10 Nov 2023 — Multicommodity Flow via Multiplicative Weights

Recall. Given sequence of payoff vectors $u_1, \ldots, u_T \in [0,1]^K$

if we define a seq. of weight vectors $w_1, \ldots, w_T \in \mathbb{R}_+^K$

and probability vectors $p_t = \frac{w_t}{\|w_t\|_1}$ by

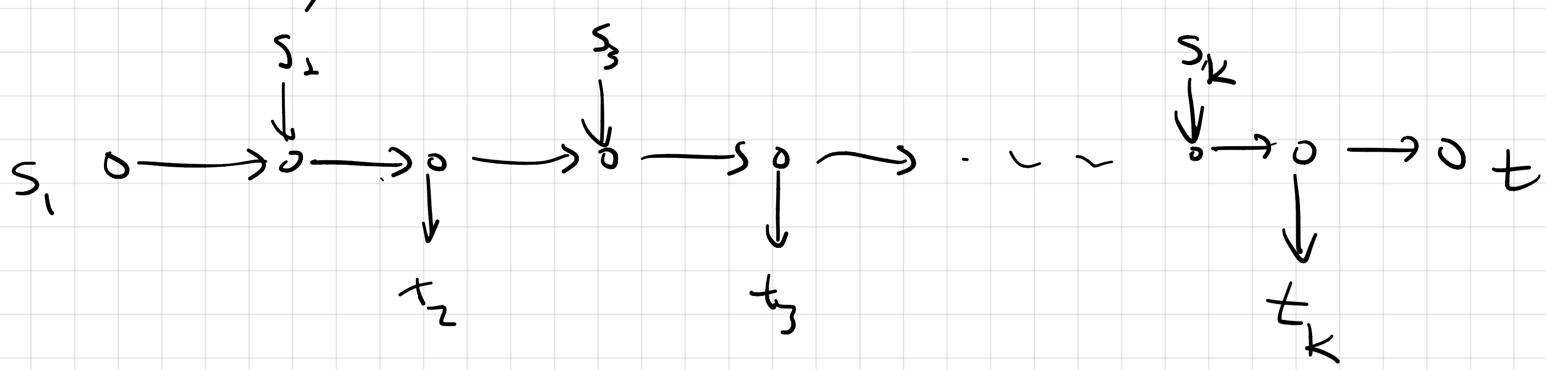
$$w_{ti} = \exp\left(\varepsilon \cdot \sum_{s<t} u_{si}\right)$$

← prob of picking i at time t is proportional to exponential of how well it did in the past

then

$$\sum_{t=1}^{T} \langle u_t, p_t \rangle \geq (1-\varepsilon) \max_i \sum_{t=1}^{T} u_{ti} - O\left(\frac{\ln K}{\varepsilon}\right).$$

Multicommodity Flow. (Max Total Throughput)



$\mathcal{P} = \{$ paths from s_i to t_i for some $i \}$

[Primal]

$$\max \sum_{P \in \mathcal{P}} x_P$$

s.t. $\sum_{P:\, e \in P} x_P \leq c_e \quad \forall e \quad [y_e]$

$x_P \geq 0$

[Dual]

$$\min \sum c_e y_e$$

s.t. $\sum_{e \in P} y_e \geq 1 \quad \forall P \in \mathcal{P},$

$y_e \geq 0 \quad \forall e$

Say opt. value of primal & dual is v^*.

[Primal]

$$\max \sum_{p \in \mathcal{P}} x_p$$

$$\text{s.t.} \quad \sum_{p:\, e \in p} x_p \le c_e \qquad \forall e \quad [y_e]$$

$$x_p \ge 0$$

[Dual]

$$\min \sum c_e y_e$$

$$\text{s.t.} \quad \sum_{e \in p} y_e \ge 1 \qquad \forall p \in \mathcal{P}$$

$$y_e \ge 0 \qquad \forall e$$

If (y_e) is feasible dual of value V,

$$\hat{y}_e = \frac{c_e y_e}{V}$$

Renormalized $(\hat{y}_e)$ satisfies

$$\sum \hat{y}_e = 1$$

$$\sum_{e \in p} \frac{\hat{y}_e}{c_e} \ge \frac{1}{V} =: u$$

$$\hat{y}_e \ge 0$$

Dual equiv. to

$$\max \quad u$$

$$\text{s.t.} \quad \sum_{e \in p} \frac{\hat{y}_e}{c_e} \ge u \qquad \forall p \in \mathcal{P}$$

$$\sum_e \hat{y}_e = 1$$

$$\hat{y}_e \ge 0$$

Kicker – goalie game:

1. Goalie moves first, announces distribution over edges e.
2. Kicker responds by choosing one path $p \in \mathcal{P}$.
3. Goalie randomly samples e from its distribution.
4. Goalie's payoff is $\frac{1}{c_e}$ if $e \in P$. Kicker's payoff $-\frac{1}{c_e}$.