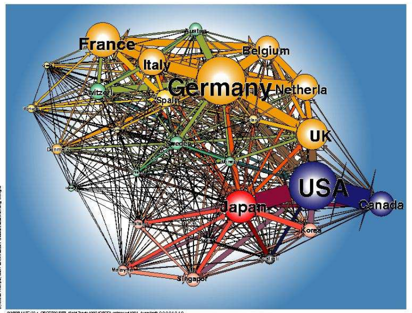
The Impact of Network Topology on Pure Nash Equilibria in Graphical Games

Bistra Dilkina, Carla Gomes and Ashish Sabharwal

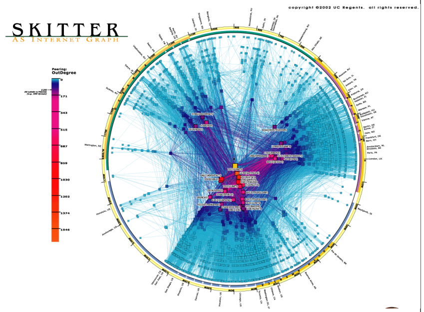
Graphical models of strategic interaction

Motivation: interactions between agents are local

International Trade



Internet Connectivity



Game-theoretic model of strategic interactions :

-Undirected graph G captures locality of interactions

-Each player p is represented by a vertex

-A player's decisions depend directly only on his neighbors in the graph, N_p

-Each player p has a set of actions A_p and a payoff matrix U_p over the actions of p and N_p

Stability in non-cooperative setting:

-Pure Nash Equilibrium (PNE)

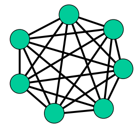
-Each player chooses an action that maximizes his payoff

-No player has an incentive to unilaterally deviate in order to increase his payoff

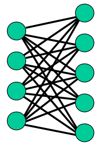
Research Question: what is the relationship between the topological properties of the network and stability?

Answer: different interaction graph topologies lead to radically different behavior

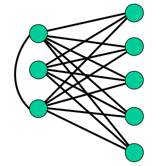
Facilitate the existence of PNE:



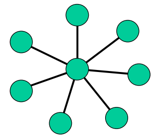
Complete graph (V_n)



Bipartite graph (X_n,Y_n)

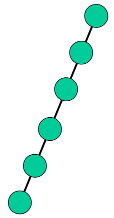


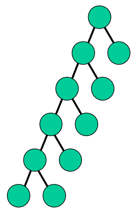
Augmented Bipartite graph (X_n,Y_n,E_{Xn})

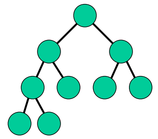


Star

Hinder the existence of PNE:







•**Theorem (Bipartite).** *Given a k-action random-payoff game on a complete bipartite graph $G_n=K(X_n,Y_n)$, $Pr[PNE]\to 1-1/e$ as $n\to\infty$.*

•**Theorem (Aug Bipartite).** *Given a k-action random-payoff game on an augmented bipartite graph $G_n=K(X_n,Y_n,E_X)$ such that $|X_n\cup Y_n|=n$, $|X_n|=m$ and $1-\left(1-\frac{1}{k^m}\right)^{k^m}$ $n/3-m\to\infty$ as $n\to\infty$, $Pr[PNE]\to$ as $n\to\infty$.*

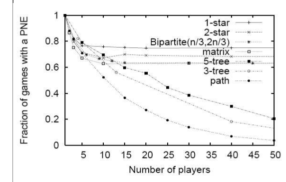
•**Corollary (Star).** *Given a 2-action random-payoff game on a star $Pr[PNE]\to 0.75$ as $n\to\infty$.*

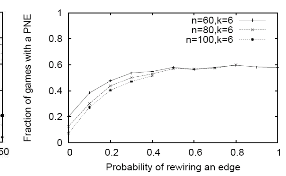
•**Theorem (Trees).** *Given a 2-action random-payoff game on a tree graph T_n with diameter that grows without bound with n, $Pr[PNE]\to 0$ as $n\to\infty$.*

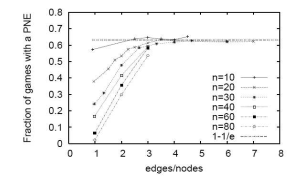
Summary of theoretical results:

Topology	Prob. of PNE
star	0.75
2-star	0.683
augmented bipartite graph	$1-\left(1-\frac{1}{2^m}\right)^{2^m}$
bipartite	$1-1/e\approx 0.632$
matrix	$1-1/e\approx 0.632$
tree	0
path	0

Experimental Results







Small World Graphs: start with a structured k-ring, for each edge with probability p rewire one endpoint randomly

G(n,m) Random graphs: construct a graph with n vertices and m edges, adding each edge between two random vertices

Summary: Shortcutting the long range dependencies between players through random re-wirings or adding random edges increases the probability of PNE