2022-12-01

1 Big ticket items

1.1 Linear algebra and calculus

- Linear algebra background (abstract and concrete)
  - Vectors, spaces, subspaces, bases
  - Interpreting matrices: operators, mappings, quadratic forms
  - Canonical forms
- Calculus with matrices
  - Sensitivity analysis and conditioning
  - Variational notation for derivatives
  - Optimization with quadratics
  - Lagrange multipliers and constraints

1.2 Matrix algebra

- Ways to write matrix-matrix products
- Blocked matrices and blocked algorithms
- Graph structures: sparse, diagonal, triangular, Hessenberg, etc
- LA structures: symmetric, skew, orthogonal, etc
- Other structure: Toeplitz, Hankel, other special matrices

1.3 The big problems

$$
\begin{aligned}
Ax &= b \\
\text{minimize } \|Ax - b\|^2 & \\
Ax &= x\lambda
\end{aligned}
$$

1.4 The big factorizations

- LU and company (LDL^T and Cholesky)
- QR (economy and full)
- SVD (economy and full)
- Schur factorization
- Symmetric eigendecomposition

1.5 Iterations

- Iterative refinement
- Stationary iterations (Jacobi, Gauss-Seidel, etc)
- Krylov subspace definition
- Approximation from a subspace and Galerkin
- Characterization of CG and GMRES

1.6 Philosophical odds and ends

- Identifying the right structure matters a lot
- We need *both* algebra and analysis
- When you don't know what else to do... eigenvalues or SVD
- I differentiate five expressions before breakfast!

2 What else?

There is a lot that I wish I could get to in a course like this. If it were a two semester course, perhaps I would! Three things come immediately to mind.

- LA for data science (c.f. CS 6241)
  - Non-negative matrix factorizations

  - Tensors and tensor factorizations
  - More on factorization-based methods in stats/ML
  - The linear algebra of multivariate normals
  - Connections to convex optimization: active sets, quadratic programming, etc

- Iterative methods (c.f. CS 6220)
  - More on multigrid and domain decomposition
  - More on other “data-sparse” matrices
  - More on elliptic PDEs, integral equations, etc

- Eigensolvers
  - More on eigensolvers (especially iterative ones)
  - Much more on perturbation theory and sensitivity analysis
  - Matrix functions, and complex analysis connections
  - Connections to control theory
  - More on orthogonal polynomials

But there is always more to learn. If the course gave you a starting point to thinking about other corners of linear algebra that you care about for your research, then it was a success.

I enjoyed the class this semester. I hope you did as well.