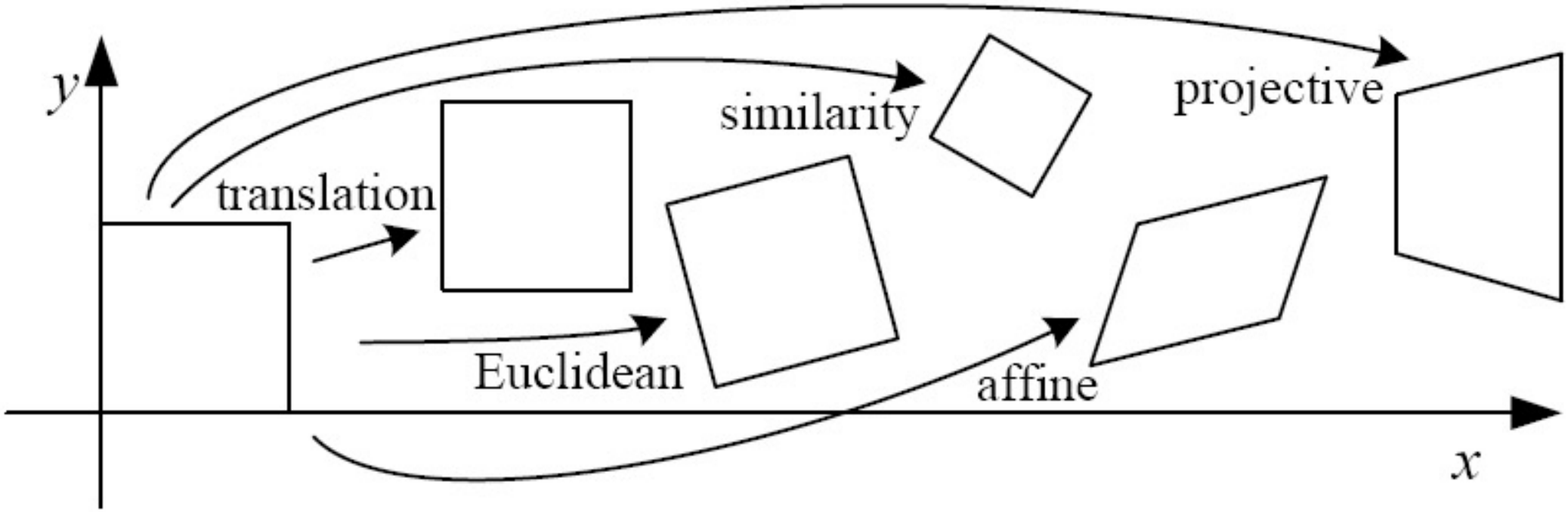
# CS5670: Computer Vision

## Image transformations and image warping



# Reading

- Szeliski: Chapter 3.6

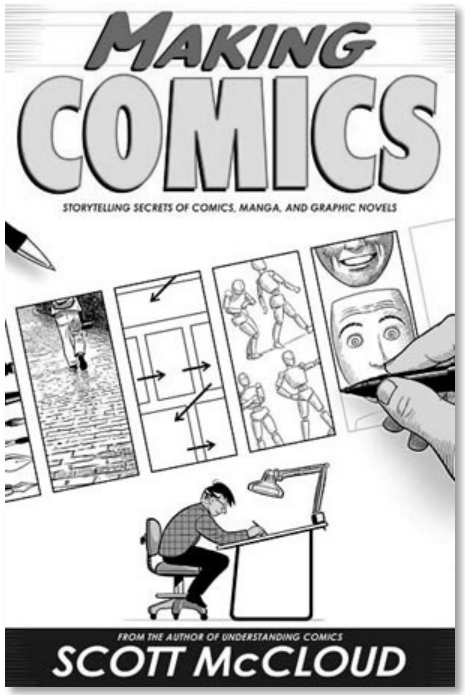
# Announcements

- Project 2 out, due Friday, February 24 by 8pm
  - Do be done in groups of 2 – if you need help finding a partner, try Ed Discussions or let us know

# Image alignment

Why don’t these image line up exactly?

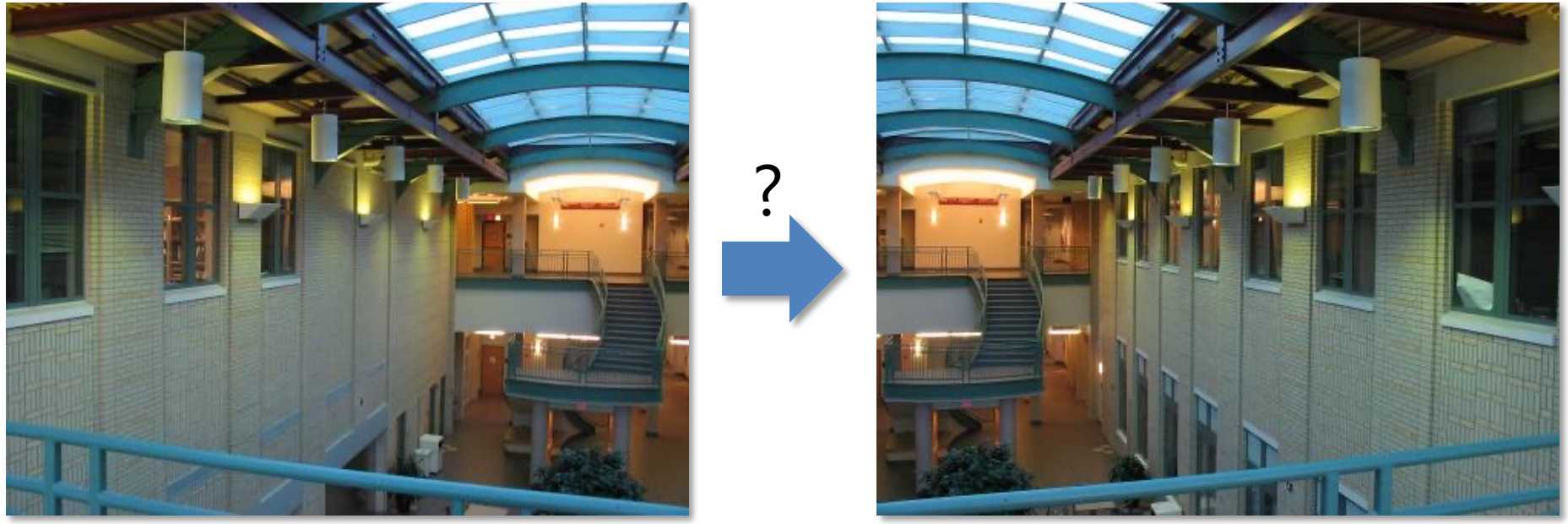
# What is the geometric relationship between these two images?




?

**Answer: Similarity transformation** (translation, rotation, uniform scale)

# What is the geometric relationship between these two images?



# What is the geometric relationship between these two images?

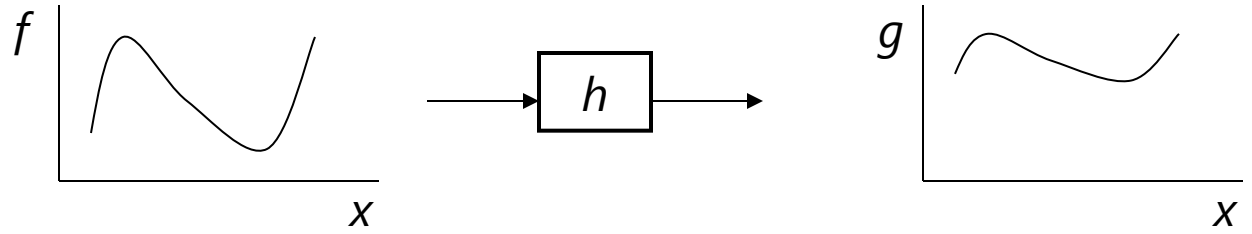
**Very important for creating mosaics!**

First, we need to know what this transformation is.

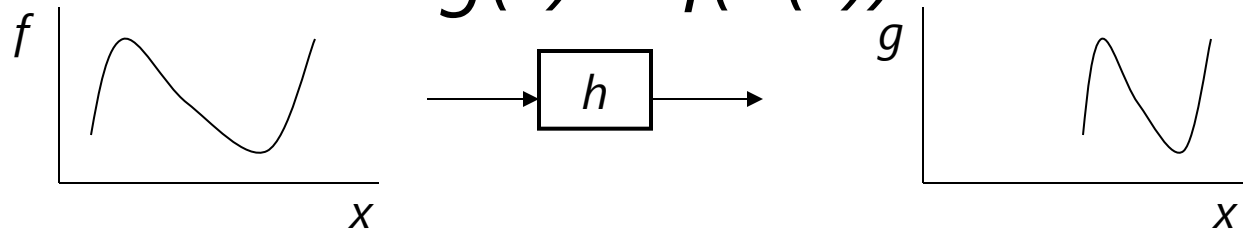
Second, we need to figure out how to compute it using feature matches.

# Image Warping

- image filtering: change *range* of image
  - $g(x) = h(f(x))$




- image warping: change *domain* of image
  - $g(x) = f(h(x))$



# Image Warping

- image filtering: change *range* of image
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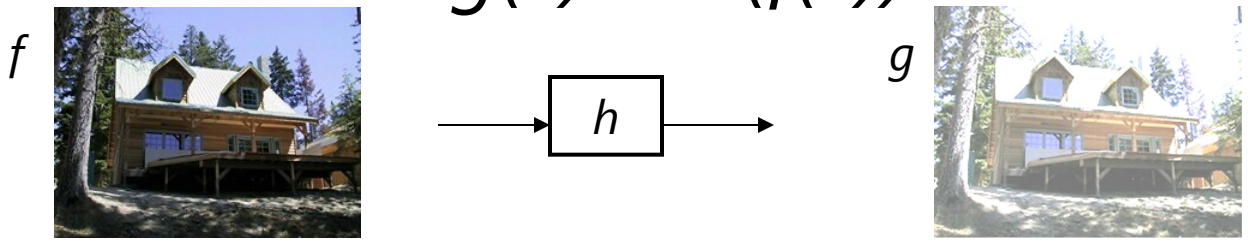


- image warping: change *domain* of image
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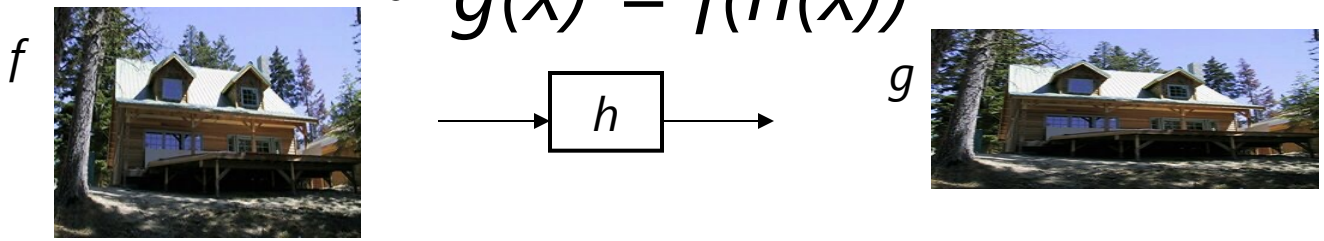


# Parametric (global) warping

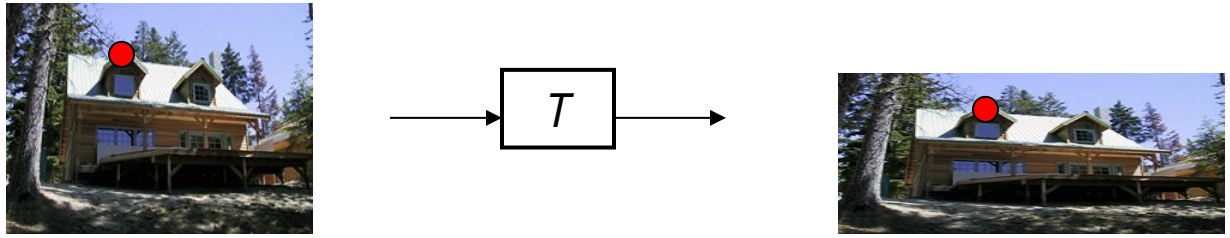
- Examples of parametric warps:

translation

rotation

aspect

# Parametric (global) warping




**p** = (x,y)

**p'** = (x',y')

- Transformation T is a coordinate-changing machine:

$$\mathbf{p}' = T(\mathbf{p})$$

- What does it mean that *T* is global?
  - Is the same for any point **p**
  - can be described by just a few numbers (parameters)
- Let's consider *linear* xforms (can be represented by a 2x2 matrix):

$$\mathbf{p}' = \mathbf{T}\mathbf{p} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Common linear transformations

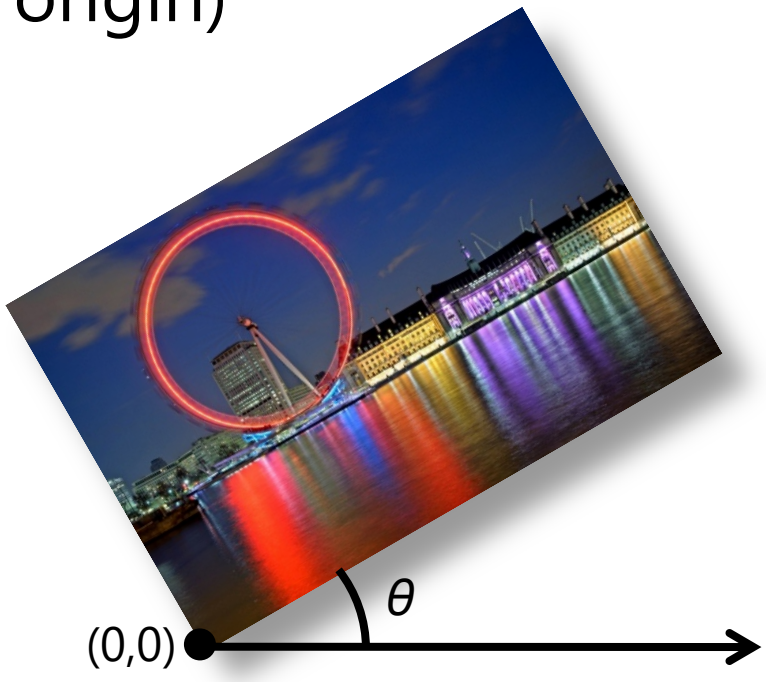
- Uniform scaling by *s*:

$$\mathbf{S} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

What is the inverse?

# Common linear transformations

- Rotation by angle $\theta$ (about the origin)




$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

What is the inverse?

For rotations:

$$\mathbf{R}^{-1} = \mathbf{R}^{T}$$

# 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

  2D mirror across Y axis?

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned}$$

  2D mirror across line y = x?

$$\begin{aligned} x' &= y \\ y' &= x \end{aligned}$$

slido

# Can a 2D mirror across the line y=x be represented with a 2x2 matrix transformation?

ⓘ Start presenting to display the poll results on this slide.

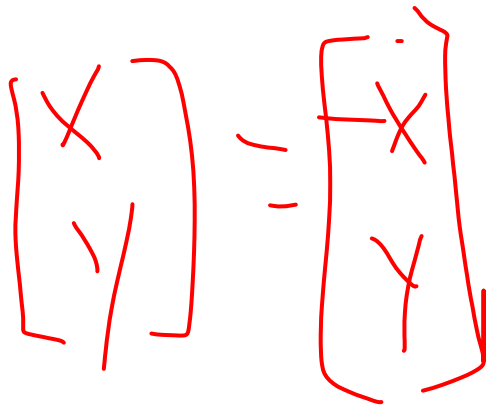
# 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D mirror across Y axis?

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned}$$

$$\mathbf{T} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



2D mirror across line y = x?

$$\begin{aligned} x' &= y \\ y' &= x \end{aligned}$$

$$\mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

# 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$
\begin{aligned}
x' &= x + t_x \\
y' &= y + t_y
\end{aligned}
$$

slido

# Can a 2D translation by (u,v) be represented with a 2x2 matrix transformation?

ⓘ Start presenting to display the poll results on this slide.

# 2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x$$
$$y' = y + t_y$$

NO!

Translation is not a linear operation on 2D coordinates

# All 2D Linear Transformations

- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

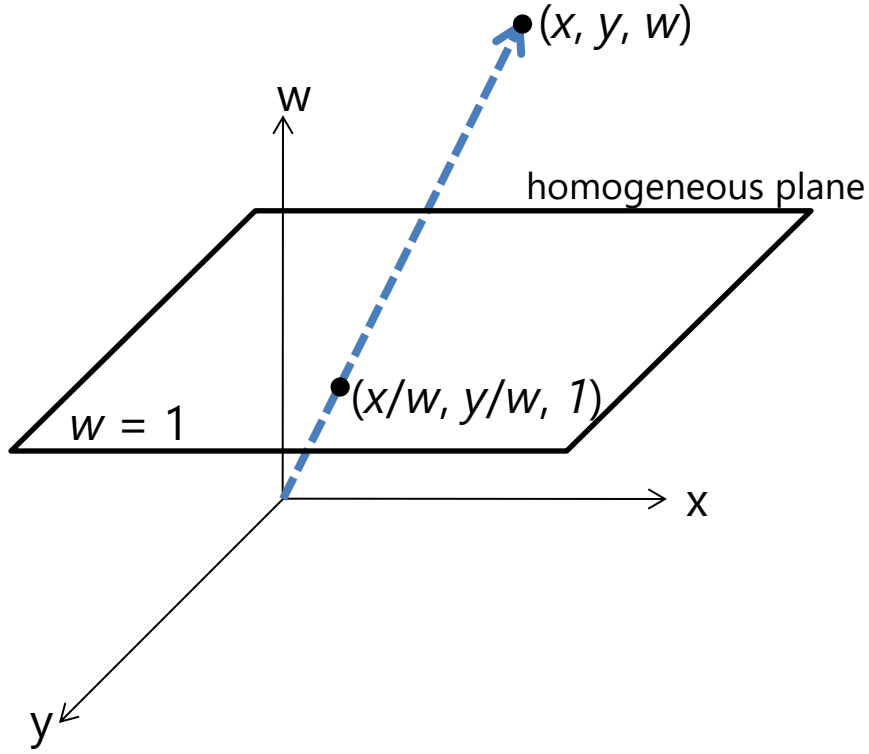
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

# Homogeneous coordinates

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates




Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

# Translation

- Solution: homogeneous coordinates to the rescue

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

# Affine transformations

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

any transformation represented by a 3x3 matrix with last row [ 0 0 1 ] we call an *affine*

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

# Basic affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_x & 0 & 0 \\ 0 & \boldsymbol{s}_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D *in-plane* rotation

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh}_x & 0 \\ \boldsymbol{sh}_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

# Affine transformations

- Affine transformations are combinations of …
  - Linear transformations, and
  - Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of affine transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

# Is this an affine transformation?

# Where do we go from here?

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

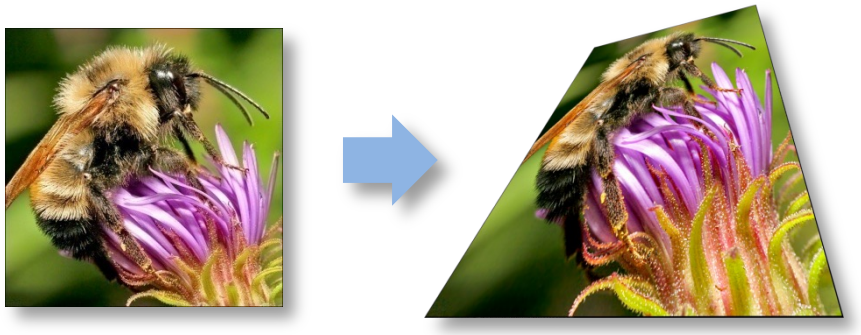
what happens when we mess with this row?

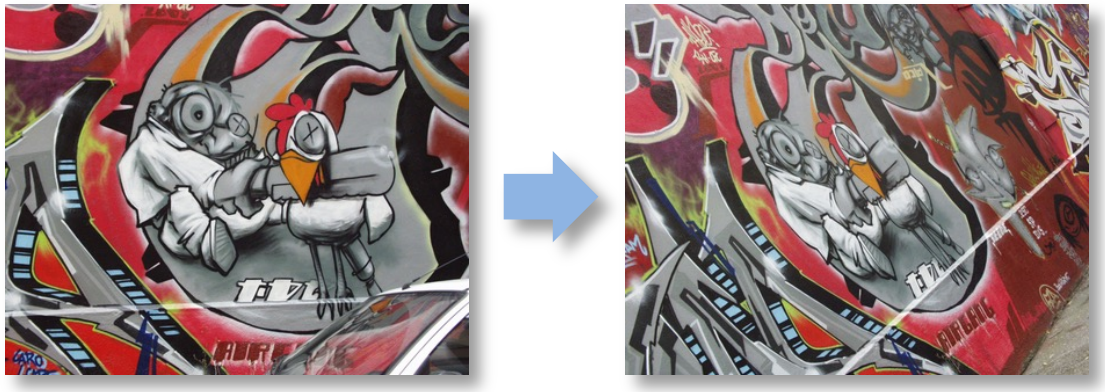
affine transformation

# Projective Transformations *aka* Homographies *aka* Planar Perspective Maps

$$\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

Called a *homography*
(or *planar perspective map*)





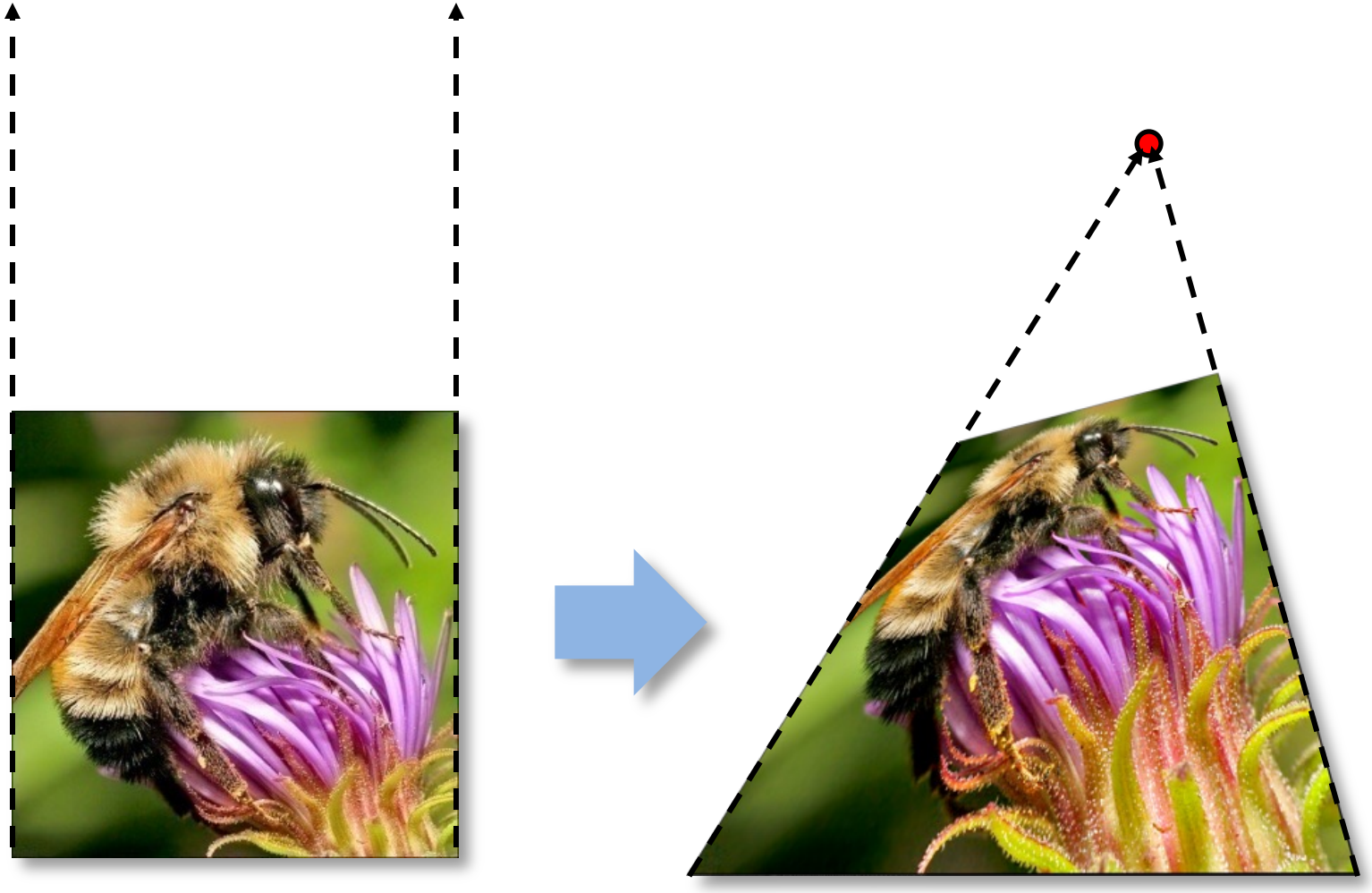
# Homographies

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

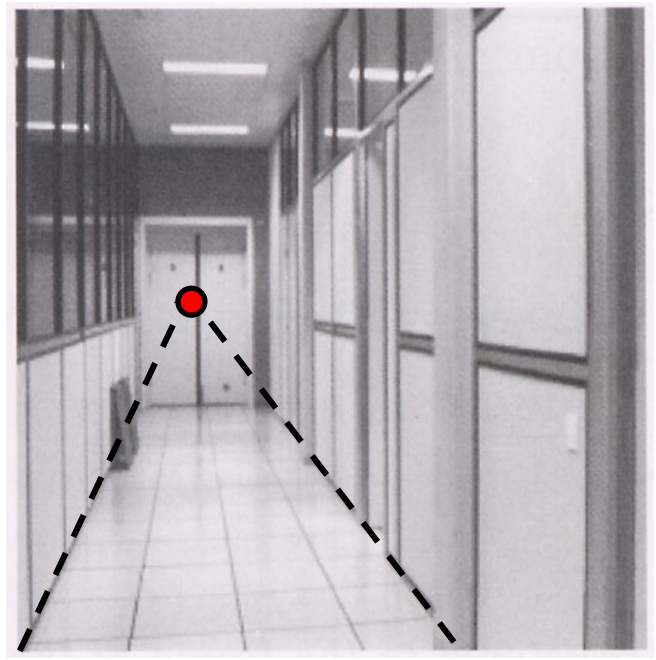
**What happens when the denominator is 0?**

$$\sim \begin{bmatrix} \frac{ax+by+c}{gx+hy+1} \\ \frac{dx+ey+f}{gx+hy+1} \\ 1 \end{bmatrix}$$

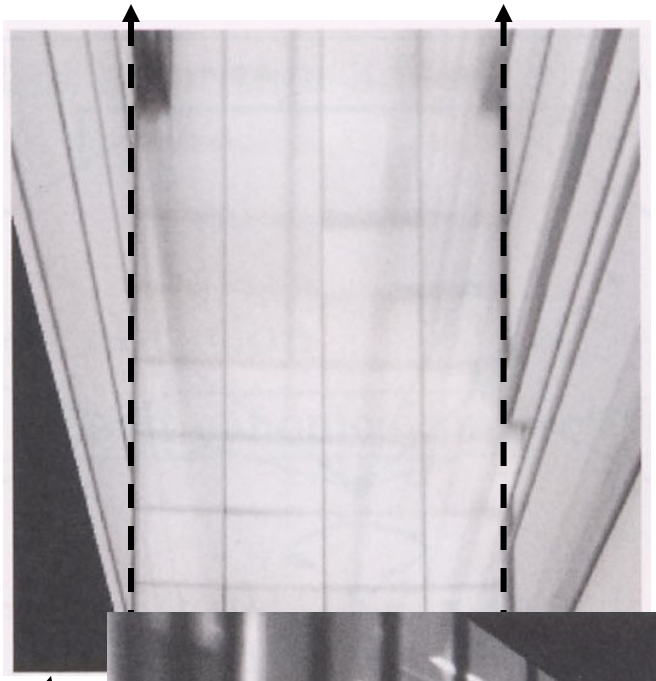
# Points at infinity



# Image warping with homographies



$\mathbf{H}_1$



$\mathbf{H}_2$

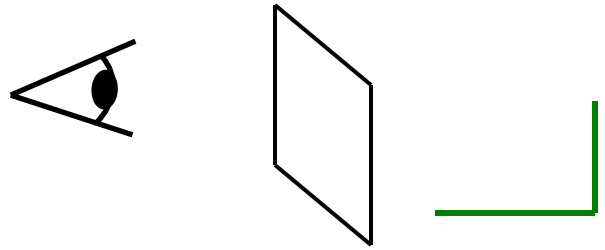


image plane in front

black area
where no pixel
maps to

# Homographies

# Homographies

- Homographies …
  - Affine transformations, and
  - Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

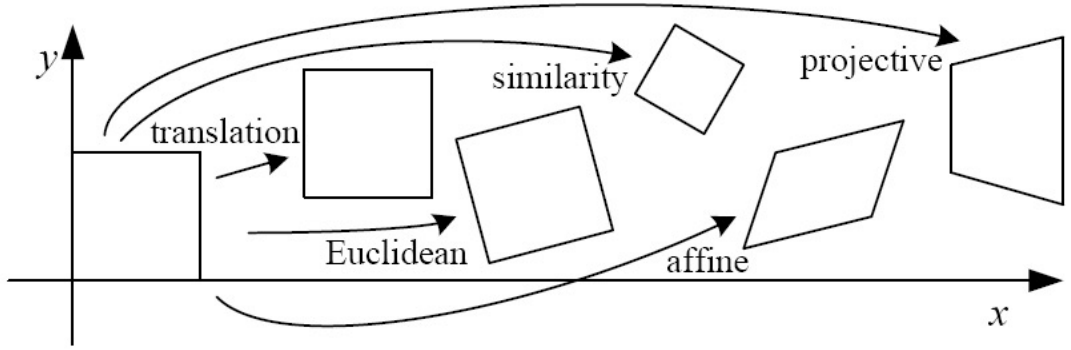
- Properties of projective transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines do not necessarily remain parallel
  - Ratios are not preserved
  - Closed under composition

# Alternate formulation for homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

where the length of the vector [$h_{00}$ $h_{01}$ … $h_{22}$] is 1

# 2D image transformations



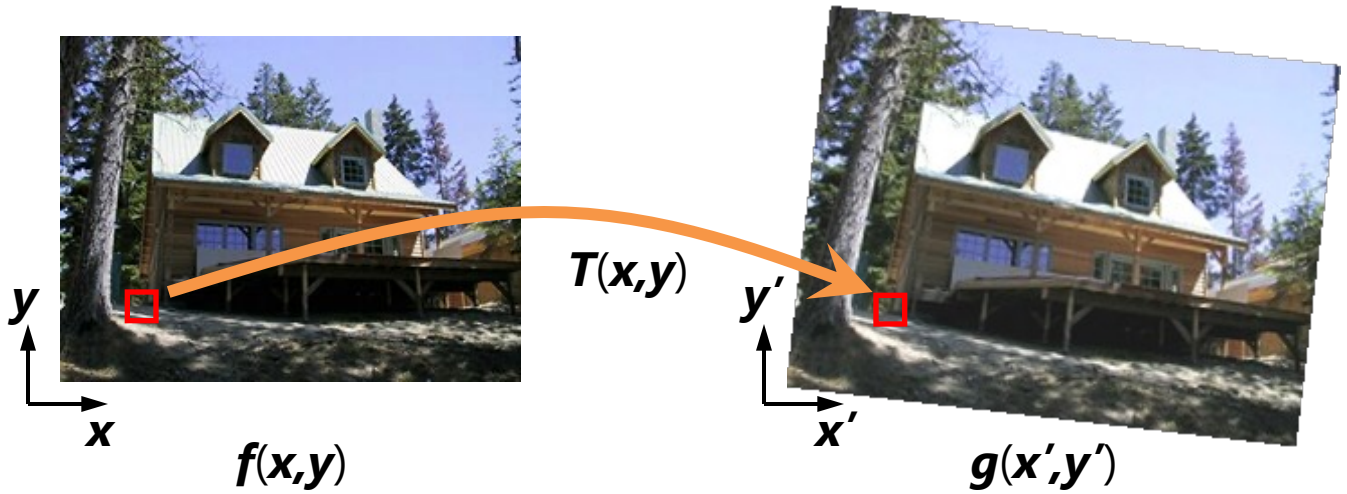

| Name | Matrix | # D.O.F. | Preserves: | Icon |
|---|---|---|---|---|
| translation | $\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$ | 2 | orientation $+\cdots$ | |
| rigid (Euclidean) | $\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$ | 3 | lengths $+\cdots$ | |
| similarity | $\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$ | 4 | angles $+\cdots$ | |
| affine | $\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$ | 6 | parallelism $+\cdots$ | |
| projective | $\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$ | 8 | straight lines | |

These transformations are a nested set of groups

- Closed under composition and inverse is a member

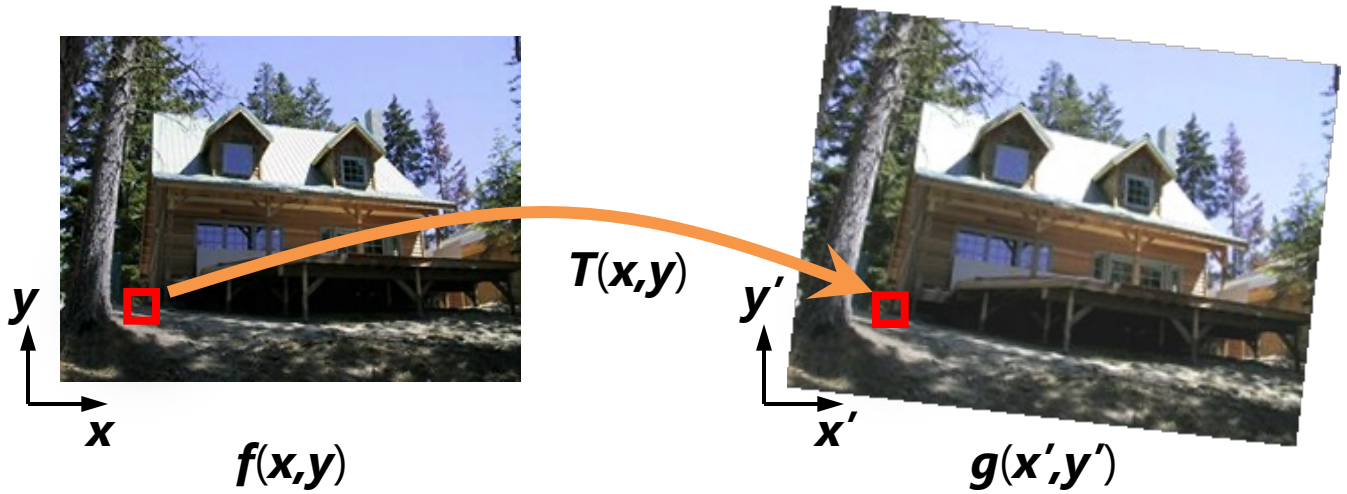
# Implementing image warping

- Given a coordinate xform ($x', y'$) = $T(x,y)$ and a source image $f(x,y)$, how do we compute a transformed image $g(x', y')$ = $f(T(x,y))$?



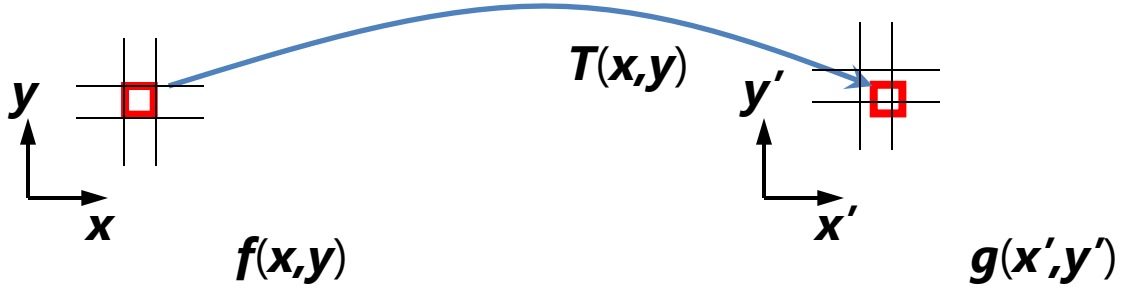
# Forward Warping

- Send each pixel ($x,y$) to its corresponding location ($x',y'$) = $T(x,y)$ in $g(x',y')$
    - What if pixel lands "between" two pixels?



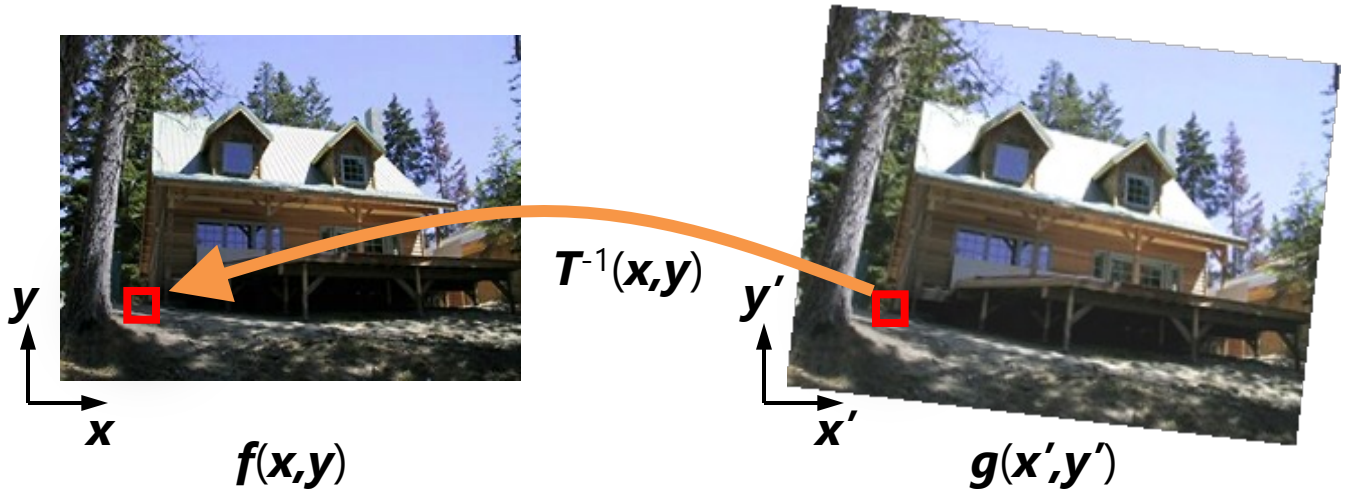
# Forward Warping

- Send each pixel (***x,y***) to its corresponding location (***x′,y′***) = ***T***(***x,y***) in ***g***(***x′,y′***)
  - What if pixel lands "between" two pixels?
  - Answer: add "contribution" to several pixels, normalize later (*splatting*)
  - Can still result in holes



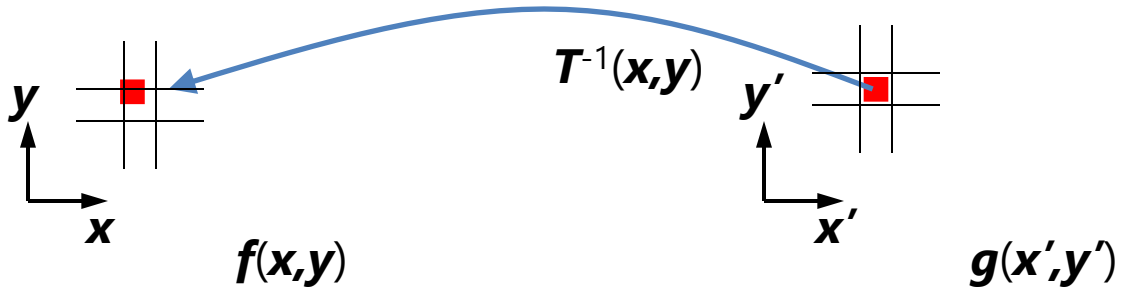
# Inverse Warping

- Get each pixel $g(x',y')$ from its corresponding location $(x,y) = T^{-1}(x,y)$ in $f(x,y)$
  - Requires taking the inverse of the transform
  - What if pixel comes from "between" two pixels?



# Inverse Warping

- Get each pixel $g(x',y')$ from its corresponding location $(x,y) = T^{-1}(x',y')$ in $f(x,y)$
  - What if pixel comes from "between" two pixels?
  - Answer: *resample* color value from *interpolated* (*prefiltered*) source image



# Interpolation

- Possible interpolation filters:
  - nearest neighbor
  - bilinear
  - bicubic
  - sinc
- Needed to prevent “jaggies” and “texture crawl”

  (with prefiltering)

# Questions?