Jan 24 The Stable Matching Problem (§1.1)

Announcements:

1. Prof. Kleinberg's OH moved to 2-3 today, Gates 317. (Generally 3-4 Weds.)

2. Waitlist questions? courses@cis.cornell.edu
   Read CS course enrollment web page.
   Open ticket, if needed, using link at bottom.

   We believe some space will open up in 4820, not enough for everyone on waitlist.

3. Homework 1 is coming Fri morning.

   By Fri morning you should contact us (Ed > email) if you're not on Canvas, Ed, Gradescope for 4820.

Algorithms in Job Markets

	MSK	MGH
Alice	10	5
Bob	16	8

2 applicants, 2 hospitals, each hiring 1,

An alternative system based on rankings.

Alice: MSK > MGH

Bob: MSK > MGH

MSK: A > B

MGH: A > B

<u>Def.</u> In a set of applicants (A) and firms (F) a <u>matching</u> is a set of ordered pairs, M, such that

(1) each pair in M has exactly one applicant, one firm

(2) each party in $A \cup F$ belongs to <u>at most one</u> pair in M.

belong to one pair ; "matched"

belong to no pair ; "free"

A perfect matching is one where every party is matched.

Assume now that each applicant has a total ordering of F and each firm has a total ordering of A. ("preferences")

If M is a perfect matching a <u>blocking pair</u> with respect to M is an (applicant, firm) pair (a,f) such that:

1. a is not matched to f in M
2. a prefers f to its partner
3. f prefers a to its partner.

A <u>stable perfect matching</u> is one without blocking pairs.

Given the participants and their pref's does there exist a stable perfect matching (is it unique?) and how to find one?

A: Yes, a stable perfect matching **always** exists.

(Gale - Shapley, 1950's)

The Proposal Algorithm

Initialize $M = \emptyset$

while $\exists$ a free firm f that hasn't yet proposed to every applicant:

  f finds its most preferred applicant that it hasn't yet proposed to, a.

  if a is free:

    insert (a, f) into M

  if a is matched to some $f' \neq f$:

    if a prefers to f to f':

      remove (a, f') from M

      insert (a, f)

    else:

      do nothing

Output M