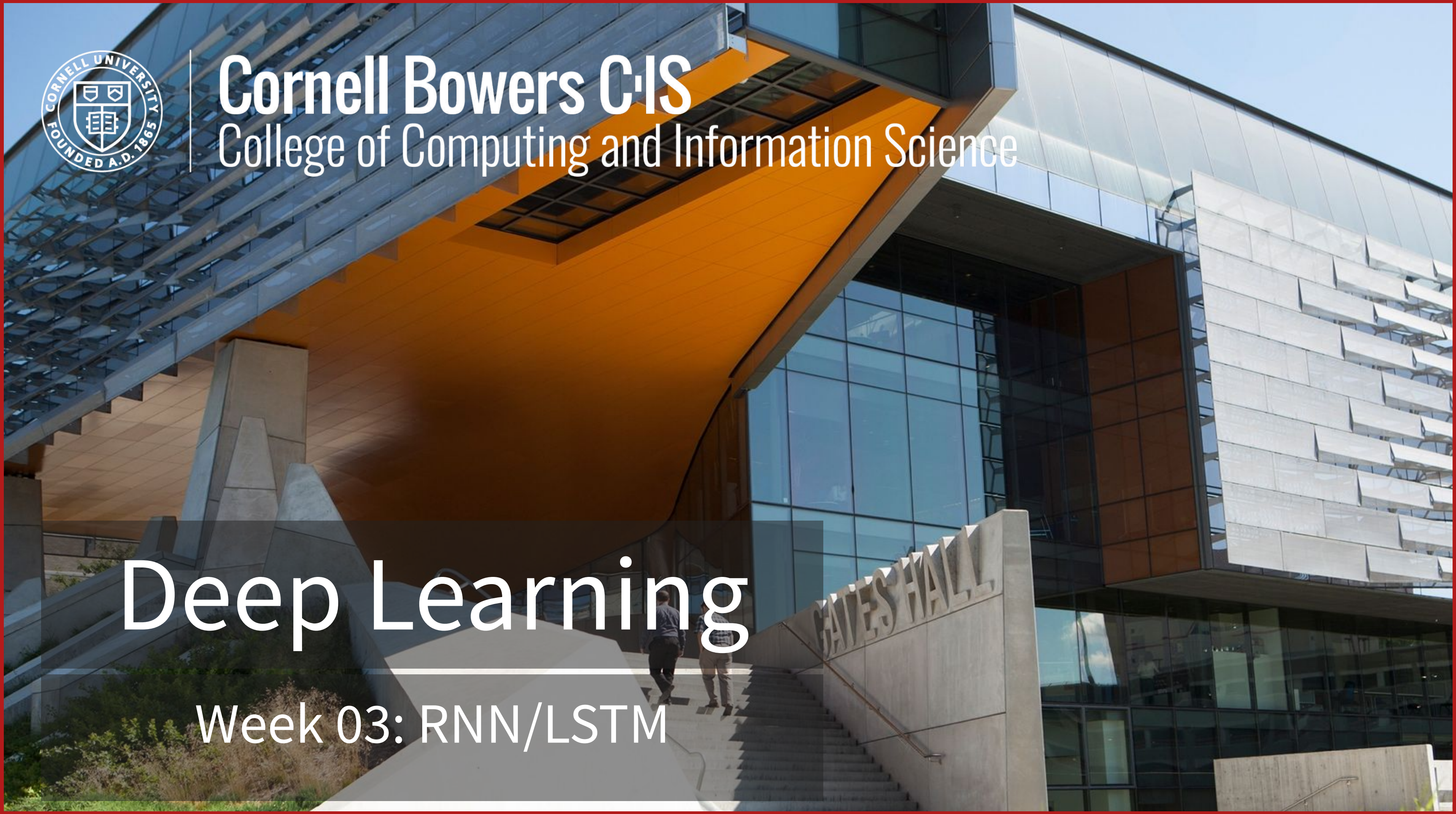

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College of Computing and Information Science

Deep Learning

Week 03: RNN/LSTM

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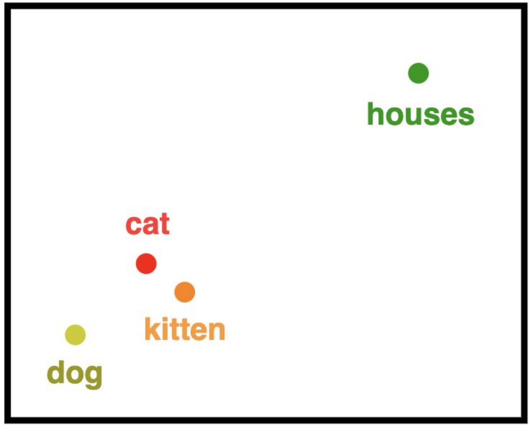
Why Do We Need Word Embeddings?

- Numerical Input
- Shows Similarity and Distance



	living being	feline	human	gender	royalty	verb	plural
cat	0.6	0.9	0.1	0.4	-0.7	-0.3	-0.2
kitten	0.5	0.8	-0.1	0.2	-0.6	-0.5	-0.1
dog	0.7	-0.1	0.4	0.3	-0.4	-0.1	-0.3
houses	-0.8	-0.4	-0.5	0.1	-0.9	0.3	0.8
man	0.6	-0.2	0.8	0.9	-0.1	-0.9	-0.7
woman	0.7	0.3	0.9	-0.7	0.1	-0.5	-0.4
king	0.5	-0.4	0.7	0.8	0.9	-0.7	-0.6
queen	0.8	-0.1	0.8	-0.9	0.8	-0.5	-0.9

embedding using features of words

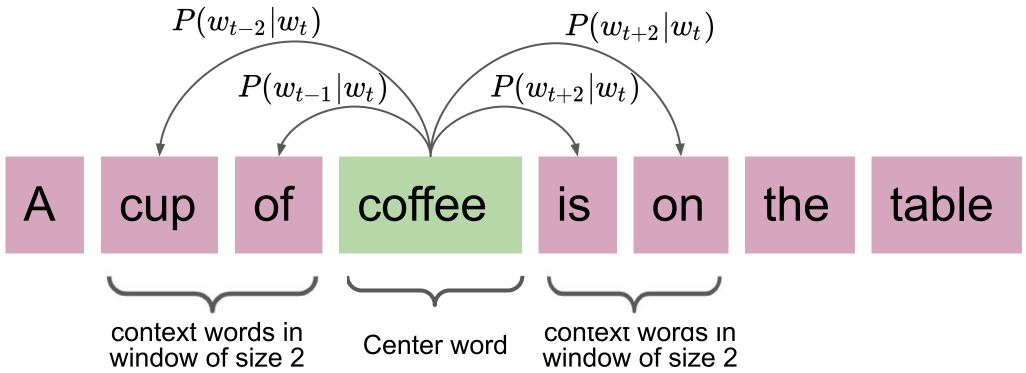
Word2Vec

- We want vectors for words so that the context of a word can suggest the vector of this word, and vice versa
- Idea: **Similar words appear in similar contexts**

A cup of **coffee** is on the table.
Coffee helps me focus.
Espresso is my favorite type of **coffee**.

Word2Vec - Training

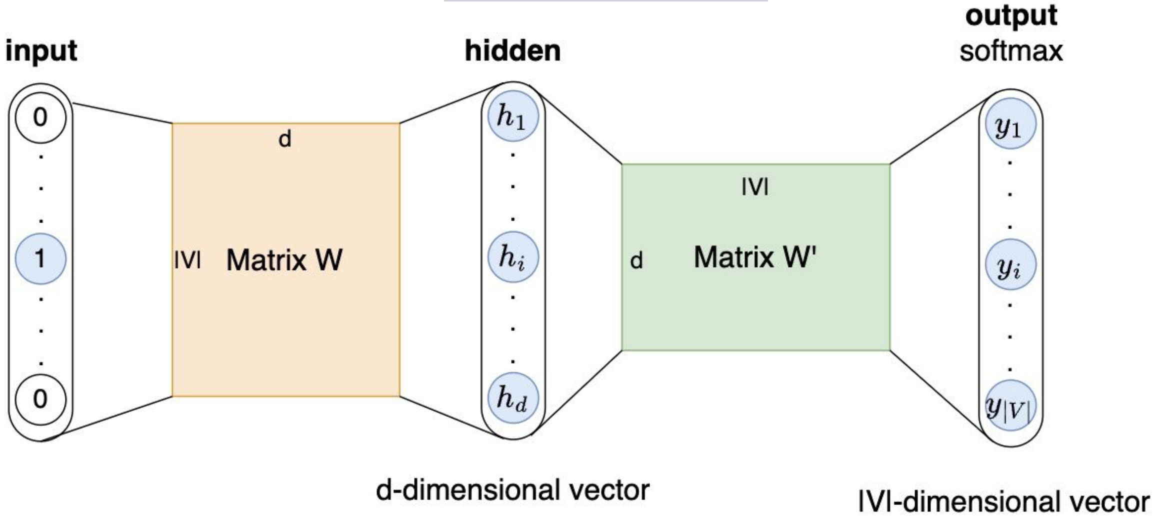
SkipGram - Predict context from target



Word2Vec Architecture - SkipGram

Predict every target word from each context word!

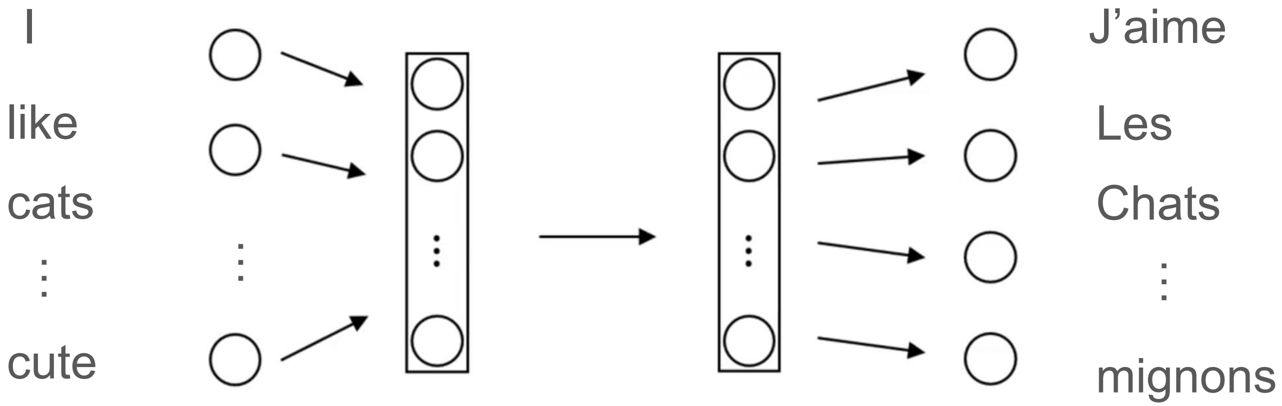
coffee → cup



Big Question: How to model sequences of words?

I	0.7	-0.1	0.4	0.3	-0.4	-0.1	-0.3	$\mathbf{v}_1 \in \mathbb{R}^d$
like	0.6	-0.2	0.8	0.9	-0.1	-0.9	-0.7	$\mathbf{v}_2 \in \mathbb{R}^d$
cats	0.6	0.9	0.1	0.4	-0.7	-0.3	-0.2	
because	0.5	0.8	-0.1	0.2	-0.6	-0.5	-0.1	$\vdots$
they	-0.8	-0.4	-0.5	0.1	-0.9	0.3	0.8	
look	0.5	-0.4	0.7	0.8	0.9	-0.7	-0.6	
cute	0.8	-0.1	0.8	-0.9	0.8	-0.5	-0.9	$\mathbf{v}_T \in \mathbb{R}^d$

How to use word vectors with neural networks?



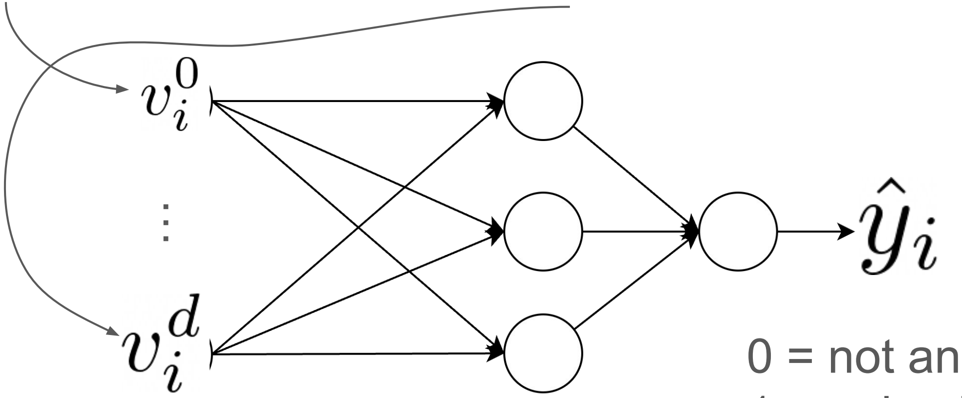
- Inputs and outputs don't have fixed lengths
- Weights are not shared

Let’s simplify!

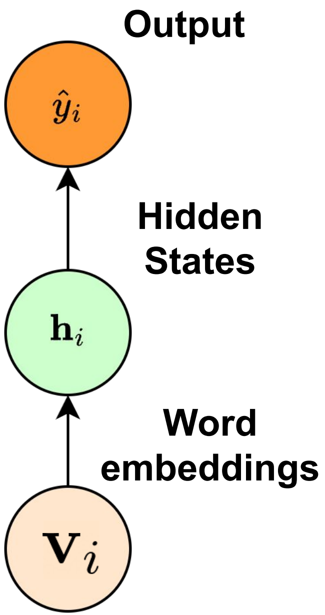
What if we have a single word and a single output?

cat	0.6	0.9	0.1	0.4	-0.7	-0.3	-0.2

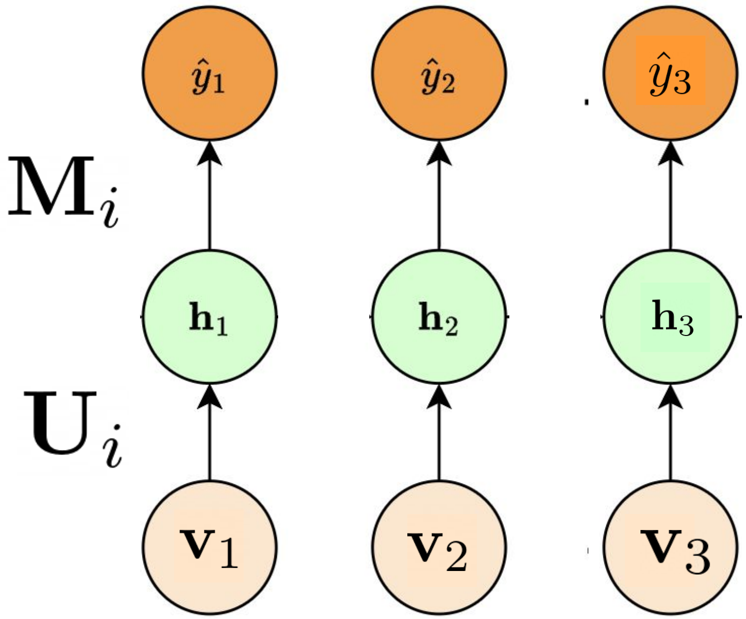
$$\mathbf{v}_i \in \mathbb{R}^d$$



0 = not animal
1 = animal



Towards RNNs



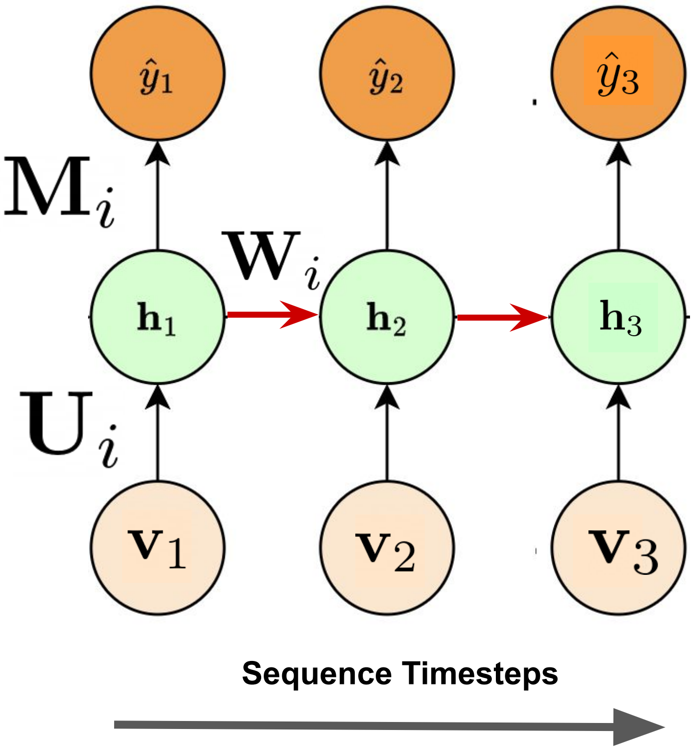
Output

$$\hat{y}_i = \mathbf{M}_i \mathbf{h}_i$$

Hidden State

$$\mathbf{h}_i = \sigma(\mathbf{U}_i \mathbf{v}_i)$$

Towards RNNs



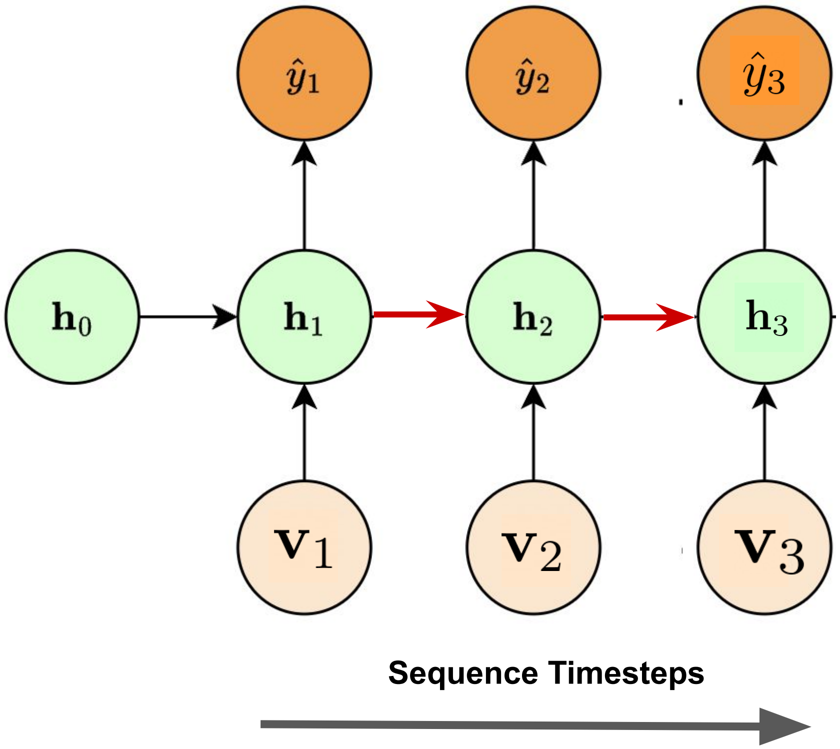
Output

$$\hat{y}_i = \mathbf{M}_i \mathbf{h}_i$$

Hidden State

$$\mathbf{h}_i = \sigma(\mathbf{U}_i \mathbf{v}_i + \mathbf{W}_i \mathbf{h}_{i-1})$$

Towards RNNs



Output

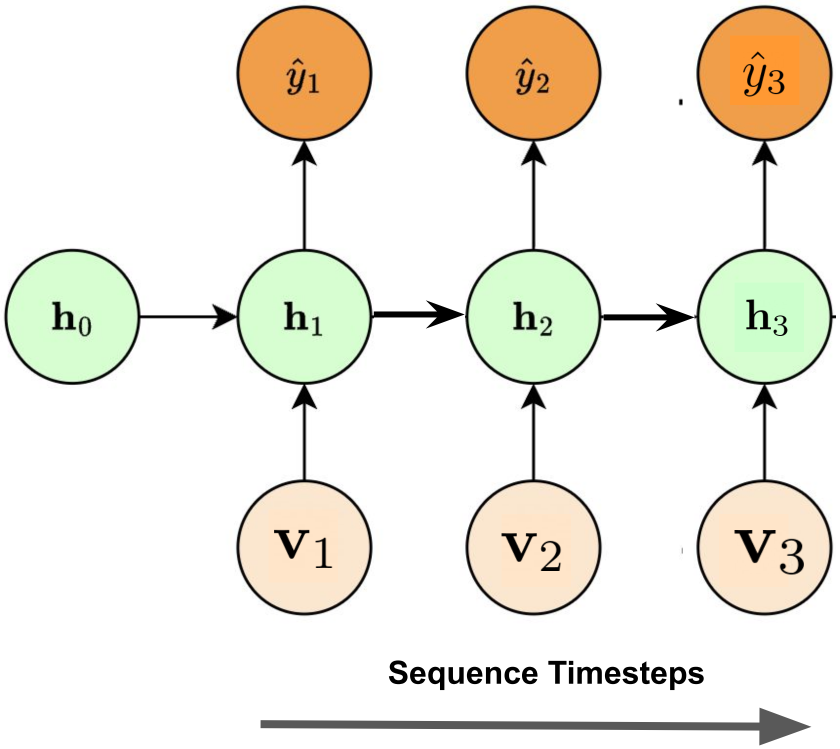
$$\hat{y}_i = \mathbf{M}_i \mathbf{h}_i$$

Hidden State

$$\mathbf{h}_i = \sigma(\mathbf{U}_i \mathbf{v}_i + \mathbf{W}_i \mathbf{h}_{i-1})$$

What’s the issue with this setup?

- Too many parameters if we have a long sequence!
- Longer sequence parameters will not receive many updates



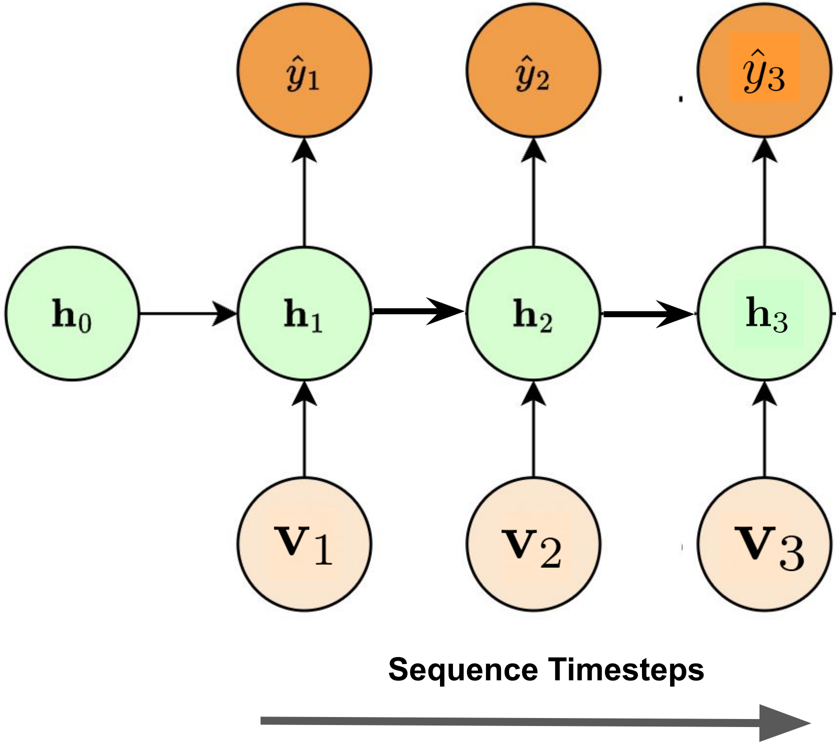
Output

$$\hat{y}_i = \mathbf{M}_i \mathbf{h}_i$$

Hidden State

$$\mathbf{h}_i = \sigma(\mathbf{U}_i \mathbf{v}_i + \mathbf{W}_i \mathbf{h}_{i-1})$$

Recurrent neural network (RNN)



Use the same parameters across different timesteps.

Output

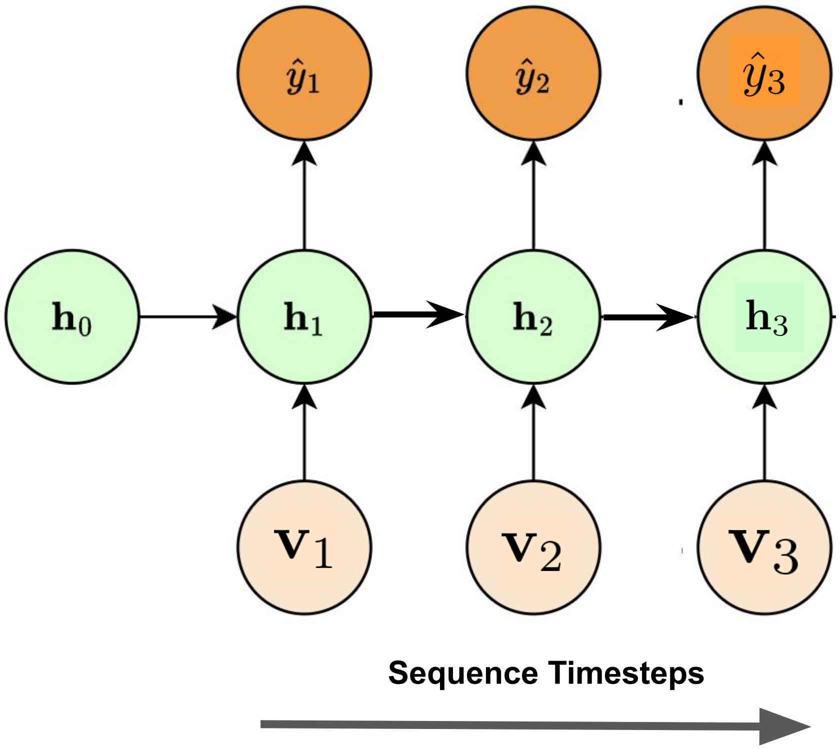
$$\hat{y}_i = \mathbf{M}\ \mathbf{h}_i$$

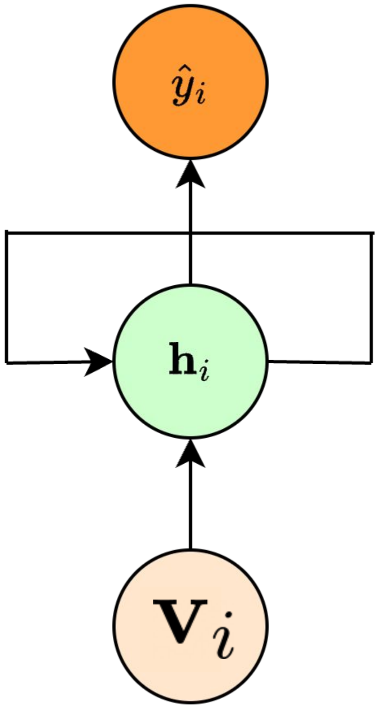
Hidden State

$$\mathbf{h}_i = \sigma(\mathbf{U}\ \mathbf{v}_i + \mathbf{W}\ \mathbf{h}_{i-1})$$

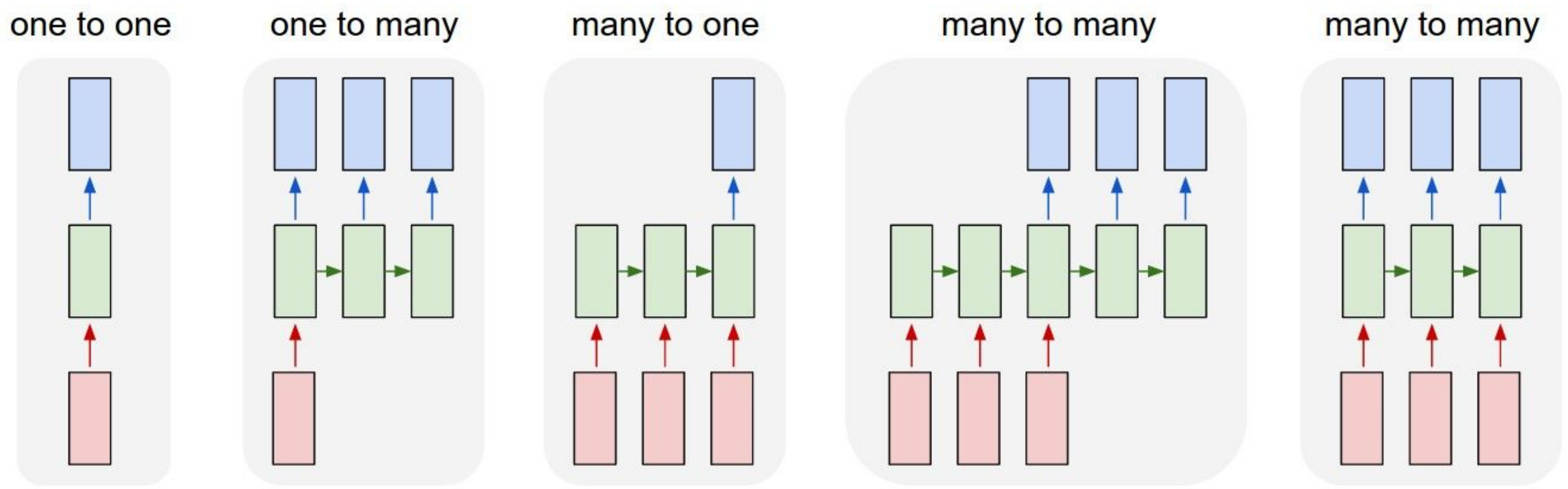
Recurrent neural network (RNN)

Use the same parameters across different timesteps.





Discuss: Which tasks can you perform with RNNs? Can you find an example of each task?



https://www.analyticsvidhya.com/blog/2021/06/time-series-analysis-recurrence-neural-network-in-python/

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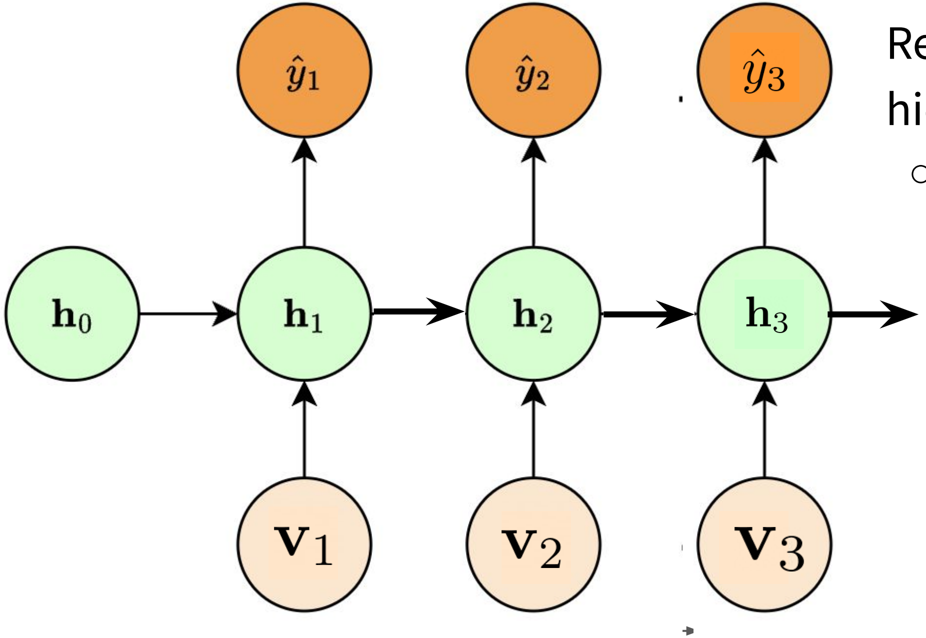
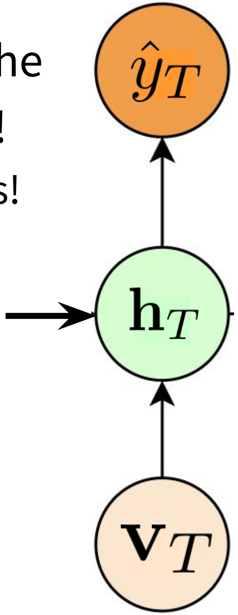
RNN: Issues under **Looooooooong** Context



Recurrent forward will **rewrite** the hidden states on every timestep!

- What will happen? Let’s discuss!



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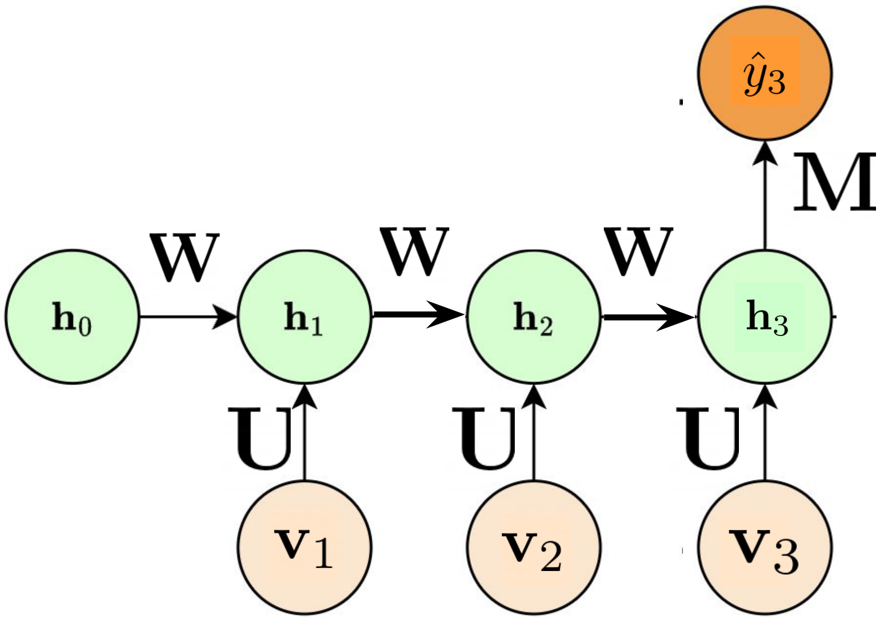
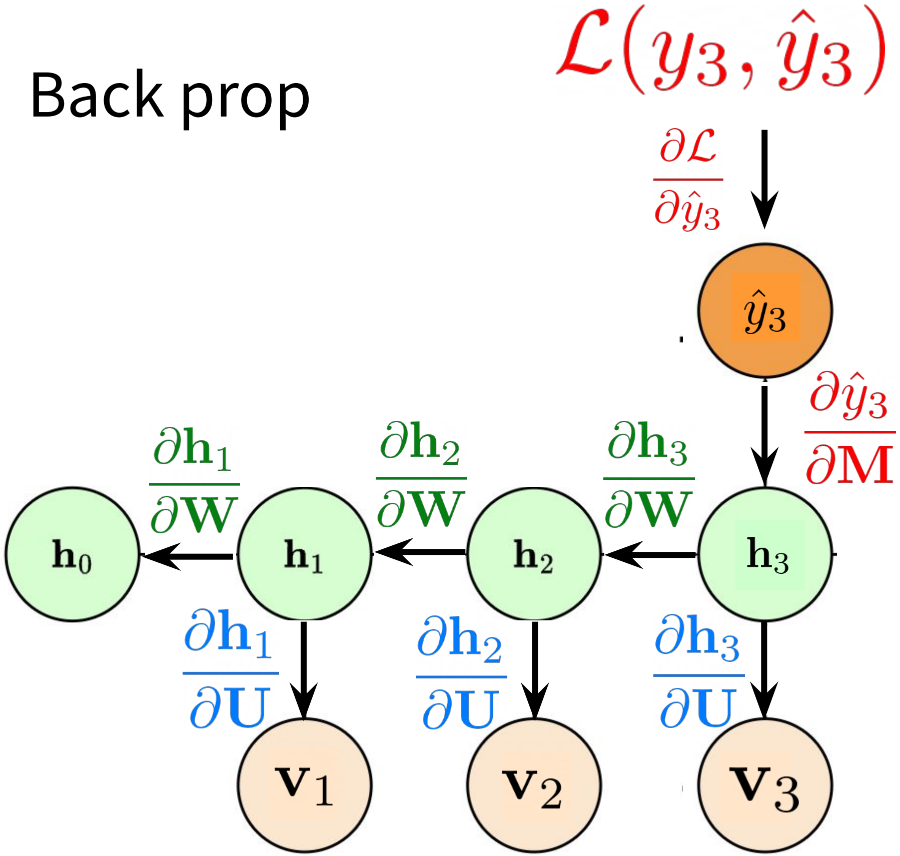
Forward prop



Back prop



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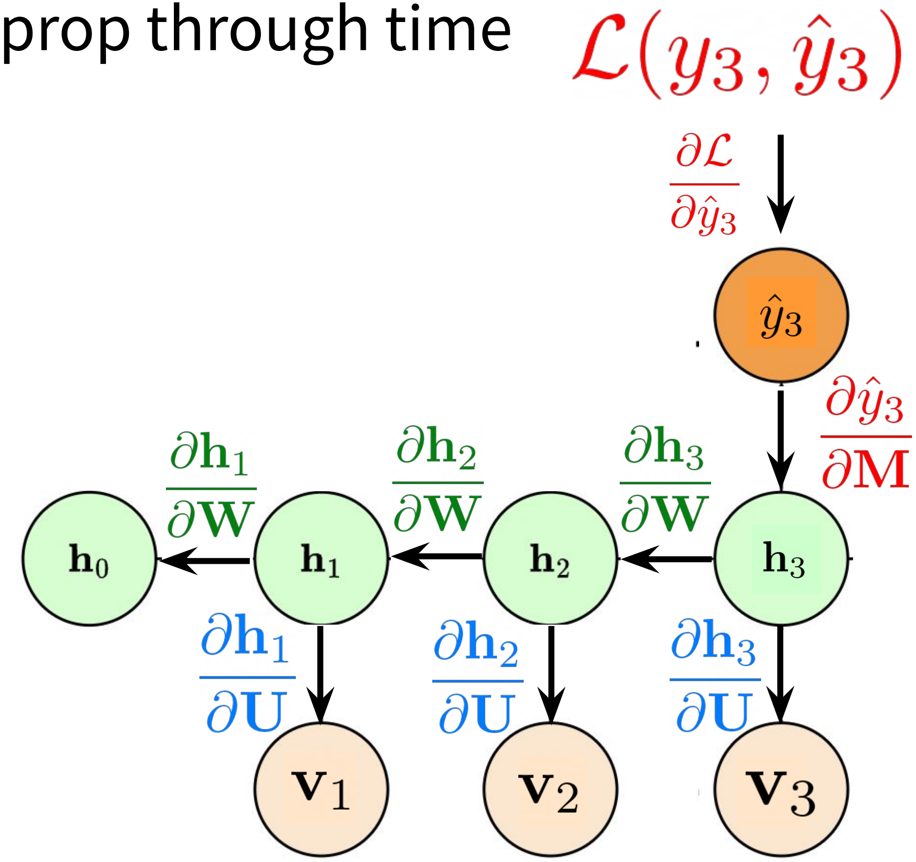
Back prop through time

- Unfold a recurrent neural network in time
- Gradients are accumulated across all time steps by applying the chain rule
- Propagate gradients backwards through time steps



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Back prop through time

$$\frac{\partial \mathcal{L}}{\partial \mathrm{M}} = \frac{\partial \mathcal{L}}{\partial \hat{y}_3} \cdot \frac{\partial \hat{y}_3}{\partial \mathrm{M}}$$

Gradients wrt W from last time step:

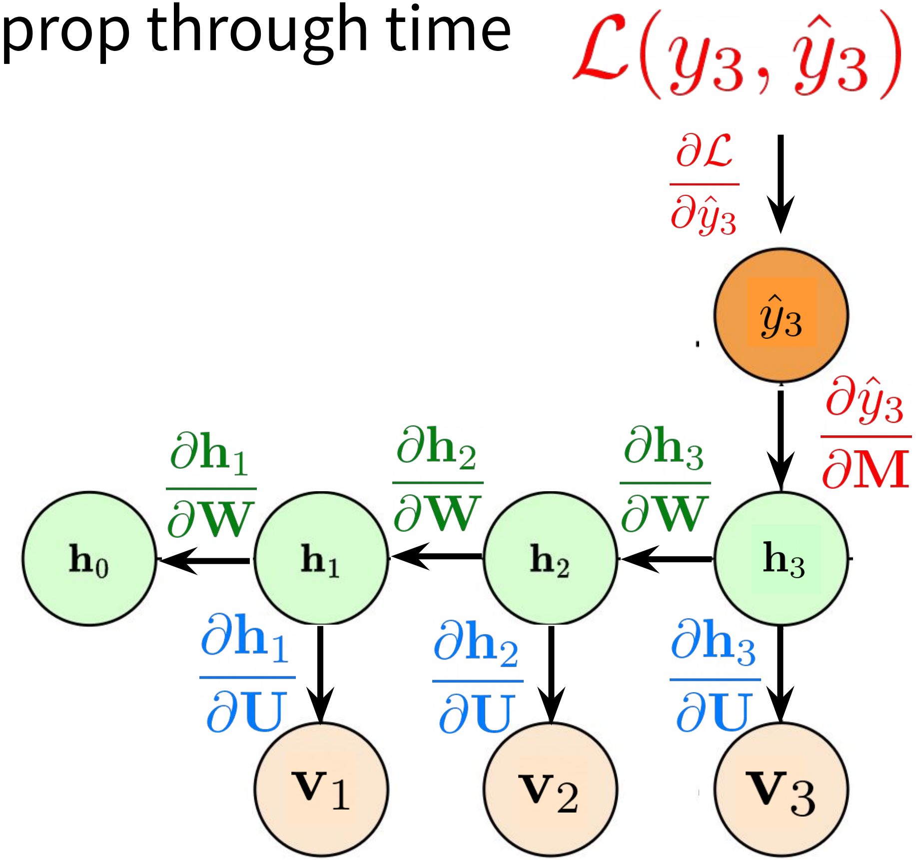
$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \frac{\partial \mathcal{L}}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial \mathbf{h}_3} \frac{\partial \mathbf{h}_3}{\partial \mathbf{W}}$$

Gradients wrt W from time step 2:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \frac{\partial \mathcal{L}}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial \mathbf{h}_3} \frac{\partial \mathbf{h}_3}{\partial \mathbf{h}_2} \frac{\partial \mathbf{h}_2}{\partial \mathbf{W}}$$

Gradients wrt W from time step 1:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \frac{\partial \mathcal{L}}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial \mathbf{h}_3} \frac{\partial \mathbf{h}_3}{\partial \mathbf{h}_2} \frac{\partial \mathbf{h}_2}{\partial \mathbf{h}_1} \frac{\partial \mathbf{h}_1}{\partial \mathbf{W}}$$



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T = 3

What is the general form of $\frac{\partial \mathcal{L}}{\partial \mathbf{W}}$ with **T = T**, at time step t?

Gradients wrt W from time step 3:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \frac{\partial \mathcal{L}}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial \mathbf{h}_3} \frac{\partial \mathbf{h}_3}{\partial \mathbf{W}}$$

Gradients wrt W from time step 2:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \frac{\partial \mathcal{L}}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial \mathbf{h}_3} \frac{\partial \mathbf{h}_3}{\partial \mathbf{h}_2} \frac{\partial \mathbf{h}_2}{\partial \mathbf{W}}$$

Gradients wrt W from time step 1:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \frac{\partial \mathcal{L}}{\partial \hat{y}_3} \frac{\partial \hat{y}_3}{\partial \mathbf{h}_3} \frac{\partial \mathbf{h}_3}{\partial \mathbf{h}_2} \frac{\partial \mathbf{h}_2}{\partial \mathbf{h}_1} \frac{\partial \mathbf{h}_1}{\partial \mathbf{W}}$$

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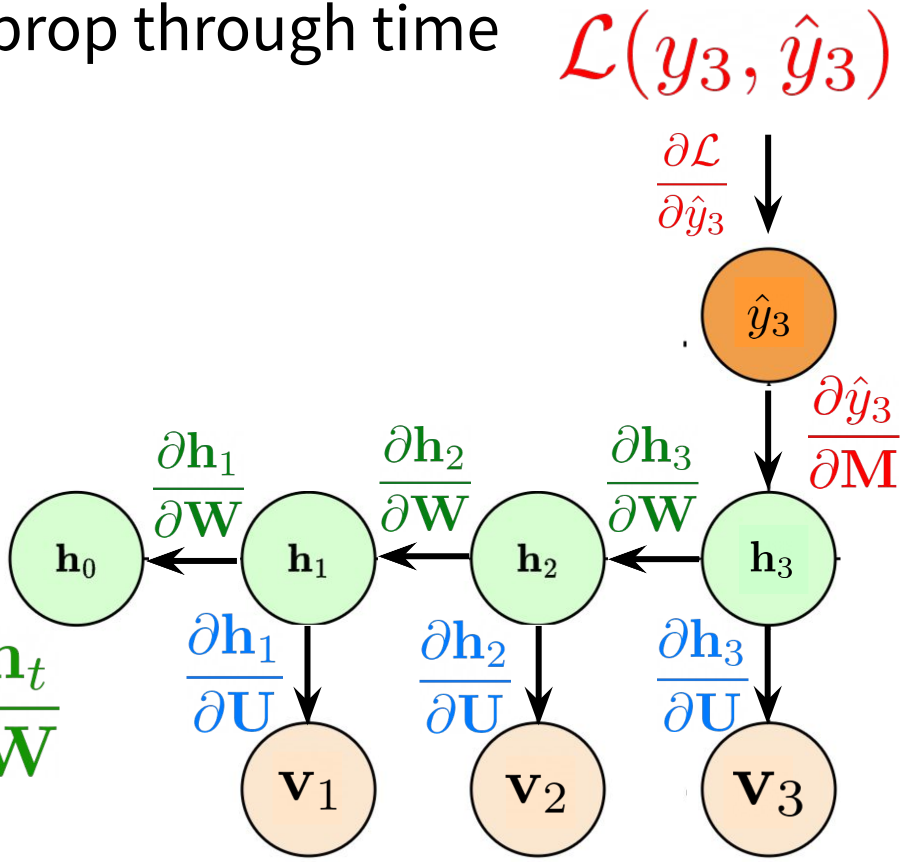
Back prop through time

Gradients wrt W from time step t:

Each timestamp contributes to the gradient!
Summing over all timestamps:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}} = \sum_{t=1}^{T} \frac{\partial \mathcal{L}}{\partial \hat{y}_T} \frac{\partial \hat{y}_T}{\partial \mathbf{h}_T} \left(\prod_{i=t}^{T-1} \frac{\partial \mathbf{h}_{i+1}}{\partial \mathbf{h}_i} \right) \frac{\partial \mathbf{h}_t}{\partial \mathbf{W}}$$



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RNN: Issues under **Loooooooong** Context

$$\frac{\partial \mathcal{L}(\hat{y}_T)}{\partial \mathbf{h}_1} = \frac{\partial \mathcal{L}(\hat{y}_T)}{\partial \mathbf{h}_T} \prod_{1<t\leq T} \frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t-1}}$$

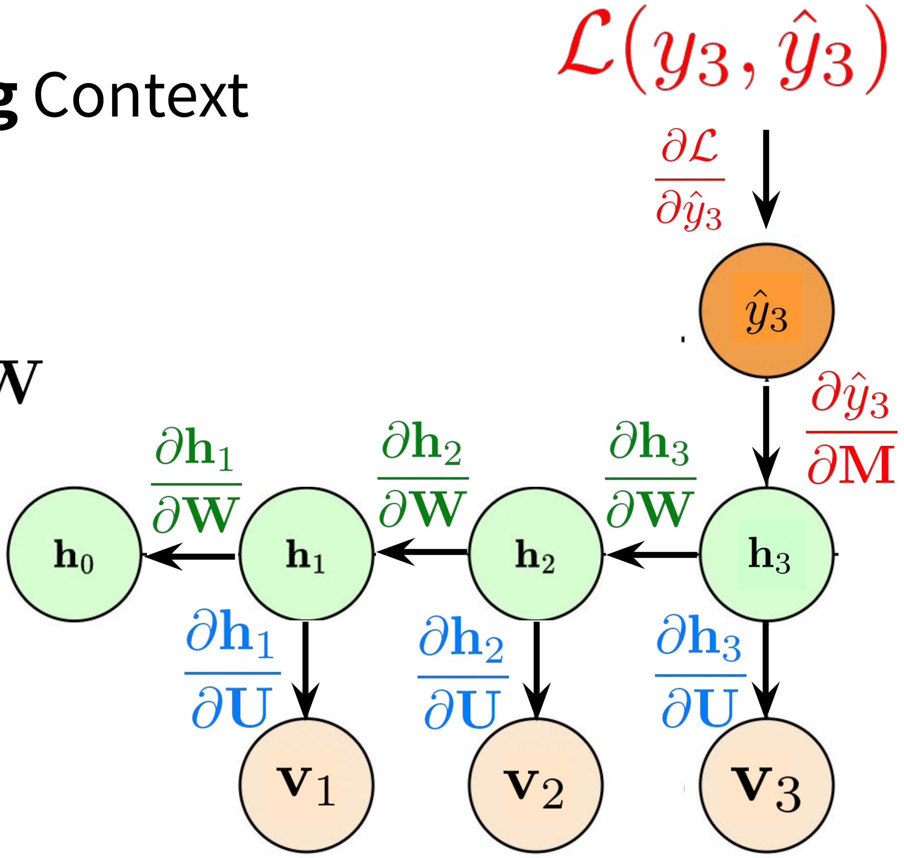
$$\frac{\partial \mathbf{h}_t}{\partial \mathbf{h}_{t-1}} = \mathrm{diag}(\sigma'(\mathbf{W}\mathbf{h}_{t-1} + \mathbf{U}\mathbf{v}_t))\mathbf{W}$$

- **Vanishing gradients: grad to *0***

If $\|\frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_{i-1}}\| < 1$ and T is large, $\|\frac{\partial \mathcal{L}(\hat{y}_T)}{\partial \mathbf{h}_1}\| \to 0.$

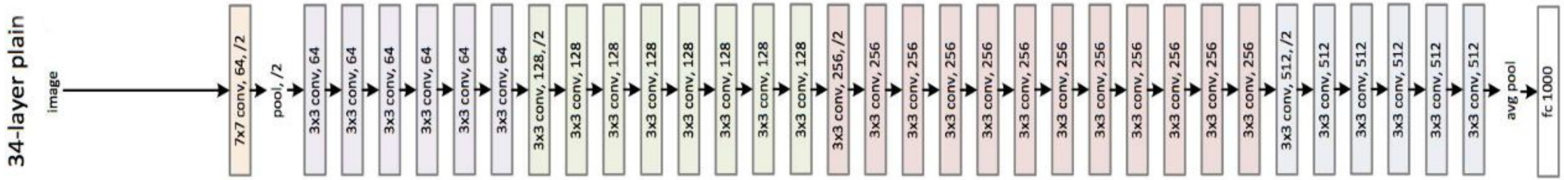
- **Exploding gradients: grad to *inf***

If $\|\frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_{i-1}}\| > 1$ and T is large, $\|\frac{\partial \mathcal{L}(\hat{y}_T)}{\partial \mathbf{h}_1}\| \to \mathrm{inf}.$

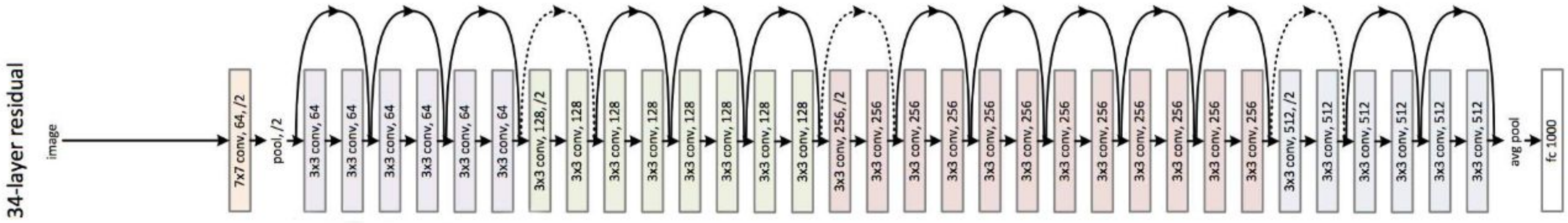


Recall: ResNet

“Plain” Network

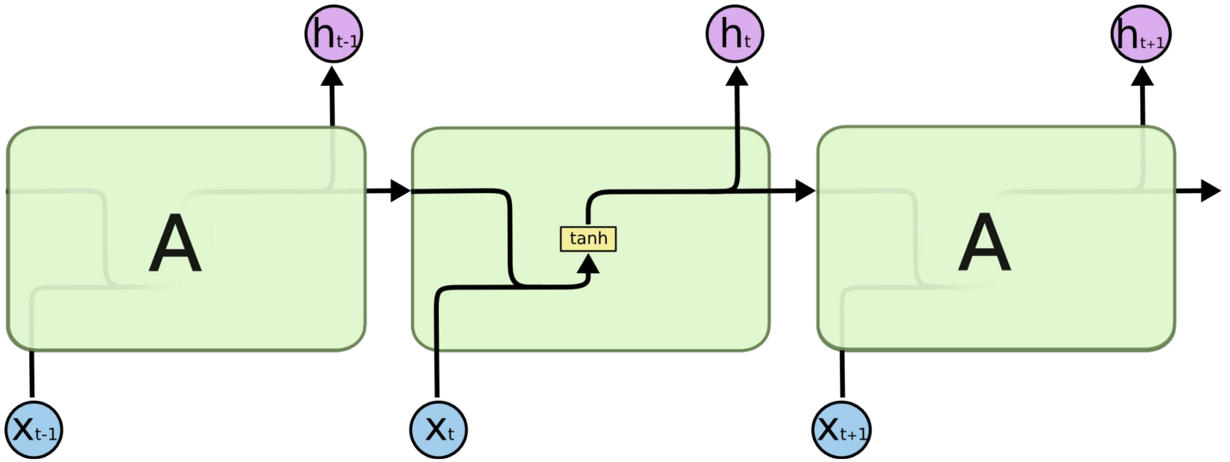


ResNet

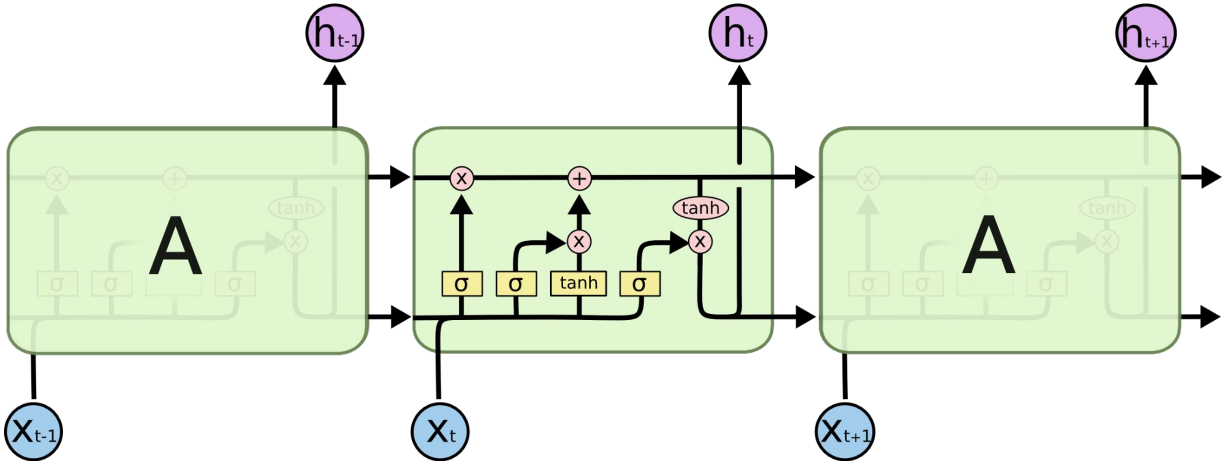


[He, Kaiming, et al. "Deep residual learning for image recognition." *Proceedings of the IEEE conference on computer vision and pattern recognition*. 2016.]

RNN

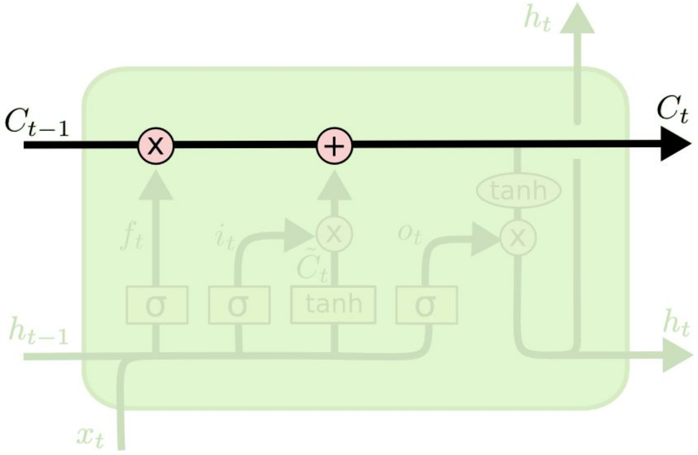


Long-short Term Memory (LSTM)



Long-short Term Memory (LSTM)

- Main idea: add a “cell” state that allows information to flow easily
    - Similar to residual connections
    - No repeated matrix multiplications!



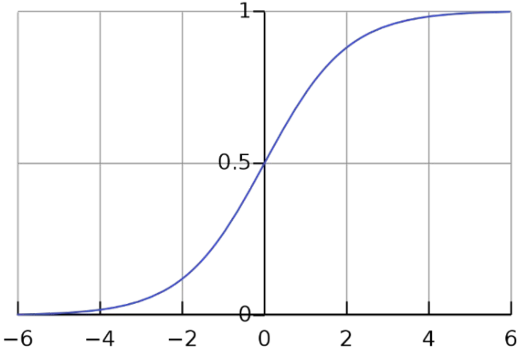
https://colah.github.io/posts/2015-08-Understanding-LSTMs/

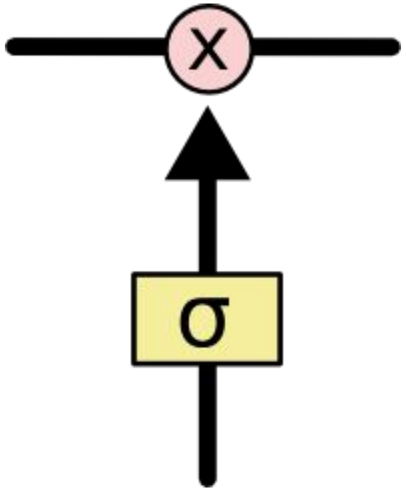
LSTMs- Gates

- Control the flow of information with “gates”
  - Element-wise product with the output of a sigmoid activation

Sigmoid Function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

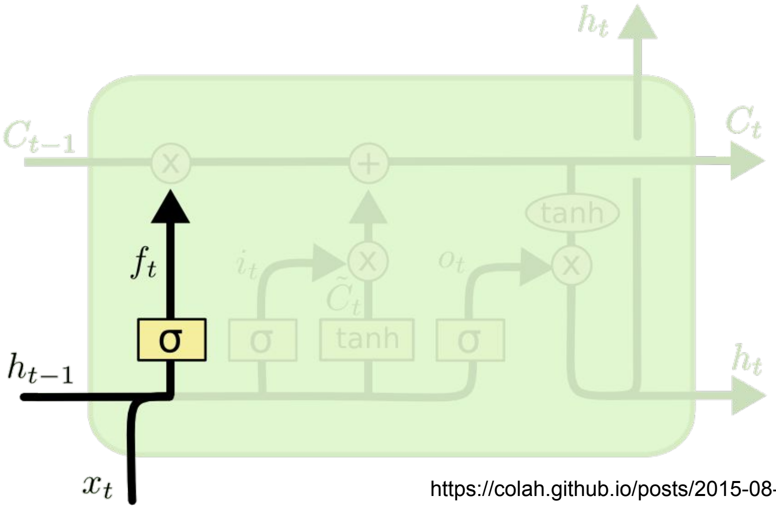




https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTMs- Forget Gate

- Forget gate- function of current input and previous hidden state
- Controls what should be remembered in the cell state

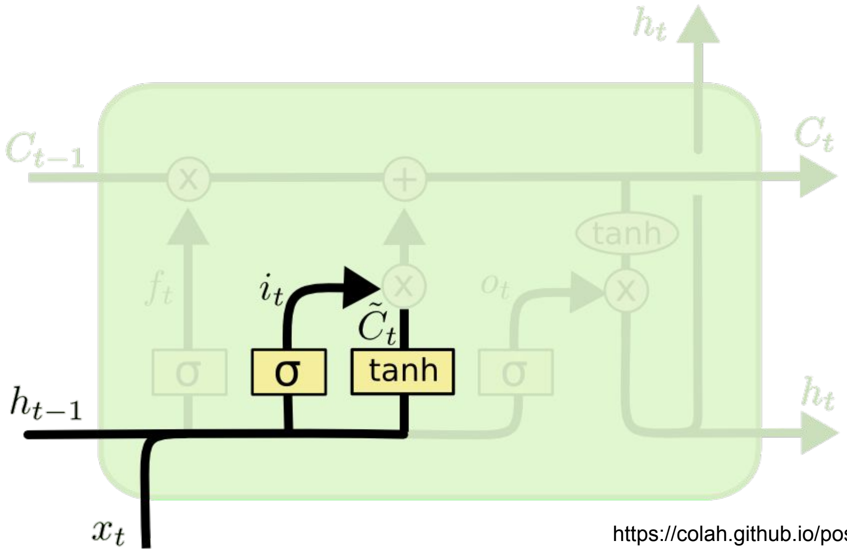


$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right)$$

https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTMs- Input Gate

- Input gate- function of current input and previous hidden state
- Decides what information to write to the cell state



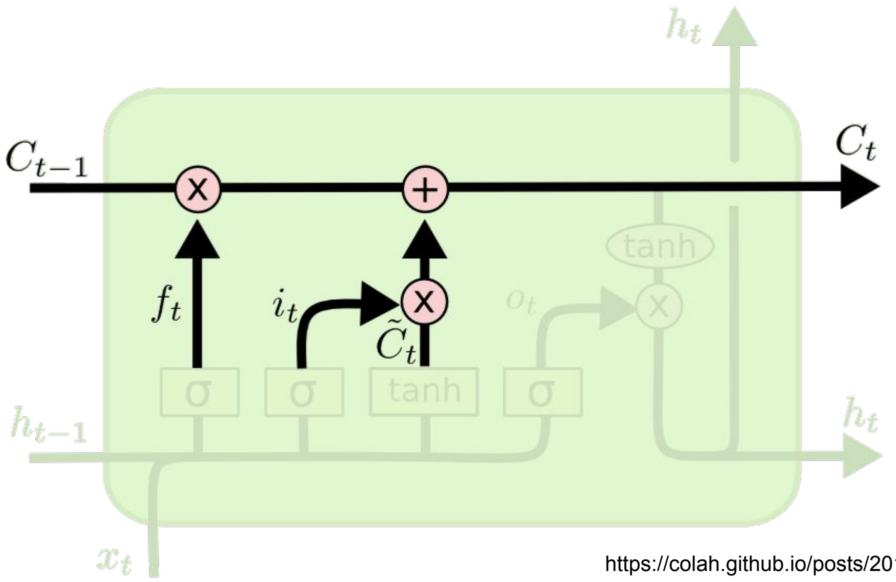
$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM- Cell Update

- Forget irrelevant information
- Add new information from the current token

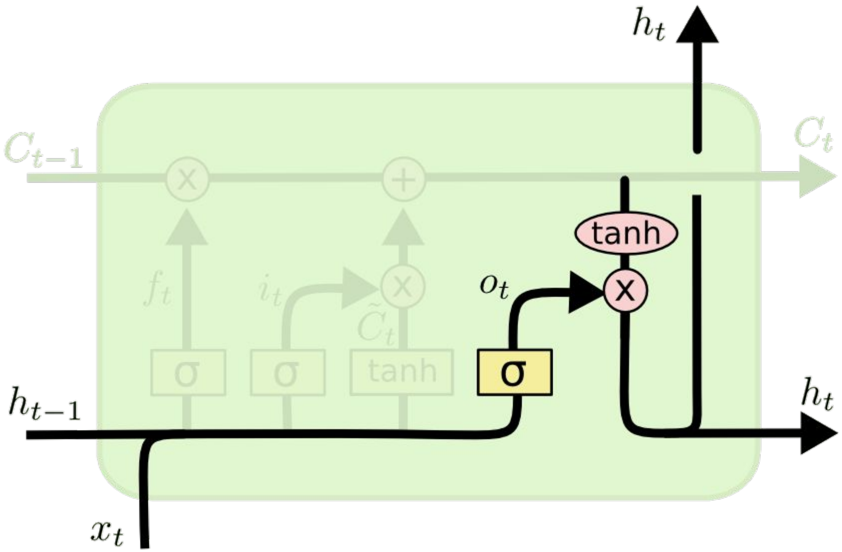


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM- Output Gate

- Output gate- function of current input and previous hidden state
- Controls flow of information from the cell state to the hidden state
- Given some weight matrix W_o, how do we write to o_t and h_t?



https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTMs

- Add a cell state to store information
  - Gradient flows along the cell state
- Update cell state with parameterized gating functions
- Performs better with long sequences

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

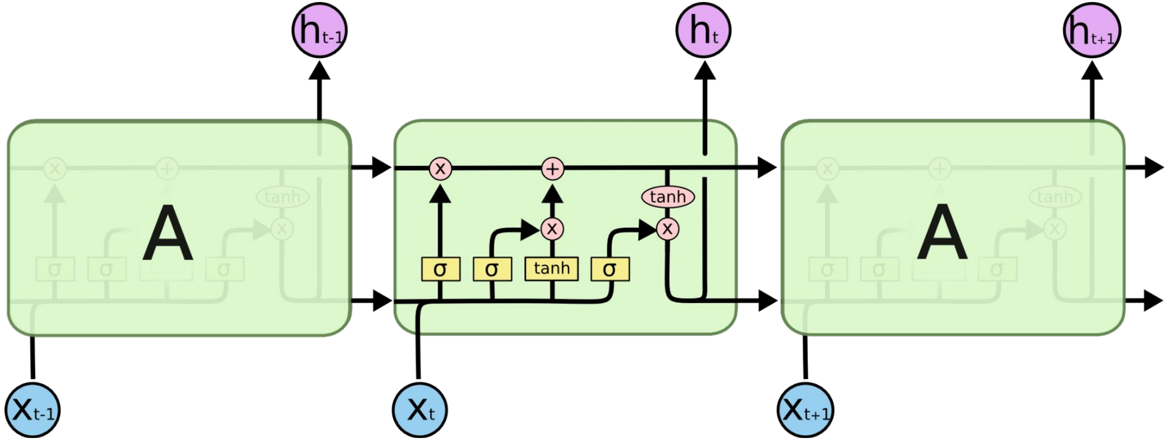
$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$o_t = \sigma\left(W_o \ [h_{t-1}, x_t] \ + \ b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$



https://colah.github.io/posts/2015-08-Understanding-LSTMs/

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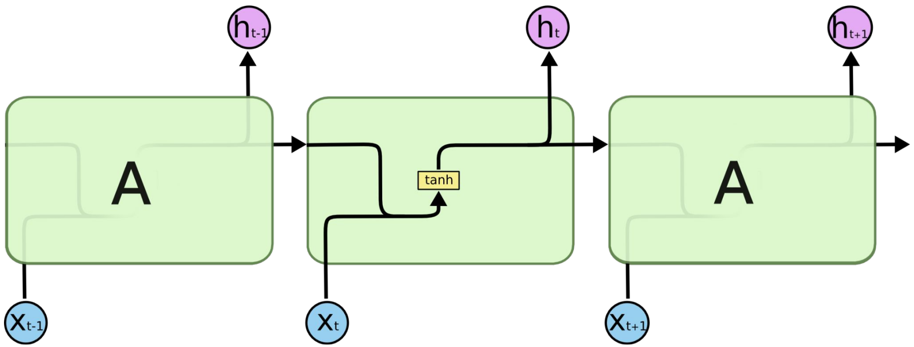
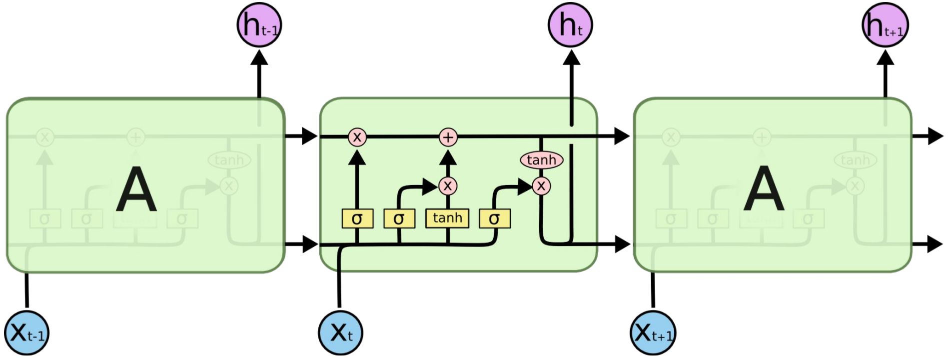
RNN vs. LSTM

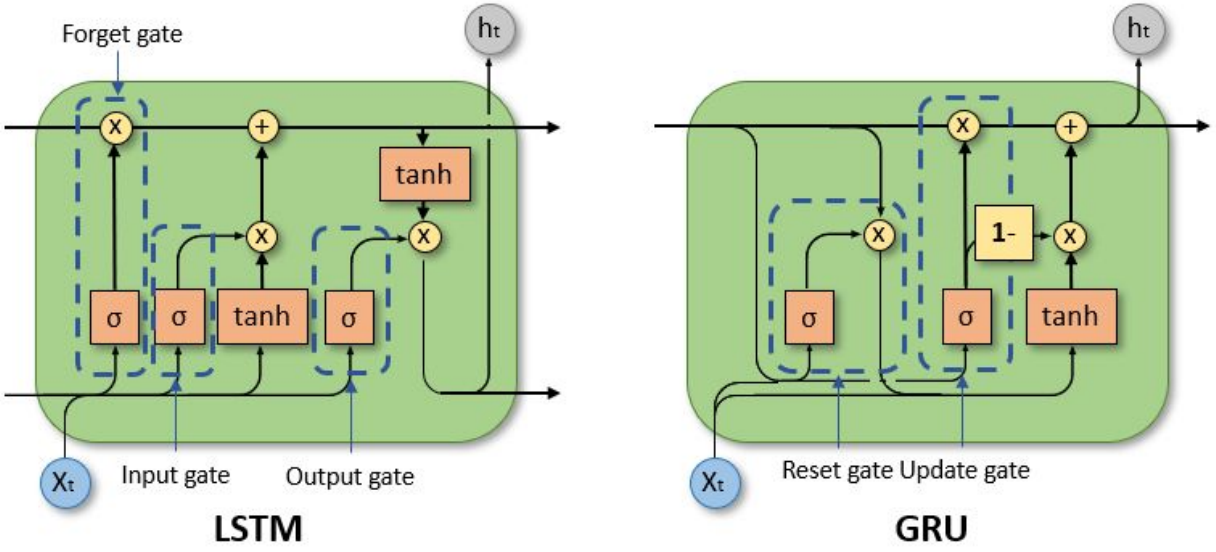
- **RNN**
  - Can be applied to variable-length sequences
  - Share parameters across time
  - Hard to train!
- **LSTM**
  - Mitigates the vanishing gradient problem with the cell state
  - Better for long sequences





https://colah.github.io/posts/2015-08-Understanding-LSTMs/

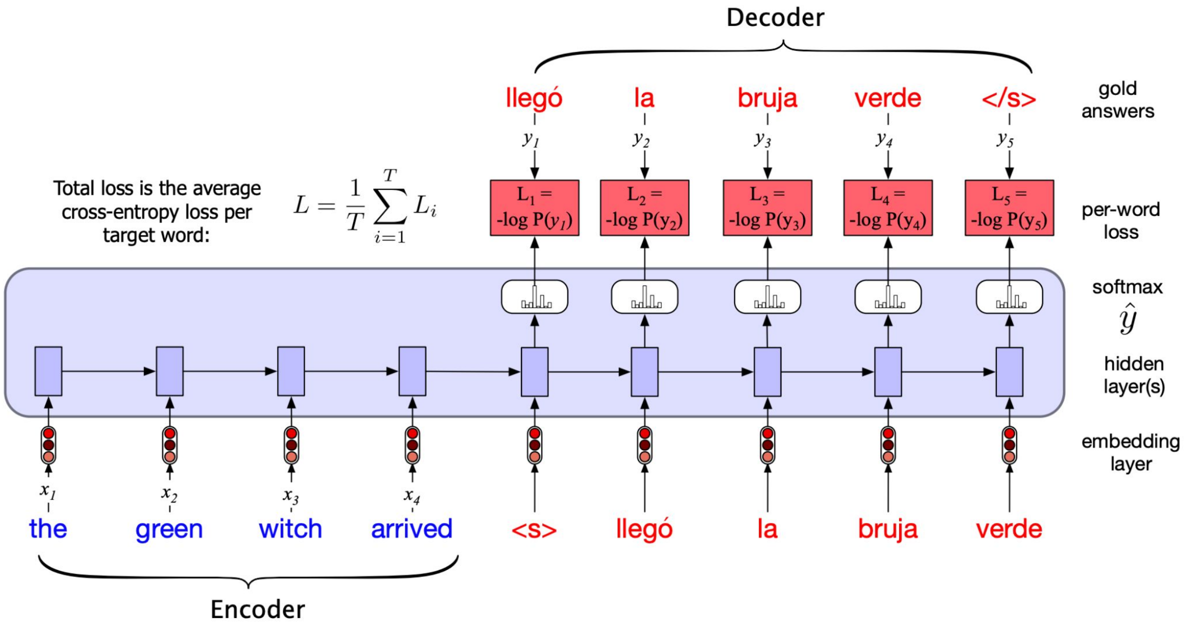
Gated recurrent units (GRUs)



https://towardsdatascience.com/illustrated-guide-to-lstms-and-gru-s-a-step-by-step-explanation-44e9eb85bf21

Sequence-to-Sequence Generation

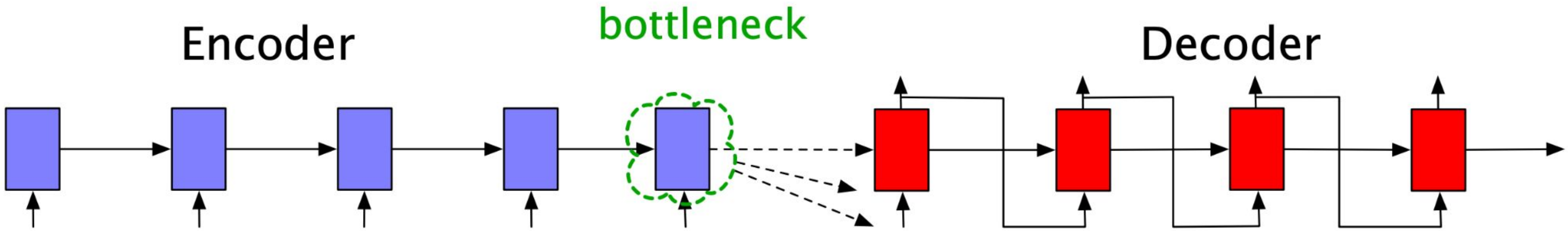
- Map some input sequence to a target sequence
- Applications:
  - Machine translation
  - News summarization
  - ChatGPT!



https://web.stanford.edu/~jurafsky/slp3/

Bottleneck Problem

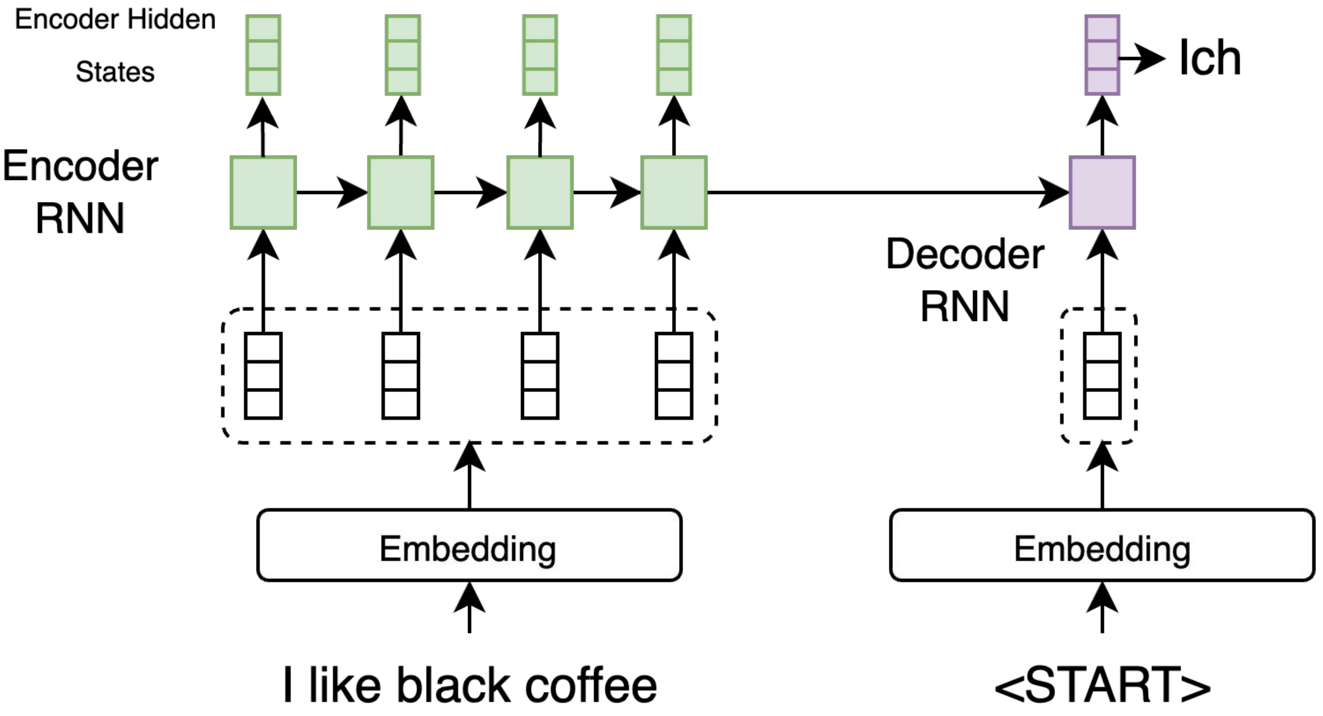
- All the information about the source sequence must be stored in a single vector
  - How to translate a long paragraph?
  - How to summarize long articles?



https://web.stanford.edu/~jurafsky/slp3/

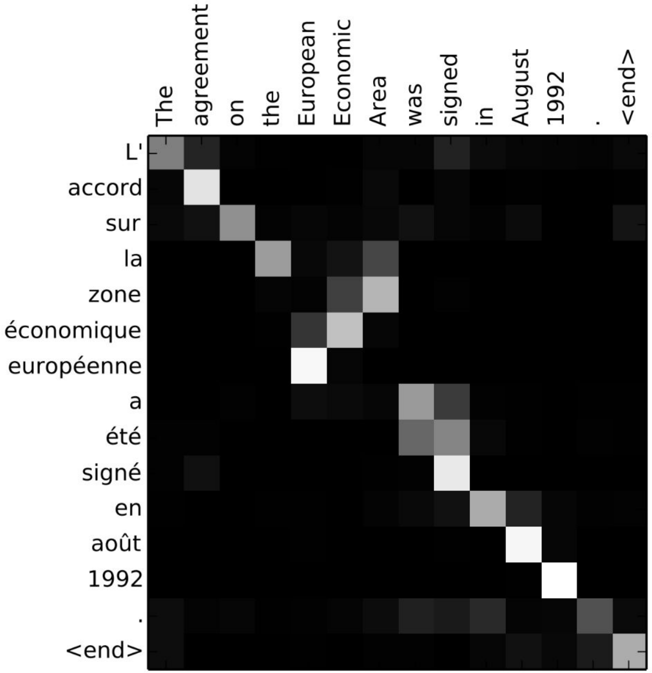
RNN for Machine Translation

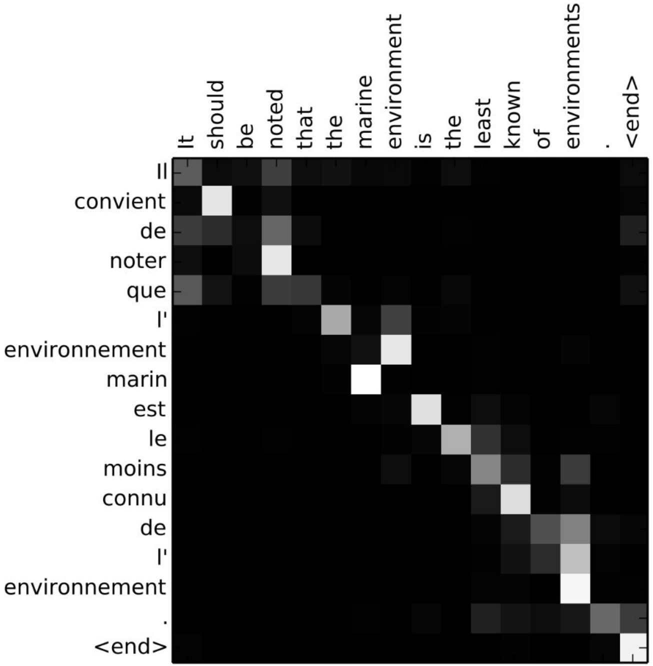
Would be nice if we could “look back” at previous hidden states



Visualizing Attention

- Plot attention weights to see where the model is “looking”
  - Learns language alignment for translation!



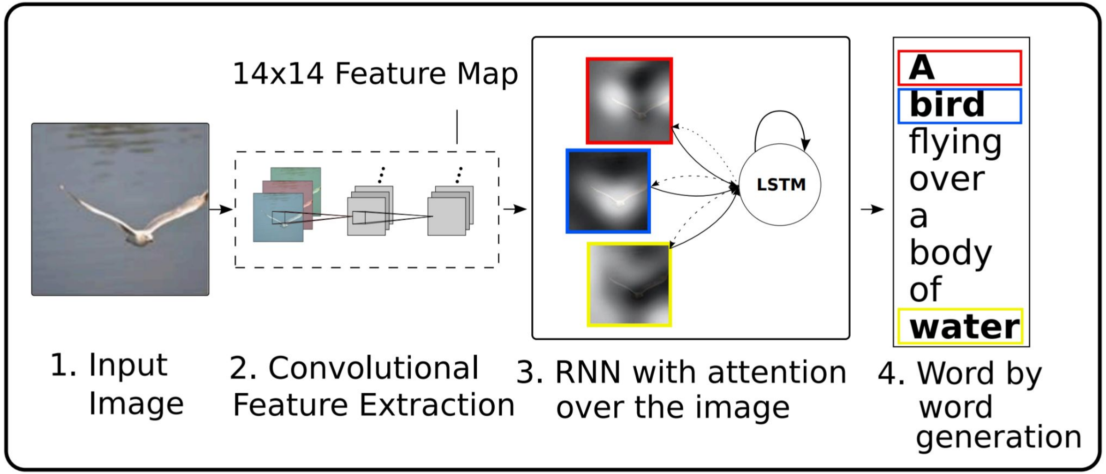


Bahdanau, Dzmitry, Kyunghyun Cho, and Yoshua Bengio. "Neural machine translation by jointly learning to align and translate."

Attention Application- Image Captioning!

- Extract image features with a CNN
- Use an LSTM with attention to generate image captions

Figure 1. Our model learns a words/image alignment. The visualized attentional maps (3) are explained in section 3.1 & 5.4

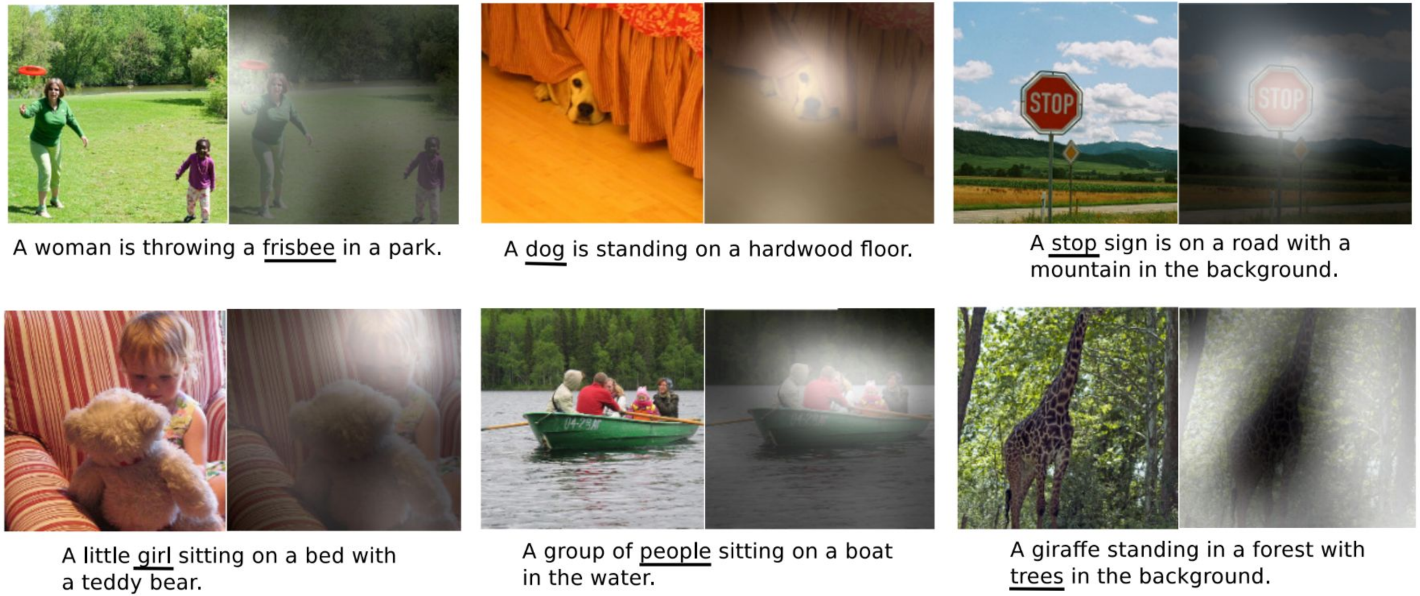


Xu, Kelvin, et al. "Show, attend and tell: Neural image caption generation with visual attention." International conference on machine learning. PMLR, 2015.

Visualize Attention Weights

- Learns to focus on relevant regions of the image

Figure 3. Examples of attending to the correct object (*white* indicates the attended regions, *underlines* indicated the corresponding word)



Xu, Kelvin, et al. "Show, attend and tell: Neural image caption generation with visual attention." International conference on machine learning. PMLR, 2015.

Recap

- RNNs can be applied to arbitrary length sequences
  - Run into vanishing/exploding gradient problems
- LSTMs add a cell state to RNNs to improve gradient flow
  - Better a handling long sequences
- Attention can look back at past feature vectors!
  - Scales better to long sequences
  - Can incorporate image features
  - Many, many more applications!