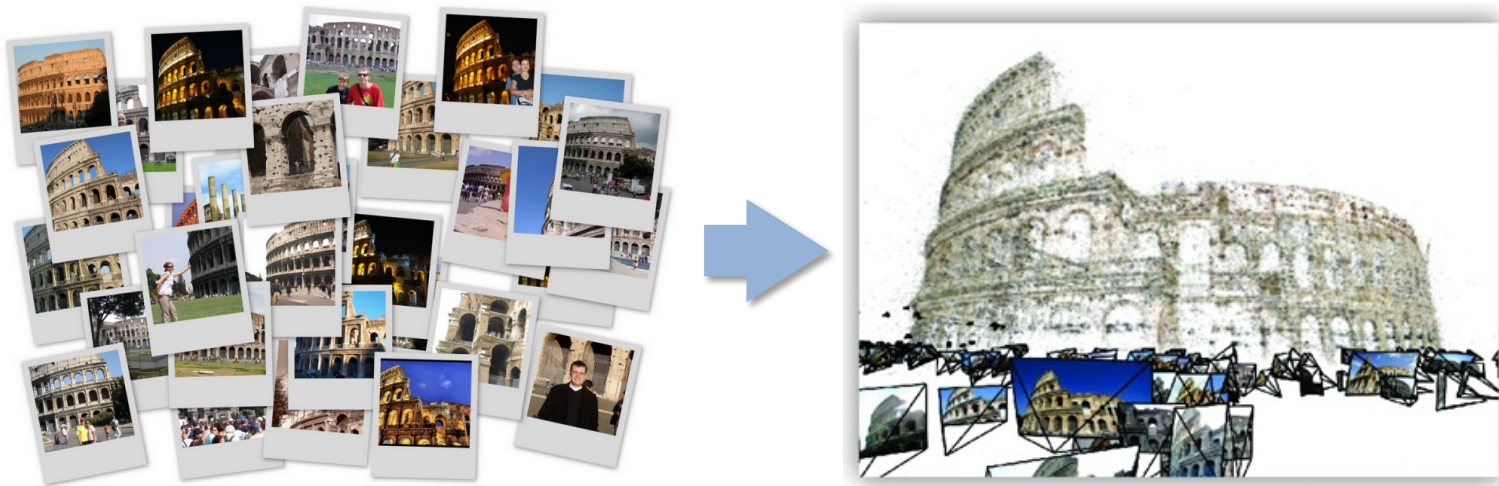


CS5670: Computer Vision

Structure from motion



Readings

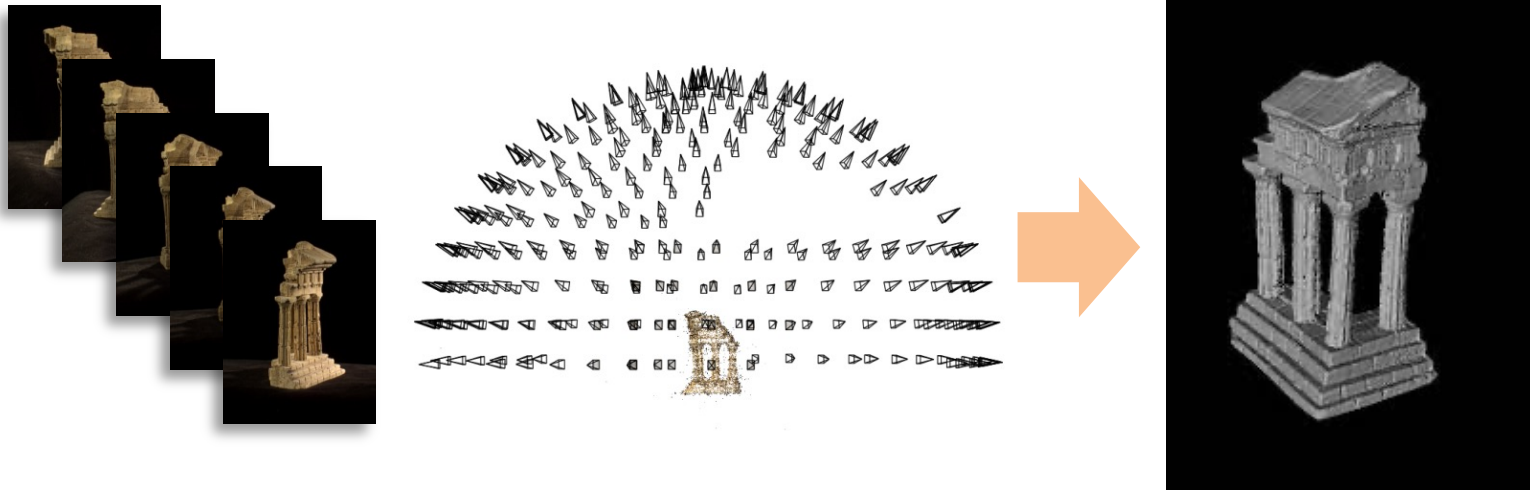
- Szeliski (2nd Edition), Chapter 11.4

Announcements

- Project 4 due tomorrow, 3/29, by 8pm
- Project 5 on NeRF coming up after Spring Break

Reminder: multi-view stereo

Problem formulation: given several images of the same object or scene, compute a representation of its 3D shape



Structure from motion

- Multi-view stereo assumes that cameras are calibrated
 - Extrinsic and intrinsic are known for all views
- How do we compute calibration if we don't know it? In general, this is called *structure from motion*

Large-scale structure from motion



Dubrovnik, Croatia. 4,619 images (out of an initial 57,845).

Total reconstruction time: 23 hours

Number of cores: 352

Two views



- Solve for Fundamental matrix / Essential matrix
- Factorize into intrinsics, rotation, and translation

What about more than two views?

- The geometry of three views is described by a $3 \times 3 \times 3$ tensor called the *trifocal tensor*
- The geometry of four views is described by a $3 \times 3 \times 3 \times 3$ tensor called the *quadrifocal tensor*
- After this it starts to get complicated...
 - Instead, we explicitly solve for camera poses and scene geometry

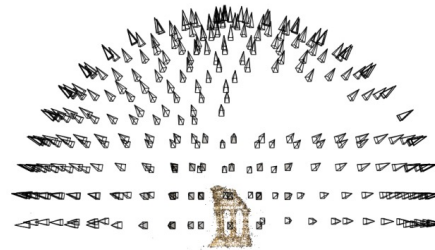
Structure from motion

- Given many images, how can we
 - a) figure out where they were all taken from?
 - b) build a 3D model of the scene?

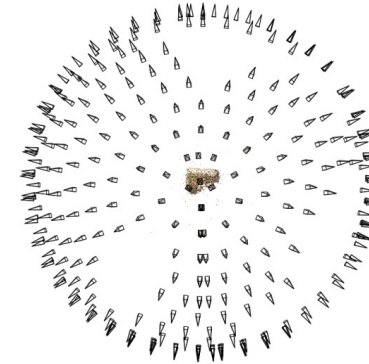


This is the **structure from motion (SfM)** problem

Structure from motion



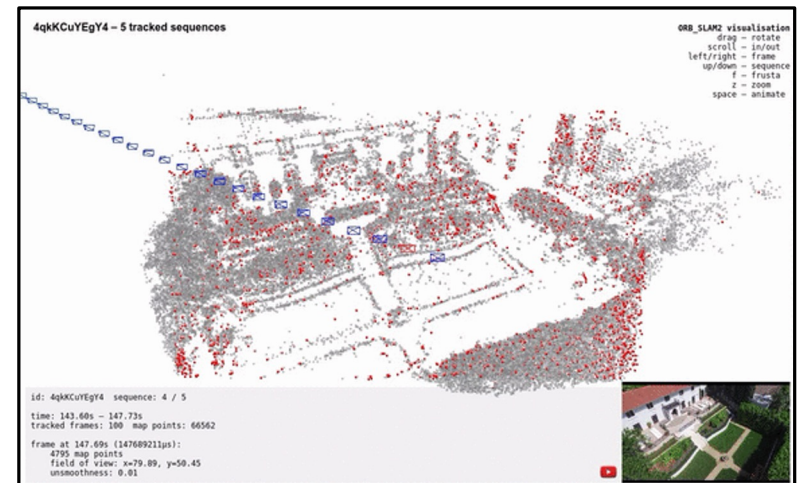
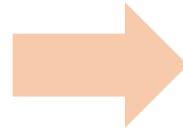
Reconstruction (side)



(top)

- Input: images with 2D points in correspondence
 $p_{ij} = (u_{ij}, v_{ij})$
- Output
 - structure: 3D location \mathbf{x}_i for each point p_i
 - motion: camera parameters $\mathbf{R}_j, \mathbf{t}_j$ & possibly \mathbf{K}_j
- Objective function: minimize *reprojection error*

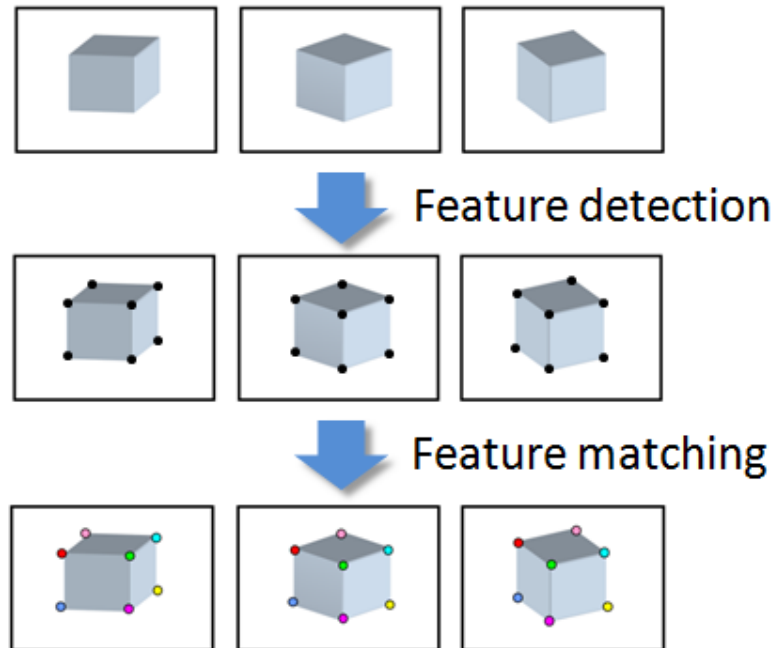
Can also compute camera poses from video (often called Visual SLAM)



What we've seen so far...

- 2D transformations between images
 - Translations, affine transformations, homographies...
- Fundamental matrices
 - Represent relationships between 2D images in the form of corresponding 2D lines
- **What's new:** Explicitly representing 3D geometry of cameras *and points*

Input

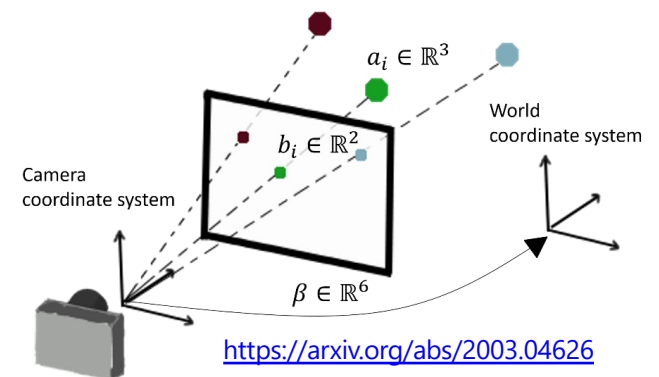


Triangulation & camera calibration

- Suppose we have known camera parameters, each of which observes a point
 - How can we compute the 3D location of that point?
 - This is called *triangulation* (known since ancient times)
- On the other hand: Suppose we have known 3D points
 - And have matches between these points and an image
 - How can we compute the camera parameters?
 - This is called *camera calibration* (or *camera resectioning*)



[Liu Hui](#) (c. 263), How to measure the height of a sea island.



Structure from motion

- What if we don't know 3D points or camera parameters?
- SfM solves both of these problems *at once*
- A kind of chicken-and-egg problem
 - (but solvable)

Photo Tourism

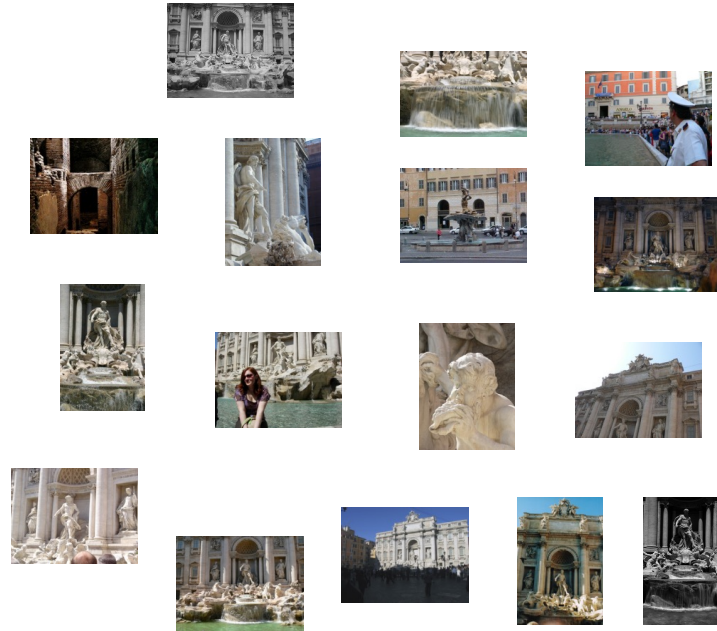


First step: how to get correspondence?

- Feature detection and matching

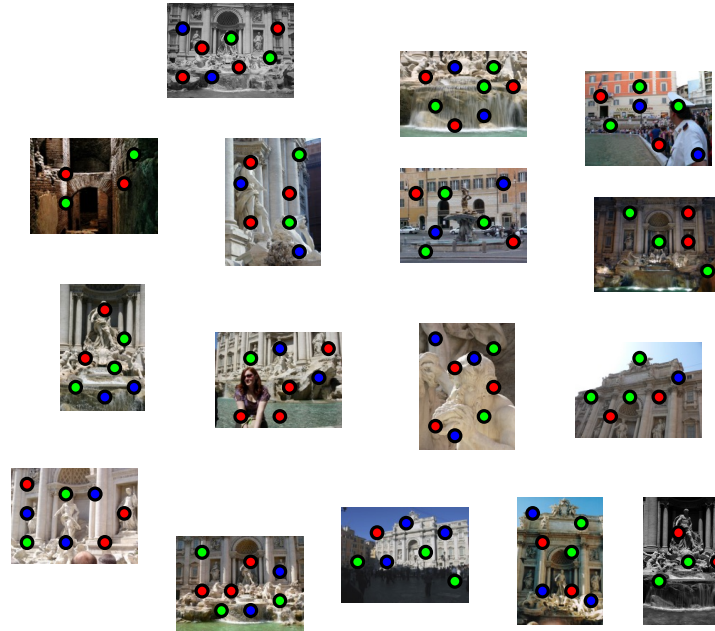
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



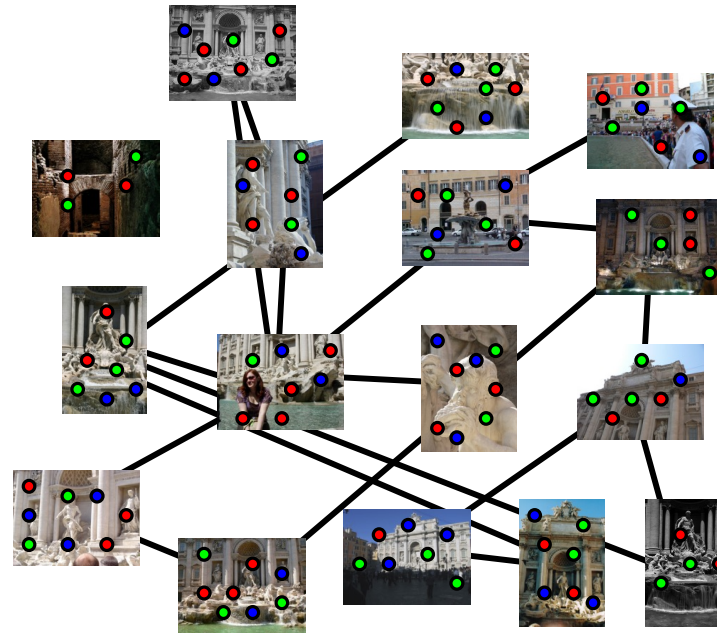
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



Feature matching

Match features between each pair of images



Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair

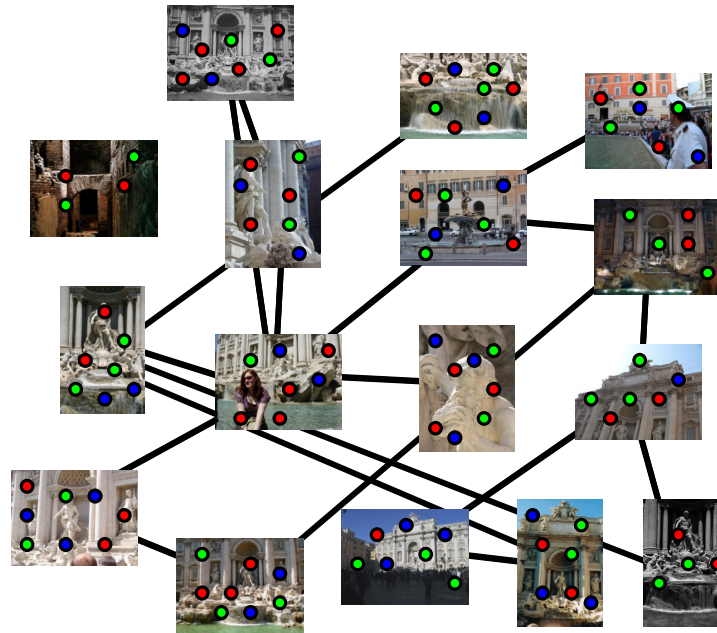
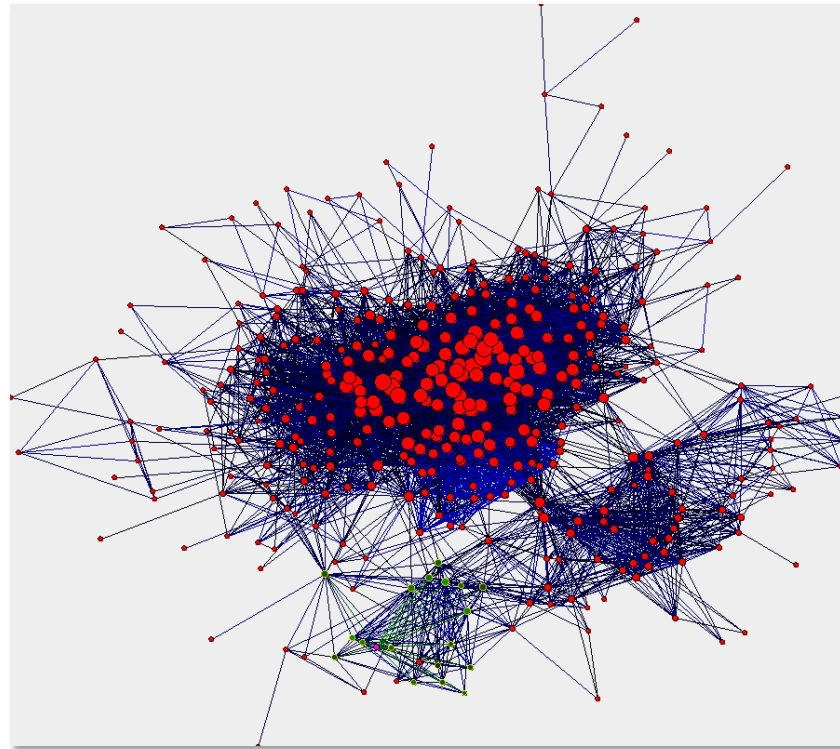


Image connectivity graph



(graph layout produced using the Graphviz toolkit: <http://www.graphviz.org/>)

Demo

- https://www.cs.cornell.edu/courses/cs5670/2023sp/demos/ImageGraphs/?dot_url=trevi.dot&list_url=url_list.txt

Correspondence estimation

- Link up pairwise matches to form connected components of matches across several images

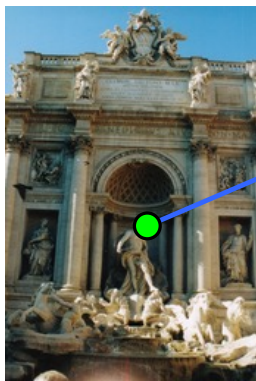


Image 1

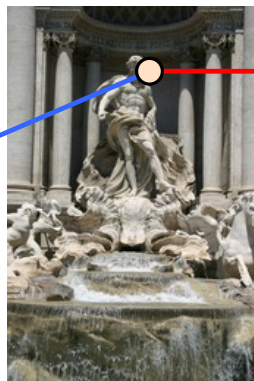


Image 2

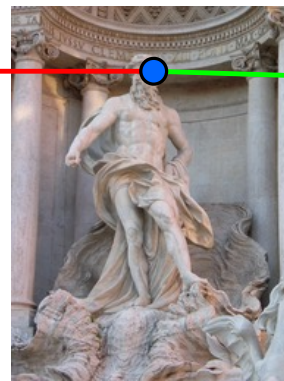


Image 3

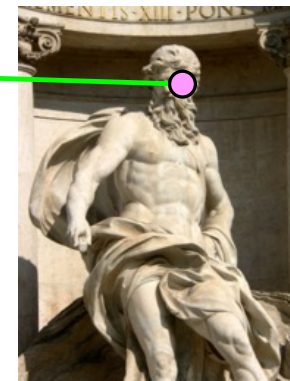
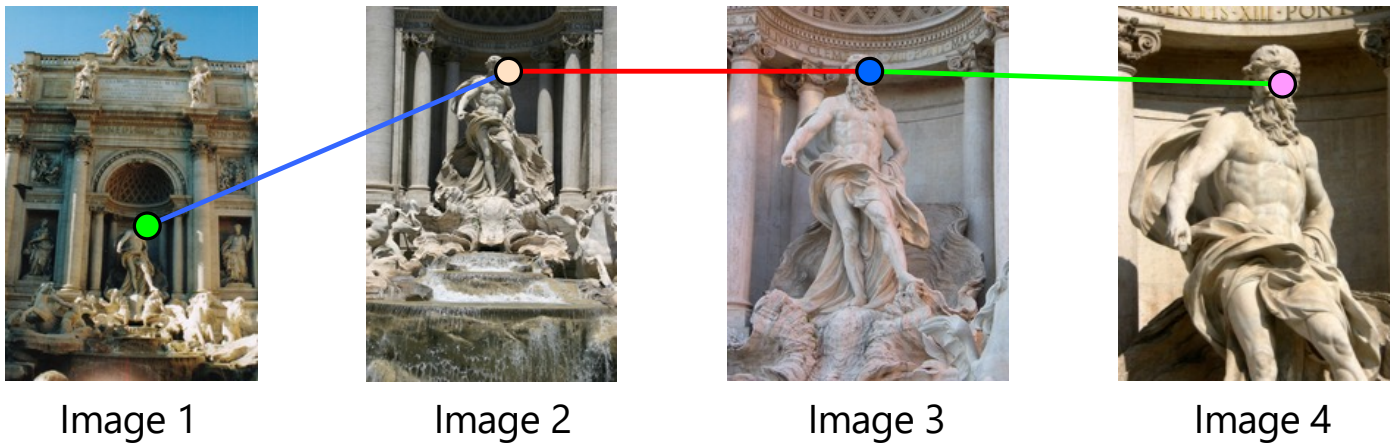
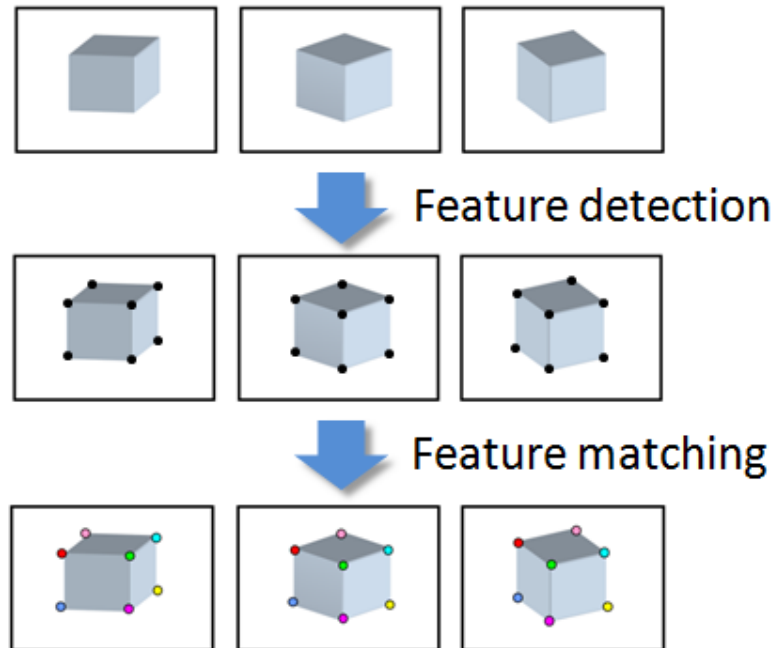


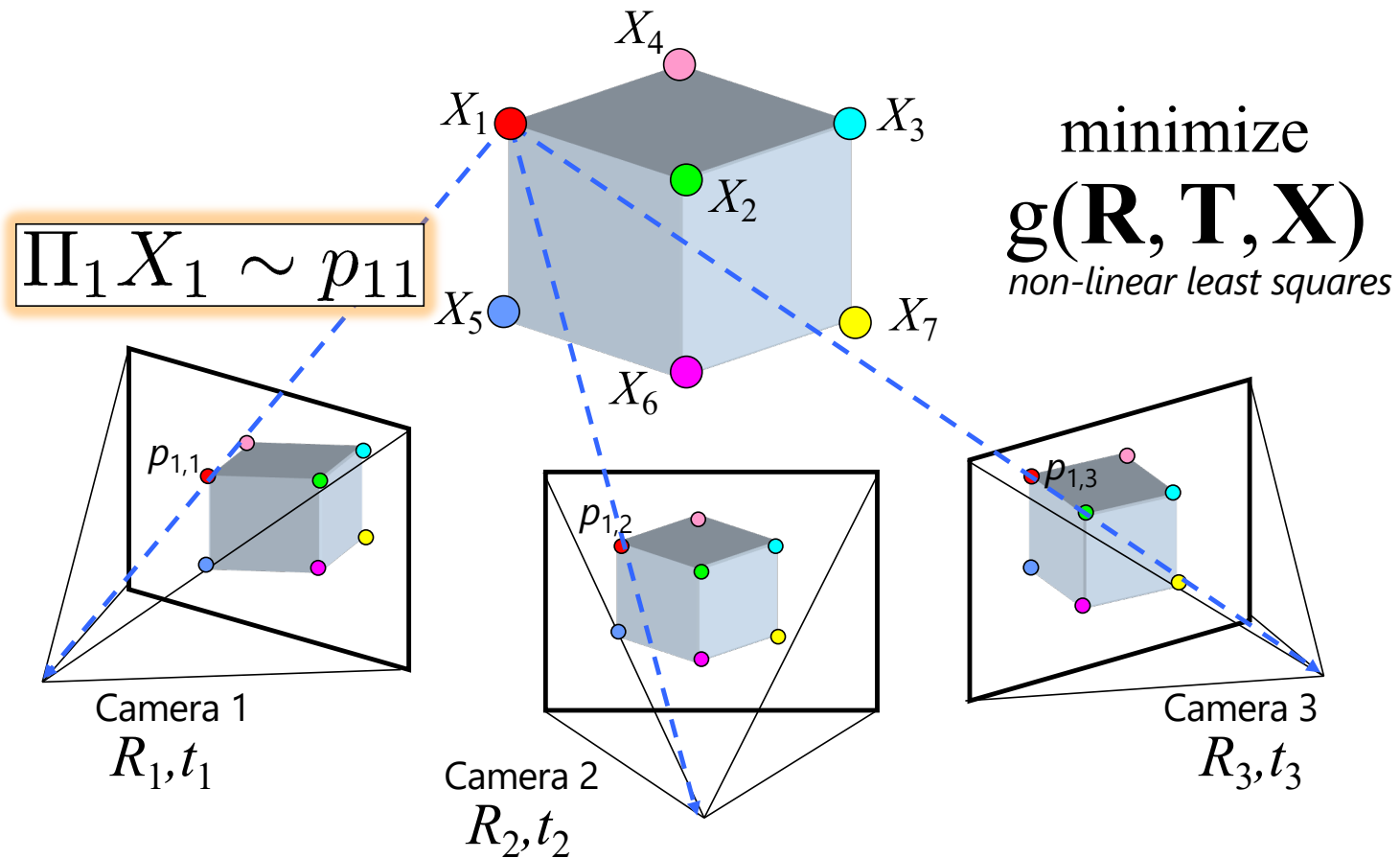
Image 4



Input to Structure from Motion



Structure from motion



Problem size

- What are the variables?
 - Cameras and points
- How many variables per camera?
 - 6 (if calibrated), more if uncalibrated
- How many variables per point?
 - 3

- Trevi Fountain collection
 - 466 input photos
 - + > 100,000 3D points
 - = very large optimization problem

Structure from motion

- Minimize sum of squared reprojection errors:

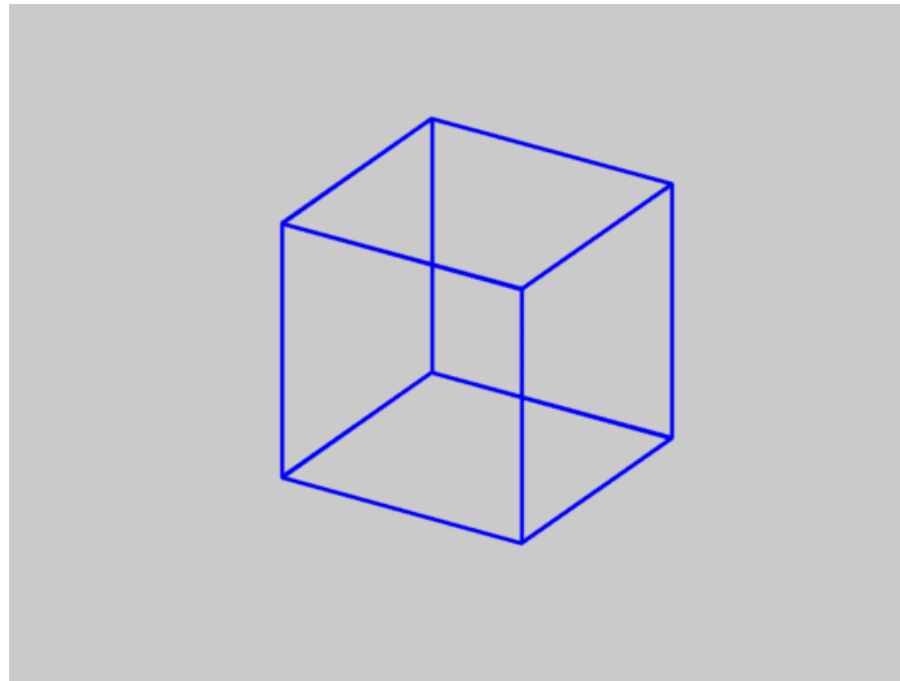
$$g(\mathbf{X}, \mathbf{R}, \mathbf{T}) = \sum_{i=1}^m \sum_{j=1}^n \underbrace{w_{ij}}_{\substack{\text{indicator variable:} \\ \text{is point } i \text{ visible in image } j?}} \cdot \left\| \underbrace{\mathbf{P}(\mathbf{x}_i, \mathbf{R}_j, \mathbf{t}_j)}_{\substack{\text{predicted} \\ \text{image location}}} - \underbrace{\begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix}}_{\substack{\text{observed} \\ \text{image location}}} \right\|^2$$

- Minimizing this function is called *bundle adjustment*
 - Optimized using non-linear least squares, e.g. Levenberg-Marquardt algorithm

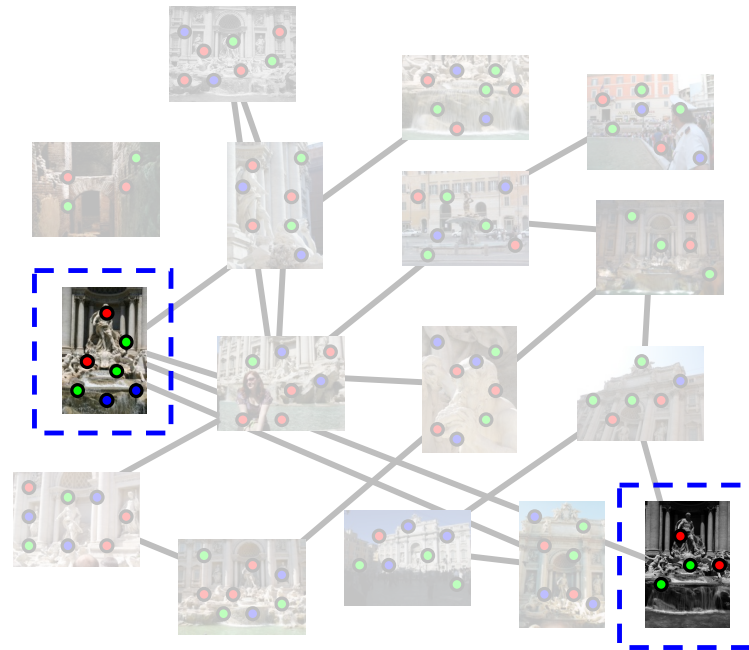
Is SfM always uniquely solvable?

Is SfM always uniquely solvable?

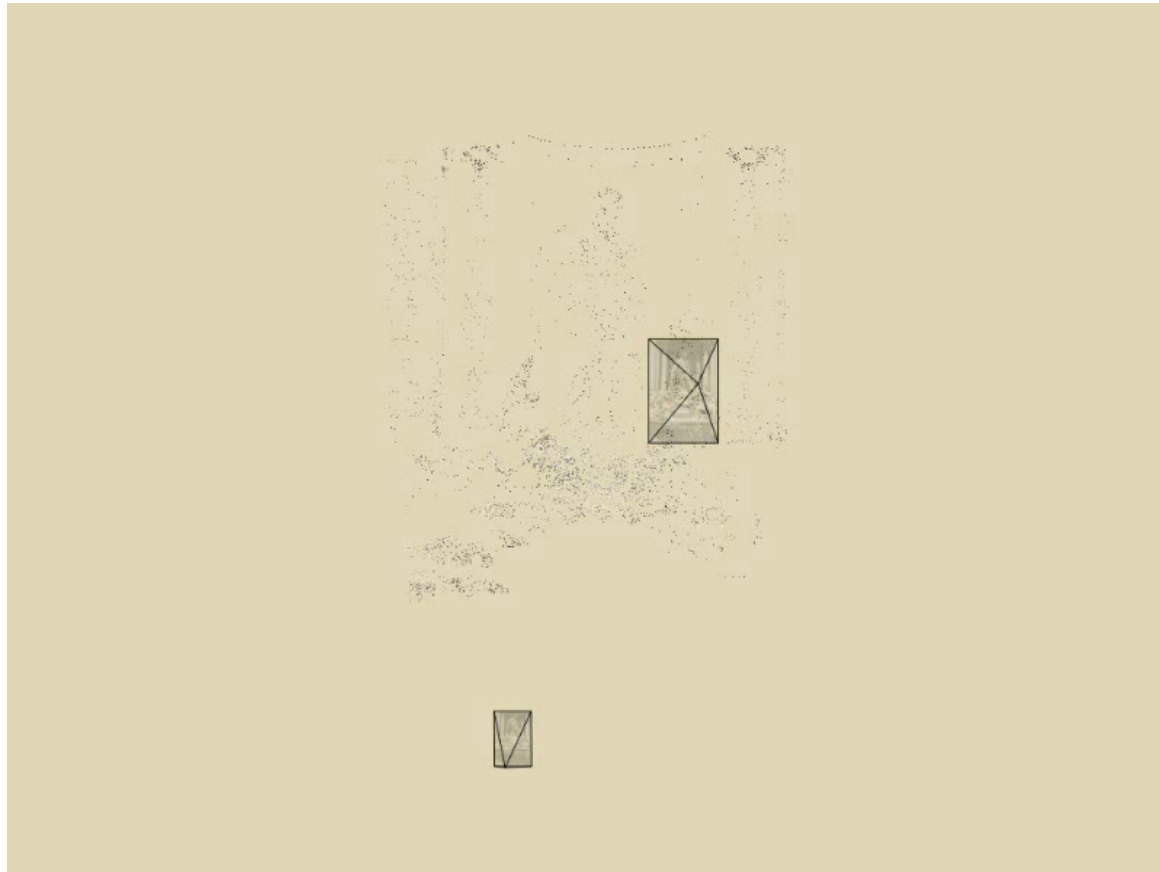
- No...



Incremental structure from motion



Incremental structure from motion



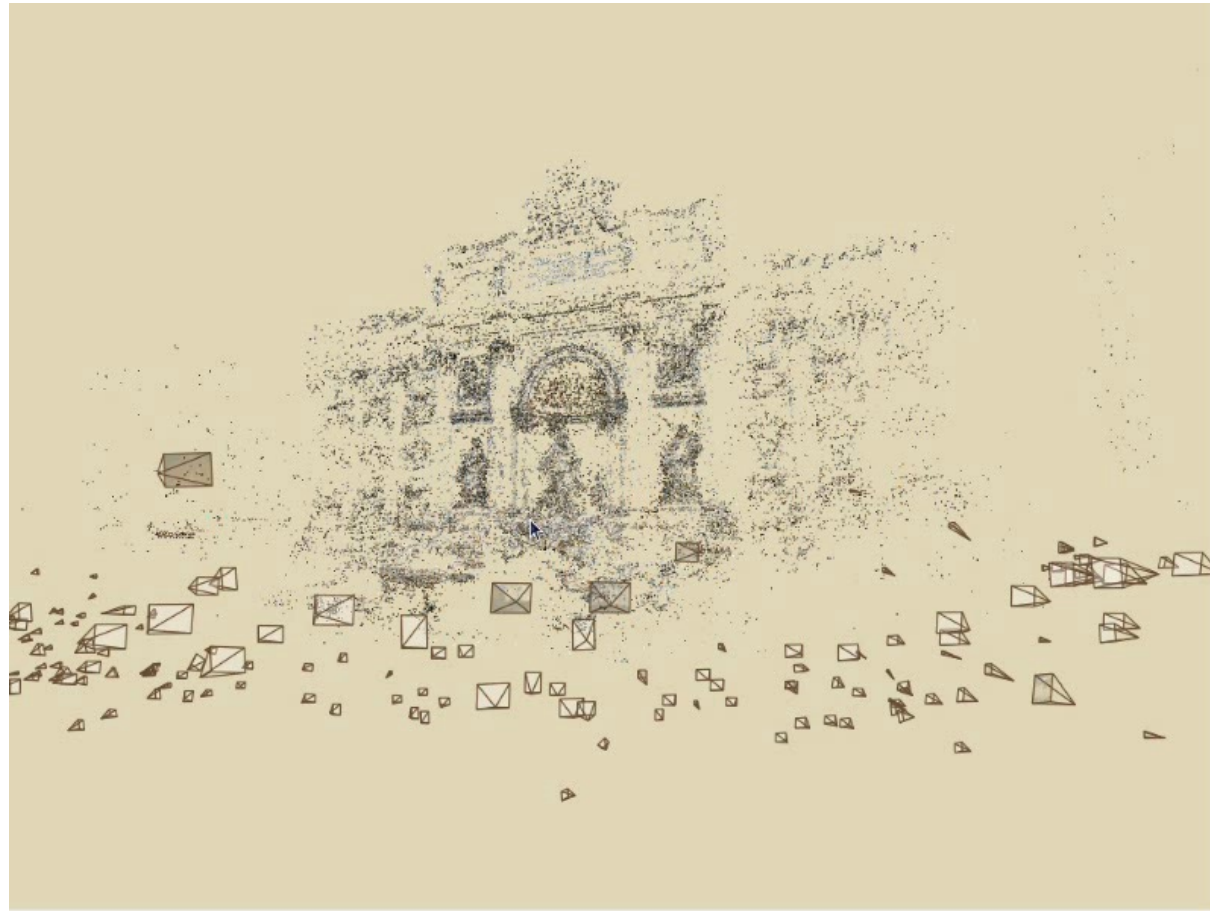
Incremental structure from motion

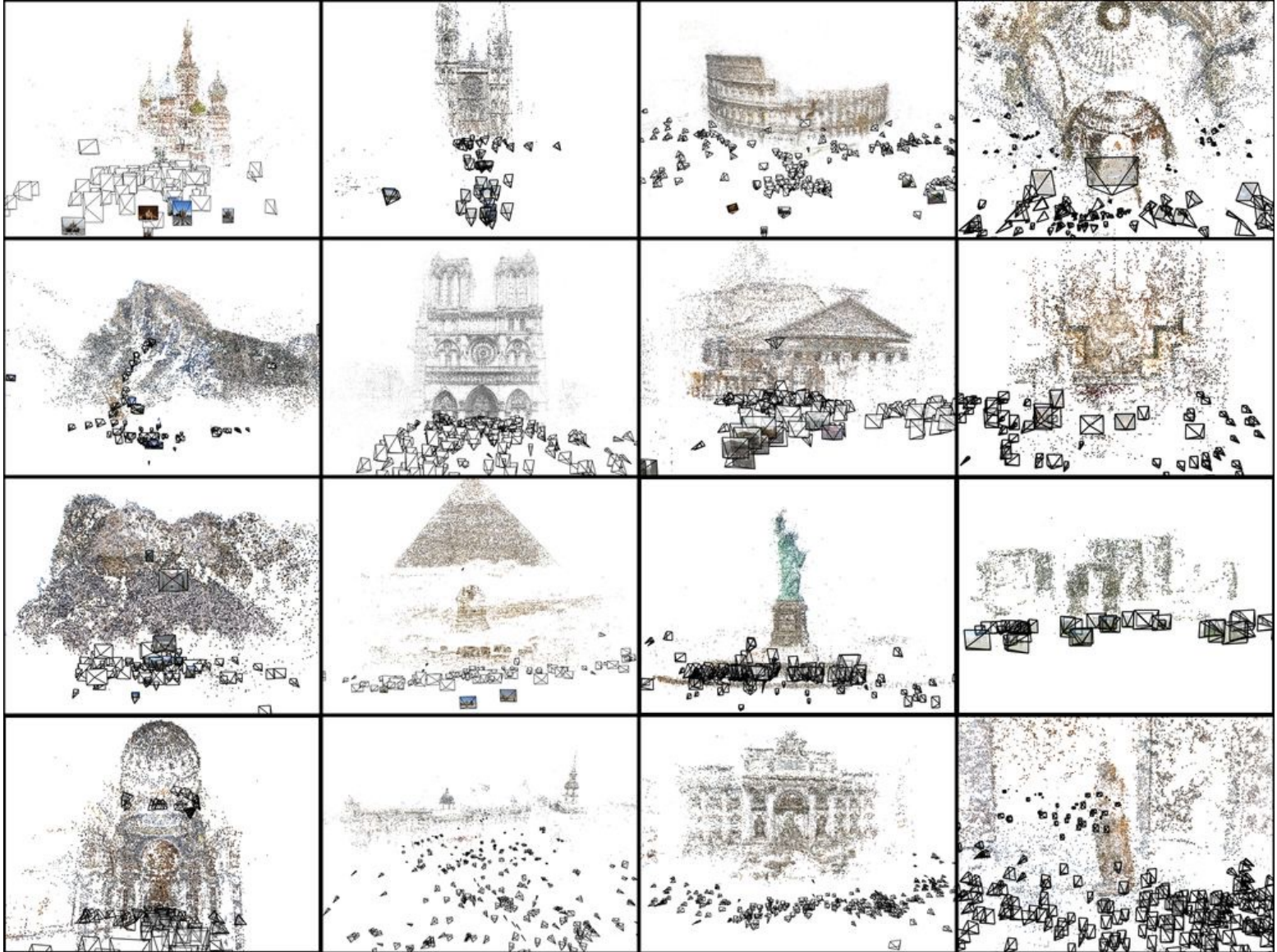


Incremental structure from motion

Time-lapse reconstruction of Dubrovnik, Croatia, viewed from above

Photo Tourism









Libration

From Wikipedia, the free encyclopedia

This article is about astronomical observations. For molecular motion, see [Libration \(molecule\)](#).

Not to be confused with [liberation](#), [libation](#), or [vibration](#).

In astronomy, **libration** is a perceived [oscillating](#) motion of [orbiting bodies](#) relative to each other, notably including the motion of the [Moon](#) relative to [Earth](#), or of [trojan asteroids](#) relative to [planets](#). Lunar libration is distinct from the slight changes in the Moon's [apparent size](#) viewed from Earth. Although this appearance can also be described as an oscillating motion, it is caused by actual changes in the physical [distance](#) of the Moon because of its [elliptic orbit](#) around Earth. Lunar libration is caused by three phenomena detailed below.

Contents [hide]

- [Lunar libration](#)
- [Trojan libration](#)
- [See also](#)
- [References](#)
- [External links](#)

Lunar libration [edit source]

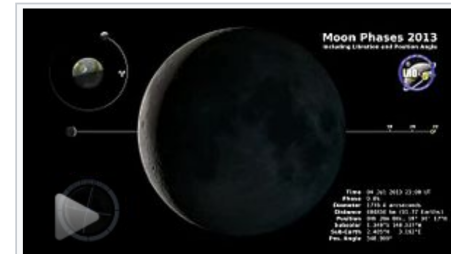
The Moon keeps one [hemisphere](#) of itself facing the Earth, due to [tidal locking](#). Therefore, humans' first view of the [far side of the Moon](#) resulted from [lunar exploration](#) on October 7, 1959. However, this simple picture is only approximately true: over time, slightly *more* than half (about 59%) of the Moon's surface is seen from Earth due to libration.^[1]

Libration is manifested as a slow rocking back and forth of the Moon as viewed from Earth, permitting an observer to see slightly different halves of the surface at different times.

There are three types of lunar libration:

- Libration in longitude* results from the [eccentricity](#) of the Moon's orbit around Earth; the Moon's rotation sometimes leads and sometimes lags its orbital position.
- Libration in latitude* results from a slight inclination (about 6.7 degrees) between the Moon's [axis of rotation](#) and the [normal](#)

<https://en.wikipedia.org/wiki/Libration>



The phase and libration of the Moon for 2013 at hourly intervals, with music, titles and supplemental graphics.



Simulated views of the Moon over one month, demonstrating librations in [latitude](#) and [longitude](#). Also visible are

Libration



COLMAP [colmap.github.io]

- Popular open-source SfM system



Sparse model of central Rome using 21K photos produced by COLMAP's SfM pipeline.

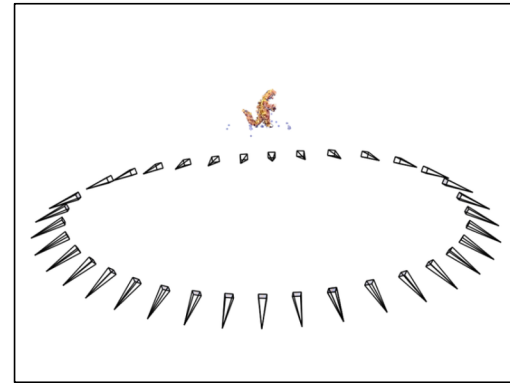
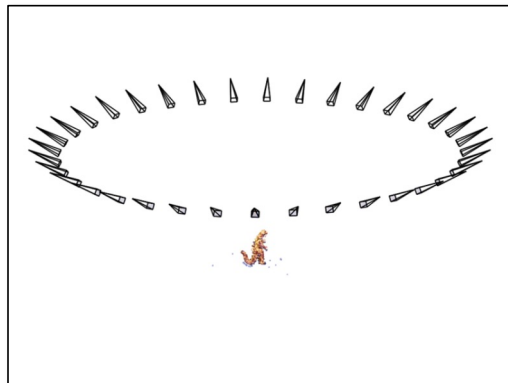


Dense models of several landmarks produced by COLMAP's MVS pipeline.

Questions?

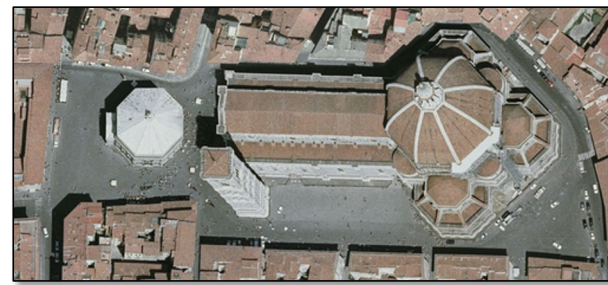
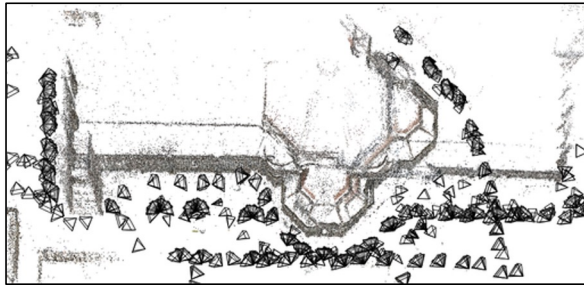
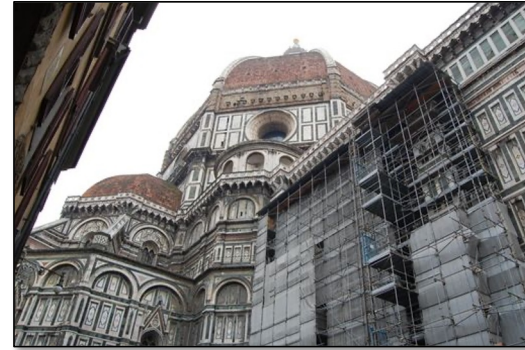
SfM – Failure cases

- Necker reversal



Structure from Motion – Failure cases

- Repetitive structures: Symmetries in man-made scenes



Visual twins (“Doppelgangers”)



Local feature matching will find lots of high-confidence matches



Doppelgangers: Learning to Disambiguate Images of Similar Structures

Ruojin Cai¹, Joseph Tung¹, Qianqian Wang¹, Hadar Averbuch-Elor², Bharath Hariharan¹, Noah Snavely¹

¹Cornell University, ²Tel Aviv University

<https://doppelgangers-3d.github.io/>

Building a dataset of Doppelgangers

Category:Arc de Triomphe de l'Étoile



From Wikimedia Commons, the free media repository

▶ Historical images of Arc de Triomphe de l'Étoile (2 C)

B

▶ Back left views of the Arc de Triomphe de l'Étoile (4 C)

▶ **Back right views of the Arc de Triomphe de l'Étoile (5 C)**

▶ Back views of the Arc de Triomphe de l'Étoile (4 C)

D

▶ Details of Arc de Triomphe de l'Étoile (7 C, 25 F)

▶ Dégradation de l'Arc de Triomphe le 1er décembre 2018 (14 F)

E

▶ Arc de Triomphe de l'Étoile special events (6 C, 44 F)

▶ Exterior of Arc de Triomphe de l'Étoile (7 C)

F

▶ Front left views of the Arc de Triomphe de l'Étoile (5 C, 1 F)

▶ **Front right views of the Arc de Triomphe de l'Étoile (5 C, 1 F)**

▶ Front views of the Arc de Triomphe de l'Étoile (7 C, 3 F)

I

▶ Interior of Arc de Triomphe de l'Étoile (1 C, 43 F)

N

▶ Arc de Triomphe de l'Étoile at night (178 F)

▶ Left side views of the Arc de Triomphe de l'Étoile (1 C, 27 F)

Arc de Triomphe [Collapse]

Triumphal arch in Paris



- image nighttime view aerial view
 view image of interior video
 3D model
 Show all

[Upload media](#)

[Wikipedia](#)

Instance of	triumphal arch
Made from material	Lutetian limestone limestone
Location	Paris, Grand Paris, Île-de-France, Metropolitan France, France
Located on street	place Charles-de- Gaulle
Architectural style	Neoclassical architecture

The Doppelgangers Dataset

- 224 scenes, 76K photos, 1M image pairs, 178K

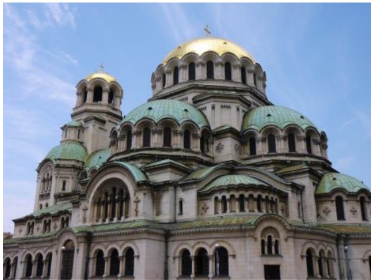
dc



(a) Arc de Triomphe



(b) Charlottenburg Palace



(c) Alexander Nevsky Cathedral



(d) Sleeping Beauty Castles

Training a classifier



Input image pair

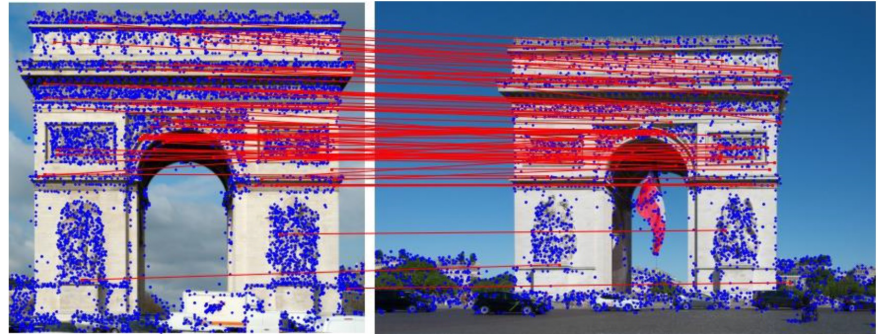
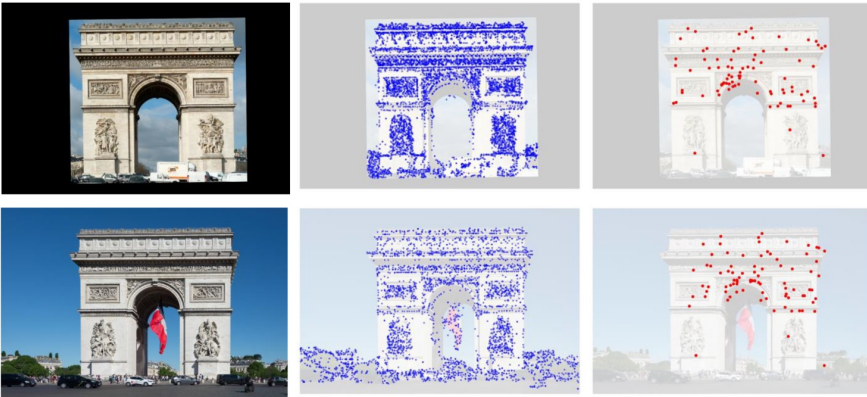
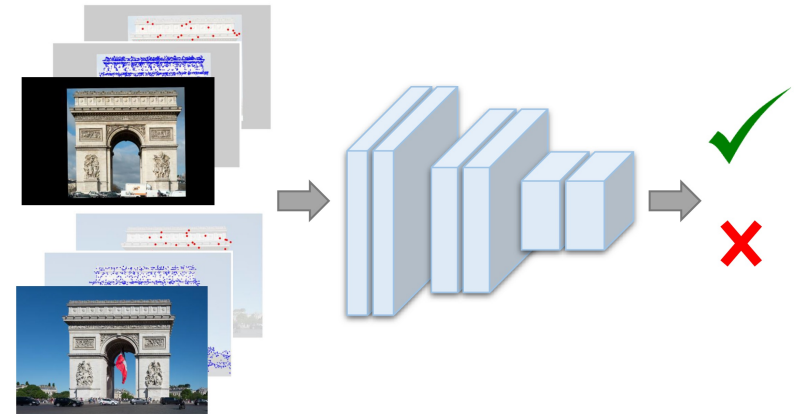


Image features and matches

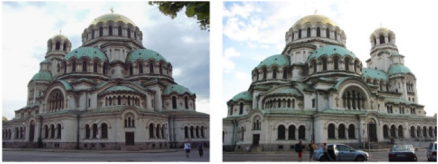


Roughly aligned images and feature/match masks



Doppelganger classifier

Alexander Nevsky Cathedral



Arc de Triomphe



Berliner Dom

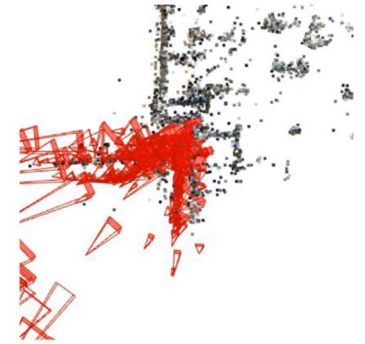
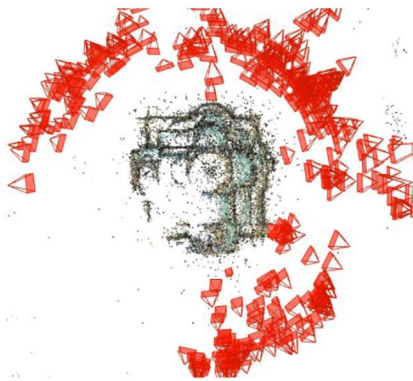


Big Ben



Images

COLMAP



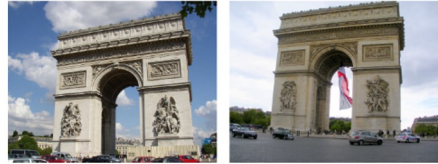
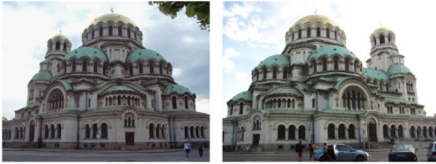
Alexander Nevsky Cathedral

Arc de Triomphe

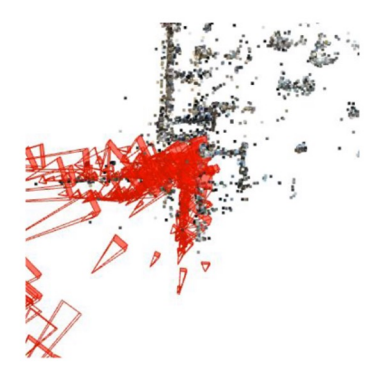
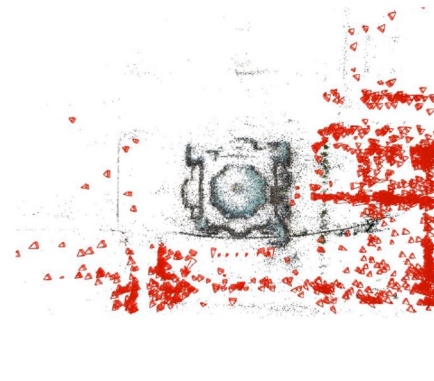
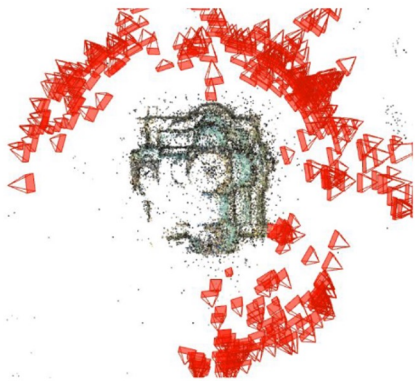
Berliner Dom

Big Ben

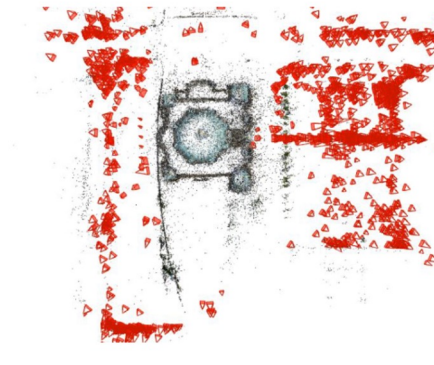
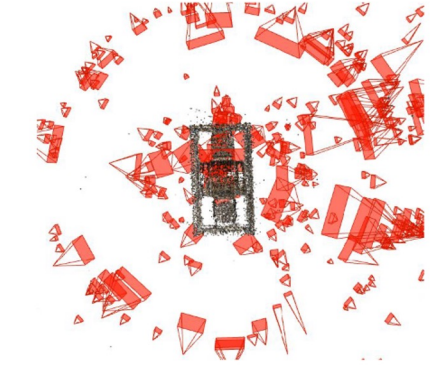
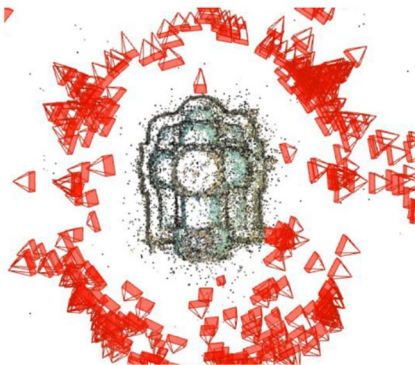
Images



COLMAP



Ours



SfM applications

- 3D modeling
- Surveying
- Robot navigation and mapmaking
- Virtual and augmented reality
- Visual effects (“Match moving”)
 - https://www.youtube.com/watch?v=RdYWp70P_kY

Applications: NeRF



Applications – Hyperlapse



<https://www.youtube.com/watch?v=SOpwHaQnRSY>

<https://www.youtube.com/watch?v=sA4Za3Hv6ng>

Applications: Visual Reality & Augmented Reality



Oculus

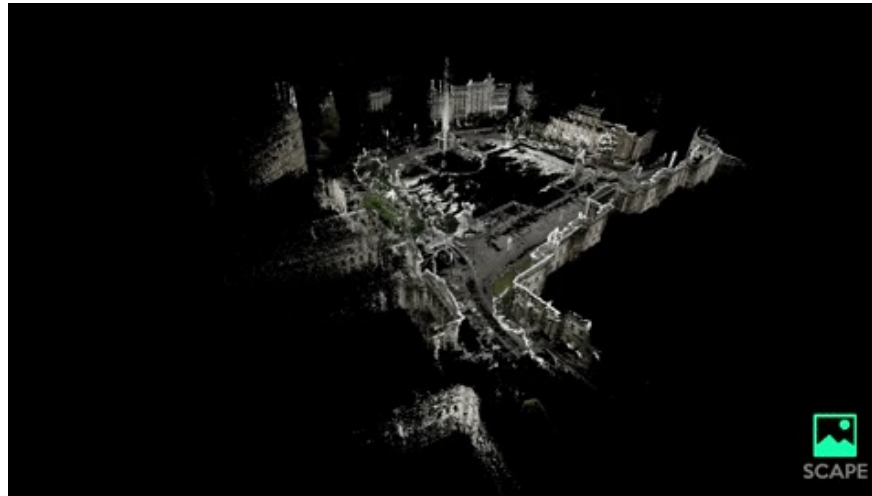
<https://www.youtube.com/watch?v=KOG7yTz1iTA>



Hololens

<https://www.youtube.com/watch?v=FMtvrTGnP04>

Applications: Visual Reality & Augmented Reality



Scape: Building the 'AR Cloud': Part Three —3D Maps, the Digital Scaffolding of the 21st Century

<https://medium.com/scape-technologies/building-the-ar-cloud-part-three-3d-maps-the-digital-scaffolding-of-the-21st-century-465fa55782dd>

Application: AR walking directions



<https://www.theverge.com/2019/8/8/20776247/google-maps-live-view-ar-walking-directions-ios-android-feature>

Questions?