac{1}{m}\sum_{j=1}^{m}(O_{i,j}E^{\text{obs}}_{i,j} + (1-O_{i,j})\hat{E}_{i,j} + \eta R_{i,j})$
3: **for** $t = T_0, \cdots, T_0 + [(T-T_0)/b]$ **do**
4: **Select:** Draw $i_t \in \arg\max_i B_i$; Draw $J_t := \text{Top}{-}b\,(R_{i_t,j} | j \in [m])$.
5: **for** $j_t \in J_t$ **do**
6: **Evaluate:** Run inference for the method-example pair (f_{i_t}, x_{j_t}), send the result to the annotator, and receive $e(f_{i_t}, x_{j_t})$; $E^{\text{obs}}_{i_t,j_t} \leftarrow e(f_{i_t}, x_{j_t})$
7: **end for**
8: **Update:** $\forall j_t \in J_t$, $O_{i_t,j_t} \leftarrow 1$; $\hat{E}, R \leftarrow \mathcal{M}(E^{\text{obs}}, O; r, K)$; $\forall f_i \in \mathcal{F}, B_i \leftarrow \frac{1}{m}\sum_{j=1}^{m}(O_{i,j}E^{\text{obs}}_{i,j} + (1-O_{i,j})\hat{E}_{i,j} + \eta R_{i,j})$.
9: **end for**
Return: $\hat{i}^* = \arg\max_i \frac{1}{m}\sum_{j=1}^{m}(O_{i,j}E^{\text{obs}}_{i,j} + (1-O_{i,j})\hat{E}_{i,j})$

($\mathcal{A}_{\text{uel}}$; Algorithm 2). This algorithm builds on UCB-E which dynamically allocates the budget to the promising methods. We estimate the upper confidence bound for each method f_i and evaluate an example x_{j_t} yet to be evaluated for the method f_{i_t} with largest upper confidence bound. However, the specification of this idea in UCB-E-LRF is different, because of our additional intermediate goal – a good estimation $\hat{E}$ for the unevaluated pairs. Suppose we have a scoring matrix estimation oracle $\mathcal{M}$: given t evaluated method-example pairs, the oracle outputs the estimated scoring matrix $\hat{E}$ and the uncertainty matrix R that quantifies the uncertainty for each estimation in $\hat{E}$. Suppose we keep updating observation matrix $O \in \{0,1\}^{n \times m}$ where $O_{i,j}$ indicates whether the method-example pair (f_i, x_j) has been evaluated. The different specifications from UCB-E are:

1. Once the method f_{i_t} is selected, unlike in UCB-E where the example x_{j_t} is uniformly drawn from the unevaluated examples, we pick the example x_{j_t} with the largest uncertainty R_{i_t,j_t} from the unevaluated examples, because evaluating this example x_{j_t} will result in the largest reduction in uncertainty for the method f_{i_t}. This allows us to better estimate the scoring matrix $\hat{E}$ with less evaluations.
2. The upper confidence bound is now computed as $\frac{1}{m}\sum_{j=1}^{m}(O_{i,j}E^{\text{obs}}_{i,j} + (1-O_{i,j})\hat{E}_{i,j} + \eta R_{i,j})$, which is a combination of the current score estimation $\frac{1}{m}\sum_{j=1}^{m}(O_{i,j}E^{\text{obs}}_{i,j} + (1-O_{i,j})\hat{E}_{i,j})$ and the uncertainty estimation $\frac{1}{m}\sum_{j=1}^{m}\eta R_{i,j}$. Notably, the current score estimation is the partially observed scores $\frac{1}{m}\sum_{j=1}^{m}O_{i,j}E^{\text{obs}}_{i,j}$ plus the estimated scores $\frac{1}{m}\sum_{j=1}^{m}(1-O_{i,j})\hat{E}_{i,j}$

The choice of scoring matrix estimation oracle. We now introduce the scoring matrix estimation oracle $\mathcal{M}$ used in this paper. The inputs of the oracle $\mathcal{M}$ are a partially observed scoring matrix $E^{\text{obs}} \in ([0,1] \cup \{?\})^{n \times m}$ and an observation matrix $O \in \{0,1\}^{n \times m}$ where $O_{i,j}$ indicates whether the method-example pair (f_i, x_j) has been evaluated. The outputs of $\mathcal{M}$ are supposed to be an estimated scoring matrix $\hat{E}$ and the corresponding uncertainty matrix R.

We define our $\mathcal{M}$ as an ensemble of K low-rank factorization (LRF) solutions [59, 14]. The low-rank factorization assumes the ground truth matrix is (approximately) low-rank with rank r. We optimize the low-rank representations $U \in \mathbb{R}^{n \times r}$ and $V \in \mathbb{R}^{m \times r}$ for following optimization problem by the *alternating least squares* method [26]:

$$\min_{U \in \mathbb{R}^{n \times r}, V \in \mathbb{R}^{m \times r}} \sum_{i \in [n], j \in [m]} O_{i,j}\left(U_i^\top V_j - E^{\text{obs}}_{i,j}\right)^2, \tag{1}$$

where U_i is the ith row of U and V_j is the jth row of V. We further produce both the estimation and the uncertainty by bootstrapping [23, 67]. In order to get a diverse ensemble of low-rank factorizations for each factorization $(U^{(k)}, V^{(k)})$, we randomly zero out 5% of the "1" in O as O_k and optimize the objective Equation 1 with E^{obs} and O_k instead. Finally we define the estimated scoring matrix $\hat{E}$ and the uncertainty matrix R as

$$\hat{E} := (\mathbf{1} - O) \circ \left(\frac{1}{K} \sum_{k=1}^{K} U^{(k)} \left(V^{(k)} \right)^\top \right), R_{i,j} := \sqrt{\frac{1}{K} \sum_{k=1}^{K} ((U^{(k)} \left(V^{(k)} \right)^\top)_{i,j} - \hat{E}_{i,j})^2} \quad (2)$$

where $\mathbf{1}$ is an all-one matrix with the shape of $n \times m$ and $\circ$ is the element-wise multiplication.

This oracle introduces four additional hyperparameters for our UCB-E-LRF algorithm, which determine performance and efficiency:

- The rank r of the low-rank factorization. The value of r adjusts the expressiveness of the factorization when fit to the data in Equation 2. Setting r requires considering the bias-variance trade-off of choosing smaller vs. larger r. The representations of smaller r inherently introduce larger approximation error, however, the representations of large r are harder to be estimated accurately given only a few observations in E^{obs}.
- The ensemble size K. Sufficiently large K is necessary to avoid additional noise when estimating $\hat{E}$ and R in Equation 2. Yet, the computational cost of the factorization scales with K and therefore K cannot be too large.
- The warm-up budget T_0. The oracle $\mathcal{M}$ requires a minimum number of initial observations in order to estimate the low-rank representation accurately. Therefore, before the active selection phase (line 3-9 in Algorithm 2), we have a warm-up phase (line 1-2 in Algorithm 2), where T_0 method-example pairs are sample uniformly at random to evaluate before moving on to the "active" phase.
- The batch size b. The oracle $\mathcal{M}$ must be fit and refit which involves optimizing the ensemble of low-rank factorizations. As the computational cost of this procedure cannot be neglected, we introduce a batch size hyperparameter. At each time step t, we select b examples J_t paired with the method i_t (line 4 in Algorithm 2), evaluate the b method-example pairs (line 6 in Algorithm 2), and update the intermediate variables with these b scores (line 8 in Algorithm 2). Notice that although increasing b is more computationally friendly as it calls the oracle $\mathcal{M}$ less, it may hurt the performance as it reduces the granularity of the decisions, making the algorithm less "active". The choice of b is ultimately a trade-off between computational cost and the performance, however, we will see in the experiments section that performance does not vary drastically for many choices of b.

We analyze these four UCB-E-LRF hyperparameters in our experiment sections, along with the uncertainty scaling η in both UCB-E and UCB-E-LRF.

4 Experiments

4.1 Datasets

To assess the performance of our algorithms under a variety of use cases, we test with three datasets AlpacaEval [44], *Grade School Math 8K (GSM8K)* [18] and *Physical Interaction: Question Answering (PIQA)*, together with different settings of the method set $\mathcal{F}$ and the scoring function e; Table 2 summarizes the statistics of each dataset along with what *method* $\mathcal{F}$ and *scoring function* e. For AlpacaEval, we design two sets of $calF$. The first set $\mathcal{F}$ contains all LLMs reported by [44]. The second $\mathcal{F}$ contains the same LLMs except for the annotator model GPT-4 Turbo. The latter $\mathcal{F}$ that does not contain GPT-4 Turbo makes the learning more challenging and interesting since the annotator, GPT-4 Turbo, is a clear winner among all LLMs. For GSM8K and PIQA, we set $\mathcal{F}$ as a set of prompts to simulate prompt engineering, or a set of LLMs with various sampling configurations, to mimic model selection and hyperparameter tuning. We provide more information about these datasets in Appendix A.1.

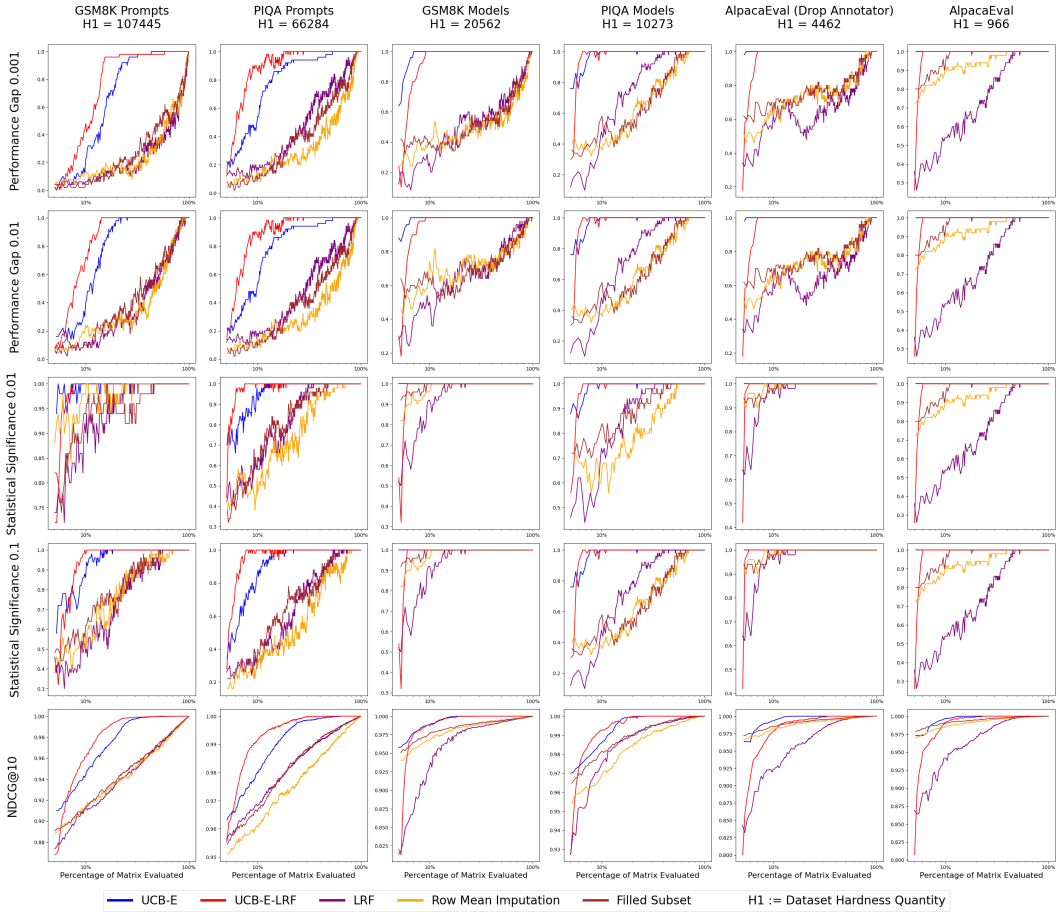

Figure 3: Comparison of baselines and our proposed algorithms on six datasets evaluated with various top 1 precision and NDCG@10. The vertical axes of all plots represent performance of a metric and horizontal axies of all plots represent the percentage of method-example pairs evaluated. All results are aggregated based on 50 trials with different random seeds. Our proposed algorithms: UCB-E and UCB-E-LRF consistently require much less evaluation to achieve the same performance as baselines.

Table 2: Statistics of each dataset used in our experiments. The examples $\mathcal{X}$ of each dataset are the questions from their respective evaluation benchmark. More details of the datasets can be found in Appendix A.1.

Dataset Name	Size $n \times m$	Method $\mathcal{F}$	Scoring Function e	H_1
AlpacaEval	154×805	Various LLMs	GPT4-turbo annotator	966
AlpacaEval (Drop Annotator)	153×805	Various LLMs excluding GPT4-turbo	GPT4-turbo annotator	4462
GSM8K Prompts	205×784	Mistral-7B [32] with different prompts	regex match with correct answer	107445
GSM8K Models	122×1000	Various LLMs and sampling configurations	regex match with correct answer	20562
PIQA Prompts	177×1546	Tulu-7B [58] with different prompts	regex match with correct choice	66284
PIQA Models	103×1000	Various LLMs and sampling configurations	regex match with correct choice	10273

4.2 Metrics

Top 1 Precision. To determine if an algorithm finds the best method, we can directly check if the algorithm's prediction matches with the best method from our empirical data. However, because every method is empirically evaluated on limited number of examples, it is possible that one method has slightly lower average performance than another method whereas if we were to evaluate on more examples, the former would achieve a higher performance. To this end, we calculate top 1 precision by first determining a set of methods we consider equally good and we check if the predicted method from an algorithm is a member of that set. We propose two ways to determine if a set of methods are

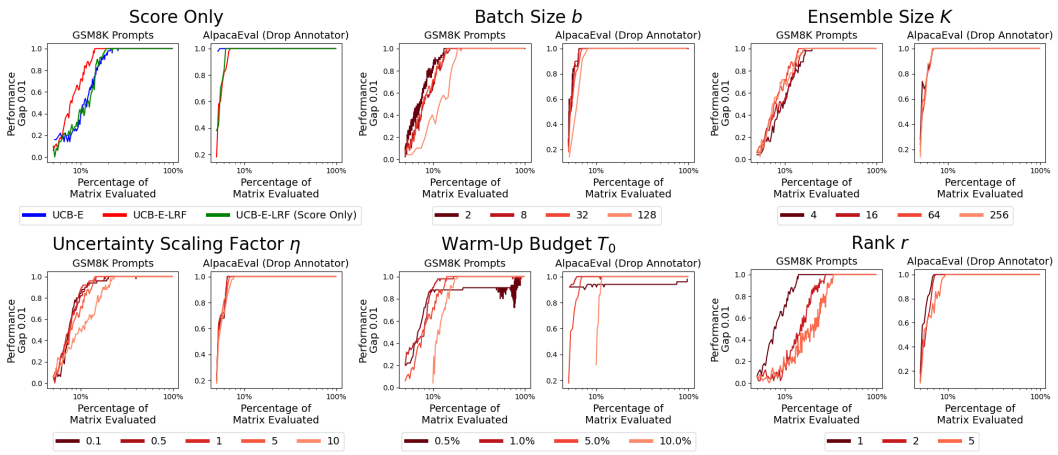

Figure 4: Ablations of effect of hyperparameters on UCB-E-LRF algorithm. The results suggest that our selected hyperparameters are efficient and effective.

equally good: *performance gap* and *statistical significance*. For *performance gap* ϵ top 1 precision, we consider all methods whose performance is within ϵ of the best empirical method to be equally good. For the *statistical significance* p top 1 precision, we perform McNemar's statistical test [47] for each method against the best empirical method. If we cannot reject the null hypothesis that the performance of one method is the same as the best empirical method up to a significance level p, then that method is considered equally good as the best method. We evaluate all baselines and our algorithms on two ϵ values $\{0.001, 0.01\}$, and two p values $\{0.01, 0.1\}$ simulating a diverse need of precision.

NDCG. Although our focus is to identify the best method quickly, it is sometimes also desirable to generally rank top-P promising methods high. We therefore evaluate with normalized discounted cumulative gain (NDCG) at P. *NDCG@*P takes as input the top-P prediction by an algorithm and the higher their true ranks are, the higher the NDCG is. We choose $P = 10$ to evaluate all algorithms.

4.3 Set-Up of Our algorithms and Baselines

We introduce the set-up of our two algorithms, UCB-E and UCB-LRF, together with three more baselines, Row Mean Imputation, Filled Subset and LRF.

UCB-E. Our proposed algorithm as shown in Algorithm 1. We use $\eta = 1$ since this consistently yields the best performance across all datasets.

UCB-E-LRF. Our proposed algorithm as shown in Algorithm 2. For all datasets, we use rank $r = 1$ for low rank factorization with an ensemble size of $K = 64$. We use 5% of data for warm up i.e. $T_0 = 0.05 \times n \times m$ and $\eta = 5$. We use a batch size of $b = 32$.

Row Mean Imputation. In this baseline, we select method-example pairs uniformly at random from all pairs. The score for each method is calculated as the average of all the scores evaluated so far for that method. The detailed algorithmic description is in Algorithm 3.

Filled Subset. Instead of randomly selecting method-example pairs, filled subset first selects an example index j uniformly at random. Then all methods are evaluated on example j. If all methods have been evaluated, the algorithm selects a new example index from remaining ones. The score for each method is calculated as the average of all the scores evaluated so far for that method. The detailed algorithmic description is in Algorithm 4. Although row mean imputation and filled subset do not have learning component, they both produce an unbiased estimate of the real score for each method at all time.

LRF. Similar to row mean imputation, LRF (low rank factorization) randomly selects method-example pairs to evaluate. For estimating the score of each method, LRF calculates the average of all scores for that method. That is, for evaluated examples, the score is the actual observed score, and for examples yet to be evaluated, the score is the estimated score from LRF. The detailed algorithmic description is in Algorithm 5.

4.4 Main Results

In Figure 3, we plot the performance of baselines and our algorithms on all six datasets (columns) as the budget T increases from 5% to 100% of the total number of method-example pairs. In each row, we evaluate the algorithms on a different metric (either top 1 precision with different ϵ, p or NDCG@10). Each line in the figure is the average result over 50 independent trials with different seeds. For example, an average top 1 precision of 0.9 indicates that 45 out of 50 trials predict the best method correctly at the budget level that its x-coordinate represents. In addition, we calculate H_1 as defined in Corollary 1 that quantifies the difficulty of finding the best method on a dataset. Intuitively, a higher H_1 value suggests that the method set $\mathcal{F}$ is large or there are many methods that have similar performance with the best one and distinguishing them can be challenging. In Appendix A.1 Figure 5, we also plot the histogram to show the distribution of performance among $\mathcal{F}$ on these datasets.

How do our algorithms compare with the baselines? As seen from Figure 3, both UCB-E and UCB-E-LRF consistently achieve high precision and NDCG with much less budget compared to the baselines. For example, on AlpacaEval (Drop Annotator), our proposed algorithms can reach precision of 1 with just 8% budget whereas the baselines require 80-90%, an order of magnitude more budget needed. These results suggest that it is entirely possible to identify the best method without exhaustively evaluating all method-example pairs. They further demonstrate that the active selection algorithms (our two algorithms) are more efficient than the non-active algorithms (three baselines). Additionally, the better NDCG performance from our proposed algorithms shows that our methods can more correctly rank top-performance methods.

How is the comparison between our two algorithms UCB-E and UCB-E-LRF? The datasets from left to right are ranked by the hardness indicated by H_1. Interestingly, we find that on easier datasets such as AlpacaEval, UCB-E performs better and saves 2-3% more on budget compared to UCB-E-LRF, while on harder datasets, such as GSM8K Prompts and PIQA Prompts, UCB-E-LRF achieves higher precision faster than UCB-E, saving about 10% in absolute budget. These observations give us a hint on what algorithm to apply in practice.

4.5 More Empirical Analysis

We provide more empirical analysis and the ablation results can be found in Figure 4.

Does H_1 correctly reflect the hardness of a dataset in the empirical experiments? Yes, going through Figure 3 from left to right, as H_1 decreases, the percentage of matrix evaluation needed to reach precision of 1 also generally decreases from more than 20% to just under 5%. Moreover, the H_1 values seem to be related to the tasks. That is, prompt engineering datasets typically have higher H_1 possibly due to the homogeneity of prompt performance with the same LLM. In contrast, datasets that benchmark different LLMs such as AlpacaEval is much easier to find the best performing model.

Score Only Ablation. To study the effect of selecting both next method and next example to evaluate using uncertainty matrix R, we consider an ablation of UCB-E-LRF by reverting the place of using R to the UCB-E: UCB-E-LRF (Score Only) where the upper confidence bound is computed by the confidence interval in original UCB-E in addition to the mean estimation through low rank factorization; the example is uniformly selected, same as UCB-E. The detailed algorithmic description can be found in Algorithm 6. We see that on the hard dataset GSM8K Prompts where UCB-E-LRF has certain benefit over UCB-E, there is a significant gap between UCB-E-LRF and UCB-E-LRF (Score Only). This means that the component of uncertainty matrix R in UCB-E-LRF is crucial to contribute the benefit over UBC-E.

Batch Size b Ablation. Intuitively, a smaller batch size is more flexible can have more fine grained selection. We experiment with $b \in \{2, 8, 32, 128\}$ and as shown in the plot, smallest batch size

gives the best performance especially on harder datasets whereas a batch size of 128 significantly degrades the performance. However, it incurs more overall computational cost due to fitting low rank factorization more often than a smaller batch size; for example when $b = 2$, it takes almost 16 times as much time as $b = 32$ to achieve 1.0 precision, which is ineffective. To achieve a balance of performance and computational cost, we use a batch size of 32 in our experiment.

Ensemble Size K Ablation. We vary the ensemble size $K \in \{4, 16, 64, 256\}$ and find that very small ensemble size gives suboptimal performance on hard datasets. There is almost no performance difference on easy dataset like AlpacaEval (Drop Annotator). The performance is robust to different K as long as it is larger than 64.

Uncertainty Scaling Factor η Ablation. We experiment with different uncertainty scaling values $\{0.1, 0.5, 1, 5, 10\}$. In Figure 4, it can be seen that a certain range of η, from 0.1 to 5, gives similar performance on both two datasets. This demonstrate the robustness of selecting η.

Warm-up Budget T_0 Ablation. Our algorithm UCB-E-LRF by default randomly evaluates 5% of the method-example pairs in the data matrix before actively selects. We analyze the effect of varying the budget T_0 among $\{0.5\%, 1\%, 5\%, 10\%\} \times n \times m$ on the algorithm performance on the two datasets. We see that a very small warm-up budget with 0.5% of data can achieve decent precision initially, but fall behind compared to larger warm-up budget as more data are evaluated. In contrast, a very large warm-up budget of 10% delays the active selection algorithm too much and also achieves suboptimal performance. We therefore use 5% as a generally strong starting point. Note that on AlpacaEval, it is possible to achieve the same performance with even smaller warm-up budget, suggesting more saving is possible.

Rank r Ablation. As discussed in the Algorithm Section, r adjusts the bias-variance trade-off. Empirically we experiment with $r \in \{1, 2, 5\}$ and find that $r = 1$ is consistently better than larger value counterparts. The results can be explained by the fact that a larger r requires more evaluated data in order to prevent overfitting, which might not have advantage in the limited budget setting. Therefore, we recommend using $r = 1$ for all datasets.

5 Related Work

Best-arm identification in multi-arm bandits. The goal of best-arm identification [6, 2] is to find the arm with the highest reward by pulling these arms and getting the feedback. By making an analogy, in our problem method f_i is the arm and the score μ_i of f_i is the reward. There are two ways to define the best-arm identification problem: fixed budget and fixed confidence. In the fixed budget setting, the budget for the arm pulls is fixed and the algorithm is designed for better chances to identify the correct best arm – our problem defined in this paper has the similar evaluation budget. UCB-E [2] and Successive Elimination [2] are two pioneering algorithms proposed for this setting, followed by a line of improvement [30, 38, 10, 39]. Qin [51] states the optimal fixed budget best-arm identification as an open problem. In the setting of fixed confidence, the algorithms [35, 37, 31] work towards fewer number of arms to guarantee the given confidence of getting the best arm. Garivier and Kaufmann [24] gives an optimal algorithm in terms of the minimum of arms to pull. Another extension beyond the setting of fixed budget or confidence is the PAC learning framework, where the target is to maximize the chance of getting an *mostly* best arm, with a tolerance of ε gap to the highest reward [36, 31, 12].

Low-rank factorization for (noisy) matrix completion. As shown in the objective function of Equation 1, low-rank factorization is a non-convex optimization problem. Therefore a line of work focus on how to solve this non-convex optimization [9, 8, 14], while another line of work [13, 7] study the approximation error between the estimated low-rank matrix and the target matrix in terms of p when assuming the observations are i.i.d. sampled with a pre-assumed chance p and the additive noise to each observation is i.i.d. Gaussian noise.

LLM performance evaluation functions. Tremendous effort has been devoted to developing effective evaluation functions to assess the quality of open-ended generations from language models. Early work in this direction such as BLEU [50] and ROUGE [46] are rule-based that use lexical

overlaps to quantify similarity between a generated response and reference. However, lexical overlaps may not align well with the underlying semantics of the text. The shortcomings of these rule-based evaluation functions motivated a line of work studying using language models [65, 55, 63] to evaluate generations. A seminal work in this area is BERTScore which uses embeddings from a BERT model [21] to compute similarity. More recently, LLM-as-a-Judge [66] proposes using instruction-tuned large language models to evaluate generations. Zheng et al. [66] finds that costly proprietary models such as GPT4 have high agreement rate with human.

LLM performance evaluation benchmarks. Diverse benchmarks have been developed to evaluate LLM performance across various domains including natural language understanding on translations and sentiment analysis [4, 54, 56], mathematical and common sense reasoning [18, 29, 3], text retrieval and question answering [61, 33, 20]. We refer readers to [11] for a more comprehensive discussion on existing benchmarks. The commonality of these benchmarks is that a ground truth answer is typically given and once the responses from LLMs are generated, they can be evaluated fairly efficiently without incurring large amount of overhead such as money and compute. Recently, a new type of benchmark or more precisely leaderboard has been created attempting to compare various LLMs to each other. The typical setup involves providing the same prompt to two LLMs and then having their responses compared side by side with another LLM, which determines which response is considered superior and thus wins. Two notable leaderboards are AlpacaEval [44] and Chatbot Arena [15]. AlpacaEval has a static prompt dataset and for each prompt and LLM, the response is compared with that of a baseline model (GPT3 or GPT4). For Chatbot Arena, the prompts are submitted by users and two random LLMs are selected to respond to the user-written prompt. For both benchmarks, to aggregate the overall performance of LLMs, average win rate or ELO score can be calculated from the pairwise comparison statistics. Since the evaluation requires using another LLM to decide the better response, the evaluation incurs additional compute and / or monetary cost than the benchmarks that evaluate models based on ground truth answers.

6 Discussion and Conclusion

One limitation of our proposed algorithms is that we assume there is a two-dimensional fixed-size matrix as our dataset for active method-example selection. In some real-world applications, new rows or columns can be gradually incorporated and the matrix size is dynamic. In other scenarios such as Chatbot Arena, instead of being able to decide what examples to select, we can only select a pair of models to compare for a user-specified example. We leave studying how to minimize budget to find the best method for these new settings as future work. Other promising extensions of our work are to apply more advanced best-arm identification algorithm in the literature, and fixed confidence setting instead of fixed budget setting.

In conclusion, we formulate the problem that when the evaluation for each model-example is resource intensive in regard to money, compute and time, given a certain budget of resource, how can we have still have a great chance to identify the best model on a specific dataset among a set of candidates. We propose two algorithms to tackle the problem, which all sequentially decide which model-example pair to evaluate next by observing the previous evaluated pairs. The first algorithm follows the idea of a classic multi-arm bandit algorithm UCB-E and enjoys the similar theoretical guarantee to lower bound the chance given any budget. The second algorithm, UCB-E-LRF extends UCB-E by leveraging the finding of approximate low-rank of the target evaluation matrix. In the experiment, we show that both algorithms introduced in our paper are significantly better than one that just uniformly samples method-example pairs. Moreover, we identified the condition when the UCB-E or UCB-E-LRF works better than the other. We do not foresee any negative societal impacts for our work.

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A Appendix

A.1 Additional Description of Datasets Used

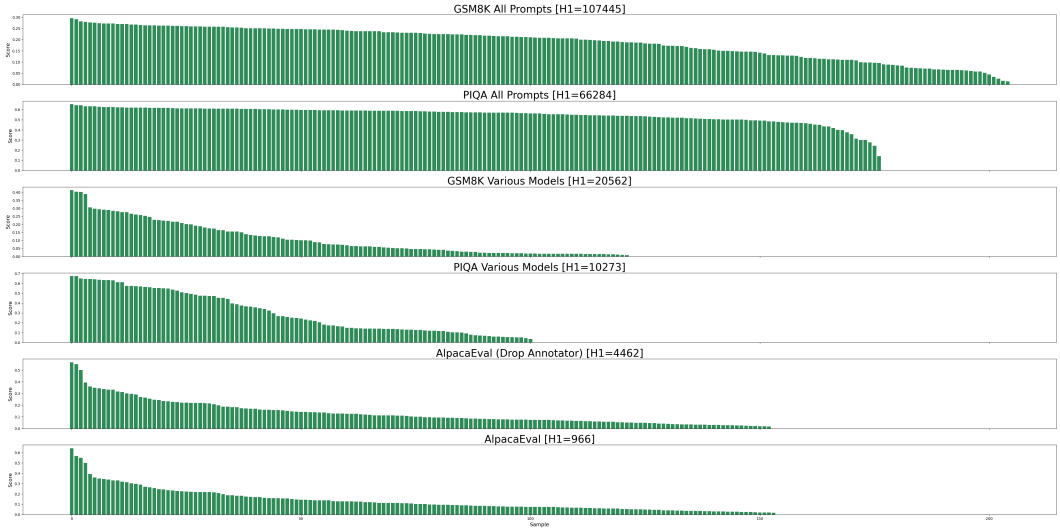

Figure 5: Histogram of model performance on all six datasets along with their H_1 values. It can be seen that datasets such as AlpacaEval have larger gap between the best and second best method leads to a much smaller H_1, making identifying the best method easier.

AlpacaEval and AlpacaEval (Drop Annotator). AlpacaEval benchmarks various models on a fixed set of 805 questions. First, for each model, the responses to these questions are collected. Then, each response is compared against that of a baseline model (GPT4-turbo) by using an LLM judge (in this case, GPT4-turbo as well). We collected the AlpacaEval dataset on May 20, 2024 from the official repository [2]. At that time, there were in total 154 models benchmarked, and hence our dataset size is 154×805. To remove the bias of favoring its own responses, we also create a derived dataset called AlpacaEval (Drop Annotator) where we drop the responses of the annotator model, which leads to a size of 153×805.

GSM8K Prompts and PIQA Prompts. To simulate a prompt engineering use case, we create two datasets: GSM8K Prompts and PIQA Prompts. For these datasets, we ask GPT4 to generate 205 and 177 prompts following the prompt engineering work from Yang et al. [60]. The LLM used to perform inference on the datasets are Mistra-7B [32] and Tulu-7B [58] respectively. We evaluate the prompts on 784 and 1546 questions on the training set (about 10% of the size of the two training sets). For scoring, we extract the final answers from the responses generated by the LLMs and compared them with the ground truth answers (for GSM8K) or ground truth choice (for PIQA).

GSM8K Models and PIQA Models. Lastly, for the model selection and hyperparameter tuning use case, we similarly create two more datasets on GSM8K and PIQA. Instead of each method being a prompt, each method is now a combination of LLM with different sampling configurations. Specifically, we use 11 public available models: GPT2 [3], GPT2-Large [4], CodeLLaMA [5], Tulu-7B

[2] https://github.com/tatsu-lab/alpaca_eval

[3] https://huggingface.co/openai-community/gpt2

[4] https://huggingface.co/openai-community/gpt2-large

[5] https://huggingface.co/codellama/CodeLlama-7b-python-hf

Algorithm 3 Row Mean Imputation ($\mathcal{A}_{rmi}(T, \mathcal{F}, \mathcal{X})$)

Input: The evaluation budget T, a set of methods $\mathcal{F}$ and a set of examples $\mathcal{X}$.

Output: The prediction $\hat{i}^*$ for best method i^*.

$\forall i$, the set of evaluated examples $S_i = \{\}$.
for $t = 1, \cdots, T$ **do**
  Select: Draw $i_t \in [n]$; Draw $j_t \in [m] \backslash S_i$.
  Evaluate: Run inference for the method-example pair (f_{i_t}, x_{j_t}), send the result to the annotator, and receive $e(f_{i_t}, x_{j_t})$; $E^{\text{obs}}_{i_t,j_t} \leftarrow e(f_{i_t}, x_{j_t})$.
  Update: $S_{i_t} \leftarrow S_{i_t} \cup \{j_t\}$.
end for

Return: $\hat{i}^* = \arg\max_i \frac{\sum_{j \in S_i} E^{\text{obs}}_{i,j}}{|S_i|}$

Algorithm 4 Filled Subset ($\mathcal{A}_{fs}(T, \mathcal{F}, \mathcal{X})$)

Input: The evaluation budget T, a set of methods $\mathcal{F}$ and a set of examples $\mathcal{X}$.

Output: The prediction $\hat{i}^*$ for best method i^*.

the set of evaluated examples (same for all methods) $S = \{\}$.
for $t = 1, \cdots, [T/n]$ **do**
  Select: $I_t = \{1, \cdots, n\}$; Draw $j_t \in [m] \backslash S$.
  for $i_t \in I_t$ **do**
    Evaluate: Run inference for the method-example pair (f_{i_t}, x_{j_t}), send the result to the annotator, and receive $e(f_{i_t}, x_{j_t})$; $E^{\text{obs}}_{i_t,j_t} \leftarrow e(f_{i_t}, x_{j_t})$
  end for
  Update: $S_t \leftarrow S_t \cup \{j_t\}$.
end for

Return: $\hat{i}^* = \arg\max_i \frac{\sum_{j \in S} E^{\text{obs}}_{i,j}}{|S|}$

[6], Tulu-2-7B [7], Gemma-7B [8], Phi2 [9], Llema-7B [10], LLaMA-2-7B [11], Mistral-7B [12] and StarCoder-7B [13]. We also have three temperature choices $\{0, 0.5, 1\}$, two maximum decoding length choices $\{128, 512\}$ and two zero-shot prompt choices (directly asking for the answer and *Let's think step by step*). The Cartesian product of all these choices give in total 132 different combinations as our methods. Depending on the dataset, some of these configurations experienced out-of-memory error on a Nvidia 3090 when we collected our data which we drop to simulate real-world scenarios. For the examples, we randomly select 1000 questions from each dataset.

A.2 Algorithmic Description of Baselines and Ablations

The algorithms for Row Mean Imputation, Filled Subset, LRF and UCB-E-LRF (Score Only) are shown in Algorithm 3, 4, 5 and 6 respectively.

[6] https://huggingface.co/TheBloke/tulu-7B-fp16
[7] https://huggingface.co/allenai/tulu-2-7b
[8] https://huggingface.co/google/gemma-7b
[9] https://huggingface.co/microsoft/phi-2
[10] https://huggingface.co/EleutherAI/llemma_7b
[11] https://huggingface.co/meta-llama/Llama-2-7b-chat-hf
[12] https://huggingface.co/mistralai/Mistral-7B-v0.1
[13] https://huggingface.co/bigcode/starcoder2-7b

Algorithm 5 LRF ($\mathcal{A}_{lrf}(T, \mathcal{F}, \mathcal{X}; \mathcal{M}, r, K, T_0, b)$)

Input: The evaluation budget T, a set of methods $\mathcal{F}$ and a set of examples $\mathcal{X}$, the LRF oracle $\mathcal{M}$, the rank r, # solutions in ensemble K, the warm-up budget T_0, the batch size b.
Output: The prediction $\hat{i}^*$ for best method i^*.

1: Uniformly sample T_0 method-example pairs from $[n] \times [m]$ and get the observed scoring matrix $E^{\text{obs}} \in ([0,1] \cup \{?\})^{n \times m}$ and the observation matrix $O \in \{0,1\}^{n \times m}$ w.r.t. these T_0 evaluations.
2: **for** $t = T_0, \cdots, T_0 + [(T - T_0)/b]$ **do**
3: **Select:** Draw I_t by sampling b distinct indices uniformly at random from $[n]$; Draw J_t by sampling b distinct indices uniformly at random from $[m]$.
4: **for** $(i_t, j_t) \in (I_t, J_t)$ **do**
5: **Evaluate:** Run inference for the method-example pair (f_{i_t}, x_{j_t}), send the result to the annotator, and receive $e(f_{i_t}, x_{j_t})$; $E^{\text{obs}}_{i_t,j_t} \leftarrow e(f_{i_t}, x_{j_t})$
6: **end for**
7: **Update:** $\forall j_t \in J_t, O_{i_t,j_t} \leftarrow 1$; $\hat{E} \leftarrow \mathcal{M}(E^{\text{obs}}, O; r, K)$.
8: **end for**
Return: $\hat{i}^* = \arg\max_i \frac{1}{m} \sum_{j=1}^{m} (O_{i,j} E^{\text{obs}}_{i,j} + (1 - O_{i,j}) \hat{E}_{i,j})$

Algorithm 6 UCB-E-LRF (Score Only) ($\mathcal{A}_{uel-so}(T, \mathcal{F}, \mathcal{X}; \mathcal{M}, r, K, T_0, b, \eta)$)

Input: The evaluation budget T, a set of methods $\mathcal{F}$ and a set of examples $\mathcal{X}$, the LRF oracle $\mathcal{M}$, the rank r, # solutions in ensemble K, the warm-up budget T_0, the batch size b, the uncertainty scaling η.
Output: The prediction $\hat{i}^*$ for best method i^*.

1: Uniformly sample T_0 method-example pairs from $[n] \times [m]$ and get the observed scoring matrix $E^{\text{obs}} \in ([0,1] \cup \{?\})^{n \times m}$ and the observation matrix $O \in \{0,1\}^{n \times m}$ w.r.t. these T_0 evaluations.
2: $\hat{E} \leftarrow \mathcal{M}(E^{\text{obs}}, O)$, $\forall i, B_i \leftarrow \frac{1}{m} \sum_{j=1}^{m} (O_{i,j} E^{\text{obs}}_{i,j} + (1 - O_{i,j}) \hat{E}_{i,j}) + \sqrt{\frac{\eta}{\sum_{j=1}^{m} (O_{i,j})}}$
3: **for** $t = T_0, \cdots, T_0 + [(T - T_0)/b]$ **do**
4: **Select:** Draw $i_t \in \arg\max_i B_i$; Draw J_t by sampling b distinct indices uniformly at random from $[m]$.
5: **for** $j_t \in J_t$ **do**
6: **Evaluate:** Run inference for the method-example pair (f_{i_t}, x_{j_t}), send the result to the annotator, and receive $e(f_{i_t}, x_{j_t})$; $E^{\text{obs}}_{i_t,j_t} \leftarrow e(f_{i_t}, x_{j_t})$
7: **end for**
8: **Update:** $\forall j_t \in J_t$, $O_{i_t,j_t} \leftarrow 1$; $\hat{E} \leftarrow \mathcal{M}(E^{\text{obs}}, O; r, K)$; $\forall f_i \in \mathcal{F}, B_i \leftarrow \frac{1}{m} \sum_{j=1}^{m} (O_{i,j} E^{\text{obs}}_{i,j} + (1 - O_{i,j}) \hat{E}_{i,j}) + \sqrt{\frac{\eta}{\sum_{j=1}^{m} (O_{i,j})}}$.
9: **end for**
Return: $\hat{i}^* = \arg\max_i \frac{1}{m} \sum_{j=1}^{m} (O_{i,j} E^{\text{obs}}_{i,j} + (1 - O_{i,j}) \hat{E}_{i,j})$