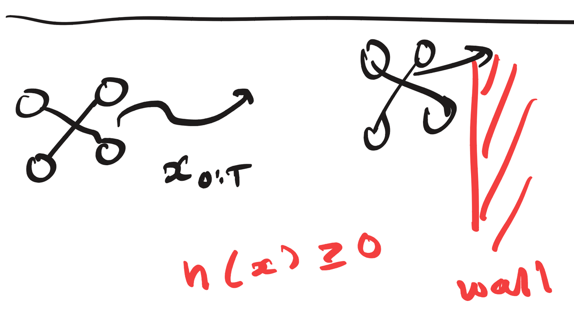
4/23: CBFs: Theory & Applications

Announcements:
- HW4 due today (4/23)
- project videos due 5/4 (mon) (no late days!)
- project report due 5/11 (mon)
  ↳ 8 pg. max, results + "lessons learned"
- guest lecture: Prof. Cauligi, 4/30

Last time:
- MBRL

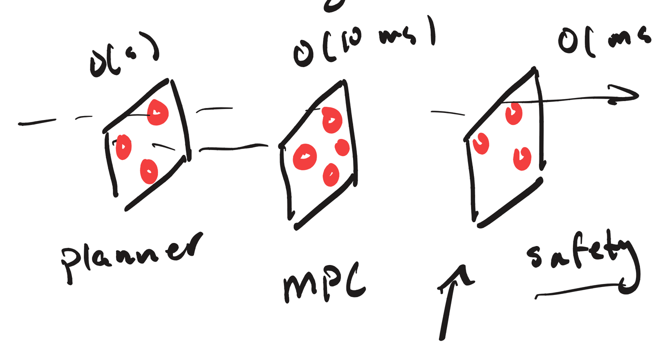
Today:
- CBFs!



$$\min_{x_{0:T}, u_{0:T-1}} J(x_{0:T}, u_{0:T-1})$$
s.t. $x_{t+1} = f(x_t, u_t)$
$h(x_t) \geq 0 \quad \forall t$ ✓
$u_t \in \mathcal{U}$

Issues: 1) horizon ("recursive feas.")
2) modeling error / mismatch

Q: How do you actually get a robot to be safe?



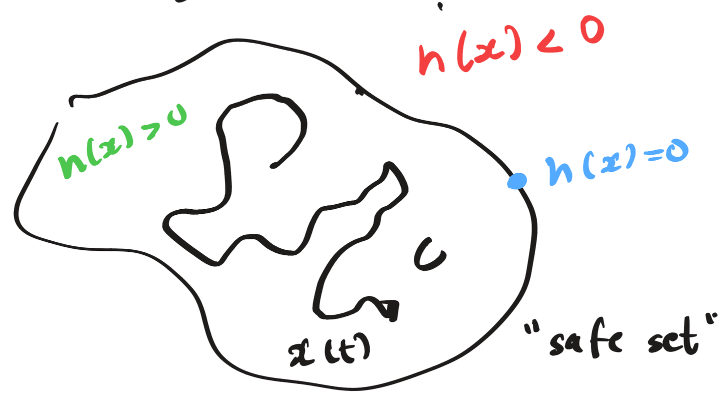
"swiss cheese"
holes don't line up
↳ enforce redundant constraints @ multiple timescales

1. Theory of CBFs
2. Applications

In this class "safety" looks like $h(x) \geq 0$
distance to obs.
distance to people
velocity limits

For CBFs/controls
$C = \{x \mid h(x) \geq 0\}$

$h_{CBF} = \min_i h_i(x)$
$h: \mathcal{X} \to \mathbb{R}$

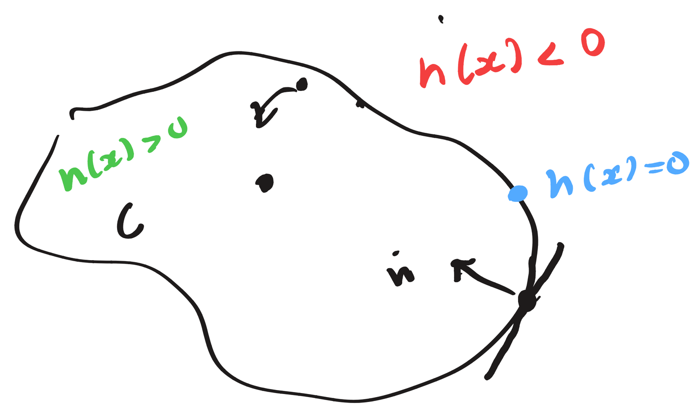


"set invariance"
↳ $x(0) \in C$ implies $x(t) \in C$ for all time

Key insight: Because $h(x)$ is one number that determines safety, we can just think about $h(t)$.

$\dot{h}(t) > 0 \Rightarrow$ becoming safer
$\dot{h}(t) < 0 \Rightarrow$ becoming less safe

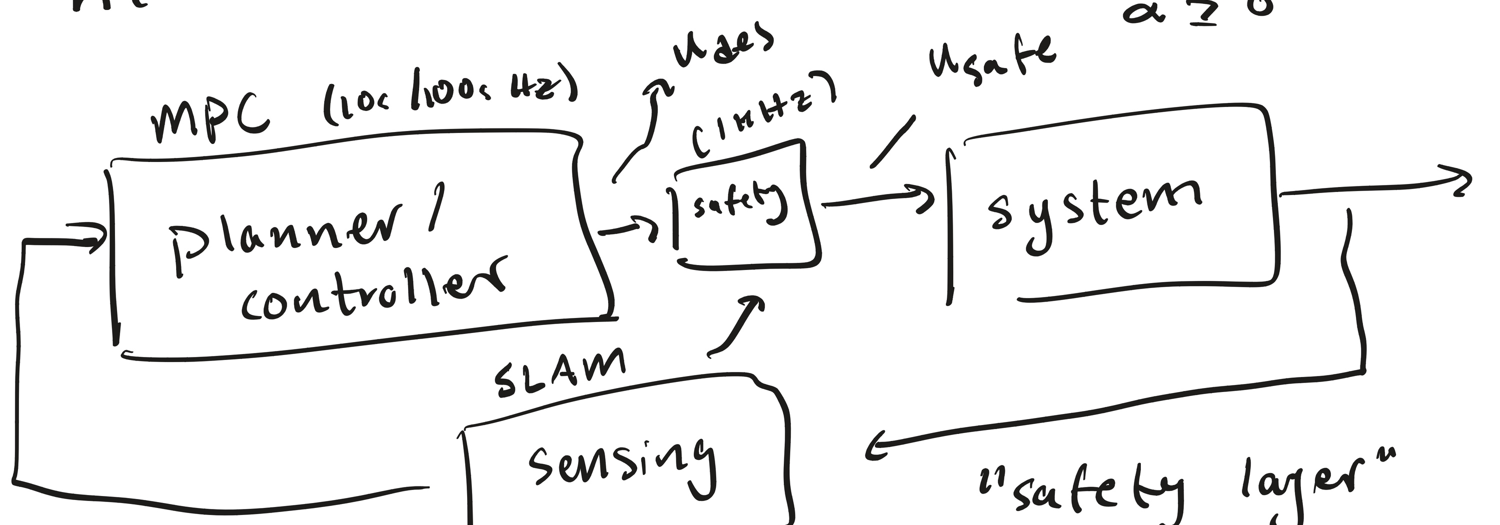
Q: For safety, what should $\dot{h}(t)$ look like?



when $h(x) = 0$,
$\dot{h}(x) \geq 0$,
when $h(x) >> 0$,
$\dot{h}(x)$ can be negative

$$\boxed{\dot{h}(x) \geq -\alpha h(x)}$$
$\alpha \geq 0$

$\dot{h}(t) = -\alpha h(t)$



"safety layer"
"safety filter"

Key idea: we want something fast that minimally changes u_{des} to enforce safety

Safety layer:

$$\boxed{\begin{aligned} \min_u \quad & \|u - u_{des}\|^2 \\ \text{s.t.} \quad & \dot{h}(x, u) \geq -\alpha h(x) \end{aligned}}$$

$$\dot{h}(x, u) = \frac{dh(x(t))}{dt} = \frac{\partial h}{\partial x}^T \dot{x}$$

Common form $\dot{x} = f(x) + g(x)u$

↳ $\dot{h}(x, u) = \frac{\partial h}{\partial x}^T (f(x) + g(x)u)$

↳ $\frac{\partial h}{\partial x}^T f(x) + \frac{\partial h}{\partial x}^T g(x) u \geq -\alpha h(x)$ "active set"

$a^T u \leq b$

$\min_u \|u_{des} - u\|^2$
s.t. $a^T u \leq b$

$$u^* = \begin{cases} u_{des}, & a^T u_{des} \leq b \\ \Pi(u_{des}), & \text{oth.} \end{cases}$$

Discrete time
$h(x_{k+1}) \geq \alpha h(x_k), \quad \alpha \in [0, 1]$

↳
$$\boxed{\begin{aligned} \min_{u_k} \quad & \|u_k - u_{des}\|^2 \\ \text{s.t.} \quad & h(f(x_k, u_k)) \geq \alpha h(x_k) \\ & u \in \mathcal{U} \end{aligned}}$$

Practical details:
i) Recursive feasibility
✓ ii) modeling / state est. error
✓ iii) $h(x)$ is an open design problem