Lecture 2: Radiometry

CS 6620, Spring 2009

Kavita Bala

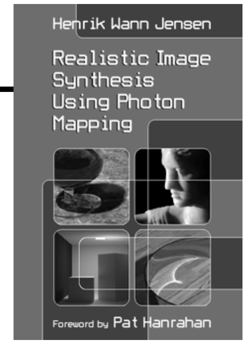
Computer Science

Cornell University

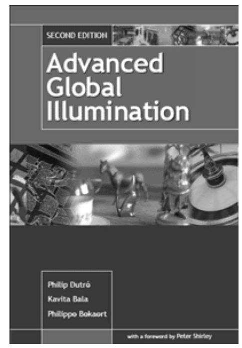
Information

- Kavita Bala (kb@cs.cornell.edu)
  - Upson 5142
- AA: Melissa Totman (mtotman@cs.cornell.edu)
- Lectures: Mon and Wed, 1:25-2:40
  - UP 111

Book



- Photon Mapping book
  - Jensen



- Advanced Global Illumination
  - Dutre, Bala, Bekaert
    2nd Edition

- Physically Based Rendering

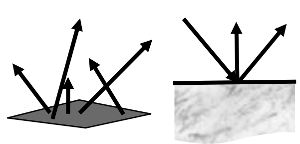
© Kavita Bala, Computer Science, Cornell University

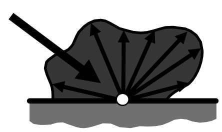
Book

- AGI book
  - Chapter 2: 2.3, 2.5, 2.6

What is the behavior of light?

- Physics of light
- Radiometry
- Material properties

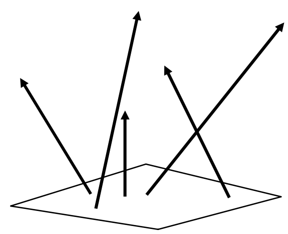




- **Rendering Equation**

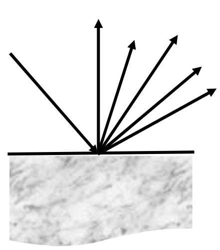
Geometric Optics: Properties

- Light travels in straight lines
- Rays do not interact with each other
- Rays have color(wavelength), intensity

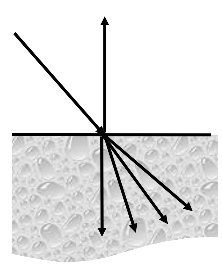


Realistic Reflections/Refractions

reflection



refraction

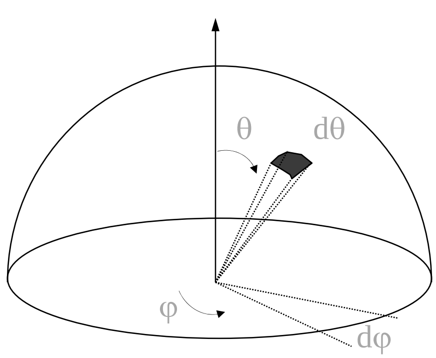


Radiometry

- Radiometry: measurement of light energy
- Defines relation between
  - Power
  - Energy
  - Radiance
  - Radiosity

Digression: Hemispheres

- Hemisphere = two-dimensional surface
- Direction = point on (unit) sphere



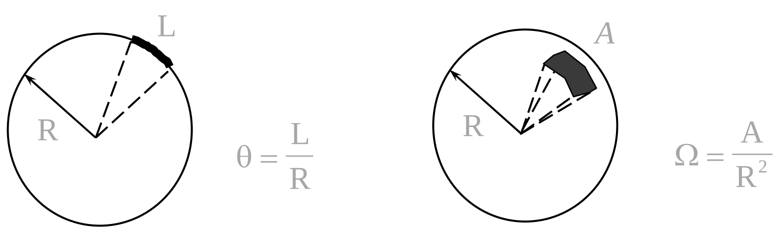
$$\theta \in [0, \frac{\pi}{2}]$$

$$\varphi \in [0, 2\pi]$$

Digression: Solid angles

2D

3D



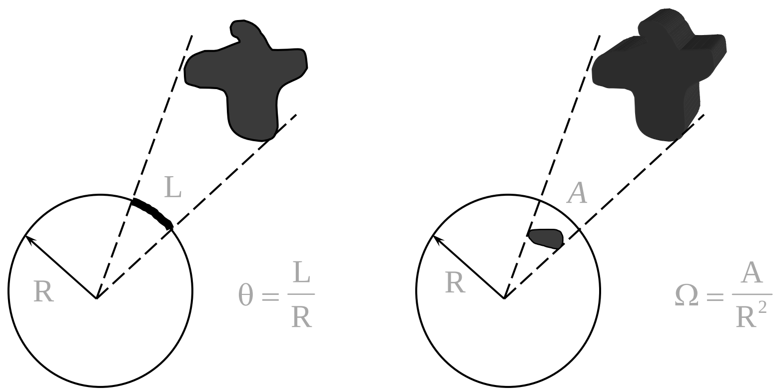
Full circle = 2π radians

Full sphere = 4π steradians

Digression: Solid angles

2D

3D



Full circle = 2π radians

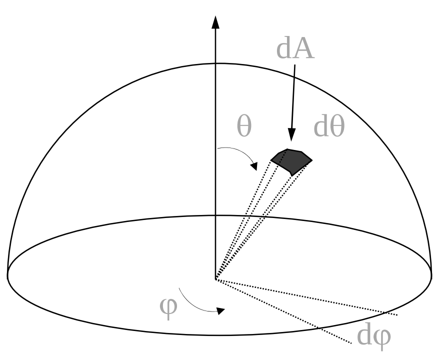
Full sphere = 4π steradians

Digression: Solid angle

- Full sphere = 4π steradian = 12.566 sr
- Dodecahedron = 12-sided regular polyhedron; 1 face = 1 sr

Hemispherical coordinates

- Direction = point on (unit) sphere



$$dA = (r \sin\theta d\varphi)(r d\theta)$$

Hemispherical coordinates

- Defined a measure over hemisphere
- $d\omega$ = direction vector
- Differential solid angle

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

Hemispherical integration

- Solid angle of hemisphere:

Hemispherical integration

- Solid angle of hemisphere:

$$\int_{\Omega_x} d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta$$
$$= \int_0^{2\pi} d\varphi \left[-\cos\theta\right]_0^{\pi/2}$$
$$= \int_0^{2\pi} d\varphi$$
$$= 2\pi$$

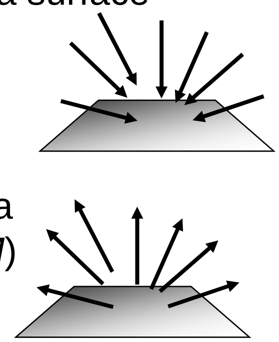
Power

- Energy: Symbol: *Q*; unit: Joules
- Power: Energy per unit time (*dQ*/*dt*)
  - Aka. "radiant flux" in this context

- Symbol: *P* or Φ; unit: Watts (Joules / sec)
  - Photons per second
  - All further quantities are derivatives of *P* (flux densities)

Irradiance

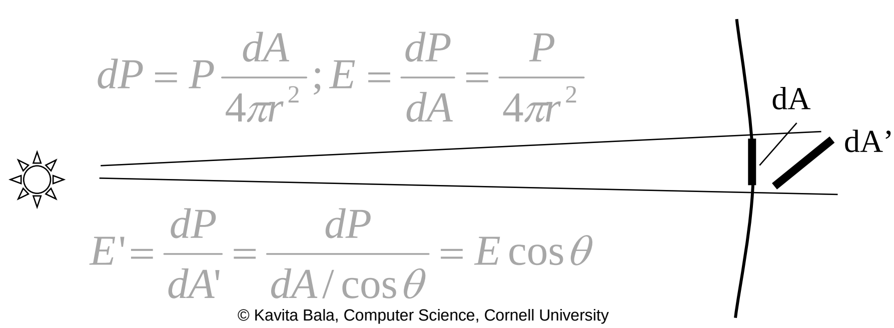
- Power per unit area (*dP*/*dA*)
  - That is, area density of power
  - It is defined with respect to a surface
- Symbol: *E*; unit: W / m^2
  - Measurable as power on a small-area detector
  - Area power density exiting a surface: *radiant exitance* (*M*) or *radiosity* (*B*)
  - Has the same units



Irradiance example

- Uniform point source illuminates small surface *dA* from distance *r*
  - Think of it as a piece of a sphere
  - Power *P* is uniformly spread over the area of the sphere

$$dP = P\frac{dA}{4\pi r^2};\ E = \frac{dP}{dA} = \frac{P}{4\pi r^2}$$



$$E' = \frac{dP}{dA'} = \frac{dP}{dA/\cos\theta} = E\cos\theta$$

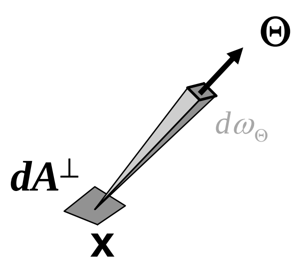
Intensity

- Power per unit solid angle (dP/dw)
  - That is, solid angle density of power
  - Normally used for point sources
- Symbol: *I*; units: W / sr
  - For uniform source $I = \frac{P}{4\pi}$

Radiance

- Radiance is radiant energy at x in direction θ: 5D function
  - $L(x \to \Theta)$: Power
    - per unit projected surface area
    - per unit solid angle

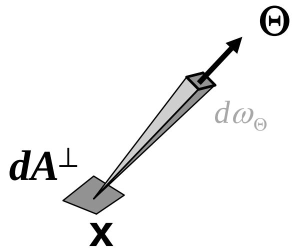
$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$



  - units: Watt / m^2.sr

Radiance

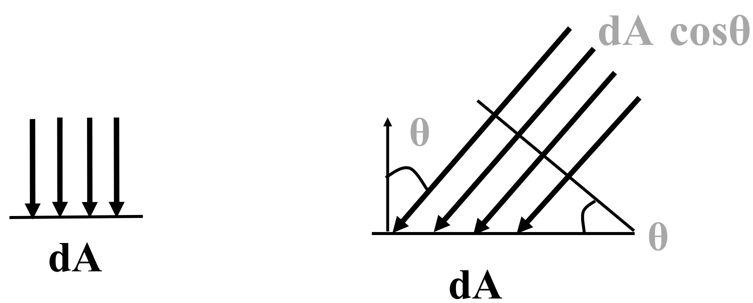
- A 2nd derivative of P: photons that
  (a) go through a little area around *x* perpendicular to Θ *and*
  (b) are traveling in directions that fall in a little solid angle around Θ
  – Irradiance per unit solid angle



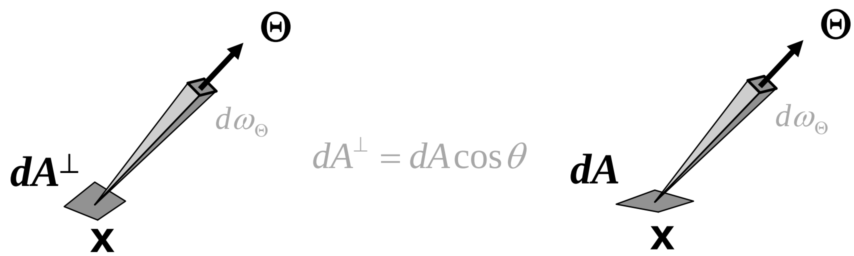
Radiance: Projected area

- $L(x \to \Theta) = \dfrac{d^2 P}{dA^\perp d\omega_\Theta}$

- Why per unit projected surface area?



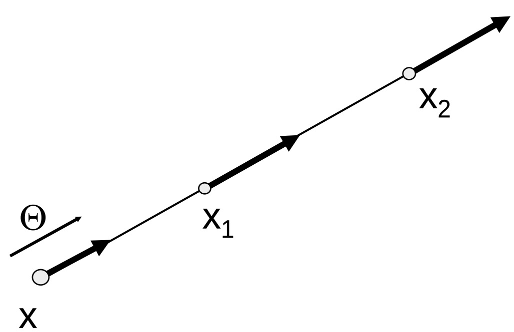
Radiance



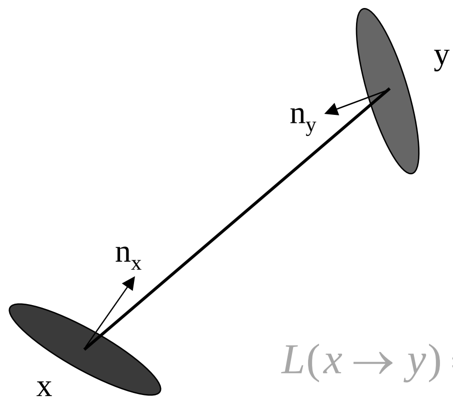
- Oblique surface "sees" same photons as a perpendicular surface
- So definition of radiance uses *A* cos θ

Why is radiance important?

- Invariant along a straight line (in vacuum)



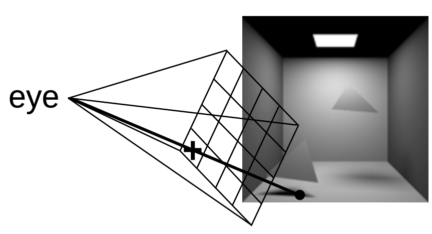
Invariance of Radiance



$$L(x \to y) = L(y \leftarrow x)$$

Why is radiance important?

- Response of a sensor (camera, human eye) is proportional to radiance



- Pixel values in image proportional to radiance received from that direction

Wavelength Dependence

- Each particle has a wavelength $E = \frac{h}{\lambda}$
- All radiometric quantities depend on wavelength
- Spectral radiance: $L(x \to \Theta, \lambda)$
- Radiance: $L(x \to \Theta) = \int L(x \to \Theta, \lambda) d\lambda$

Relationships

- Radiance is the fundamental quantity

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

- Power:

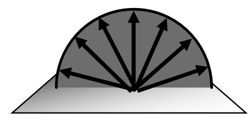
$$P = \int_{Area} \int_{Solid\ Angle} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

- Radiosity:

$$B = \int_{Solid\ Angle} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta}$$

Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere



$$L(x \rightarrow \Theta) = \frac{d^2P}{dA^{\perp} d\omega_{\Theta}}$$

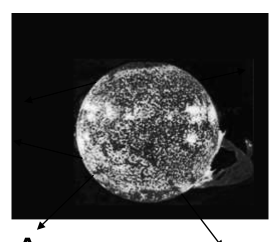
$$P = \int_{Area} \int_{Solid\ Angle} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

$$= L \int_{Area} dA \int_{Solid\ Angle} \cos\theta \cdot d\omega_{\Theta}$$

$$= L \cdot Area \cdot \pi$$

Sun Example: radiance

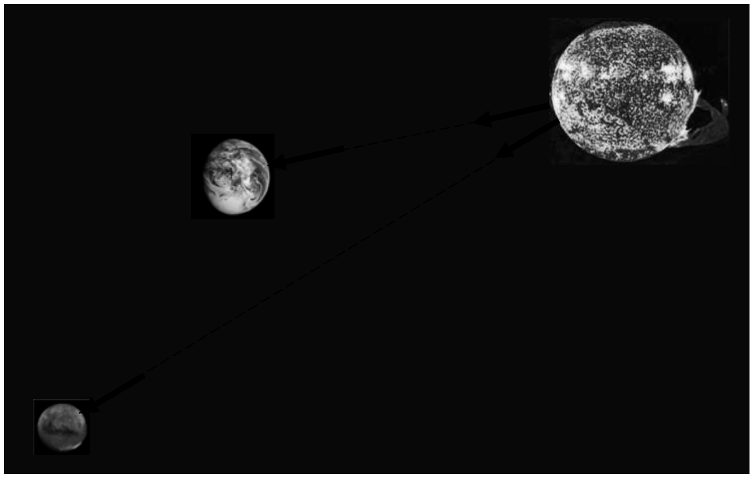
- Power: 3.91 x 10^{26} W
- Surface Area: 6.07 x 10^{18} m^2



- Power   = Radiance.Surface Area.π
- Radiance = Power/(Surface Area.π)

- Radiance = 2.05 x 10^7 W/ m^2.sr

Sun Example

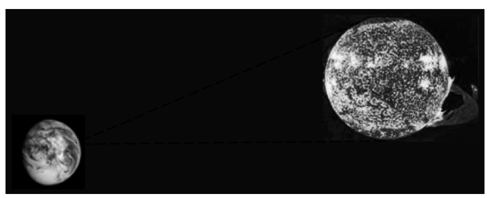


Same radiance on Earth and Mars?

Sun Example: Power on Earth

- Power reaching earth on a 1m^2 square:

$$P = L \int_{Area} dA \int_{Solid\ Angle} \cos\theta \cdot d\omega_\Theta$$

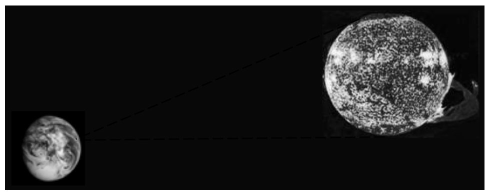


- Assume $\cos\theta = 1$ (sun in zenith)

$$P = L \int_{Area} dA \int_{Solid\ Angle} d\omega_\Theta$$

Sun Example: Power on Earth

Power = Radiance.Area.Solid Angle

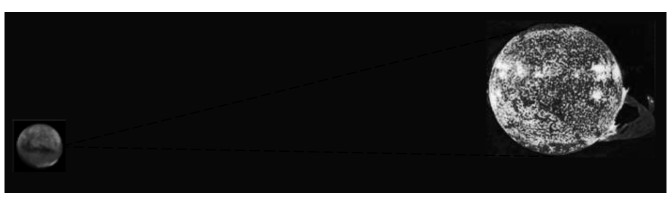


Solid Angle = Projected Area$_{Sun}$/(distance$_{earth_sun}$)^2
= $6.7\ 10^{-5}$ sr

P = (2.05×10^7 W/ m^2.sr) x (1 m^2) x ($6.7\ 10^{-5}$ sr)
= 1373.5 Watt

Sun Example: Power on Mars

Power = Radiance.Area.Solid Angle



Solid Angle = Projected Area$_{Sun}$/(distance$_{mars_sun}$)^2
= $2.92\ 10^{-5}$ sr

P = (2.05×10^7 W/ m^2.sr) x (1 m^2) x ($2.92\ 10^{-5}$ sr)
= 598.6 Watt

Relationships

- Radiance is the fundamental quantity

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

- Power:

$$P = \int_{Area} \int_{Solid\ Angle} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

- Radiosity:

$$B = \int_{Solid\ Angle} L(x \rightarrow \Theta) \cdot \cos\theta \cdot d\omega_{\Theta}$$