CS485 Spring 2007
Homework 3

Due Date: Feb 9 2007

1. Imagine performing a series of independent sucess/failure experiments (e.g. biased coin flips, or decisions whether or not to have a candy this second). Probability of each success is p, and we perform n trials. We know that the number of successes is binomially distributed, with parameters n and p.
   (a) Define a random variable X that describes the number of unsuccessful trials before the first success (or end of the trials). Give the possible values of the random variable (its support), and probabilities of these values $P[X = ?]$.
   (b) Show that the probabilities sum up to 1.
   (c) (bonus) What happens when $n \to \infty, p \to 0, np \to \lambda$? Hint: look at $P[X \leq \alpha n]$ for fixed α.
2. Write a program that performs experiments from problem 1. Fix $n = 20, p = \frac{1}{4}$. Every experiment gives you one number, value of X (e.g. number of trials before you first flipped "heads")
   (a) Perform the experiment 100 times and plot the histogram of values of X. That is one batch of experiments.
   (b) Now do 100 batches of these experiments, in each measuring the average value. This gives you 100 numbers. Plot a histogram of them.
   (c) What shape would you expect for the histogram of the averages? Why?
3. Look at when cycles occur in $G(n, p)$ as more and more edges are added. Start with an empty graph on n vertices, and add a random edge one at a time. Stop when a cycle occurs, and record how many edges are present (let's call this number $c(n)$). Repeat several times and compute averages of $c(n)$ for $n = 10, 20, 40, 80$. How does it change with n?
4. Develop a simple proof that in $G(n, p)$, as p increases, the small components of size 2 and greater disappear (merge into the giant component) before the isolated vertices disappear.