

1. If Σ contains at least one element, then Σ^* is infinite. **true**
2. There are strings in Σ^* of infinite length. **false**—The set Σ^* is the set of all finite-length strings over the alphabet Σ (K, Def. 2.1, p. 8). For example,

$$\{a, b\}^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, baa, abb, bab, bba, bbb, aaaa, aaab, \dots\}.$$

This set contains infinitely many strings and there is no finite bound on their length; nevertheless, every element is of finite length. The length of a string $x \in \Sigma^*$ is the number of letters in it and is denoted $|x|$. For example, $|aaab| = 4$.

3. Set intersection is associative. **true**
4. $A \cup \emptyset = \emptyset$. **false**
5. $A\emptyset = \emptyset$. **true**—Formally, AB is defined to be the set of all strings xy obtained by concatenating a string x from A with a string y from B . If $B = \emptyset$, then there are no strings $y \in B$, so there are no possible such concatenations xy . Thus $A\emptyset = \emptyset$.
6. The complement of $\{a\}^*$ in $\{a, b\}^*$ is $\{b\}^*$. **false**—The set $\{a, b\}^*$ is the set of all strings of a 's and b 's. The set $\{a\}^*$ is the set of all strings consisting only of a 's. A string does not consist only of a 's iff it contains at least one b . The complement of $\{a\}^*$ in $\{a, b\}^*$ is therefore the set of strings of a 's and b 's containing at least one b . This is the set $\{a, b\}^* \{b\} \{a, b\}^*$.
7. There is exactly one string in Σ^* of length 0. **true**
8. $\emptyset = \varepsilon$. **false**— \emptyset is a set, ε is a string. **Type error!**
9. $\emptyset = \{\varepsilon\}$. **false**—The set \emptyset is the empty set. It containing no elements. The set $\{\varepsilon\}$ contains one element, namely ε , the null string.
10. $\{ab, a\}\{ba, a\} = \{abba, aba, aa\}$. **true**