NetId _____ Name _____

Fifty minutes, open book and notes, no electronic devices. No collaboration allowed. Write answers in the exam book, not on this sheet. Indicate clearly which is your answer. Show all work for partial credit. Write your name and netId on this sheet and the exam book, and pass them in together. **Good luck!**

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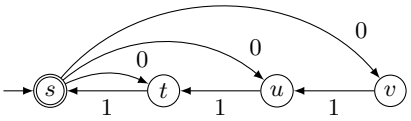
1. (7 points) True or false?

true	*false*	
☐	☐	All finite subsets of Σ^* are regular.
☐	☐	All complements of finite subsets of Σ^* are regular.
☐	☐	$\{a\}^* \subseteq \{aa\}^*$.
☐	☐	A homomorphism $h : \Sigma^* \to \Gamma^*$ is completely determined by its action on single letters.
☐	☐	$\varnothing = \varepsilon$.
☐	☐	$\varnothing = \{\varepsilon\}$.
☐	☐	For all $A, B \subseteq \Sigma^*$, if A and B each have 2 elements, then AB has 4 elements.

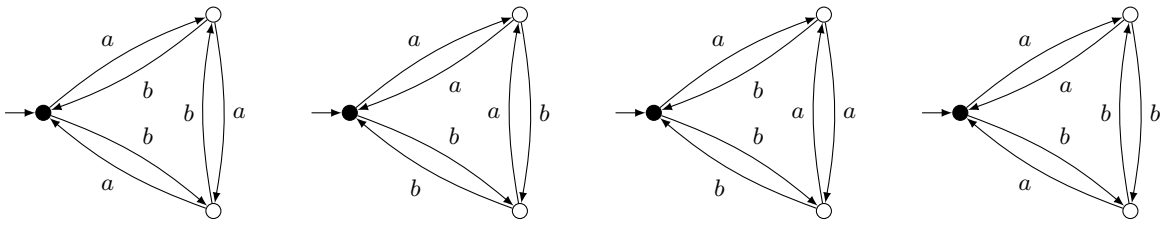
2. (10 points) Here is an NFA equivalent to the regular expression $(01 + 011 + 0111)^*$:



Convert this automaton to an equivalent deterministic one using the subset construction. Show clearly which subset of $\{s, t, u, v\}$ corresponds to each state of the deterministic automaton. Omit inaccessible states.

3. (8 points) Match each DFA with an equivalent regular expression. Black states indicate accept states.

(a) (b) (c) (d)



(i) $(a(bb)^*(a + ba) + b(bb)^*(a + ba))^*$
(ii) $(a(aa)^*(b + ab) + b(aa)^*(b + ab))^*$
(iii) $(a(ab)^*(b + aa) + b(ba)^*(a + bb))^*$
(iv) $(a(ba)^*(a + bb) + b(ab)^*(b + aa))^*$

More—go on to next page

4. (12 points) Consider the regular set R represented by the regular expression $a^*b^* + b^*a^*$. Recall the definition of the Myhill-Nerode equivalence relation

$$x \equiv_R y \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \forall z \in \Sigma^* \ (xz \in R \Leftrightarrow yz \in R) \ .$$

   (a) Draw the minimal DFA for R. (It has six states.)

   (b) How many equivalence classes of $\equiv_R$ are there? Why?

   (c) Give a regular expression describing each of the equivalence classes of $\equiv_R$. (*Hint.* ε and aa^* are two of them.)

5. (10 points) Prove that the set $\{a^n b^m c^k \mid n, m, k \geq 0, \ n + m = k\}$ is nonregular.

End of exam