# 15 March 2024

Plan.

- Baseball Elimination
- Announcements
- Image Segmentation

# Baseball Elimination Problem

| Teams | Wins | |
|---|---|---|
| BOS | ~~90~~ | 92 |
| NYY | ~~88~~ | 89 |
| BAL | ~~86~~ | 87 |
| TB | 91 | 91 |

Games

(BOS, NYY) ✓
(BOS, TB) ✓
(TB, BAL) X
(NYY, TB) X

Bos ? ✓

# Baseball Elimination Problem

BOS ?

| Teams | Wins |
|---|---|
| BOS | 90 |
| NYY | 88 |
| BAL | 86 |
| TB | 91 |

**Games**

(BOS, NYY)
(BOS, TB)
(TB, BAL)
(NYY, TB)

NYY ?

| Teams | Wins | |
|---|---|---|
| BOS | 90 | ~~2~~ |
| NYY | ~~88~~ | 90 |
| BAL | ~~86~~ | 87 |
| TB | 90 | 2 |

**Games**

(BOS, NYY) ✓
(BOS, TB)
(~~TB~~, BAL)
(NYY, TB) ✓

# Baseball Elimination Problem

Given:

- * List of teams $\langle t_0, \dots, t_k \rangle$
- * Current standings $\langle w_0, \dots, w_k \rangle$

  $w_i$ = current # of wins by $t_i$

- * Remaining games $\langle g_1, \dots, g_n \rangle$

$$g_j = \underbrace{(t_i, t_k)}_{\text{Game } g_j \text{ between } t_i \text{ and } t_k.}$$

**Question:** Can $t_0$ finish with the most wins?

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Given:

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- * Current standings $\langle w_0, \dots, w_k \rangle$
- * Remaining games $\langle g_1, \dots, g_n \rangle$

**Question:** Can $t_0$ finish with the <u>most</u> <u>wins</u>?

**Observation 0.** WLOG assume $t_0$ has no more games.

# Baseball Elimination Problem

Given:
- List of teams $\langle t_0, \ldots, t_k \rangle$
- Current standings $\langle w_0, \ldots, w_k \rangle$
- Remaining games $\langle g_1, \ldots, g_n \rangle$

**Question:** Can $t_0$ finish with the most wins?

**Observation 0.** WLOG assume $t_0$ has no more games.

- Search through all games involving $t_0$
- Assign $t_0$ the win. (i.e. $w_0 \leftarrow w_0 + 1$)
- Remove game

[Preprocessing takes $O(n)$ time]

# Baseball Elimination Problem

Given:

- List of teams $\langle t_0, \dots, t_k \rangle$
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- Remaining games $\langle g_1, \dots, g_n \rangle$

**Question:** Can $t_0$ finish with the most wins?

**Observation:** Every remaining game results in an additional win for some team.

# Baseball Elimination Problem

Given:
- List of teams $\langle t_0, \dots, t_k \rangle$
- Current standings $\langle w_0, \dots, w_k \rangle$
- Remaining games $\langle g_1, \dots, g_n \rangle$

Question: Can $t_0$ finish with the most wins?

Observation. Every remaining game results in an additional win for some team.

Can we allocate all of these wins (i.e. games) such that $t_0$ is the leader?

| | |
|---|---|
| BOS | 90 |
| NYY | 88 |
| BAL | 86 |
| TB | 91 |

(BOS, NYY)
(BOS, TB)
(TB, BAL)
(NYY, TB)

| | |
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~~(BOS, NYY)~~

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(TB, BAL)

(NYY, TB)

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~~(BOS, NYY)~~
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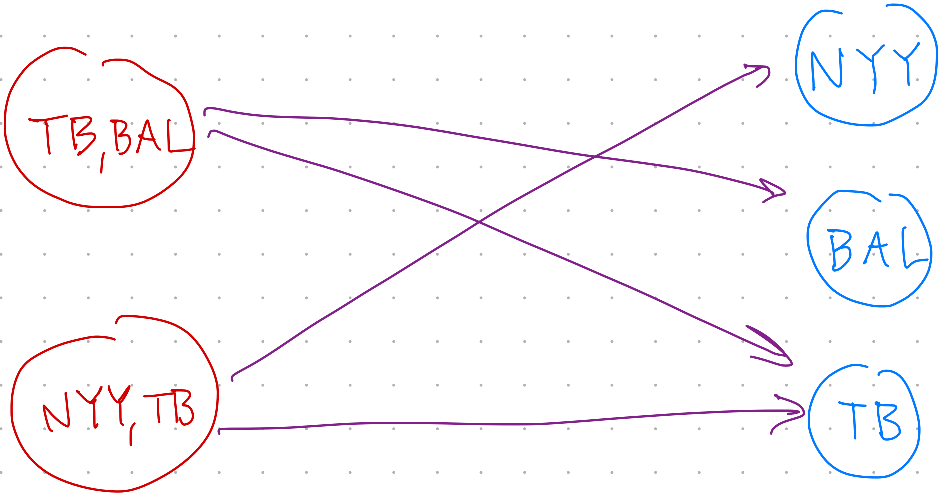

| | |
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| ~~BOS~~ | ~~90~~ 92 |
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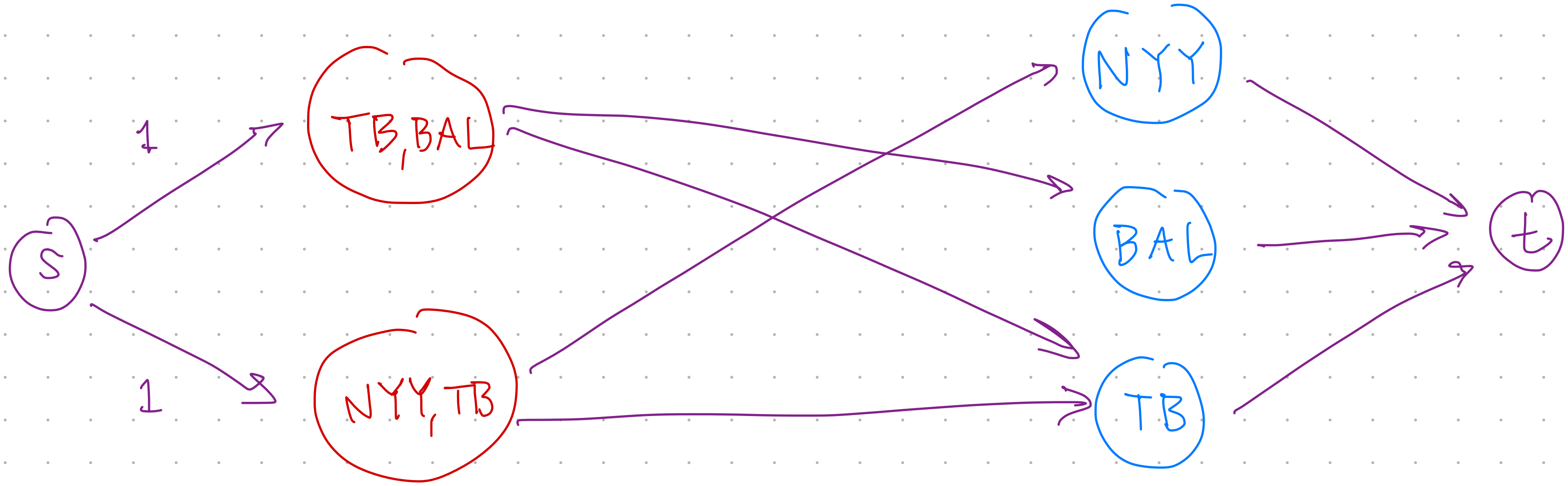

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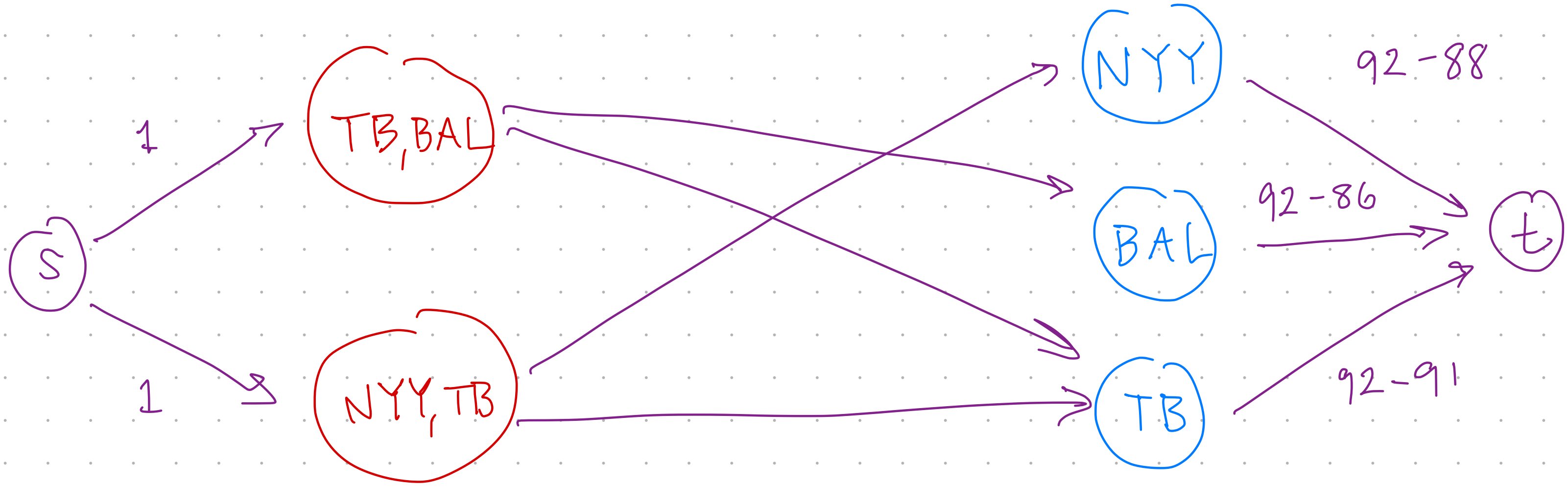

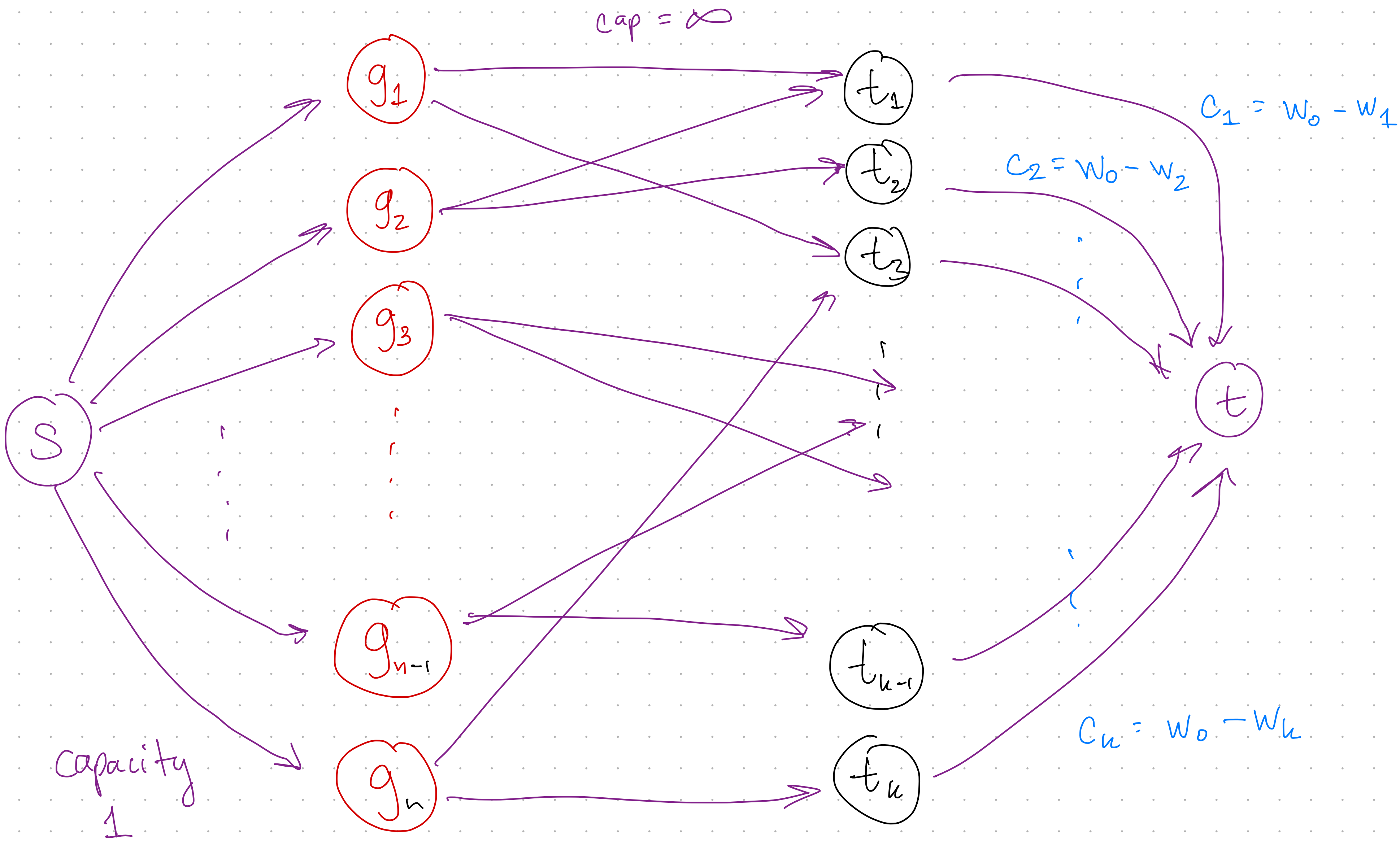

$$\boxed{C_i = W_0 - W_i}$$

← Max # of additional wins s.t. $W_i + C_i \leq W_0$.

**Correctness.** The max flow in $G$ equals $n$ if and only if $t_0$ can finish with the most wins after the remaining $n$ games.

$(\Longrightarrow)$ If the max flow is $n$, there is an "allocation" of wins s.t. $t_0$ finishes in 1st.

$(\Longleftarrow)$ If the max flow is $< n$. $t_0$ cannot finish w/ the most wins.

($\Rightarrow$) Consider a flow $f$ of $n$ units

↳ Devise an "allocation" of wins to teams as follows.

For each unit of $g_j \rightarrow t_i$ flow,

Assign 1 additional win to $t_i$

$w_i \leftarrow w_i + 1$

---

($\Rightarrow$) Consider a flow $f$ of $n$ units

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For each unit of $g_j \rightarrow t_i$ flow,

Assign 1 additional win to $t_i$

---

By capacity constraints: team $t_i$ is "allocated" at most $c_i = W_0 - w_i$ units.

$\Rightarrow$ if team $t_i$ wins each game allocated then $w_i + c_i \leq W_0$.

$(\Rightarrow)$ Consider a flow $f$ of $n$ units

↳ Devise an "allocation" of wins to teams as follows.

For each unit of $g_j \rightarrow t_i$ flow,

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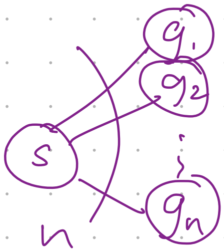
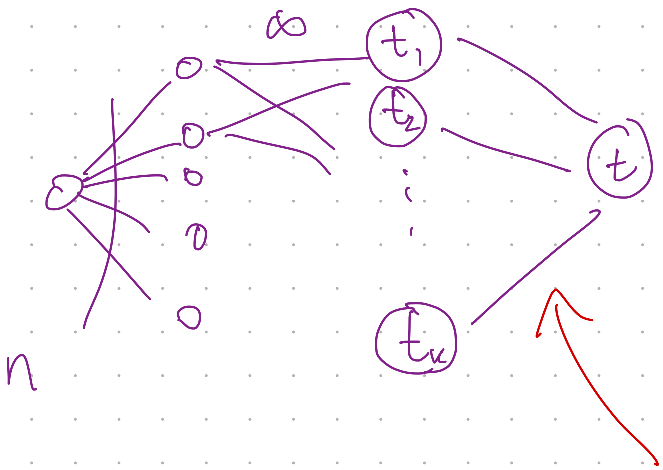
By capacity constraints: team $t_i$ is "allocated" at most $c_i = w_0 - w_i$ units.

$\Longrightarrow$ if team $t_i$ wins each game allocated then $w_i + c_i \leq w_0$.

By construction: $n$ units of flow covers all $n$ remaining games



($\Leftarrow$) Suppose max flow $< n$.



min cut must include some $(t_i, t)$

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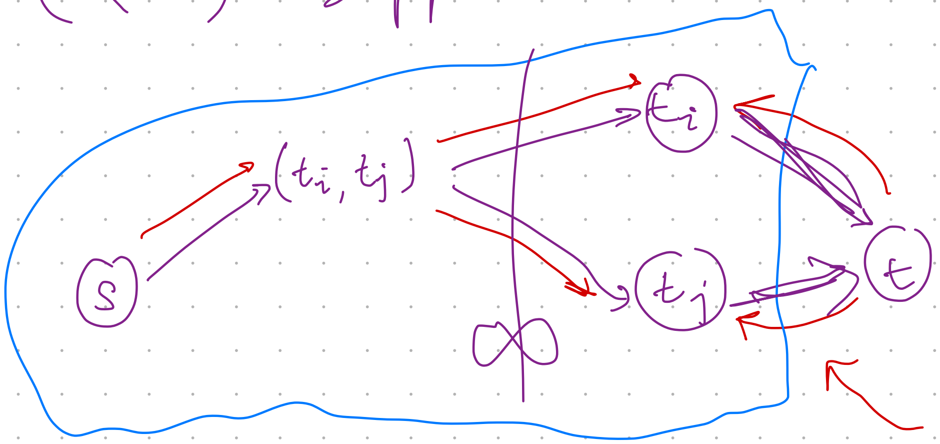


min cut must include some $(t_i, t)$ $(t_j, t)$

$\Rightarrow$ Cannot allocate a win for each game w/o violating team capacity constraints.

$\Downarrow$

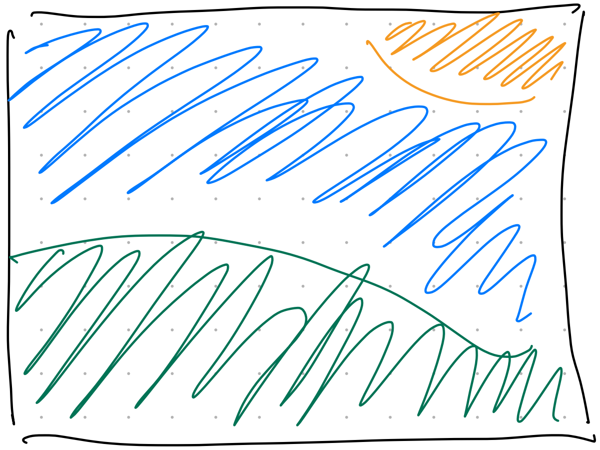
So some $t_i$ ends with

$> w_i + c_i = w_o$ wins.

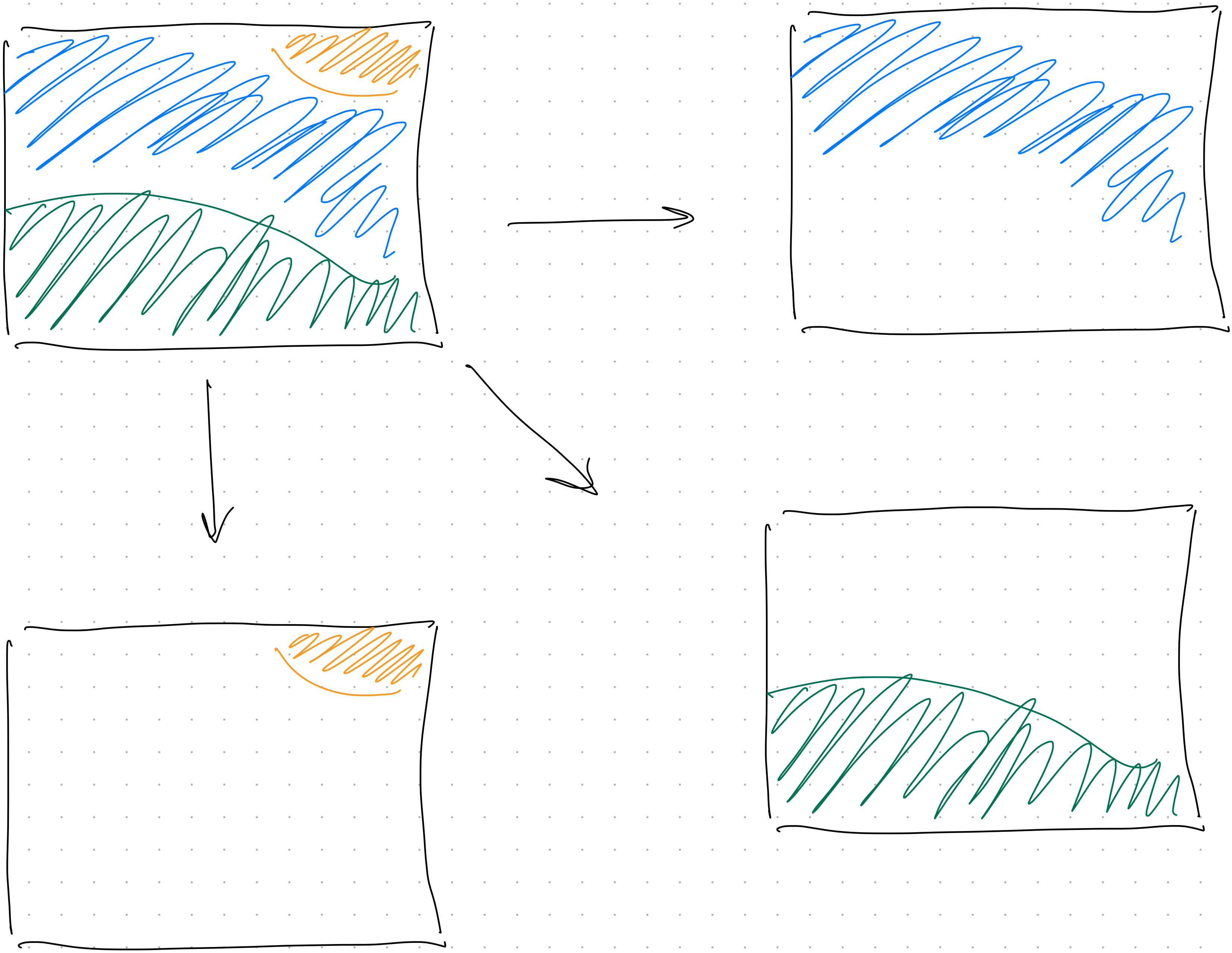
## Announcements.

HW 6 - Released after lecture

# Image Segmentation

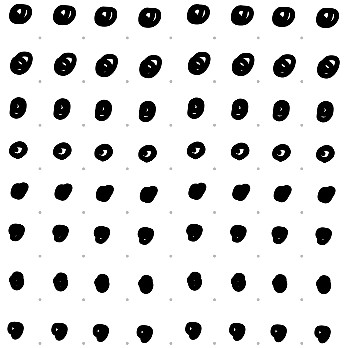


# Image Segmentation



# Segmentation Problem

Given: Pixels in a grid



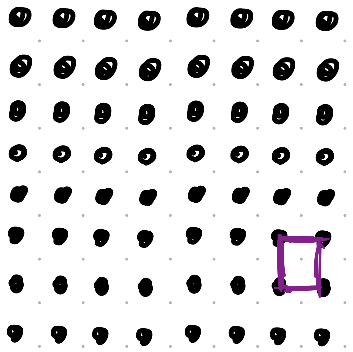
For each pixel p

$f_p$ = Foreground likelihood

$b_p$ = Background likelihood

# Segmentation Problem

Given: Pixels in a grid



For each pixel p

$f_p \equiv$ Foreground likelihood

$b_p \equiv$ Background likelihood

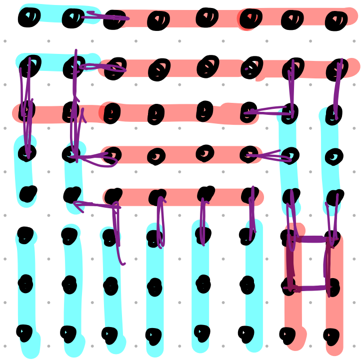
For each pair of neighboring pixels p, q

$s_{pq} \equiv$ Separation penalty

↳ suffered if p in foreground
and q in background (or vice versa)

# Segmentation Problem

Given: Pixels in a grid



For each pixel $p$

$f_p \equiv$ Foreground likelihood

$b_p \equiv$ Background likelihood

For each pair of neighboring pixels $p,q$

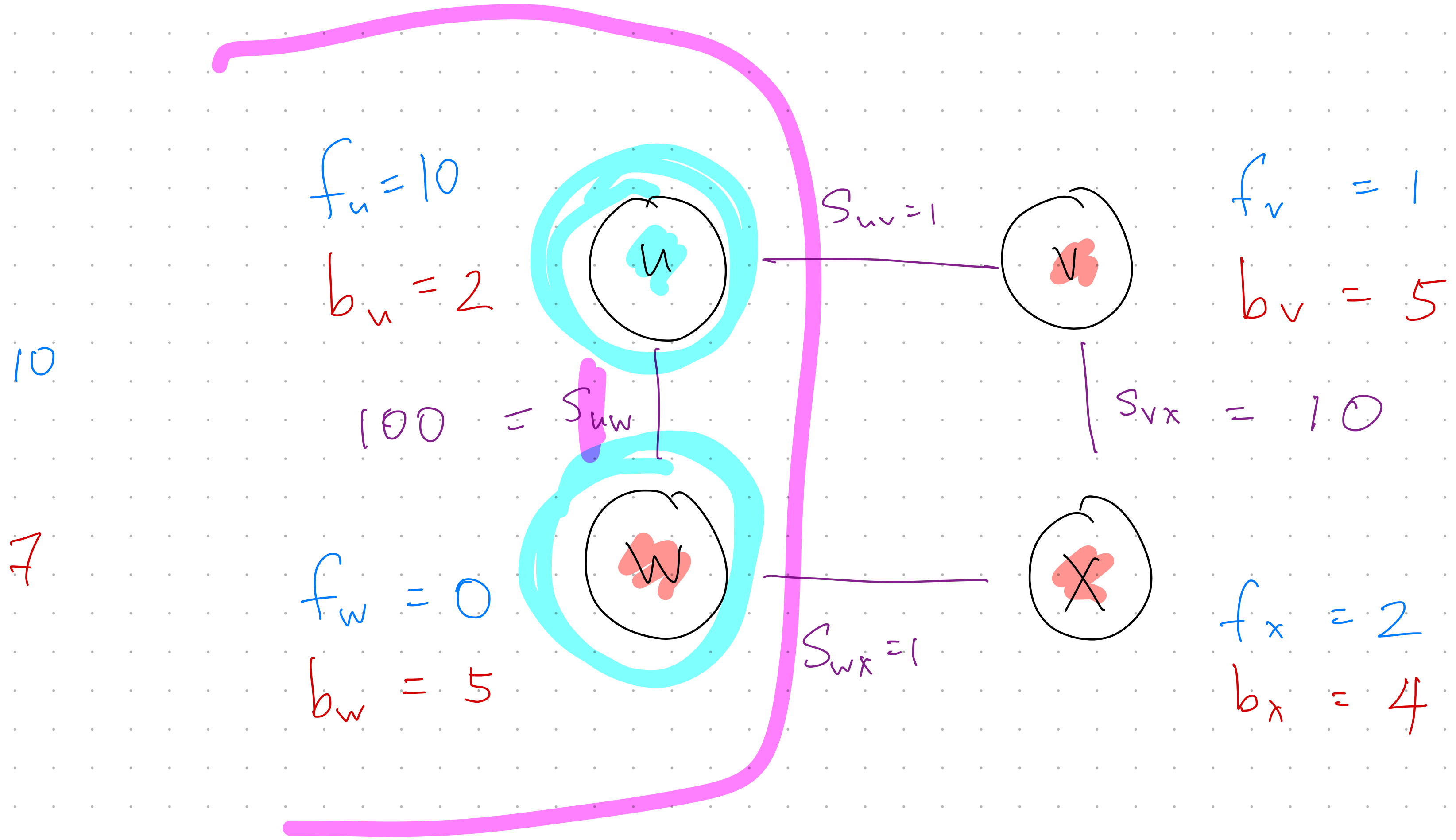
$s_{pq} \equiv$ Separation penalty

$s_{pq} = 0$ if $p,q$ are not neighbors.

Find partition of pixels $(F, B)$ maximizing

$$\sum_{p \in F} f_p + \sum_{q \in B} b_q - \sum_{\substack{p \in F \\ q \in B}} s_{pq}$$

$$\underset{F,B}{\text{Maximize}} \quad \sum_{p\in F} f_p + \sum_{q\in B} b_q - \sum_{\substack{p\in F\\ q\in B}} s_{pq}.$$



**Observation.** Maximizing "Goodness"

$$\|$$

Minimizing "Badness"

---

**Goal.** Maximize over $F, B$

$$\sum_{p \in F} f_p + \sum_{q \in B} b_q - \sum_{\substack{p \in F \\ q \in B}} s_{pq}.$$

**Observation.** Maximizing "Goodness"

$$\|$$

Minimizing "Badness"

---

**Goal.** Maximize $\sum_{p \in F} f_p + \sum_{q \in B} b_q - \sum_{\substack{p \in F \\ q \in B}} S_{pq}$.

$F, B$

$$\sum_{v \in \text{Pixels}} f_v = \sum_{p \in F} f_p + \sum_{q \in B} f_q$$

$$\sum_{v \in \text{Pixels}} b_v = \sum_{p \in F} b_p + \sum_{q \in B} b_q$$

**Observation.** Maximizing "Goodness"

$$\|$$

Minimizing "Badness"

---

**Goal.** Maximize$_{F,B}$ $\displaystyle\sum_{p\in F} f_p + \sum_{q\in B} b_q - \sum_{\substack{p\in F\\ q\in B}} s_{pq}$.

$$\sum_{p\in F} f_p = \sum_{v\in \text{Pixels}} f_v - \sum_{q\in B} f_q$$

$$\sum_{q\in B} b_q = \sum_{v\in \text{Pixels}} b_v - \sum_{p\in F} b_p$$

Observation. Maximizing "Goodness"

||

Minimizing "Badness"

---

Goal. $$\max_{F,B} \sum_{p\in F} f_p + \sum_{q\in B} b_q - \sum_{\substack{p\in F\\ q\in B}} s_{pq}.$$

$$\sum_{p\in F} f_p = \sum_{v\in \text{Pixels}} f_v - \sum_{q\in B} f_q$$

$$\sum_{q\in B} b_q = \sum_{v\in \text{Pixels}} b_v - \sum_{p\in F} b_p$$

---

Maximize $$\cancel{\sum_{v\in \text{Pixels}} (f_v + b_v)} \underbrace{- \sum_{q\in B} f_q - \sum_{p\in F} b_p - \sum_{\substack{p\in F\\ q\in B}} s_{pq}}_{}$$

**Observation.**

Maximizing "Goodness"
||
Minimizing "Badness"

---

**Goal.**

$$\underset{F,B}{\text{Maximize}} \; \cancel{\sum_{v \in \text{Pixels}} (f_v + b_v)} - \sum_{q \in B} f_q - \sum_{p \in F} b_p - \sum_{\substack{p \in F \\ q \in B}} s_{pq}$$

No dependence of
(F, B)

**Observation.** Maximizing "Goodness"

||

Minimizing "Badness"

**Goal.**

$$\max_{F,B} \cancel{\sum_{v \in \text{Pixels}} (f_v + b_v)} - \sum_{q \in B} f_q - \sum_{p \in F} b_p - \sum_{\substack{p \in F \\ q \in B}} s_{pq}$$

$$\Rightarrow \max_{F,B} - \sum_{q \in B} f_q - \sum_{p \in F} b_p - \sum_{\substack{p \in F \\ q \in B}} s_{pq}$$

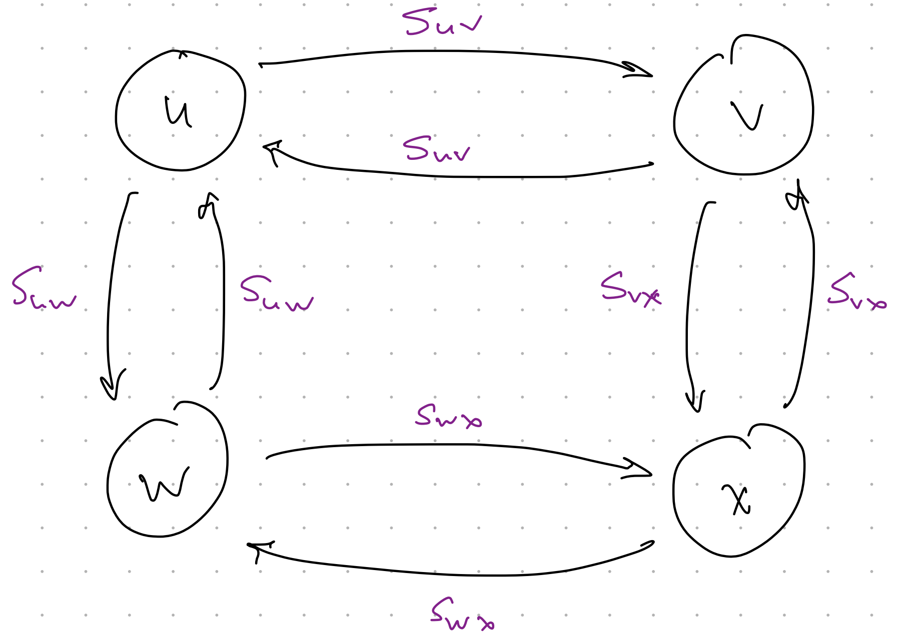
$$\Rightarrow \min_{F,B} \sum_{q \in B} f_q + \sum_{p \in F} b_p + \sum_{\substack{p \in F \\ q \in B}} s_{pq}$$

Find a partition of pixels to

$$\min_{F, B} \sum_{q \in B} f_q + \sum_{p \in F} b_p + \sum_{\substack{p \in F \\ q \in B}} S_{pq}$$

Find a partition of pixels to

$$\underset{F,\,B}{\text{Minimize}} \quad \sum_{q \in B} f_q + \sum_{p \in F} b_p + \sum_{\substack{p \in F \\ q \in B}} S_{pq}$$



Find a partition of pixels to

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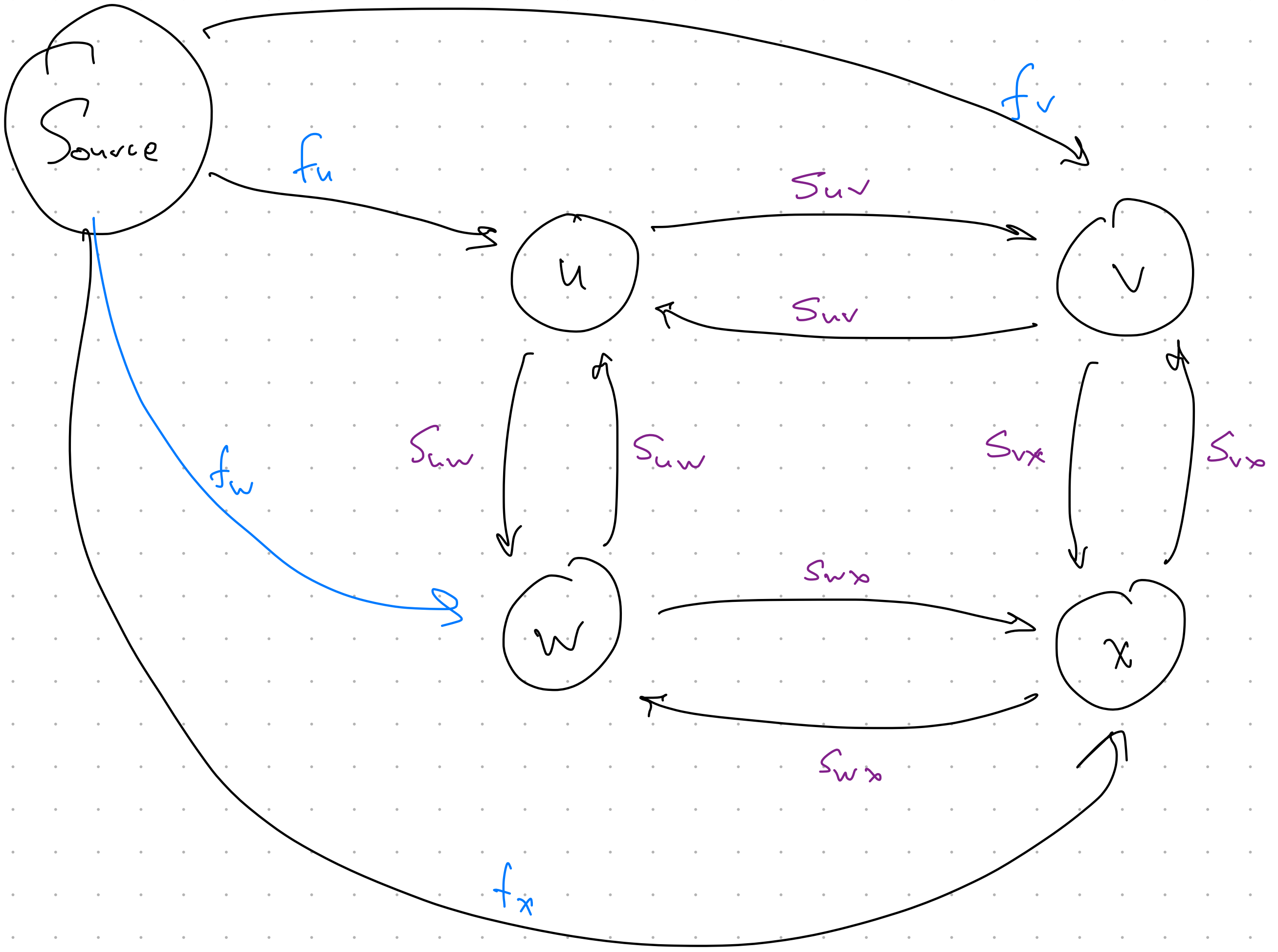
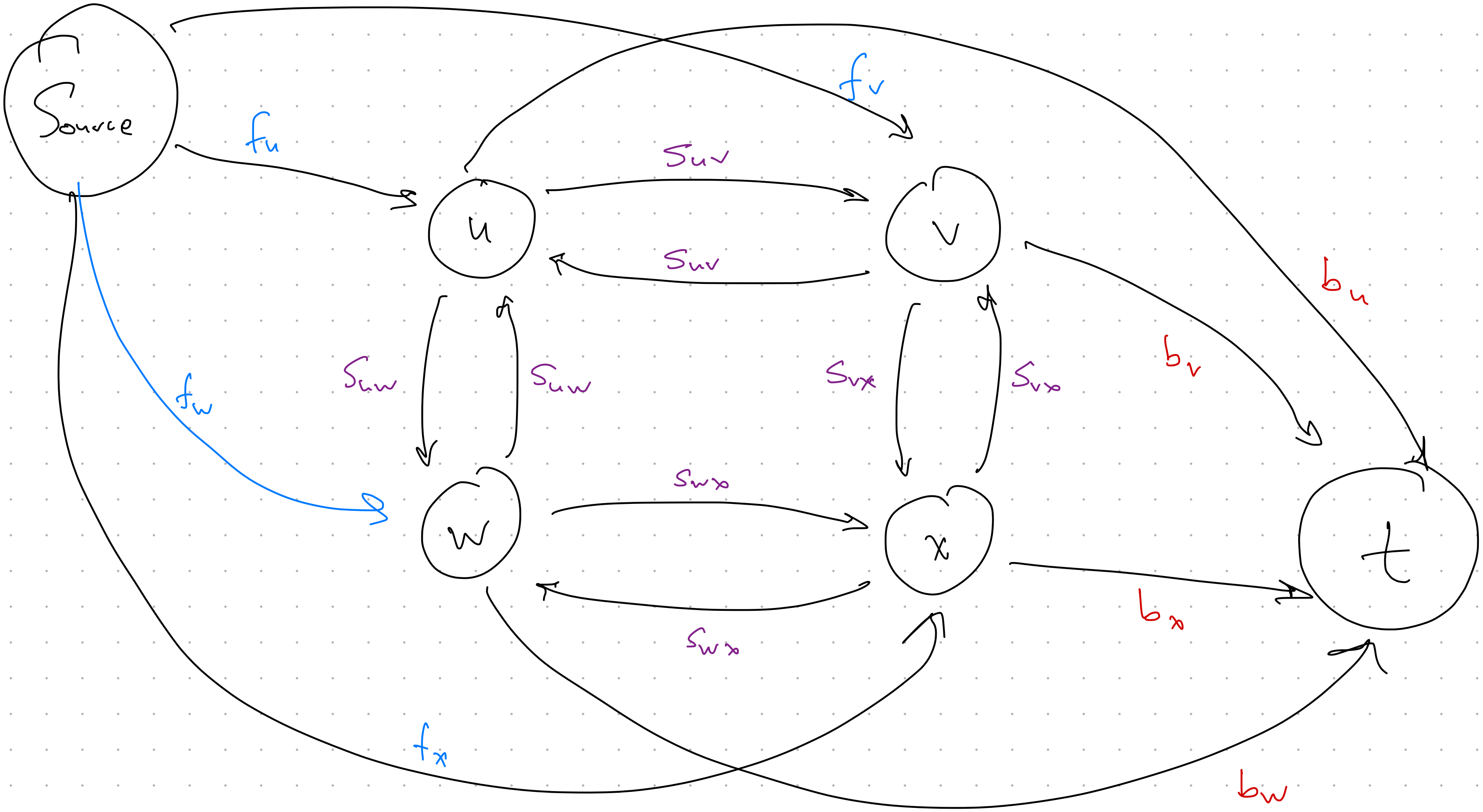


Find a partition of pixels to

Minimize $\sum_{q \in B} f_q + \sum_{p \in F} b_p + \sum_{\substack{p \in F \\ q \in B}} S_{pq}$

F, B



Find a partition of pixels to

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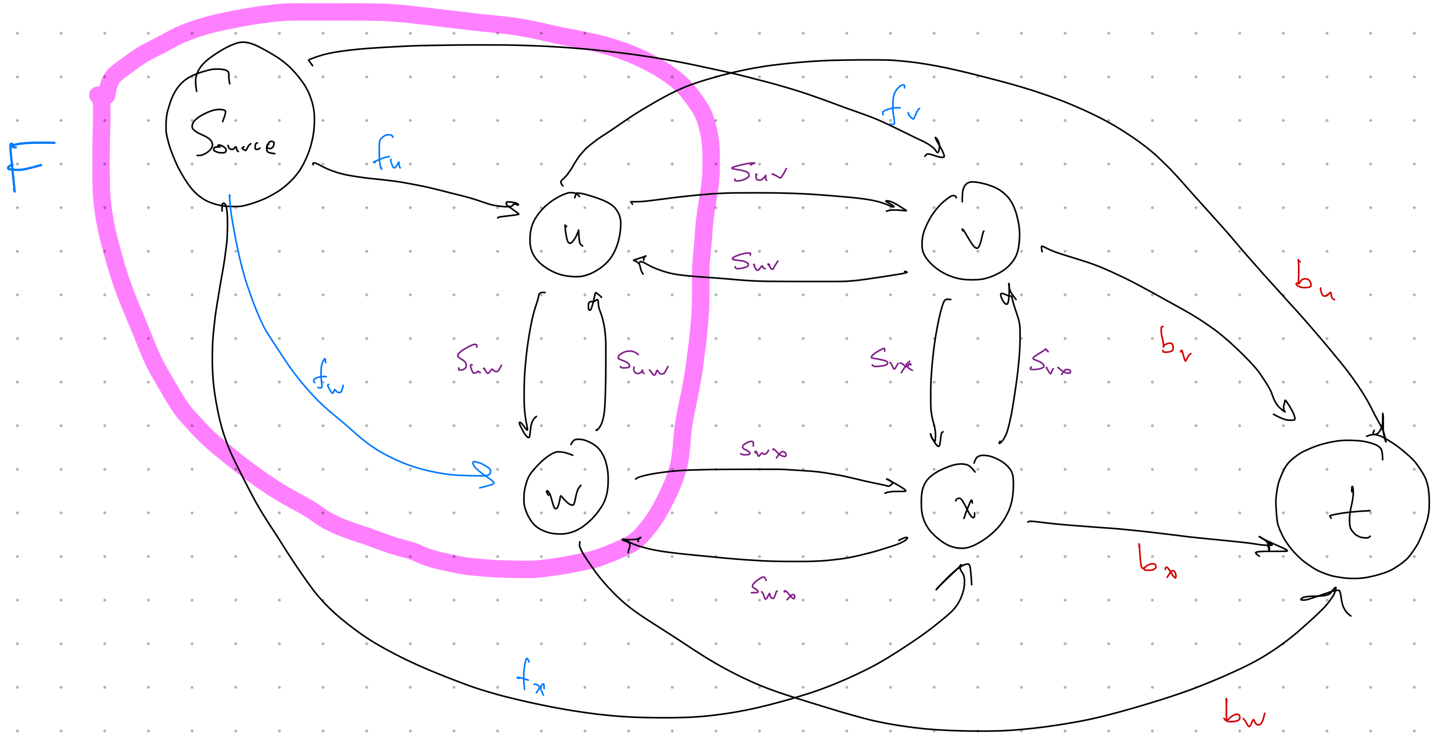

Which edges are cut?

$(s,q)$ for $q \in B$

$(p,t)$ for $p \in F$

$(p,q)$ for $p\in F$, $q\in B$.