Lecture 3: Rendering Equation

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Radiometry

- Radiometry: measurement of light energy
- Defines relation between
  - Power
  - Energy
  - Radiance
  - Radiosity

Hemispherical coordinates

- Defined a measure over hemisphere
- $d\omega$ = direction vector
- Differential solid angle

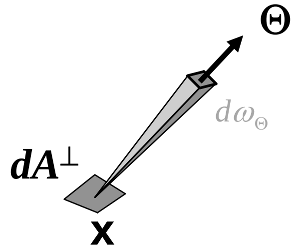
$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

Radiance

- Radiance is radiant energy at x in direction θ: 5D function
  - $L(x \to \Theta)$: Power
    - per unit projected surface area
    - per unit solid angle

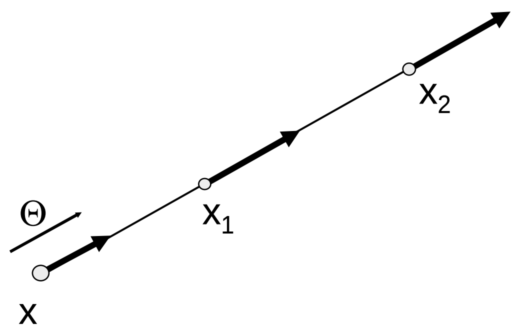
$$L(x \to \Theta) = \frac{d^2P}{dA^{\perp} d\omega_{\Theta}}$$

  - units: Watt / m^2.sr



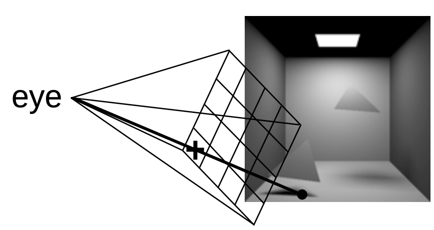
Why is radiance important?

- Invariant along a straight line (in vacuum)



Why is radiance important?

- Response of a sensor (camera, human eye) is proportional to radiance



- Pixel values in image proportional to radiance received from that direction

Relationships

- Radiance is the fundamental quantity

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

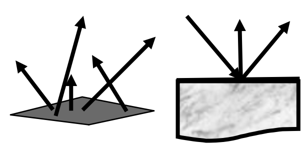
- Power:

$$P = \int_{Area} \int_{Solid\ Angle} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

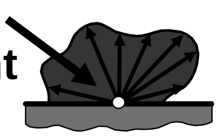
- Radiosity:

$$B = \int_{Solid\ Angle} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta}$$

Outline

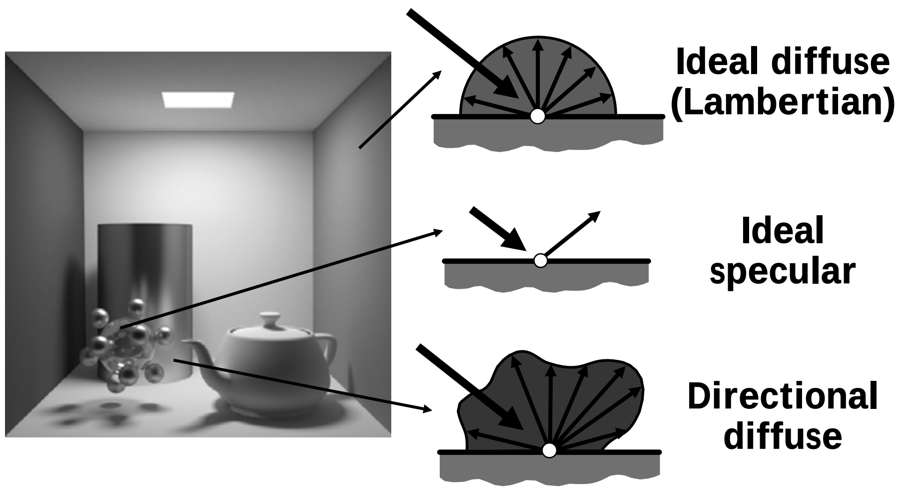


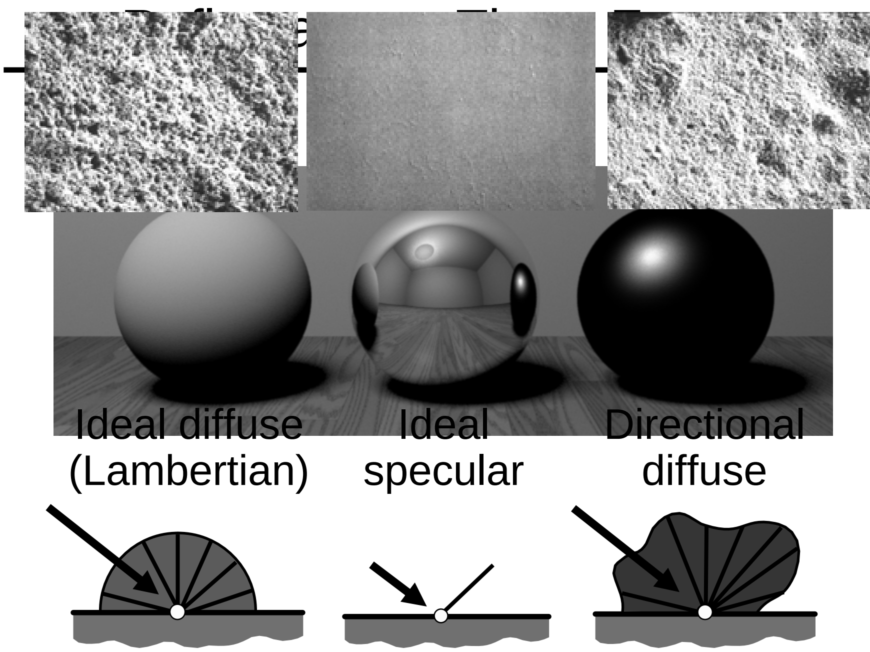
- Light Model
- Radiance
- **Materials: Interaction with light**



- Rendering equation

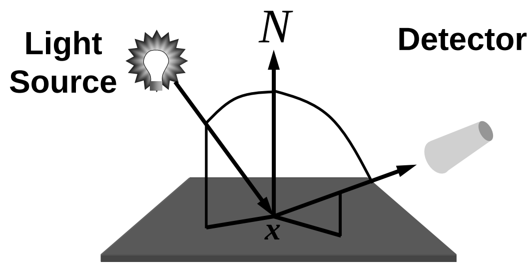
Materials - Three Forms





BRDF

- Bidirectional Reflectance Distribution Function



$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)}$$

$\frac{W}{m^2 sr}$ $\frac{W}{m^2}$

Definition of BRDF

- 6D function? 4D function?
  - Why?

- Wavelength-dependent

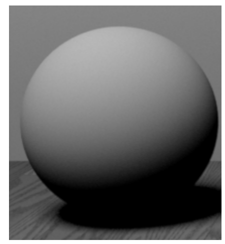
$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)}$$

$$= \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi)\cos(N_x, \Psi)d\omega_\Psi}$$

BRDF special case: ideal diffuse

Pure Lambertian

$$f_r(x, \Psi \to \Theta) = \frac{\rho_d}{\pi}$$



$$\rho_d = \frac{Energy_{out}}{Energy_{in}} \qquad 0 \le \rho_d \le 1$$

Properties of the BRDF

- Reciprocity:

$$f_r(x, \Psi \to \Theta) = f_r(x, \Theta \to \Psi)$$

- Therefore, notation: $f_r(x, \Psi \leftrightarrow \Theta)$
- Important for bidirectional tracing

Properties of the BRDF

- Bounds:

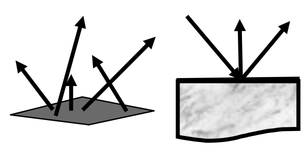
$$0 \leq f_r(x, \Psi \leftrightarrow \Theta) \leq \infty$$

- Energy conservation:

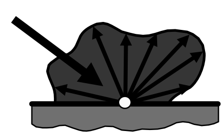
$$\forall \Psi \quad \int_{\Theta} f_r(x, \Psi \leftrightarrow \Theta) \cos(N_x, \Theta) d\omega_{\Theta} \leq 1$$

Outline

- Light Model



- Radiance
- Materials: Interaction with light



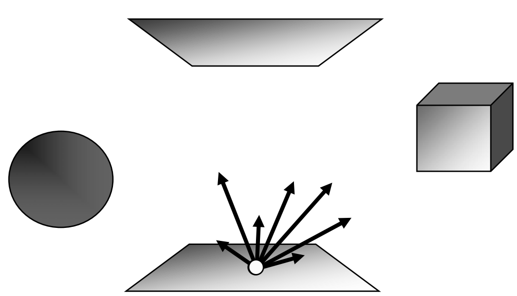
- Rendering equation

Light Transport

- Goal
  - Describe steady-state radiance distribution in scene
- Assumptions:
  - Geometric Optics
  - Achieves steady state instantaneously
- Related:
  - Neutron Transport (neutrons)
  - Gas Dynamics (molecules)

Radiance represents equilibrium

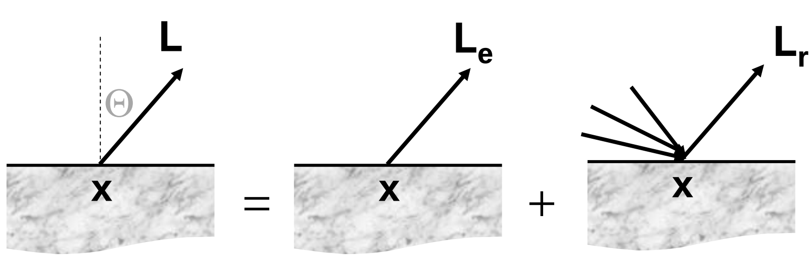
- Radiance values at all points in the scene and in all directions expresses the equilibrium
- 4D function: only on surfaces



Rendering Equation (RE)

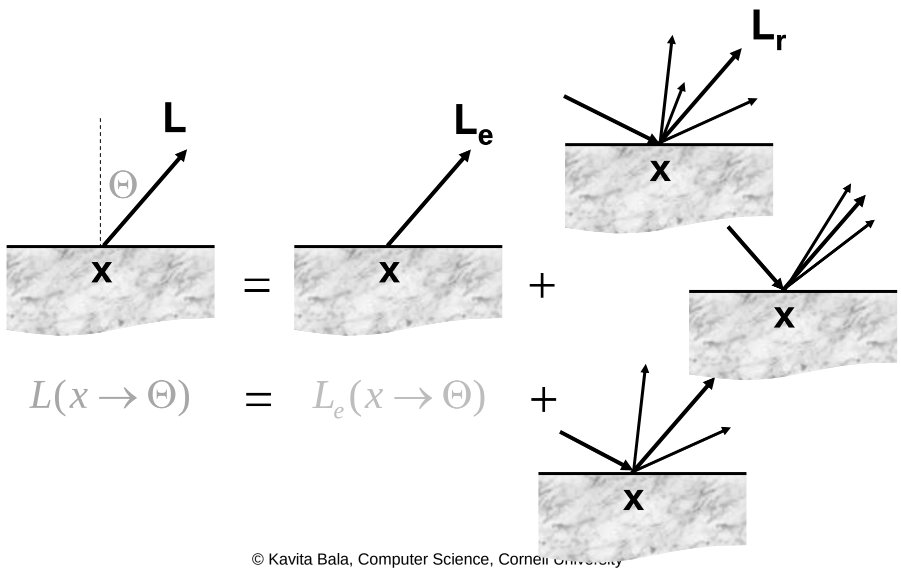
- RE describes energy transport in scene
- Input
  - Light sources
  - Surface geometry
  - Reflectance characteristics of surfaces
- Output: value of radiance at all surface points in all directions

Rendering Equation



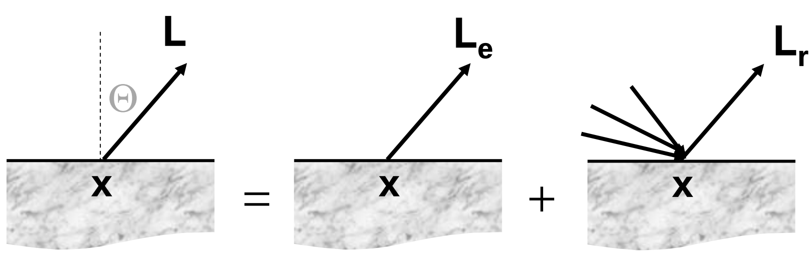
$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + L_r(x \rightarrow \Theta)$$

Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) +$$

Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) \cdots$$

Rendering Equation

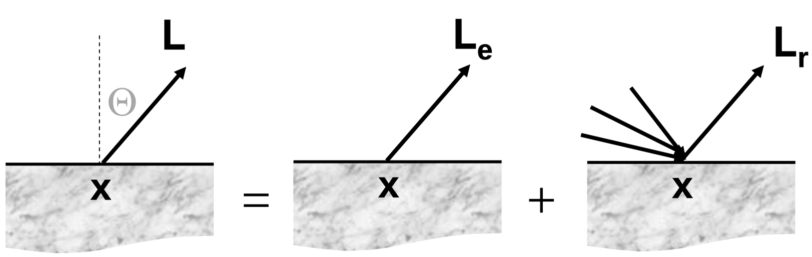
$$f_r(x, \Psi \leftrightarrow \Theta) = \frac{dL(x \to \Theta)}{dE(x \leftarrow \Psi)}$$

$$dL(x \to \Theta) = f_r(x, \Psi \leftrightarrow \Theta) dE(x \leftarrow \Psi)$$

$$dL(x \to \Theta) = f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

$$L_r(x \to \Theta) = \int_{hemisphere} f_r(x, \Psi \leftrightarrow \Theta) L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi$$

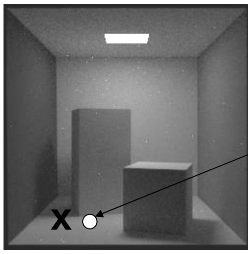
Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi) d\omega_\Psi$$

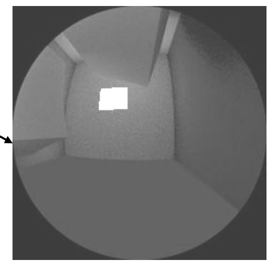
- **Applicable for each wavelength**

Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{hemisphere} L(x \leftarrow \Psi)\, f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N_x}, \Psi)\, d\omega_\Psi$$

incoming radiance

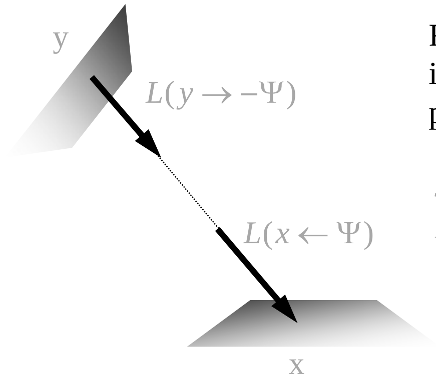


Summary

- Geometric Optics
- Goal:
  - to compute steady-state radiance values in scene
- Rendering equation:
  - mathematical formulation of problem that global illumination algorithms must solve

RE: Area Formulation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

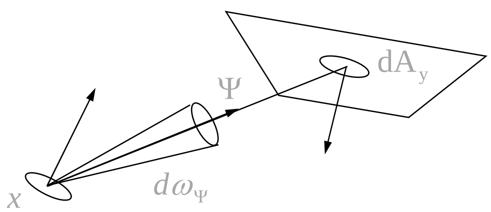


Ray-casting function: what is the nearest visible surface point seen from x in direction Ψ?

$$y = vp(x, \Psi)$$
$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

Rendering Equation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos\theta_y}{r_{xy}^2}$$

Rendering Equation: visible surfaces

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

Coordinate transform ↓

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{y \text{ on all surfaces}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi)\ \cos\theta_x \cdot \frac{\cos\theta_y}{r_{xy}^2} \cdot dA_y$$

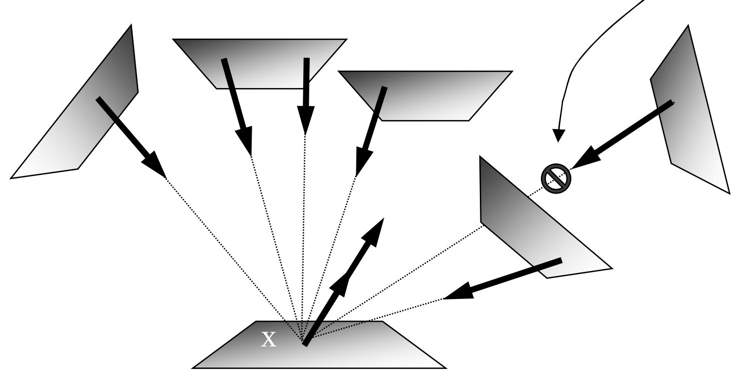
$$y = vp(x, \Psi)$$

Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function

Rendering Equation: all surfaces

$$L(x \to \Theta) = L_e(\ldots) + \int_A f_r(\ldots) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x, y) \cdot dA_y$$



Two forms of the RE

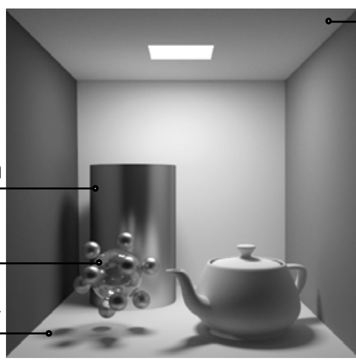
- Hemisphere integration

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

- Area integration (over polygons from set A)

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_A f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y) \cdot dA_y$$

Lighting Cues



Global Illumination is important for realism

Light Sources and Reflection Models

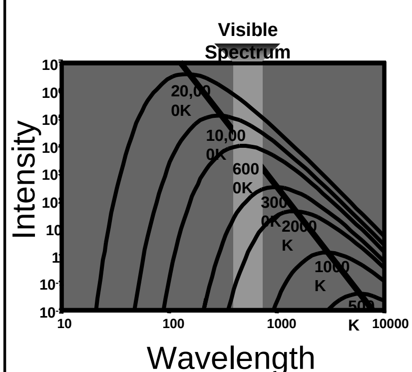
Outline

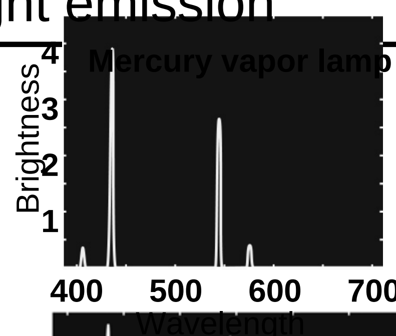
- Light sources
  - Light source characteristics
  - Types of sources
- Light reflection
  - Physics-based models
  - Empirical models

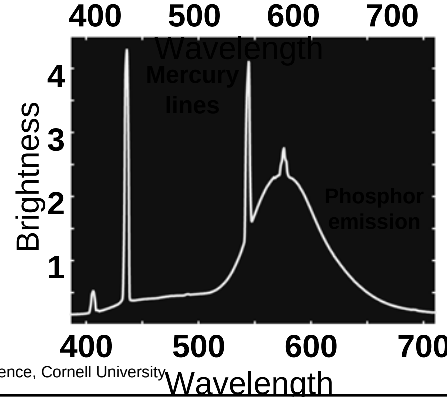
Sources of light radiation

- Thermal radiation ("blackbody")
  - Sun, tungsten & tungsten-halogen lamps; arc lamps
- Electric discharge
  - gas discharge lamps (neon, sodium, mercury vapor)
  - arc lamps, fluorescent lamps
- Other phenomena
  - fluorescence (fluorescent lamps, fluorescent dyes)
  - phosphorescence (CRTs); LEDs; lasers

Examples of light emission



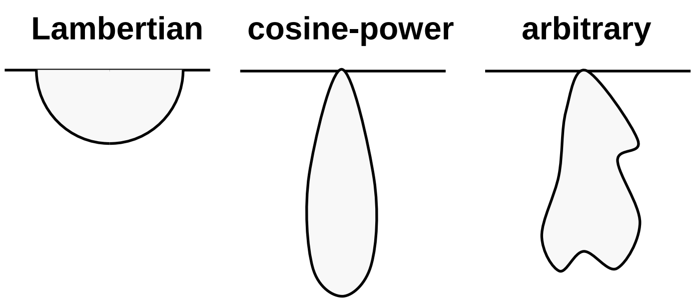




Modeling luminaires

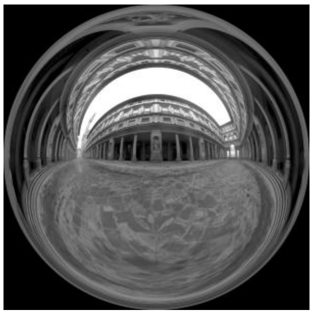
- Spectral distribution
  - Determined by physics of source
  - Generally tabulated, often RGB used
- Spatial distribution
  - Modeled as point or simple area light
  - Also light probes create high dynamic range inputs
- Directional distribution
  - Often shaped by reflectors
  - Tabulated when necessary, cosine lobe is common approximation

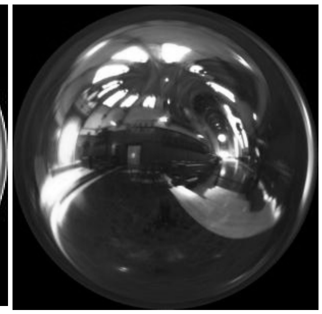
Directional distributions



Lighting w/ Environment Maps

- High lighting complexity





- Rich: captures real world

Image-based lighting

- Acquiring lighting information of real scenes
  - Image-based techniques
- Use light probe
- Varying exposure