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Growth model ~~for~~ with preferential attachments : (Review)

δ edges one endpoint at edges we added
other end with probability of degree of vertex

$d_i(t)$ degree of i^{th} vertex at time t

Probability that edge connected to vertex i = $\delta \underbrace{\dfrac{d_i(t)}{2\delta t}}_{\text{normalizing term}}$

$$\frac{\partial}{\partial t} d_i(t) = \frac{d_i(t)}{2t}$$

rate of change of degree of vertex i

Solution to the equation $d_i(t) = a t^{1/2}$.
↓
what happened at t_i's of i

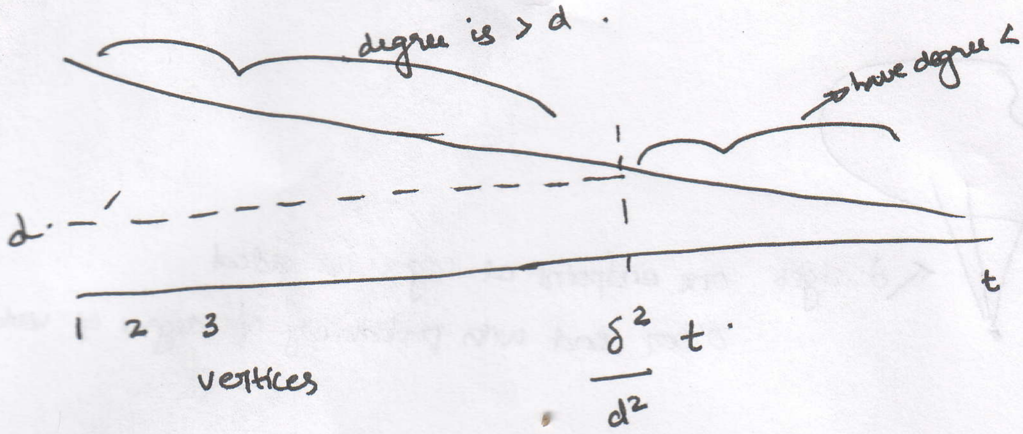
$$\underbrace{d_i(t)}_{\text{degree of } i^{th} \text{ vertex at time } t} = \delta \sqrt{t/t_i}$$

Probability of degree distribution :-

Claim $d_i(t) < d$ provided $\delta \sqrt{\dfrac{t}{t_i}} < d$. $\boxed{d \rightarrow \text{just any no.}}$

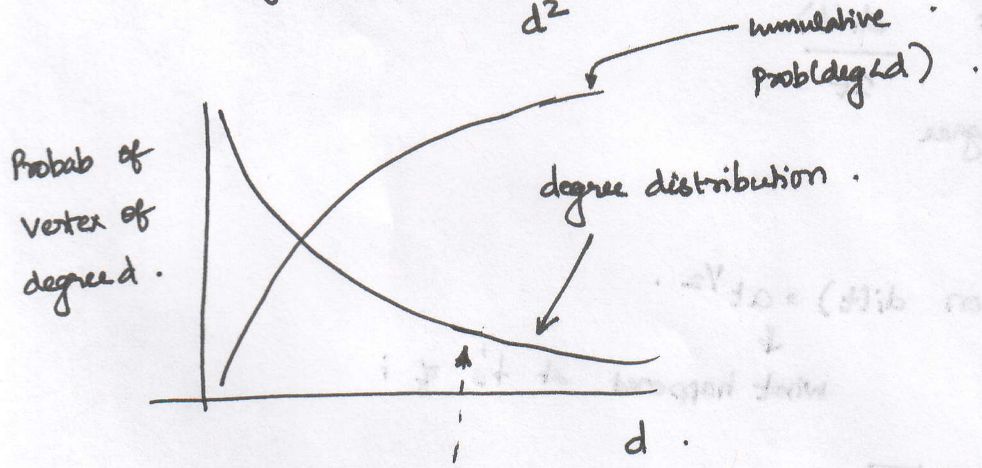
$$t_i > \frac{\delta^2}{d^2} t.$$

early in time expected degree is high!



If I want to pick a vertex at random and want to know if degree is $< d$ is $1 - \frac{\delta^2}{d^2}$.

$$\text{Prob}(\deg < d) = 1 - \frac{\delta^2}{d^2}$$



$$\frac{\partial}{\partial d}\left(1 - \frac{\delta^2}{d^2}\right) = \frac{2\delta^2}{d^3}$$

power law degree distribution with exponent of degree 3

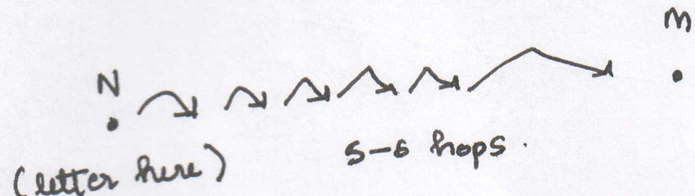
[WWW → exponent 2.85 close to this]

Small World graphs.

Watts & Strogatz → Collective dynamics of small world networks.

John Kleinberg → small world phenomenon. } most of the topics
an 'algorithmamic perspective.' from this.

Stanley Milgram → contacted people letter name and occupation M

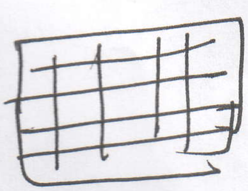


diameter is not very large. [people known to each other on first name basis]

how do we go about finding these short paths. without looking at the full graphs.

In small worlds

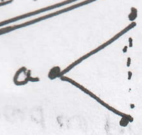
structured graphs.
avg path $n/4$ not $\log n$.



atleast n steps.

pick each vertex @ uniformly at random → diameter log n.

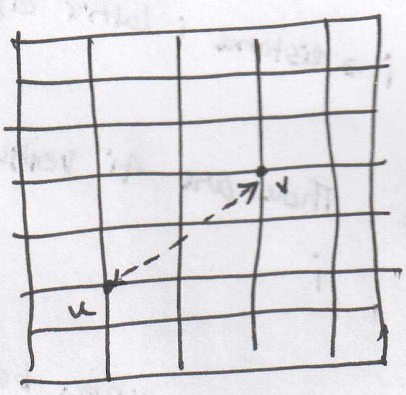
II prop



→ existence of Δ's.

→ high prob of finding them.

Model (Kleinberg) nxn grid.



directed model.

u, v → @ random.

Prob of edge u,v is proportional to $d^{-r}(u,v)$
⇓
distance.

family of graphs if r=0 all endpoints all equally likely.

⋮

as r → ∞ only grid edges.

Theorem 1 :- If r=2, ∃ efficient algorithm to find short path.

[Time in polynomial in length log n].

&

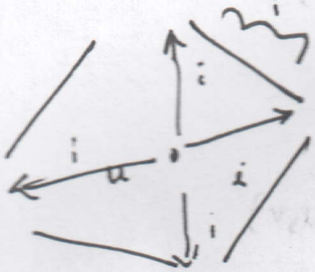
Thm 2 & 3 → on r<2 and r>2: If r≠2 but no local algorithm to find them efficiently.

Probability of edge from u to v is ^Proportional to the ~~distance~~ $d^{-2}(u,v)$

Prob $\dfrac{d^{-2}(u,v)}{\sum_{w \neq u} d^{-2}(u,w)}$ $\Rightarrow$ bound the denominator as we want lower bound on prob.

To do this we need an upperbound on denominator.

$i \Rightarrow$ distance i lattice edges.



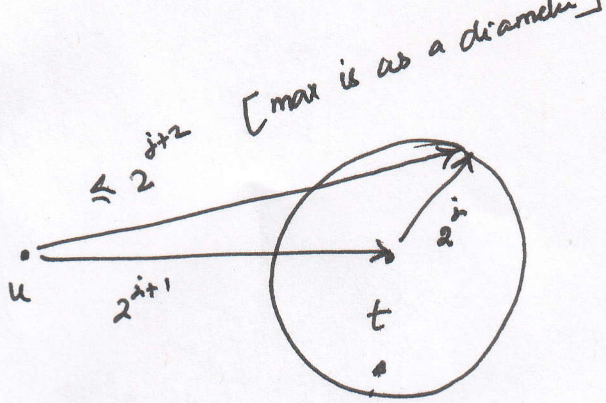
each side has i vertices. There are $4i$ vertices at distance i

If u is near bound or i is very large then $4i$ is upper bound.

$$\sum d(u,w)^{-2} \leq \underbrace{\sum_{i=1}^{2n} 4i}_{\text{no. of vertices}} \frac{1}{i^2}$$

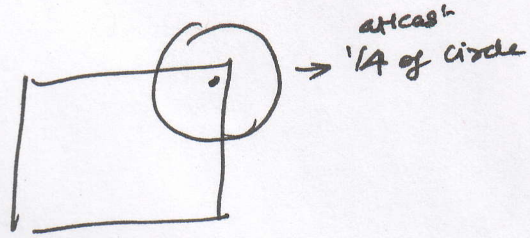
$$\leq 4 \sum_{i=1}^{2n} \frac{1}{i} \quad \left[\int \frac{1}{x} = \log x \right]$$

$\exists$ constant c $\leq c \log n$.



~~to~~ think of it as long circles / There are atleast 2^{2j-1} vertices.

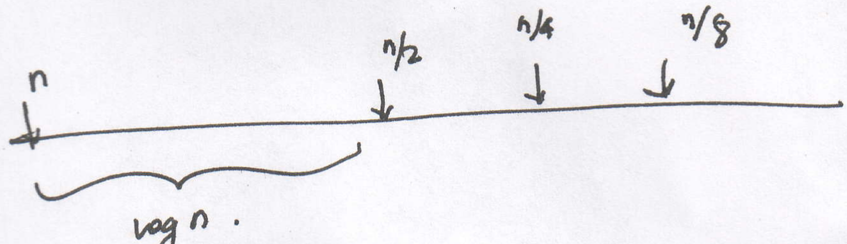
$$\sum_{i=1}^{2^j} i = \frac{2^j(2^j+1)}{2} \le 2^{2j-1}$$



Start at some source. of the many option which is going to get me to the destination.

phase j distance remaining is $(2^{j+1}, 2^j]$

So the max no. of phases is $\log n$.



If expected time in phase is $\log n$ then total expected time is $\log^2 n$.