Lecture 9: Monte Carlo Rendering

CS 6620, Spring 2009

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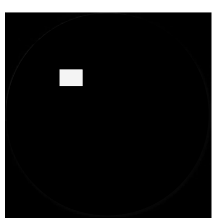
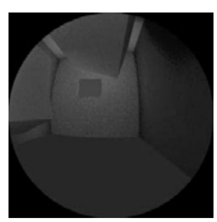
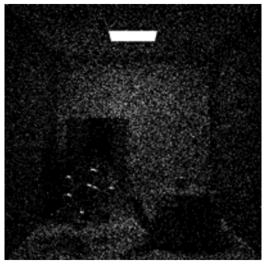
Next Event Estimation

$$L(x \rightarrow \Theta) = L_e + L_{direct} + L_{indirect}$$

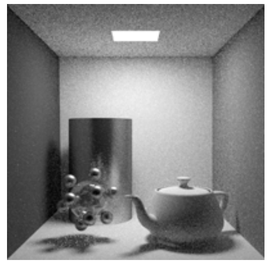
$= L_e + \int_{\Omega_x}$  $\cdot f_r \cdot \cos + \int_{\Omega_x}$  $\cdot f_r \cdot \cos$

- So … sample direct and indirect with separate MC integration

Comparison



Without N.E.E.



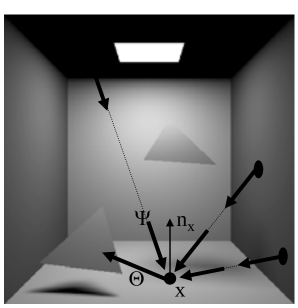
With N.E.E.

16 samples/pixel

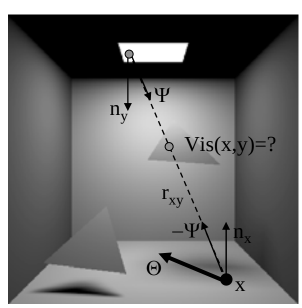
Direct Illumination

$$L(x \to \Theta) = \int_{A_{source}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L(y \to \Psi) \cdot G(x,y) \cdot dA_y$$

$$G(x,y) = \frac{\cos(n_x, \Theta)\cos(n_y, \Psi) Vis(x,y)}{r_{xy}^2}$$

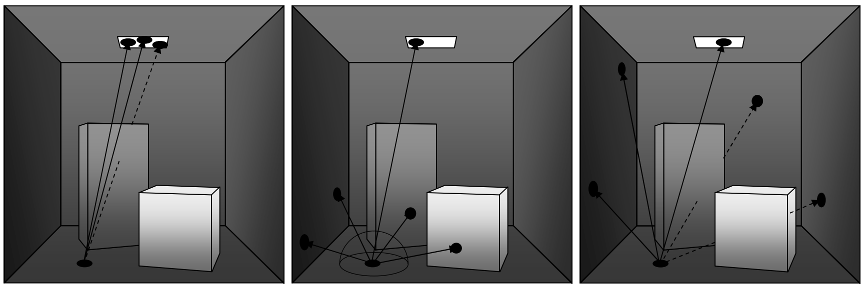


hemisphere integration



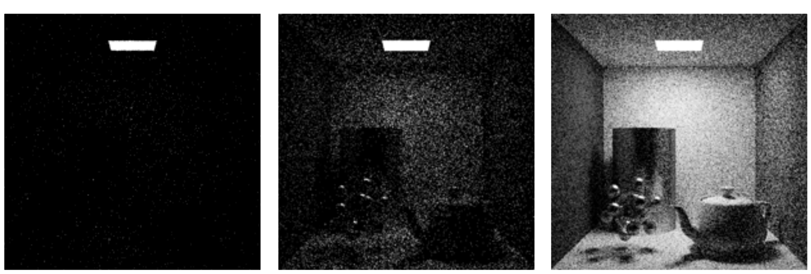
area integration

Direct path generators



Light source sampling	Hemisphere sampling	Surface sampling
- L_e non-zero - 1 visibility term in estimator	- L_e can be 0 - no visibility in estimator	- L_e can be 0 - 1 visibility term in estimator

Alternative direct paths



1 path / point | 16 paths / point | 256 paths / point

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Parameters

- How many paths ("shadow-rays")?
  - Total?
  - Per light source? (~intensity, importance, …)
- How to distribute paths within light source?
  - Uniform, Solid angle, area
  - What about light distribution?

Parameters

- Multiple lights
  - Uniform
  - Proportional to power
  - Proportional to area

$$E(x) = \frac{1}{N}\sum_{i=1}^{N} \frac{f_r L_{source} \cos\theta_x \cos\theta_{\bar{y}_i}}{r^2_{x\bar{y}_i}} Vis(x, \bar{y}_i) \frac{1}{p_L(k_i)\, p(y_i \mid k_i)}$$

Formulation over all lights

- When M is large, each direct lighting sample is very expensive
- We would like to importance sample the lights
- Instead of M integrals

$$L(x \to \Theta) = \sum_{i=1}^{M} \int_{A_{source}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L_{source}(y \to -\Psi) \cdot G(x, y) \cdot dA_y$$

- Formulation over 1 integration domain

$$L(x \to \Theta) = \int_{A_{all\ lights}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L_{source}(y \to -\Psi) \cdot G(x, y) \cdot dA_y$$

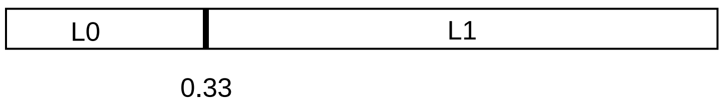
How to sample the lights?

- A discrete pdf $p_L(k_i)$ picks the light k_i
- A surface point is then picked with pdf $p(y_i|k_i)$
- Estimator with N samples:

$$E(x) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_r L_{source} G(x, \bar{y}_i)}{p_L(k_i)\, p(y_i \mid k_i)}$$

Example for 2 lights

- Light 0 has power 1, Light 1 has power 2
- Using power for pdf:
  - $p_L(L_0) = 1/3$, $p_L(L_1) = 2/3$



- Overall pdf $p(y) = \frac{1}{3} p_{L_0}(y) + \frac{2}{3} p_{L_1}(y)$

Example for 2 lights

- Pick a random value: ξ_0
- If $\xi_0 < \frac{1}{3}$
- Sample Light 0 and compute estimate e0
- Overall estimate is $\frac{e_0}{\frac{1}{3}}$

Example for 2 lights

- If

$$\frac{1}{3} \leq \xi_0 < 1$$

- Sample Light 1 and compute estimate e1

- Overall estimate is $\frac{e_1}{\frac{2}{3}}$

How to sample light?

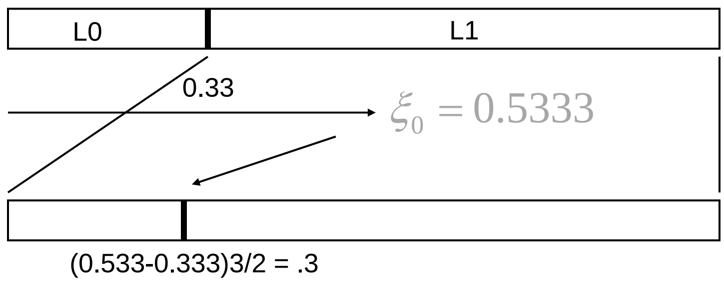
- Once light is picked, can pick two random numbers ξ_1, ξ_2 according to $p_{L0}(y)$, $p_{L1}(y)$

- To decrease variance we should reuse ξ_0

- But, already used information in ξ_0 to pick the light

Example for 2 lights

- Rescale ξ_0 $\qquad \xi'_0 = \frac{3}{2}(\xi_0 - \frac{1}{3})$



- Use (ξ'_0, ξ_1) to pick samples on light 1

Strategies for picking light

– Uniform $p_L(k) = \frac{1}{M}$

– Area $p_L(k) = \frac{A_k}{\sum A_k}$

– Power $p_L(k) = \frac{P_k}{\sum P_k}$

Don’t take visibility into account

Direct Illumination

$$L(x \to \Theta) = \int_{A_{source}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L(y \to \Psi) \cdot G(x, y) \cdot dA_y$$

$$G(x, y) = \frac{\cos(n_x, \Theta)\cos(n_y, \Psi) Vis(x, y)}{r_{xy}^2}$$

Research on many lights

- Ward '91
- Shirley, Wang, Zimmerman '94
- Fernandez, Bala, Greenberg '02
- Wald and Slusallek '03
- Walter et al. '05

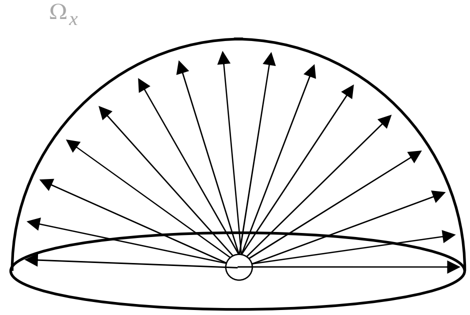
Stochastic Ray Tracing

- Sample area of light source for direct term
- Sample hemisphere with random rays for indirect term
- Optimizations:
  - Stratified sampling
  - Importance sampling
  - Combine multiple probability density functions into a single PDF

Sampling strategies

- Uniform sampling over the hemisphere

$$L(x \to \Theta) = \int_{\Omega_x} L(x \leftarrow \Psi) \cdot f_r(\Psi \leftrightarrow \Theta) \cdot \cos(\Psi, n_x) \, d\omega_\Psi$$

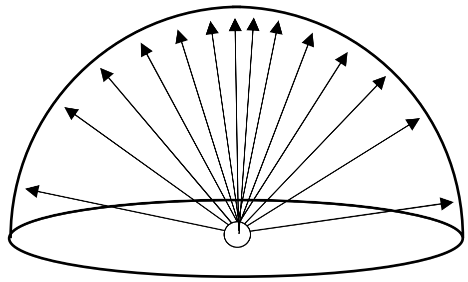


$$p(\Theta) = 1/(2\pi)$$

Sampling strategies

- Sampling according to the cosine factor

$$L(x \to \Theta) = \int_{\Omega_x} L(x \leftarrow \Psi) \cdot f_r(\Psi \leftrightarrow \Theta) \cos(\Psi, n_x) \cdot d\omega_\Psi$$

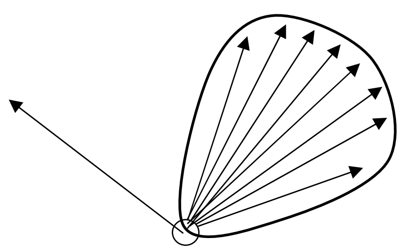


$$p(\Theta) = \cos\theta / \pi$$

Sampling strategies

- Sampling according to the BRDF

$$L(x \to \Theta) = \int_{\Omega_x} L(x \leftarrow \Psi) \cdot f_r(\Psi \leftrightarrow \Theta) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$



$$p(\Theta) \sim f_r(\Theta \leftrightarrow \Psi)$$

Example: sample according to BRDF

- Discrete pdf q_1, q_2, q_3 $\quad q_1 + q_2 + q_3 = 1$

$$L_{indirect} = L_{diffuse} + L_{specular}$$

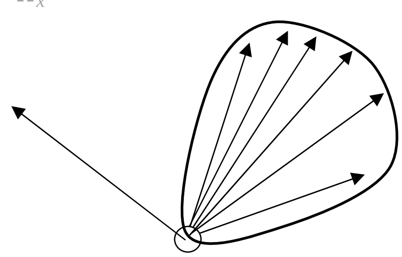
$$f_r = k_d + k_s \cos^n(\Psi, \Theta_s)$$

$$\langle L_{indirect} \rangle = \begin{cases} \dfrac{L(x \leftarrow \Psi_i) k_d \cos(N, \Psi_i)}{q_1 p_1(\Psi_i)} \mid \xi < q_1 \\ \dfrac{L(x \leftarrow \Psi_i) k_s \cos^n(\Theta_s, \Psi_i) \cos(N, \Psi_i)}{q_2 p_2(\Psi_i)} \mid q_1 \le \xi < q_1 + q_2 \\ 0 \mid otherwise \end{cases}$$

Sampling strategies

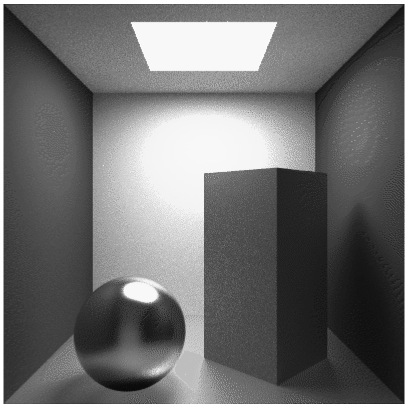
- Sampling according to the BRDF times the cosine

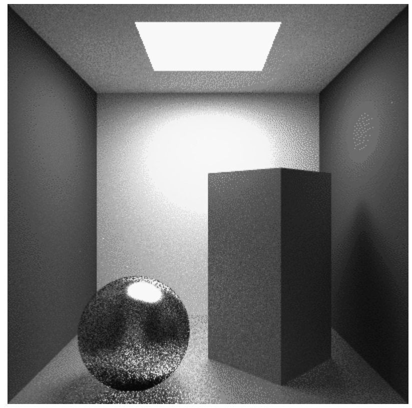
$$L(x \to \Theta) = \int_{\Omega_x} L(x \leftarrow \Psi) \cdot f_r(\Psi \leftrightarrow \Theta) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$



$$p(\Theta) \sim f_r(\Theta \leftrightarrow \Psi) \cos\theta$$

Comparison





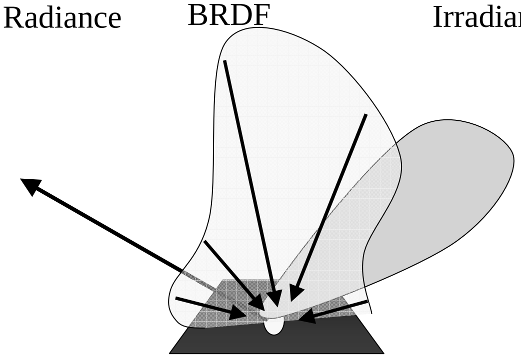
With importance sampling (brdf on sphere)

Without importance sampling

Multi-Importance-Sampling

$$L(x \to \Theta) = \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \, L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

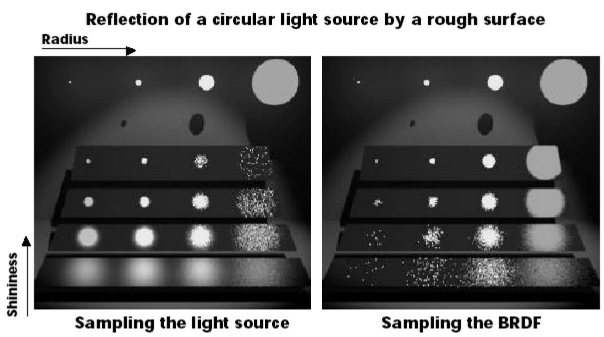
Radiance BRDF Irradiance



Importance Sampling

- Say we want to sample according to cosine term, BRDF, ….
- How do we blend the different sampling algorithms together?

Example



- Want to merge both techniques of sampling
  - How?

Balance Heuristic

- Two sampling techniques: j^{th} sample
  - $X_{1,j}$ with pdf $p_1(x)$, $X_{2,j}$ with pdf $p_2(x)$
  - Estimator Y_j for j^{th} sample

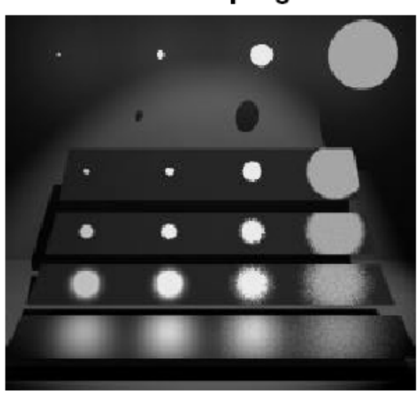
$$Y_{1,j} = \frac{f(X_{1,j})}{p_1(X_{1,j})} \qquad Y_{2,j} = \frac{f(X_{2,j})}{p_2(X_{2,j})}$$

$$Y_j = w_1 Y_{1,j} + w_2 Y_{2,j}$$

$$w_1(x) = \frac{p_1(x)}{p_1(x) + p_2(x)} \qquad w_2(x) = \frac{p_2(x)}{p_1(x) + p_2(x)}$$

Multiple Importance Sampling

Combine both sampling methods



From Veach and Guibas
Read Chapter 9, Veach

Efficiency

$$Efficiency \propto \frac{1}{Variance \bullet Cost}$$

- Some techniques:
  - Importance sampling
  - Sampling patterns
    - Stratified, Quasi-Monte Carlo
  - Many others

Indirect paths

```
indirectIllum (x, theta)
       est_rad = 0;
       if (no absorption) {
         for (i=0; i<n; i++)
              sample direction psi on hemisphere;
              y = trace (x, psi);
              est_rad +=(radiance(y,-psi)*BRDF*cos())/p(psi);
       est_rad = est_rad / n;
       return(est_rad/(1-absorption));


Compute radiance (x, dir){
  estRadiance = Le (x, dir);
  estRadiance += directIllum (x, dir);
  estRadiance += indirectIllum (x, dir);
  return estRadiance;
}
```