Tell whether the following subsets of $\{a,b\}^*$ are regular or nonregular.

1. $\{a^n b^m \mid n = 2m\}$ **nonregular**
2. $\{a^n b^{2m} \mid n \geq 0 \text{ and } m \geq 0\}$ **regular**
3. $\{a^n b^m c^n \mid n \geq 0 \text{ and } m \geq 0\}$ **nonregular**
4. $\{x \in \{a,b\}^* \mid x \text{ contains more } a\text{'s than } b\text{'s}\}$ **nonregular**
5. $\{a^n b^m \mid n \neq m\}$ **nonregular**
6. $\{a^n b^{n+4810} \mid n \geq 0\}$ **nonregular**

Of the following two subsets of $\{a,b,c\}^*$, one is regular and the other is nonregular. Which is which?

7. $\{xcy \mid x,y \in \{a,b\}^*,\ \#a(x) = \#b(y)\}$ **nonregular**
8. $\{xy \mid x,y \in \{a,b\}^*,\ \#a(x) = \#b(y)\}$ **regular**

For 7, recall that the regular sets are closed under intersection and homomorphic image. If you intersect 7 with a^*cb^*, then delete the c's with a homomorphism $h(a) = a$, $h(b) = b$, $h(c) = \varepsilon$, you get your favorite nonregular set $\{a^n b^n \mid n \geq 0\}$. So 7 must not be regular.

The set 8 is in fact just $\{a,b\}^*$. That is, *every* string $z \in \{a,b\}^*$ can be expressed as xy with $\#a(x) = \#b(y)$ for some x, y. Suppose $|z| = n$. For $0 \leq i \leq n$, let x_i be the prefix of z of length i, and let y_i be the suffix of z of length $n - i$; that is, $z = x_i y_i$ with $|x_i| = i$ and $|y_i| = n - i$. Let $f(i) = \#a(x_i) - \#b(y_i)$, $0 \leq i \leq n$. Then $f(0) = \#a(\varepsilon) - \#b(z) \leq 0$, $f(n) = \#a(z) - \#b(\varepsilon) \geq 0$, and f changes by $+1$ or -1 in each step as i goes from 0 to n. There must be an i for which $f(i) = 0$, that is, $\#a(x_i) = \#b(y_i)$.