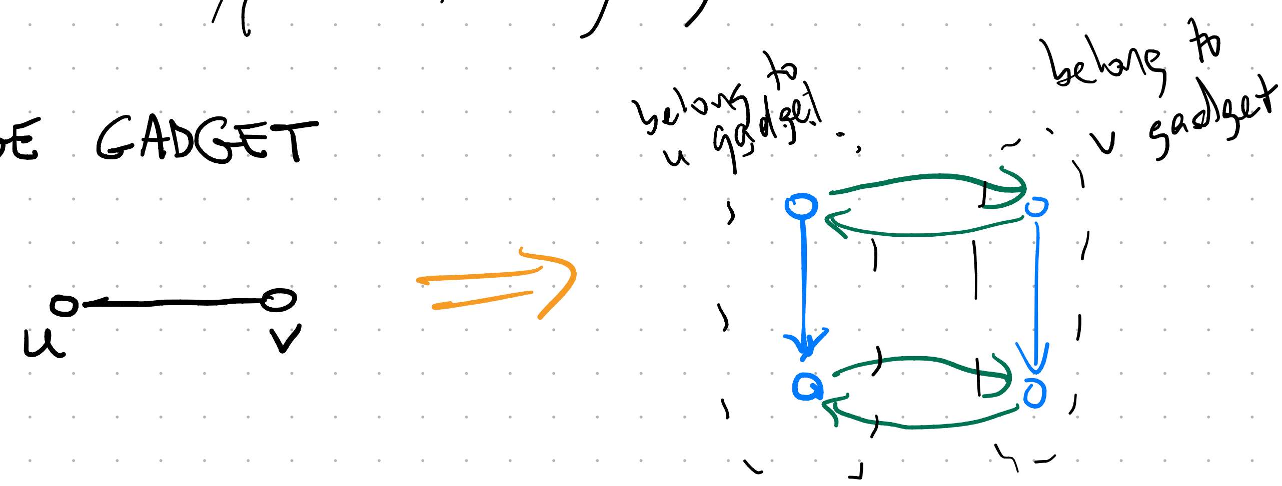
25 Mar 2024 — Hamiltonian Cycle Continued

Announcement: Early Q1 hand-in: Tues. 4pm

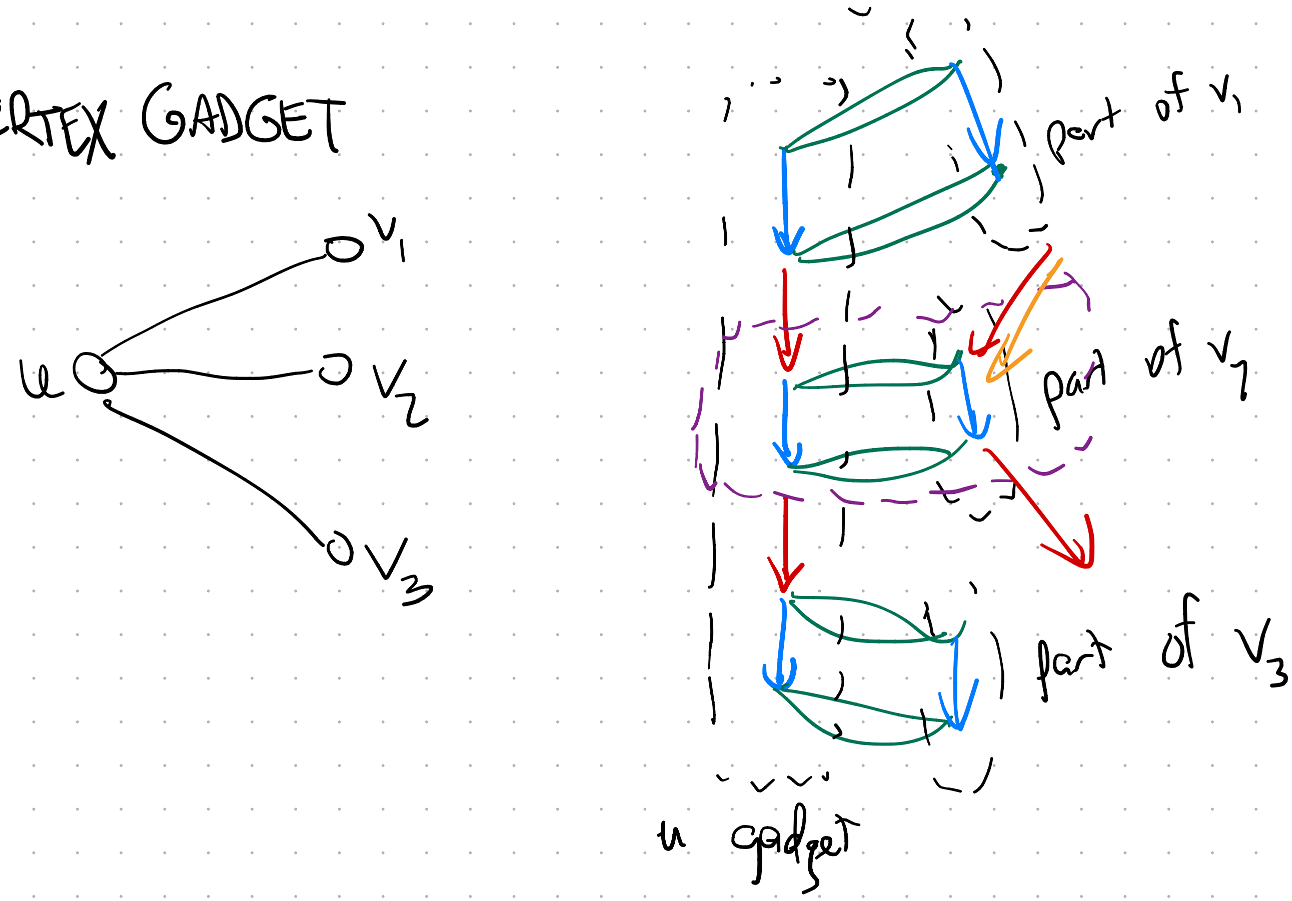
RECALL. Reducing VERTEX COVER to HAM CYCLE

Will represent G as a HAM CYCLE input with 3 types of gadgets.

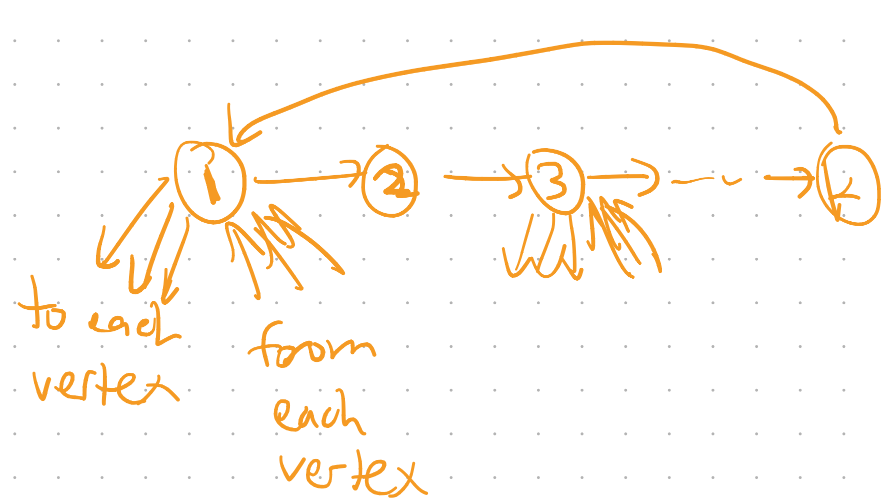
EDGE GADGET



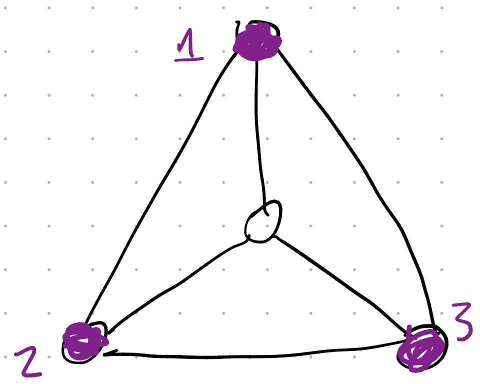
VERTEX GADGET

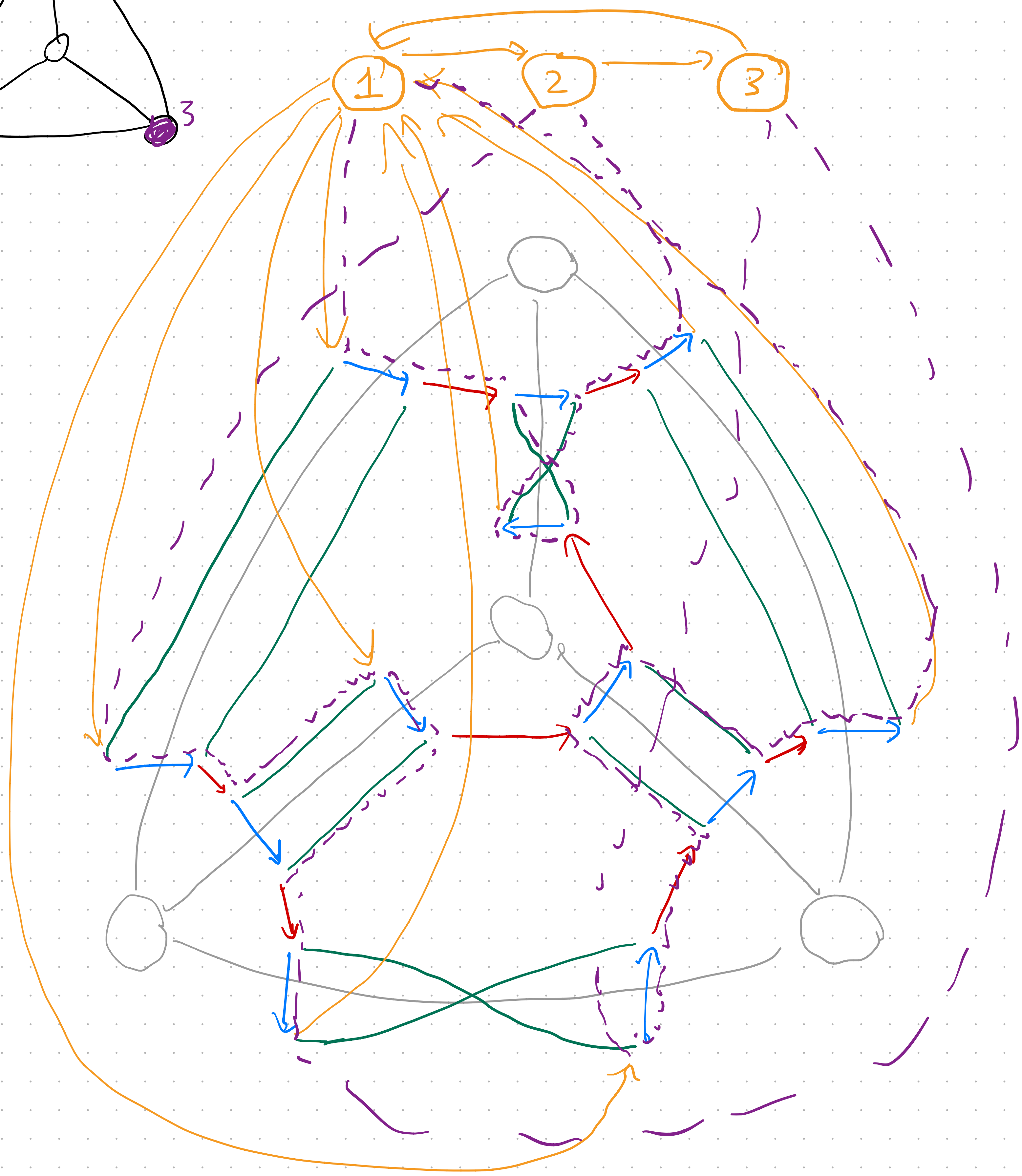


COUNTER GADGET

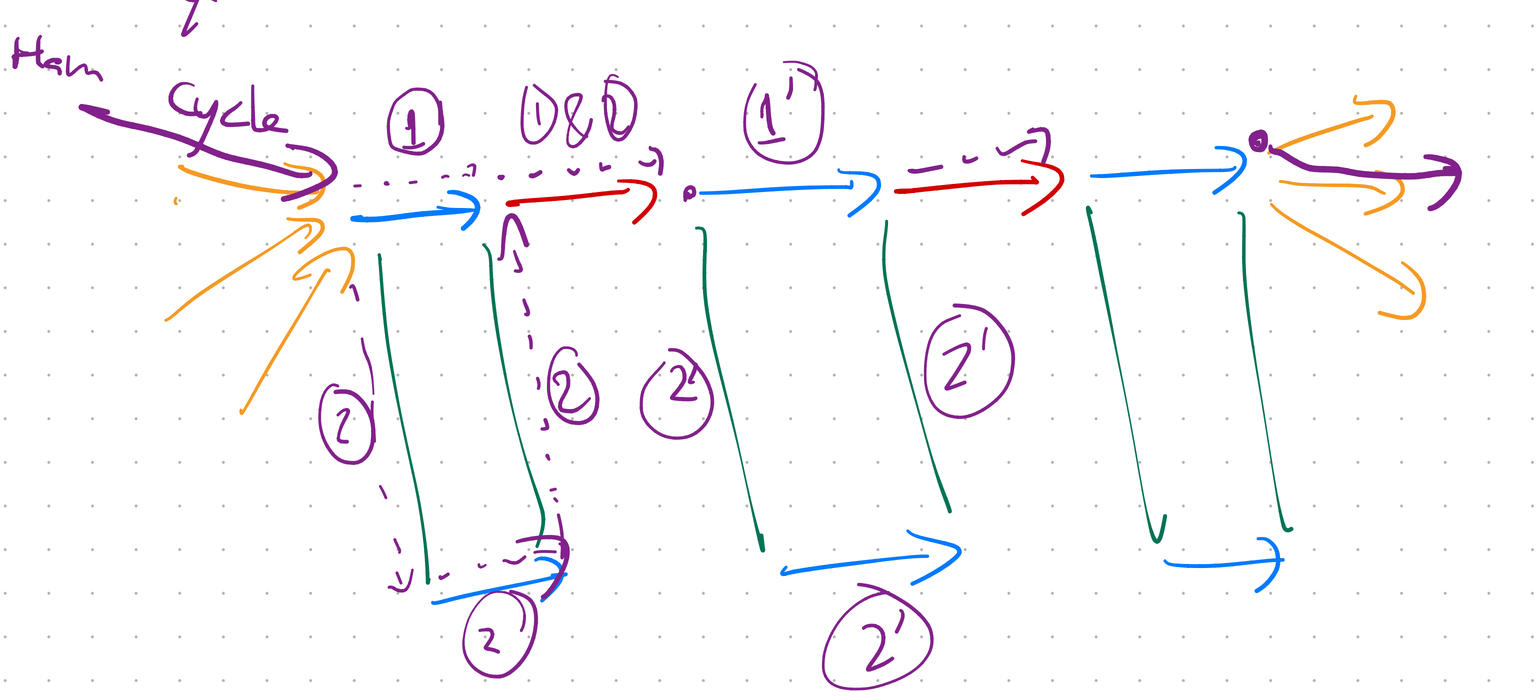


k = 3





If you enter a vtx gadget on an orange edge, you must use each of its red edges,



and exit on another orange.

If you use <u>any</u> red edge of a vertex gadget, you must use <u>all</u> of them, and you must enter and exit that vtx using orange edges.

2 types of vertices:

(A) The cycle uses all of their red edges and 2 of their orange edges.

(B) The cycle uses none of their red/orange edges.

$\leq k$ vertices of type A.

To get into/out of an edge gadget one must cross a red/orange edge, hence visiting the 4 vertices of the edge gadget requires having a type A endpoint.

When trying to reduce Problem A to Problem B, think about matching up "variables" and "constraints."

Variable type

vertex cover: 2-valued ($v \in S$ or $v \notin S$)

Ham. cycle: 2-valued ($e \in C$ or $e \notin C$)

3 SAT : 2-valued ($x_i = T$ or $x_i = F$)

(Graph 3-colorability has 3-valued variables.)

Constraints

vertex cover: One global counting constraint ($\leq k$)

many 2-variable constraints (v OR w)

Ham cycle: one global connectivity constraint.

many local counting constraints (in-degree = 1, out-degree = 1)

3SAT: many 3-variable constraints

The "alternating shared-fate gadget"

(3-dim matching is another example)

Useful when reducing FROM a problem whose variables are in unbounded # constrs. to those where they are in just a few.