

26 Aug 2024

Introduction

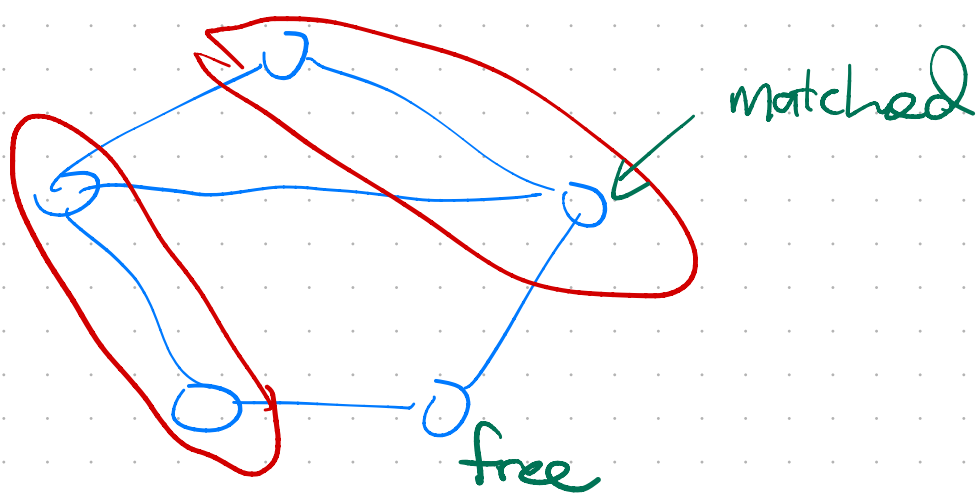
Max Bipartite Matching

<https://www.cs.cornell.edu/courses/cs6820/2024fa>

Office hrs: Gates 317 Weds 3:30-5:00

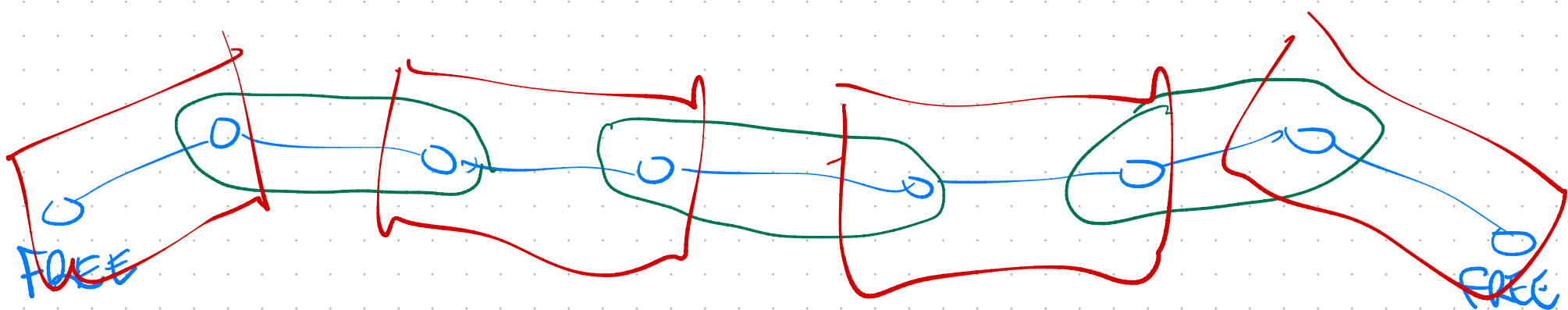
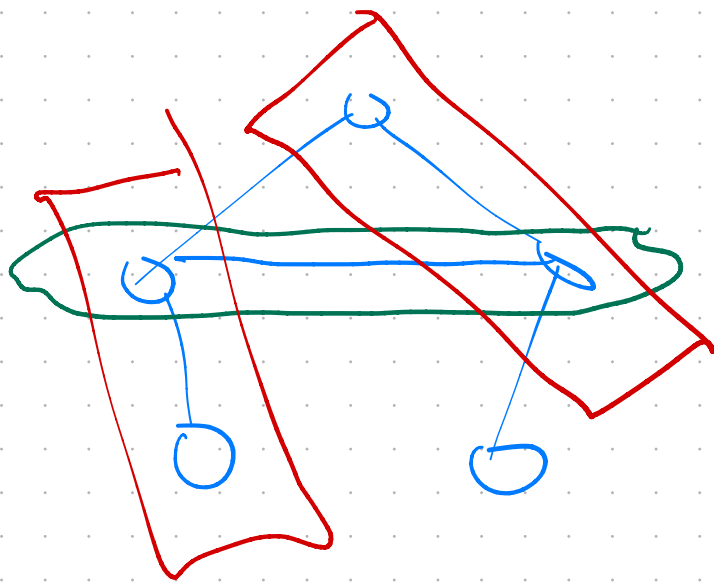
Graphs and Matchings

Def. A matching in an undirected graph is a subset of edges such that every vertex belongs to either 1 (matched) or \emptyset (free) edges of the matching.



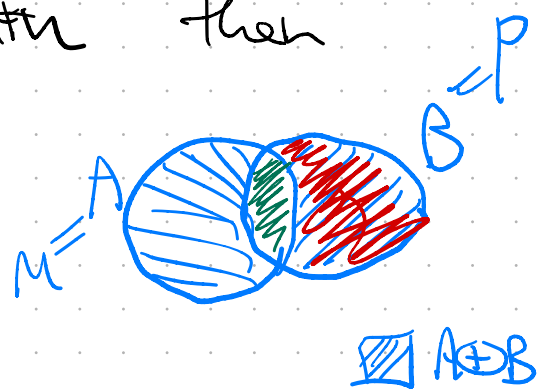
The maximum matching problem: given graph G , find matching with as many edges as possible.

Given a graph G and a suboptimal matching M , how to improve it?



Def. An M -augmenting path in G (when G is a graph, M is a matching) is a path P in G that starts and ends at free vertices and alternates between edges in M and not in M .

If P is an M -augmenting path then $M \oplus P$ is a matching, and $|M \oplus P| = |M| + 1$.



Naive max - matching algorithm.

1. Initialize $M = \emptyset$
2. While G contains an M -augmenting path P , update $M \leftarrow M \oplus P$.
3. Output M .

Why correct?

Lemma: If M_0 and M_1 are matchings in G with $|M_0| < |M_1|$ then $M_0 \oplus M_1$ contains an M_0 -augmenting path.