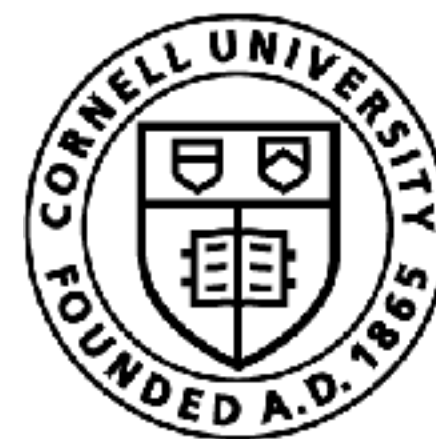


# Conquering Motion Planning via Sampling and Search

Sanjiban Choudhury



Cornell Bowers CIS  
**Computer Science**

# Recap

We saw how LQR gives us the optimal policy for linear, quadratic costs

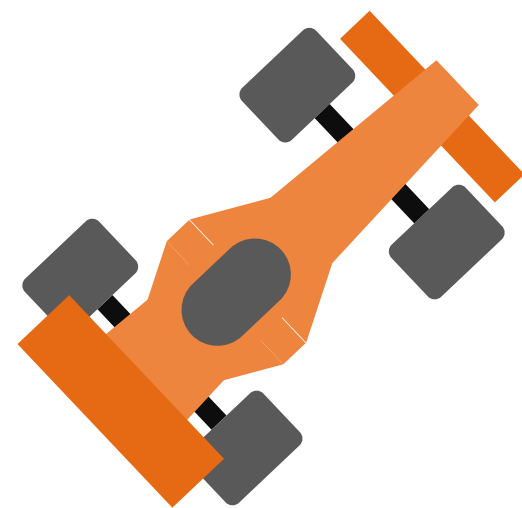
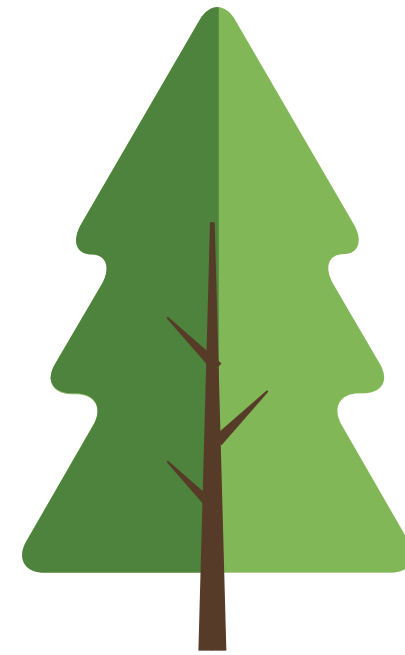
But how can we use LQR for general problems?

# LQR for a *non-linear, non-quadratic* MDP



Cost

$$\exp(- (x - x_{tree})^2 - (y - y_{tree})^2)$$



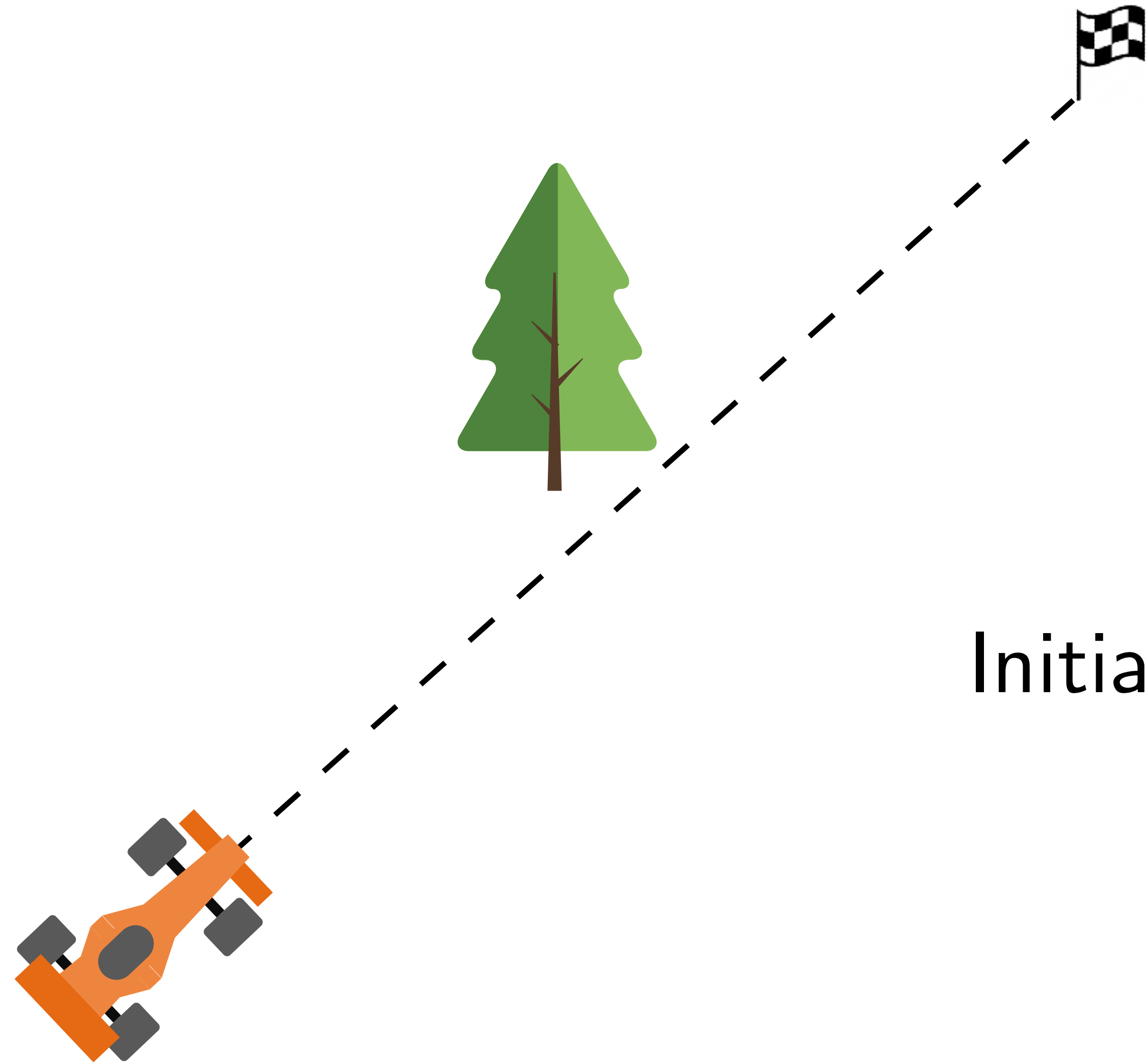
$$\dot{x} = u_s \cos \theta$$

$$\dot{y} = u_s \sin \theta$$

$$\dot{\theta} = u_\omega.$$

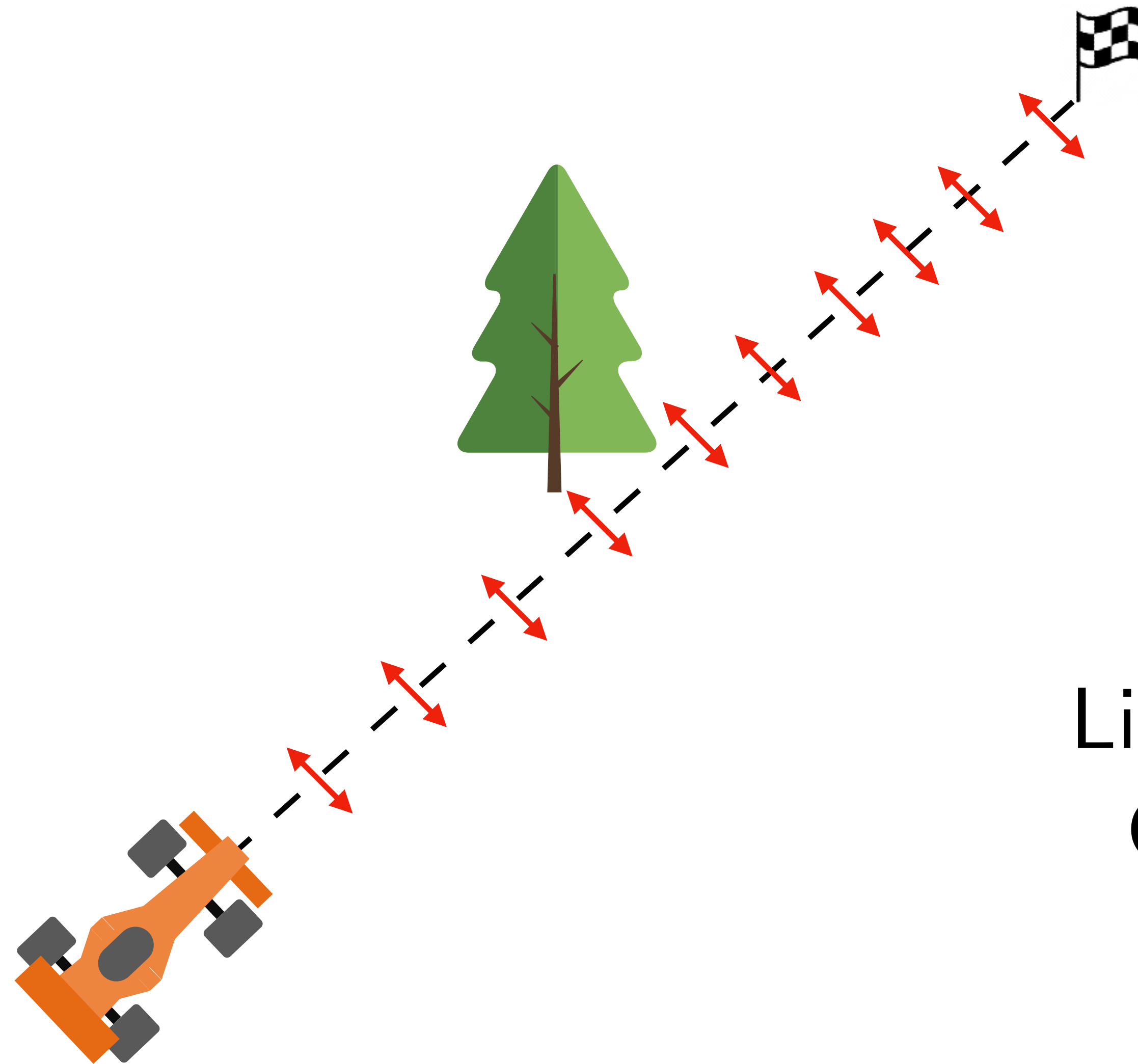
Dynamics

# LQR for a *non-linear, non-quadratic* MDP



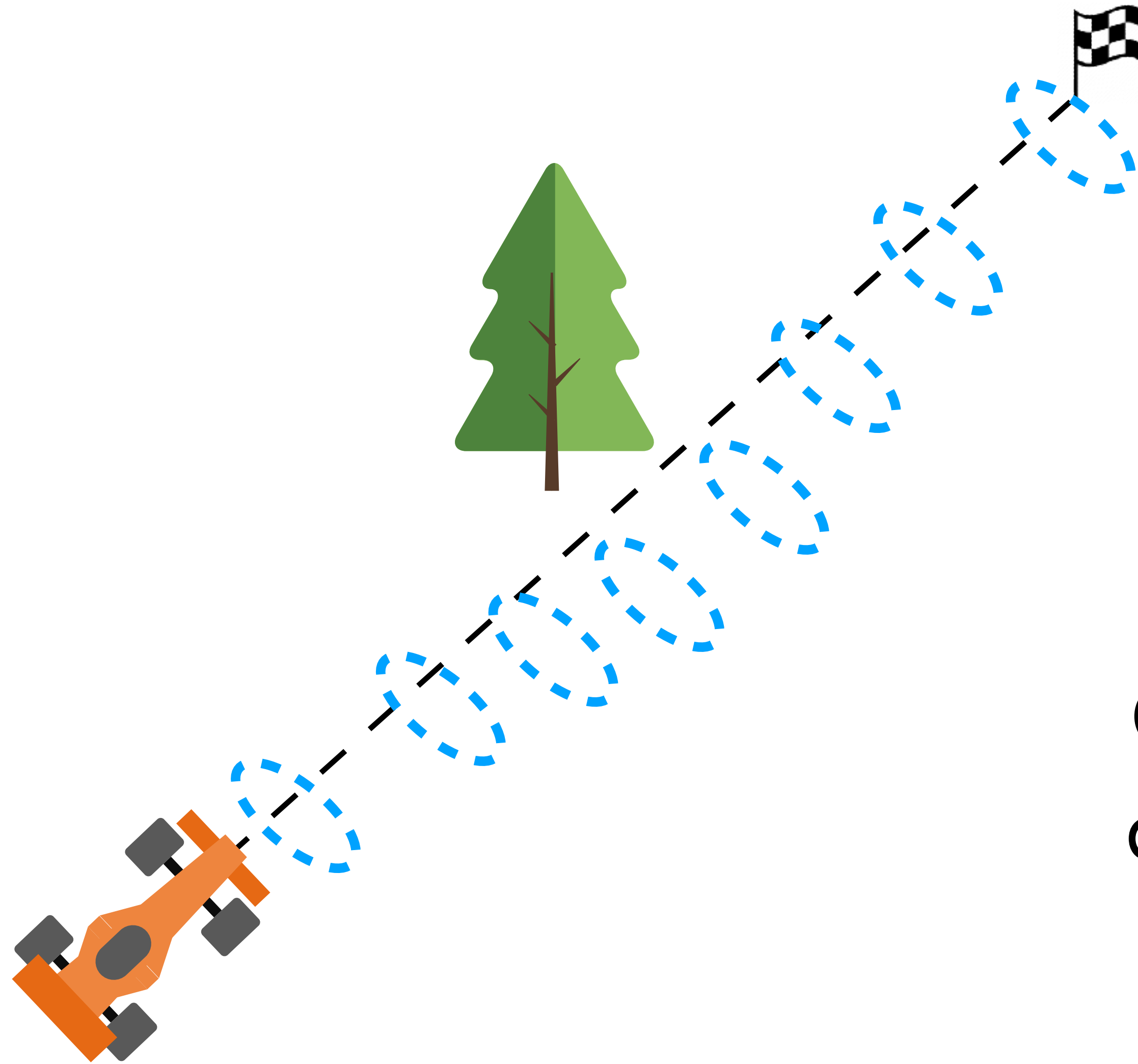
Initialize with a sequence  
of actions

# LQR for a *non-linear, non-quadratic* MDP



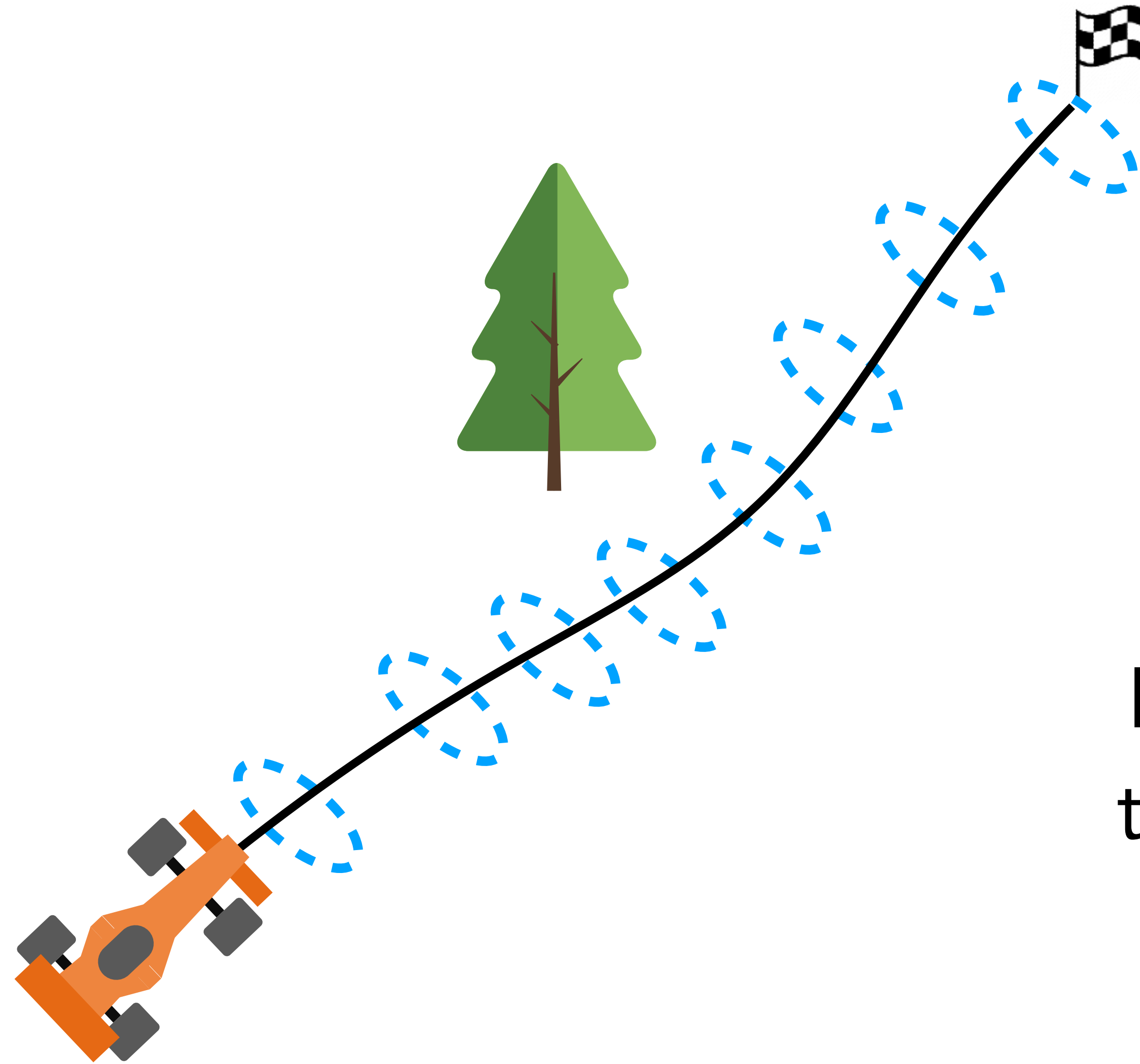
Linearize dynamics,  
Quadraticize costs

# LQR for a *non-linear, non-quadratic* MDP



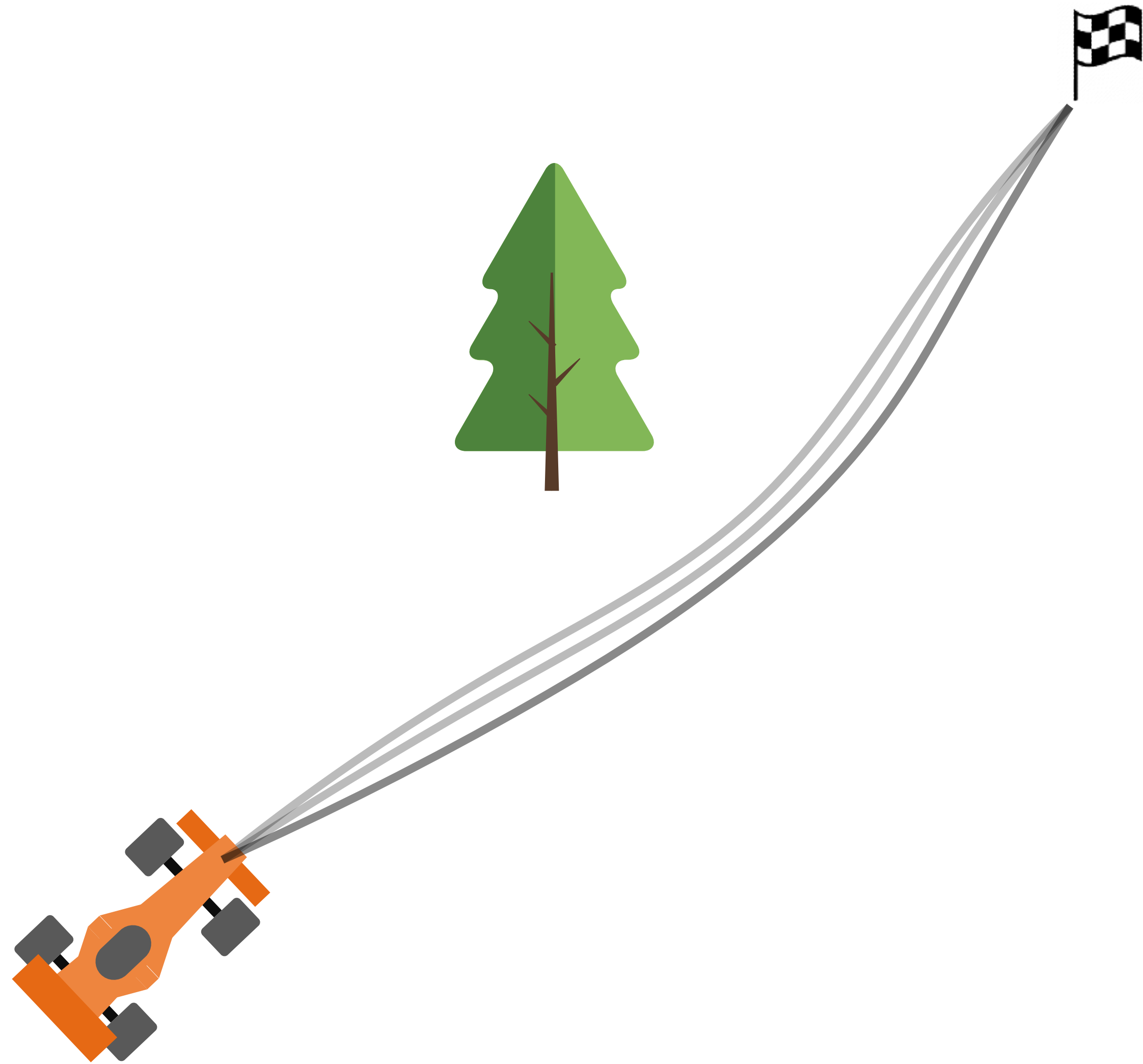
Call LQR to get quadratic values

# LQR for a *non-linear, non-quadratic* MDP



Execute LQR policy  
to get new sequence  
of actions

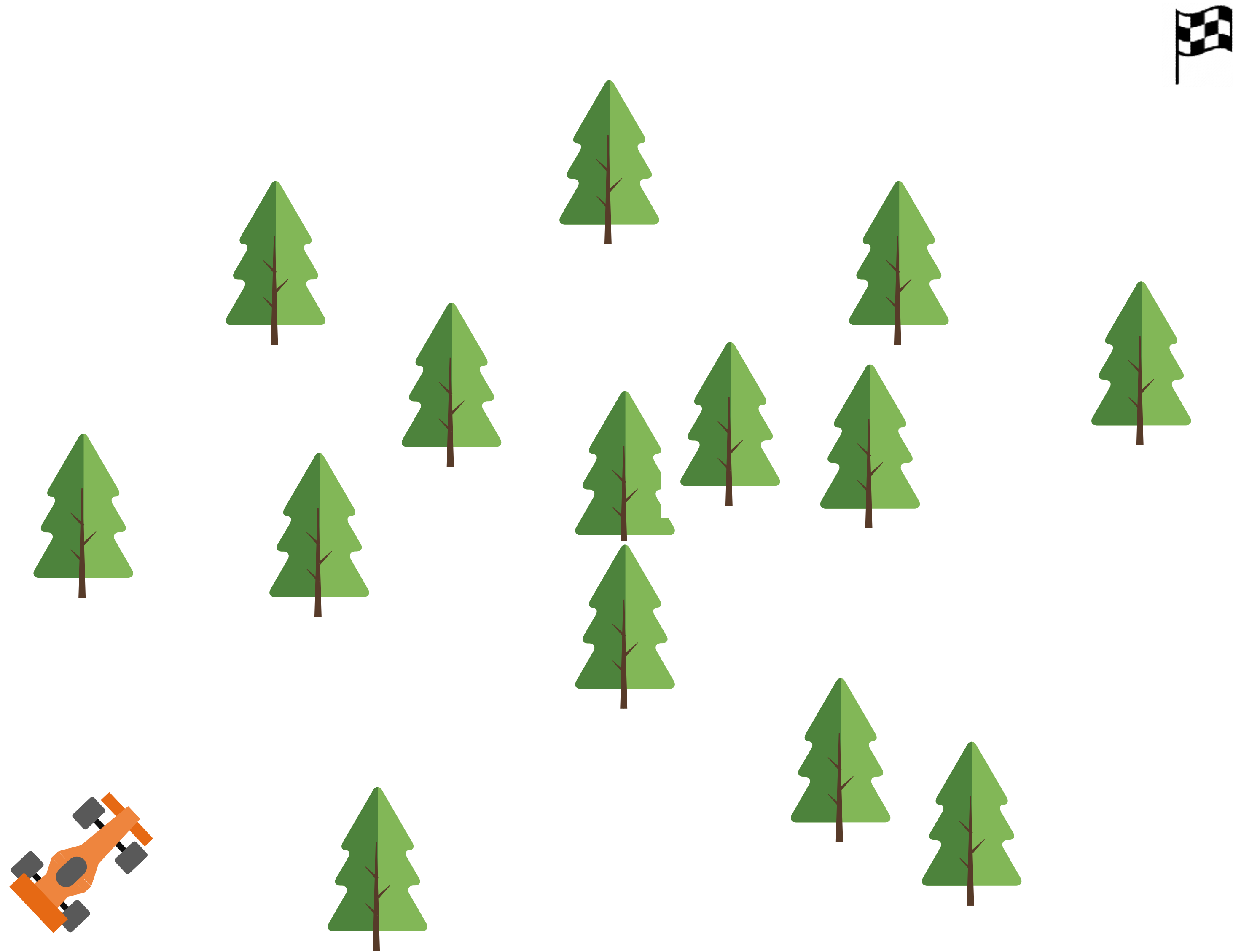
# LQR for a *non-linear, non-quadratic* MDP



Repeat the process  
till convergence!

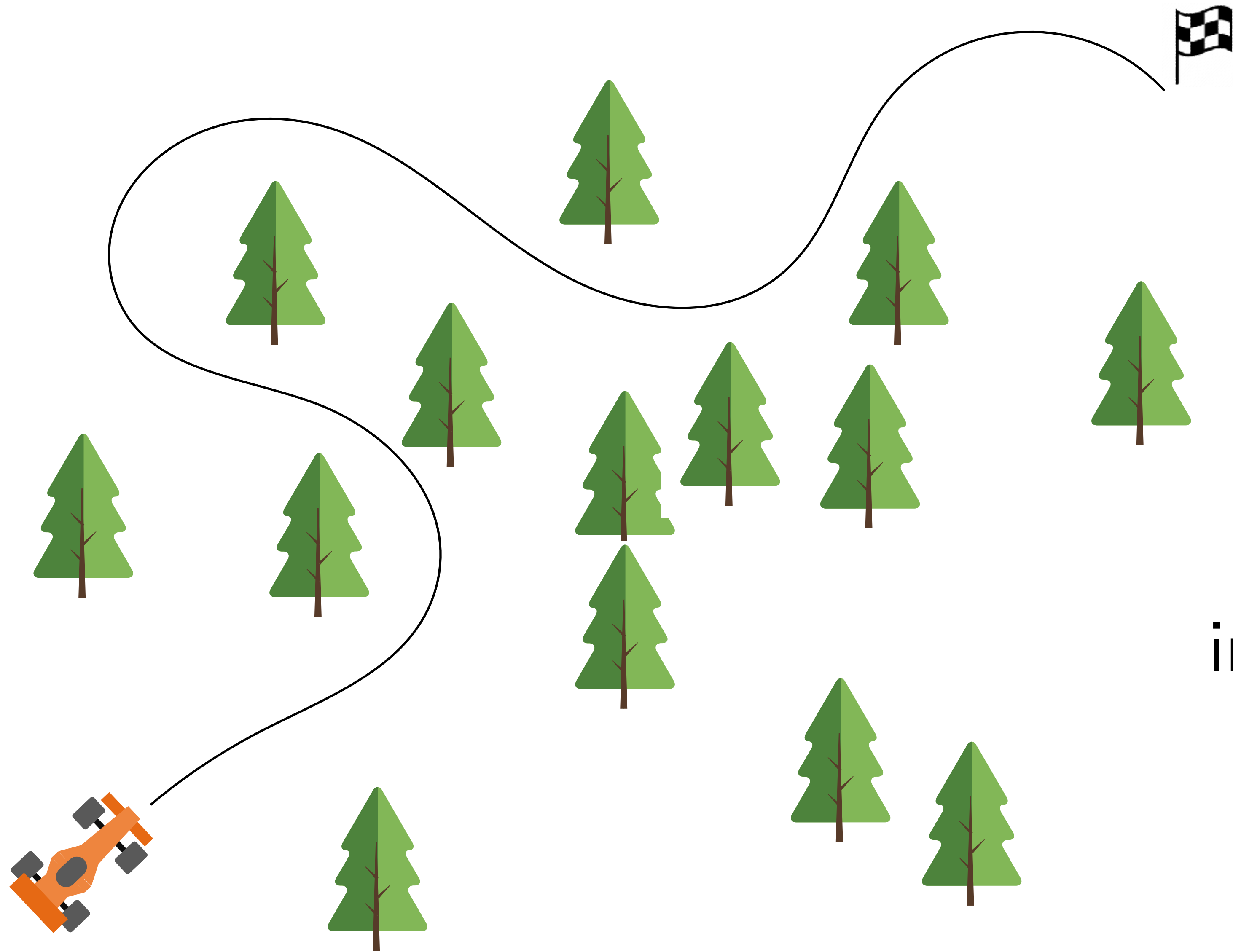


# But what happens when we have lots of trees?



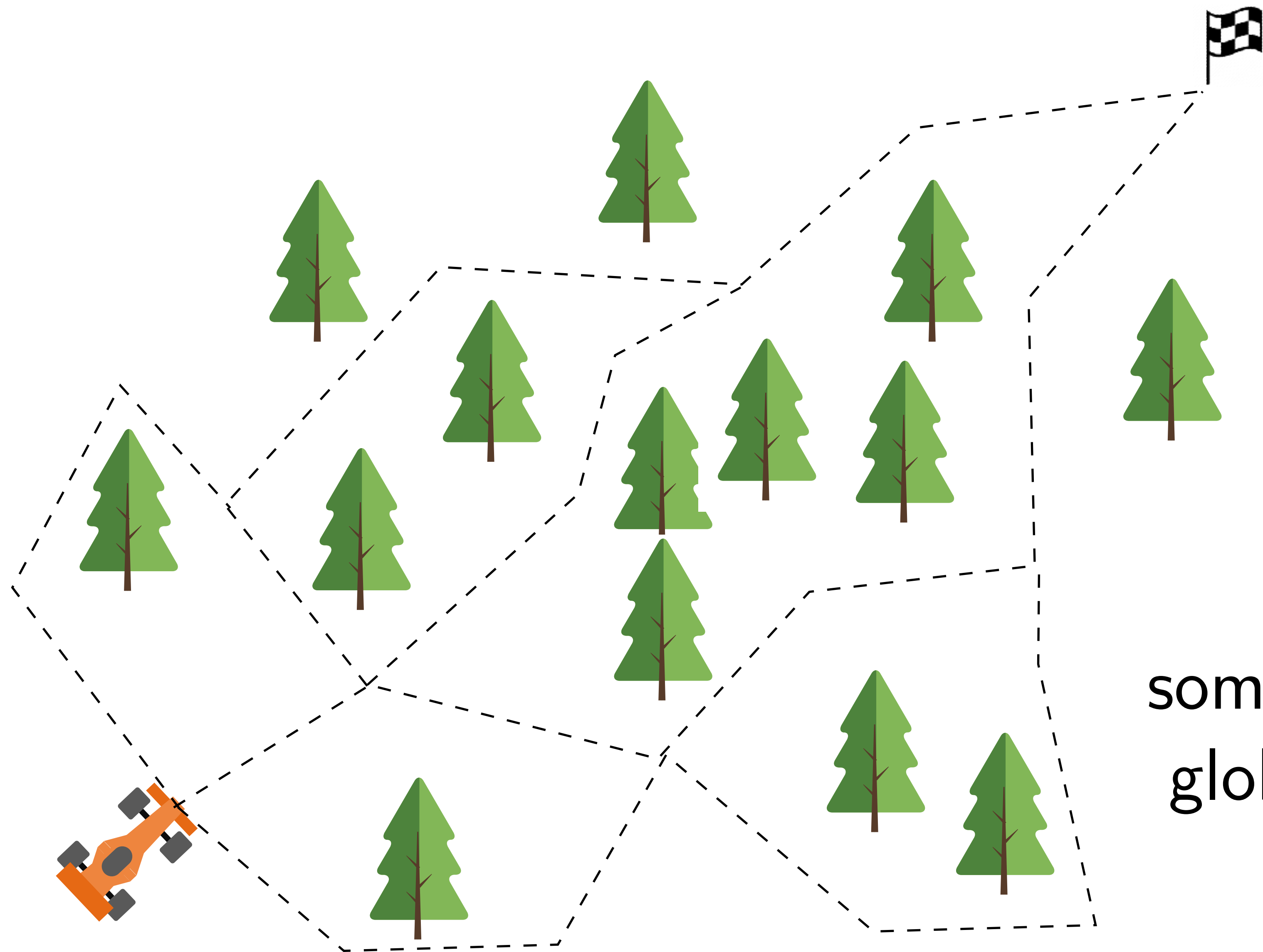
Many local optima!

# But what happens when we have lots of trees?



If we initialize LQR  
in a bad local basin,  
it finds  
a bad local optima

# But what happens when we have lots of trees?



Instead we need something that can search globally to initialize LQR

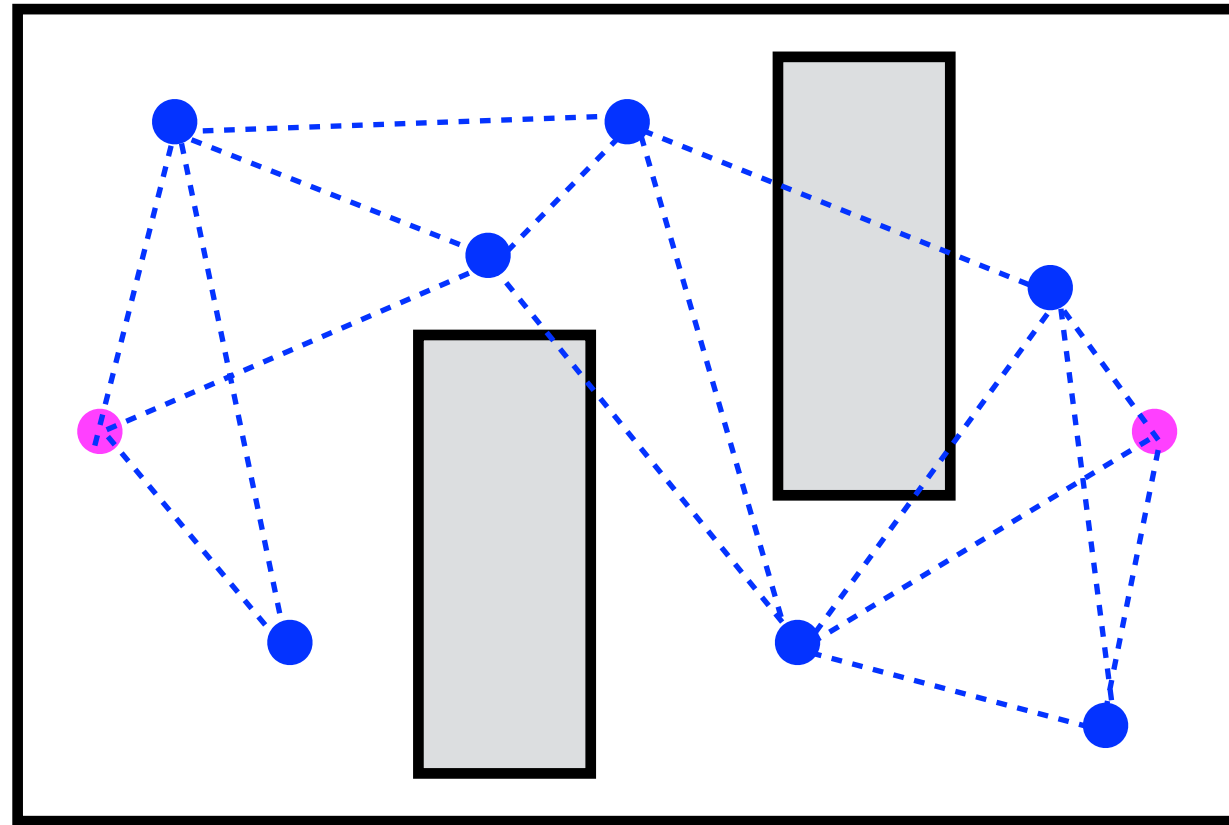
# The Problem with General MDPs

LQR reasons locally

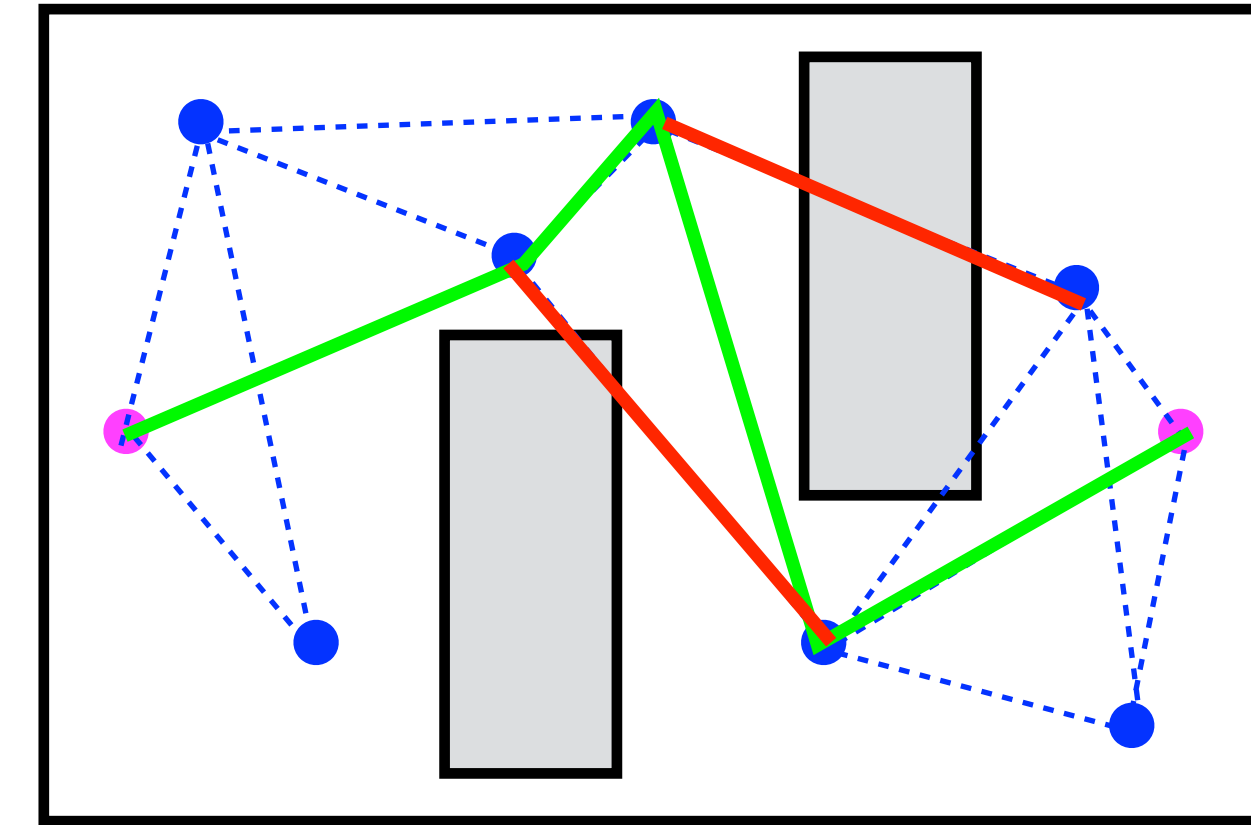
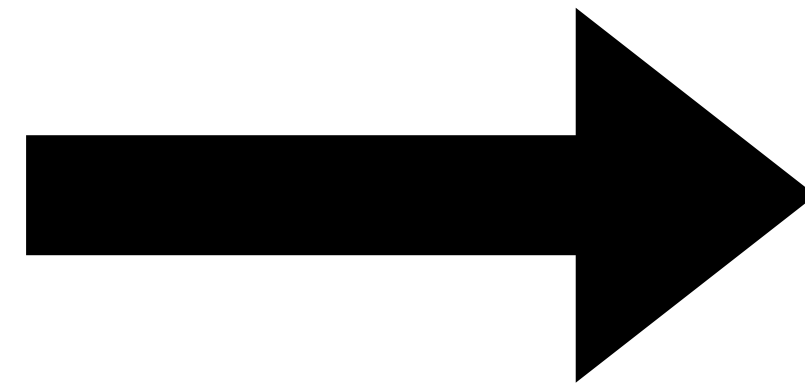
We need to combine it with something that reasons globally

This global reasoning is typically done by **motion planning**

# General framework for motion planning



Create a graph

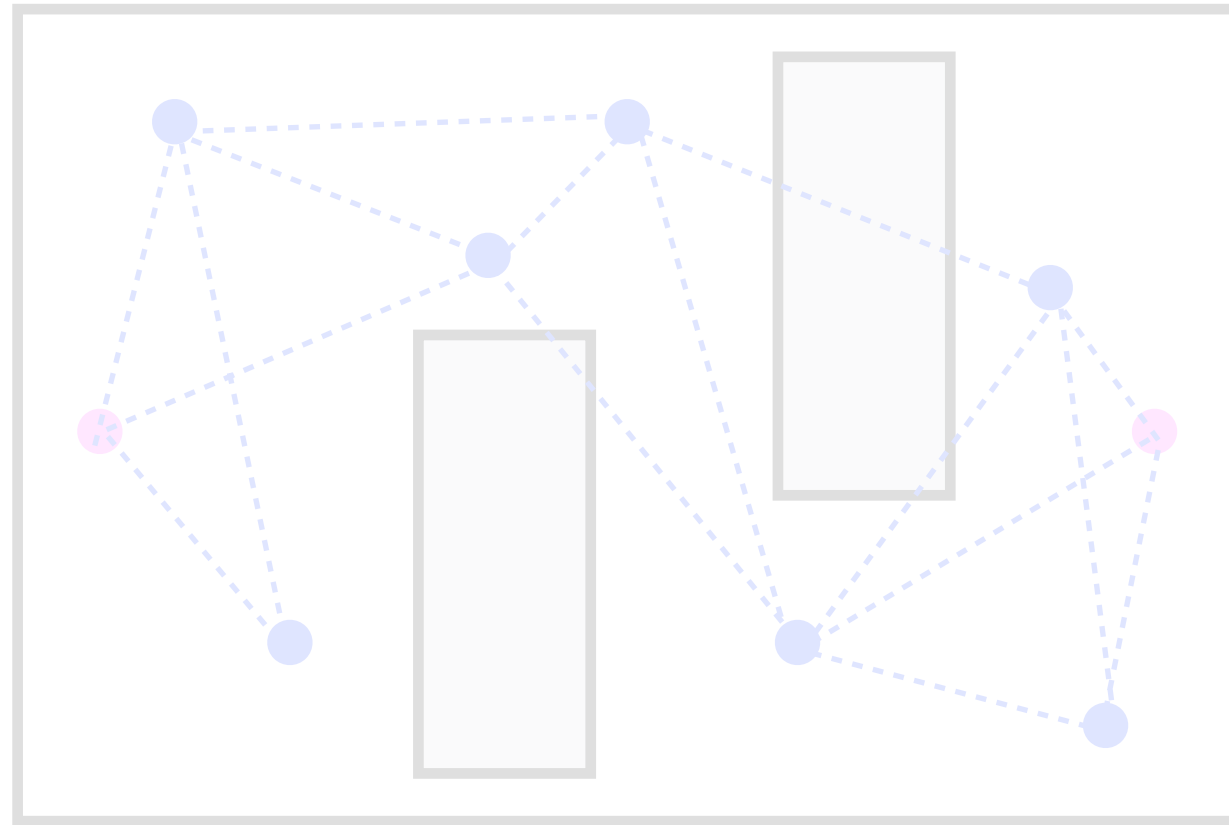


Search the graph

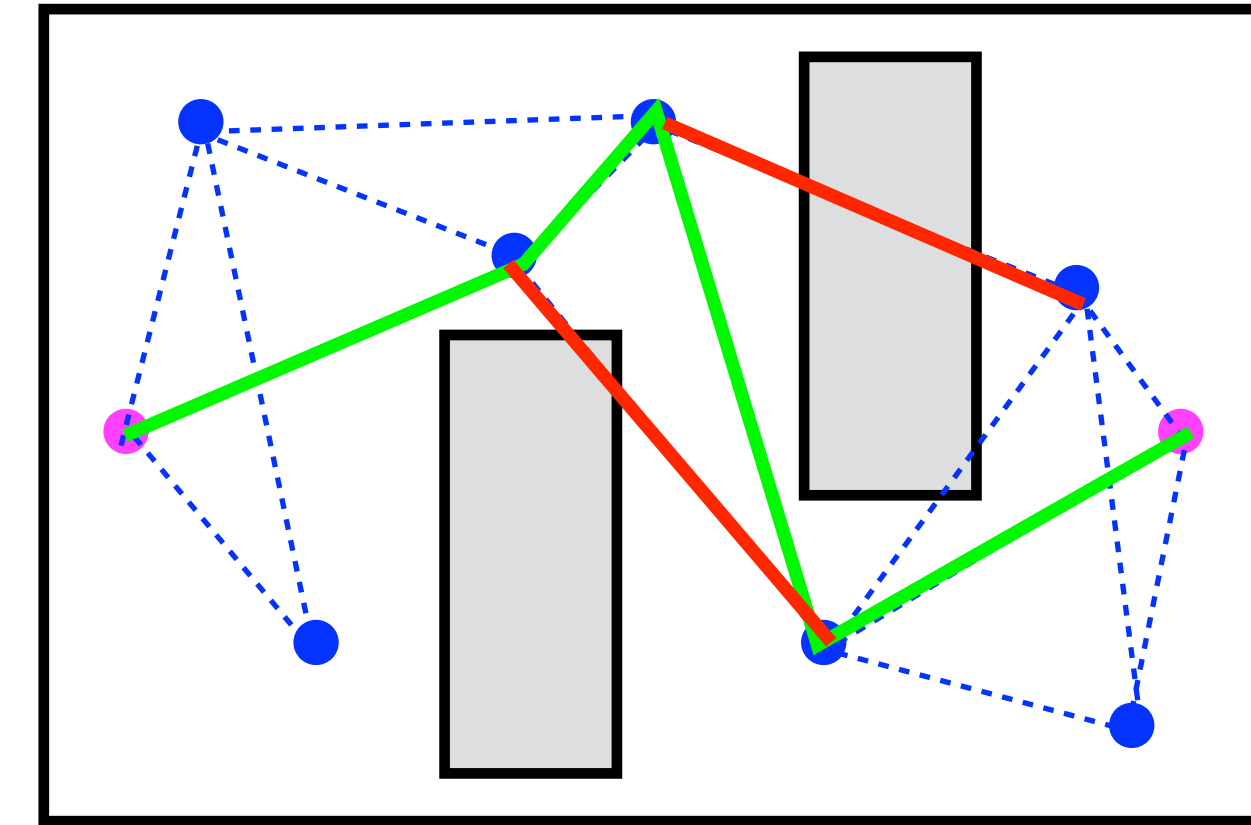
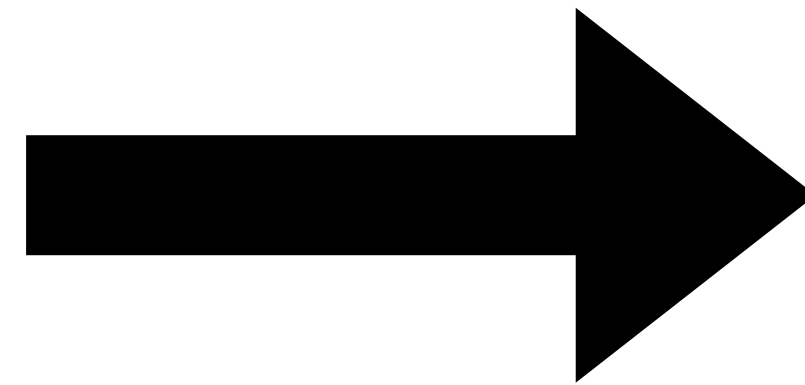


Interleave

# General framework for motion planning



Create a graph



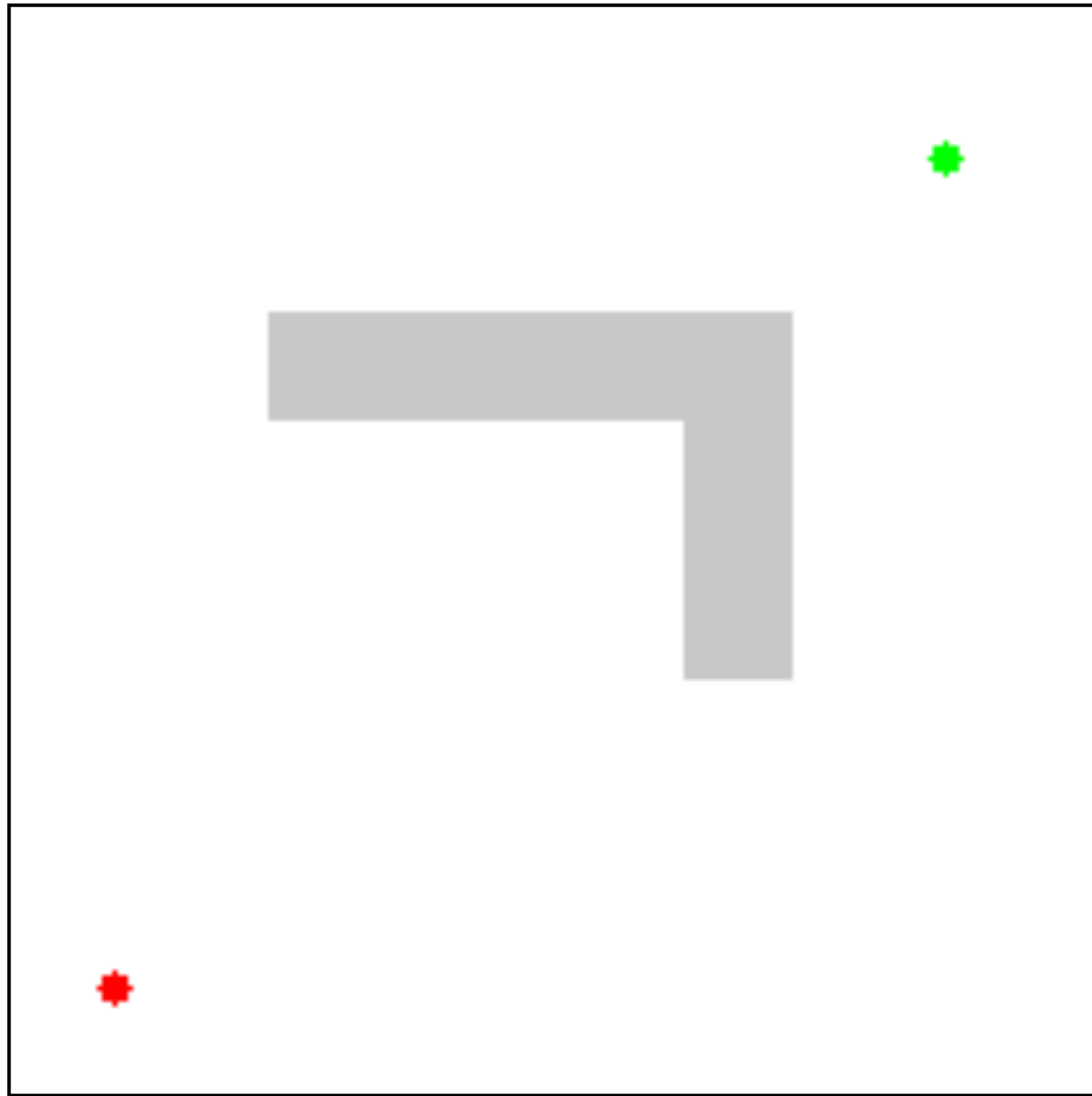
Search the graph



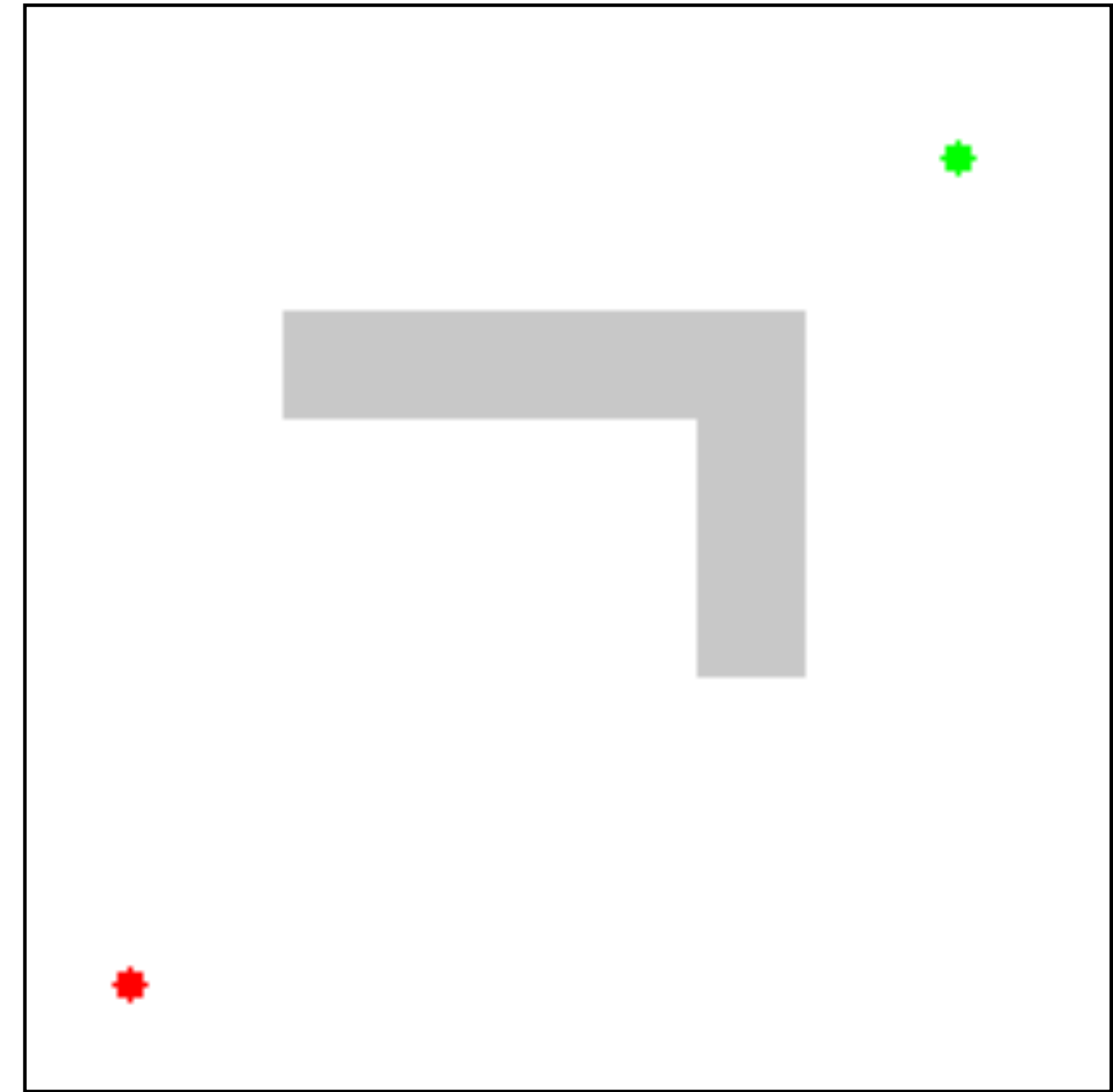
Interleave



# How can we make this search faster?



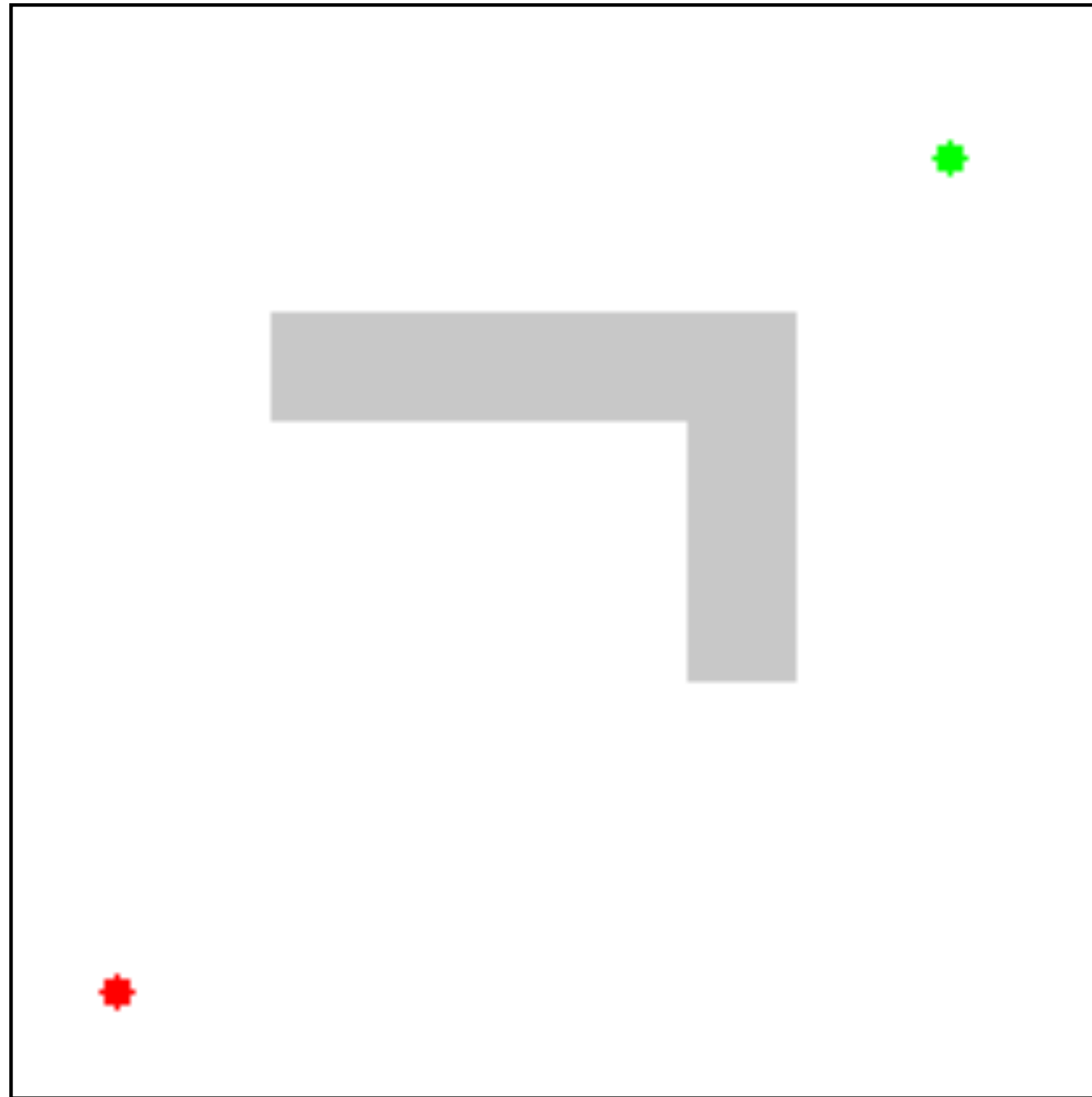
Dijkstra



A\* with heuristic!



# What can we prove about $A^*$



$A^*$  with heuristic!

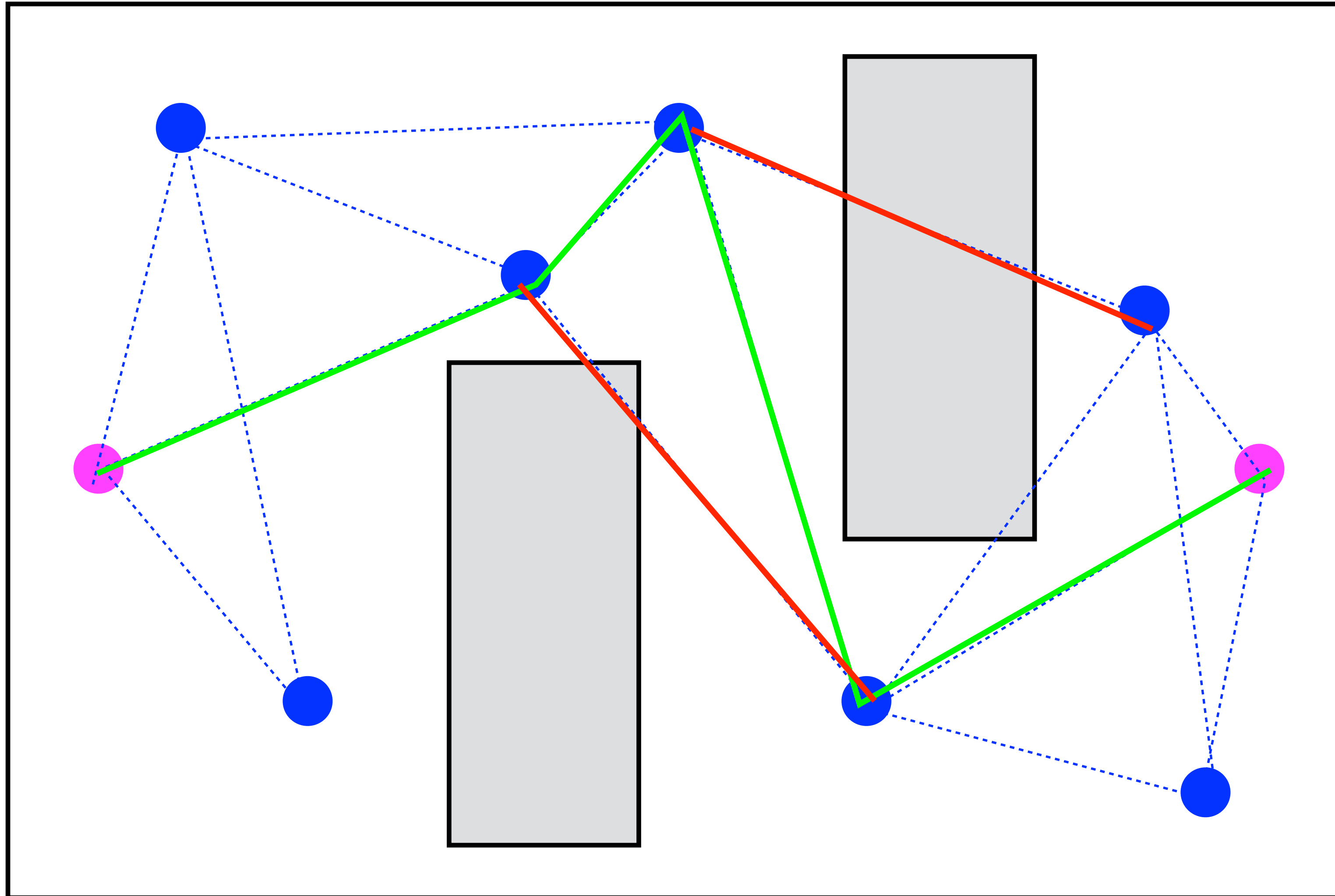
1.  $A^*$  gives us the optimal path  
*(If heuristic is admissible)*

2.  $A^*$  expands the  
optimal number of vertices  
*(If heuristic is consistent)*

But is the **number of expansions** really what we want to minimize in **motion planning**?

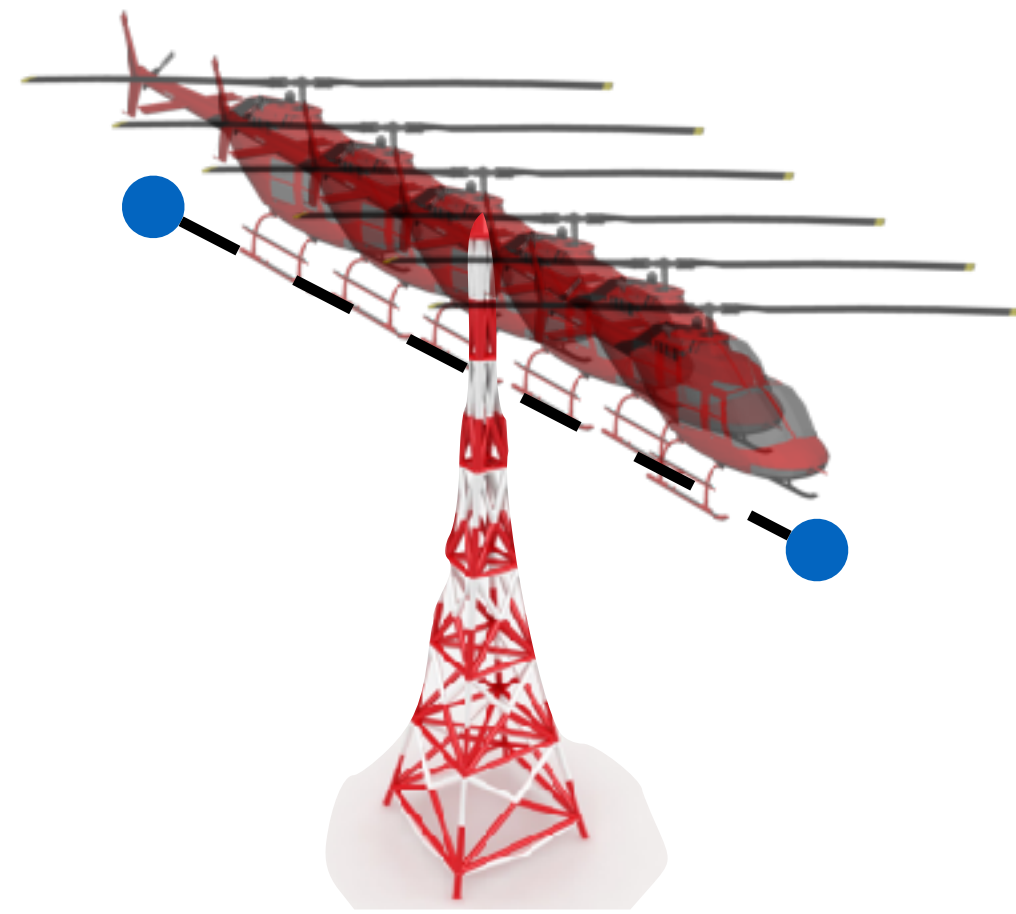
What is the most expensive step?

# Edge evaluation is the most expensive step

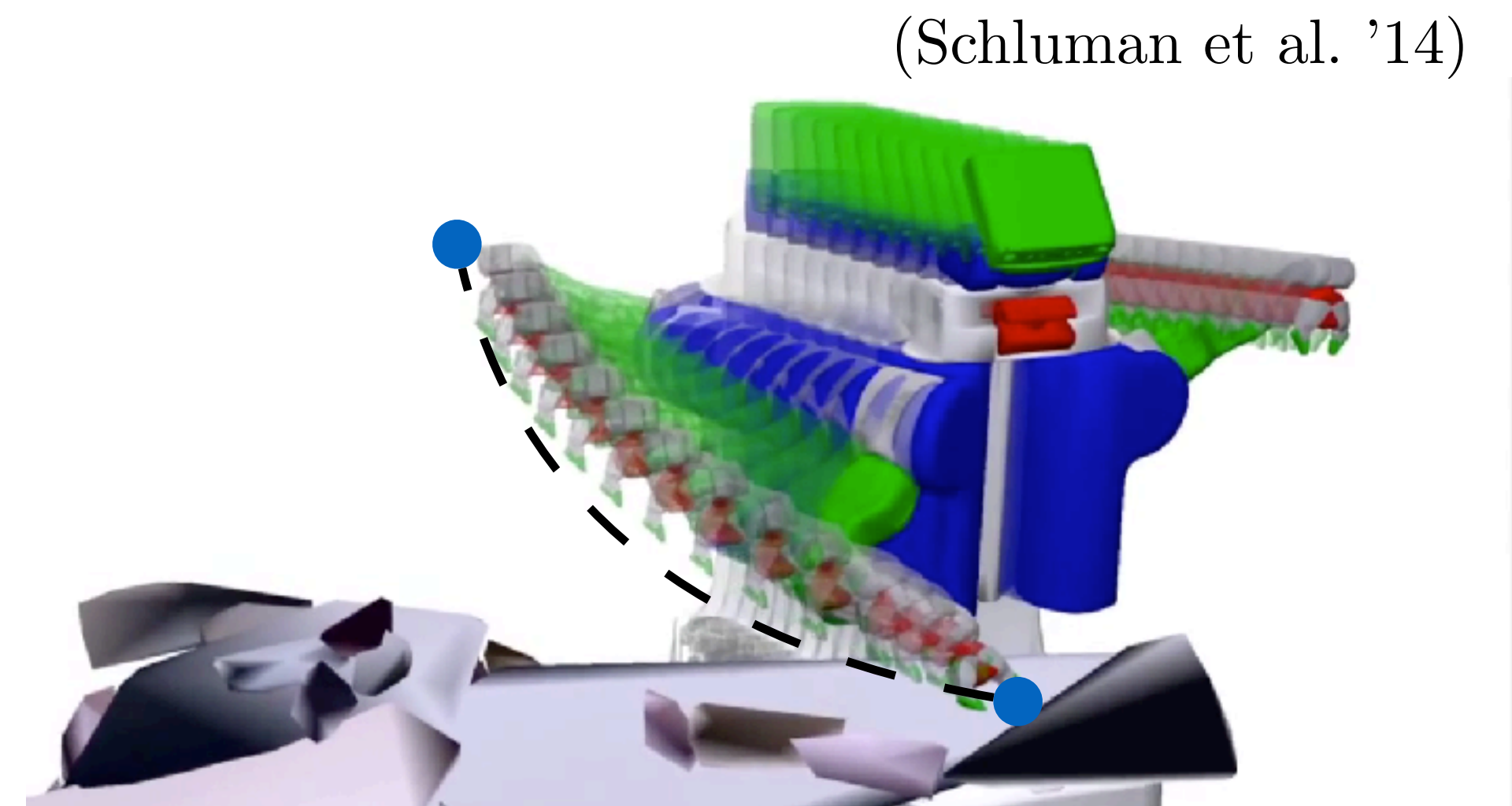


Why?

# Edge evaluation requires expensive collision checking

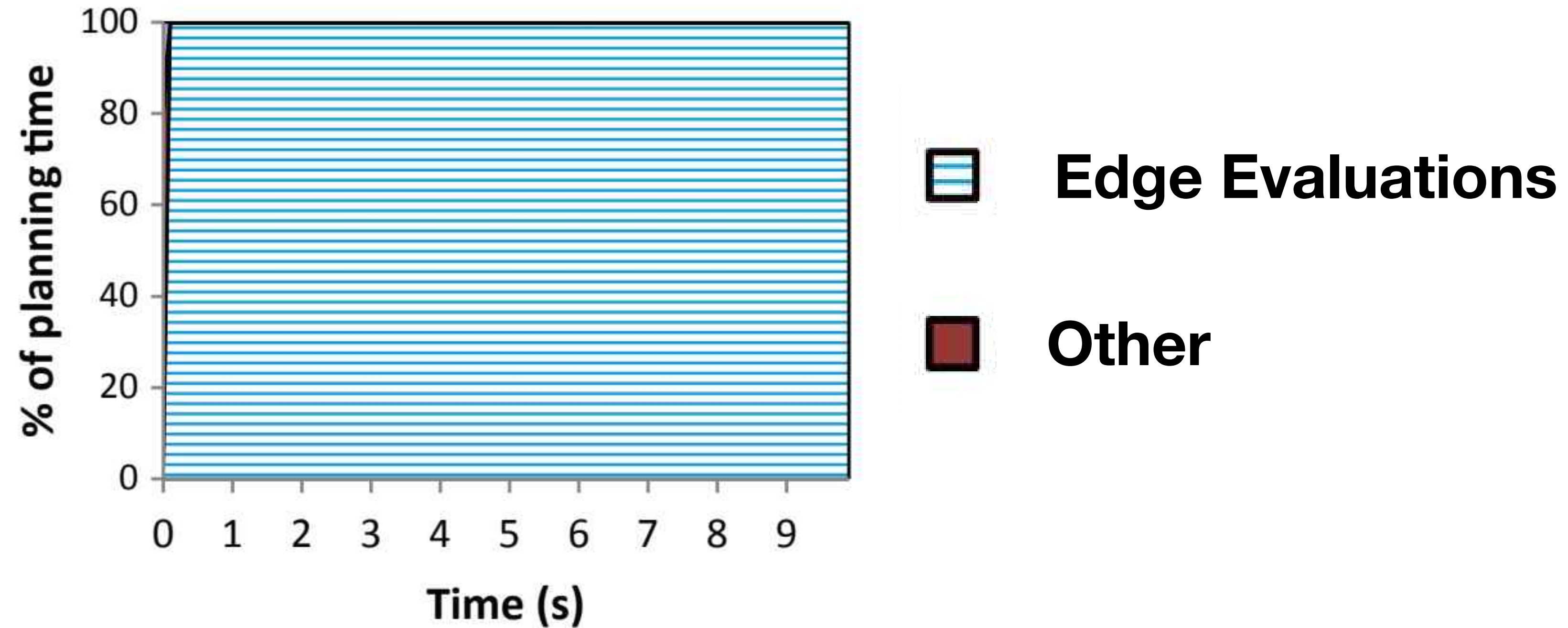


Check if helicopter intersects with tower



Check if manipulator intersects with table

# Edge evaluation **dominates** planning time



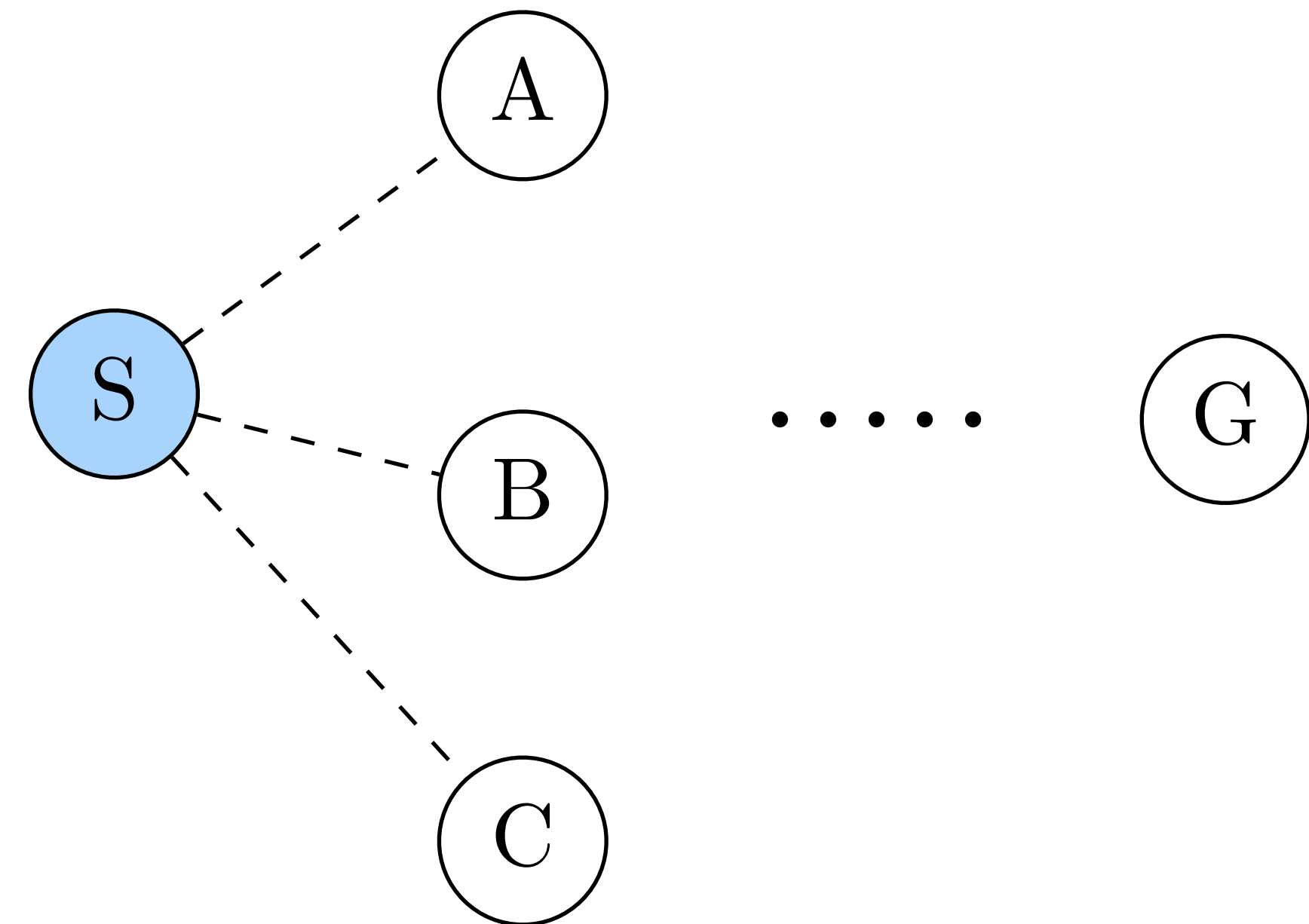


How do we modify  $A^*$  search to minimize edge evaluation?



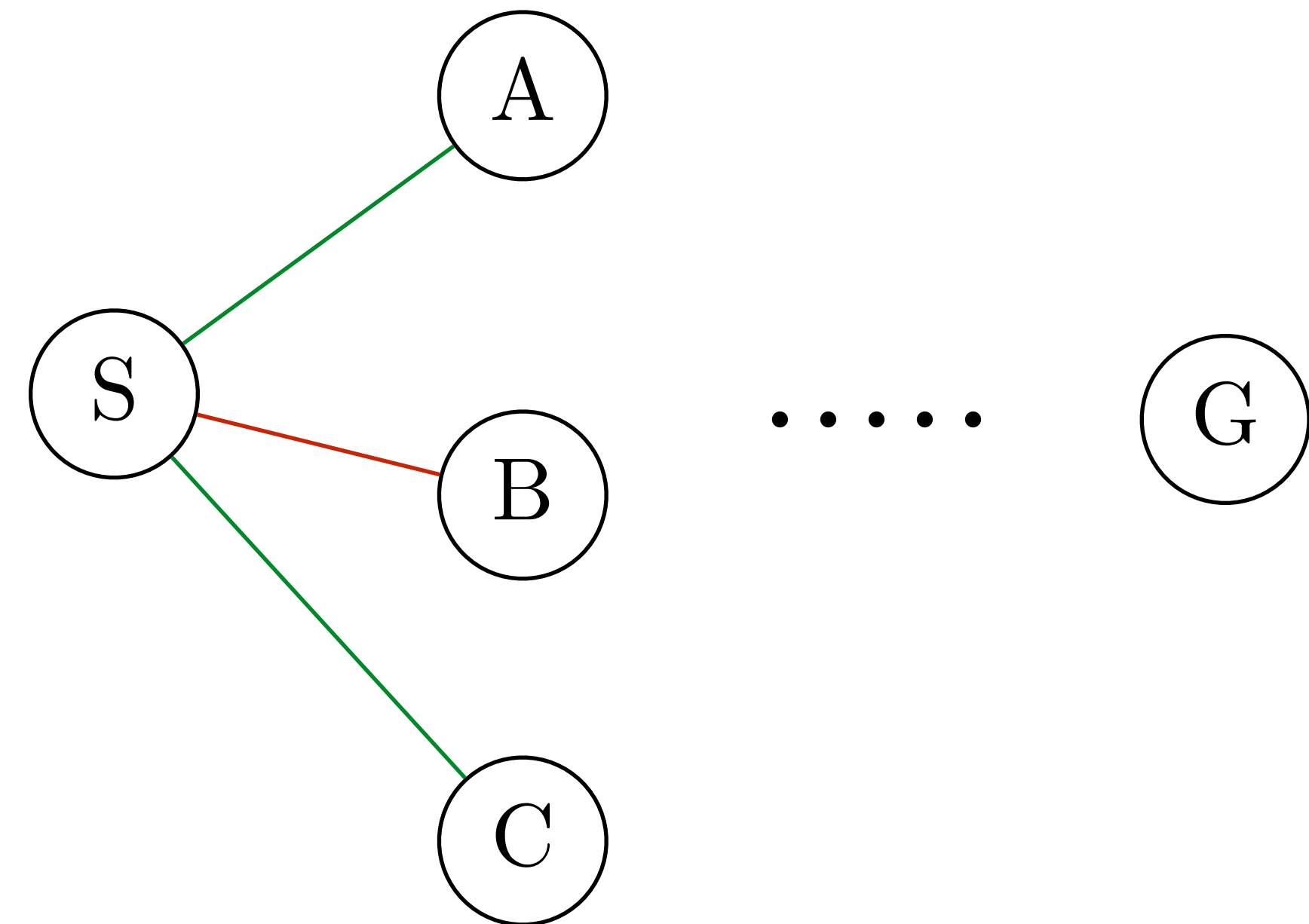
# Let's revisit Best First Search

Element (Node)	Priority Value (f-value)
Node S	$f(S)$



# Let's revisit Best First Search

Element (Node)	Priority Value (f-value)
<del>Node S</del>	<del>f(S)</del>
Node A	f(A)
Node C	f(C)



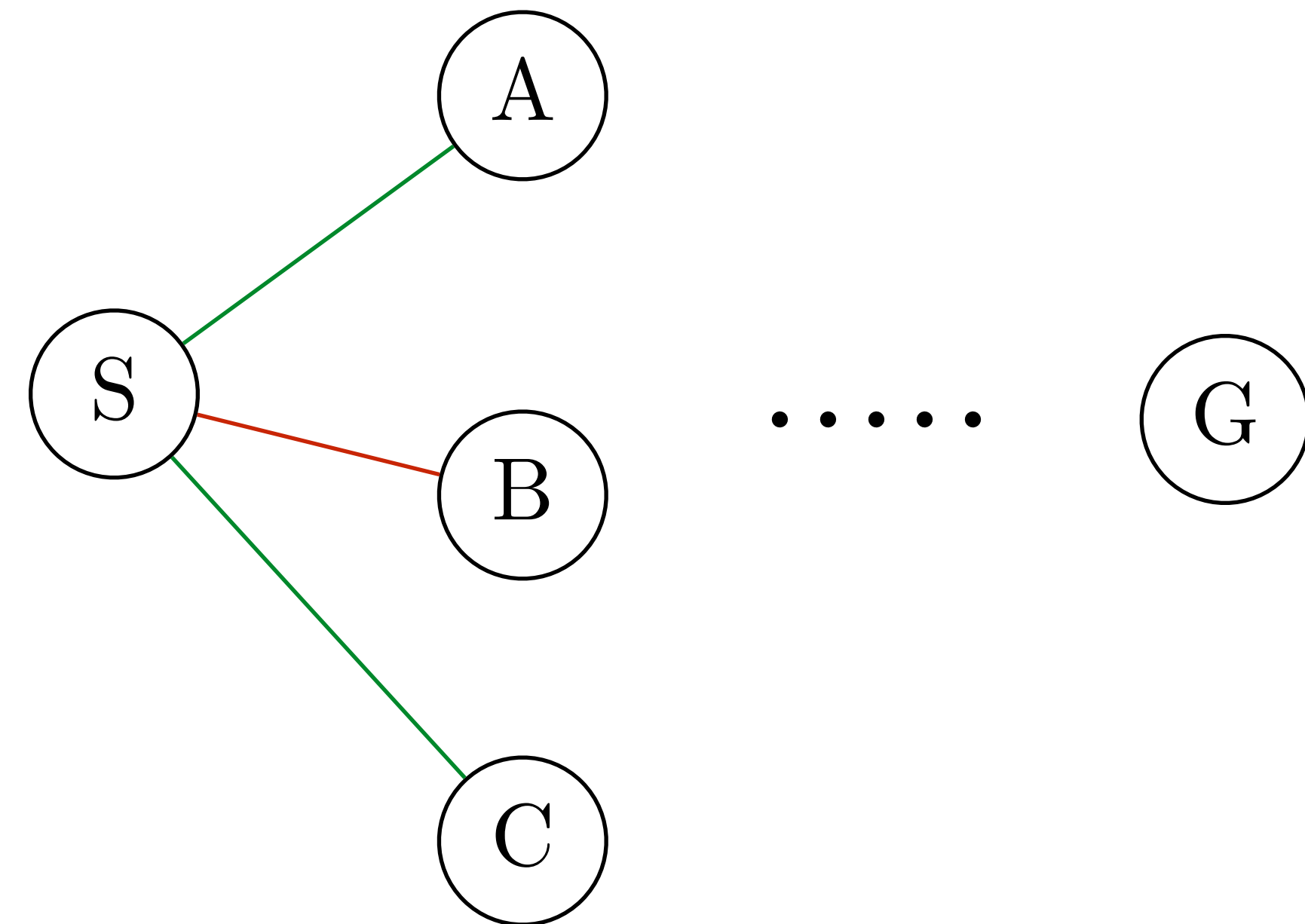
Evaluate edges  $(S,A)$ ,  $(S,B)$ ,  $(S,C)$



# What if we never use C?

## Wasted collision check!

Element (Node)	Priority Value (f-value)
<del>Node S</del>	<del>f(S)</del>
Node A	f(A)
Node C	f(C)



# The Virtue of Laziness

Take the thing that's **expensive**  
(collision checking)  
and  
**procrastinate** as long as possible  
till you have to evaluate it!

What is the laziest that we can  
be?

# LazySP

(Lazy Shortest Path)

Dellin and Srinivasa, 2016

First Provably Edge-Optimal A\*-like Search Algorithm

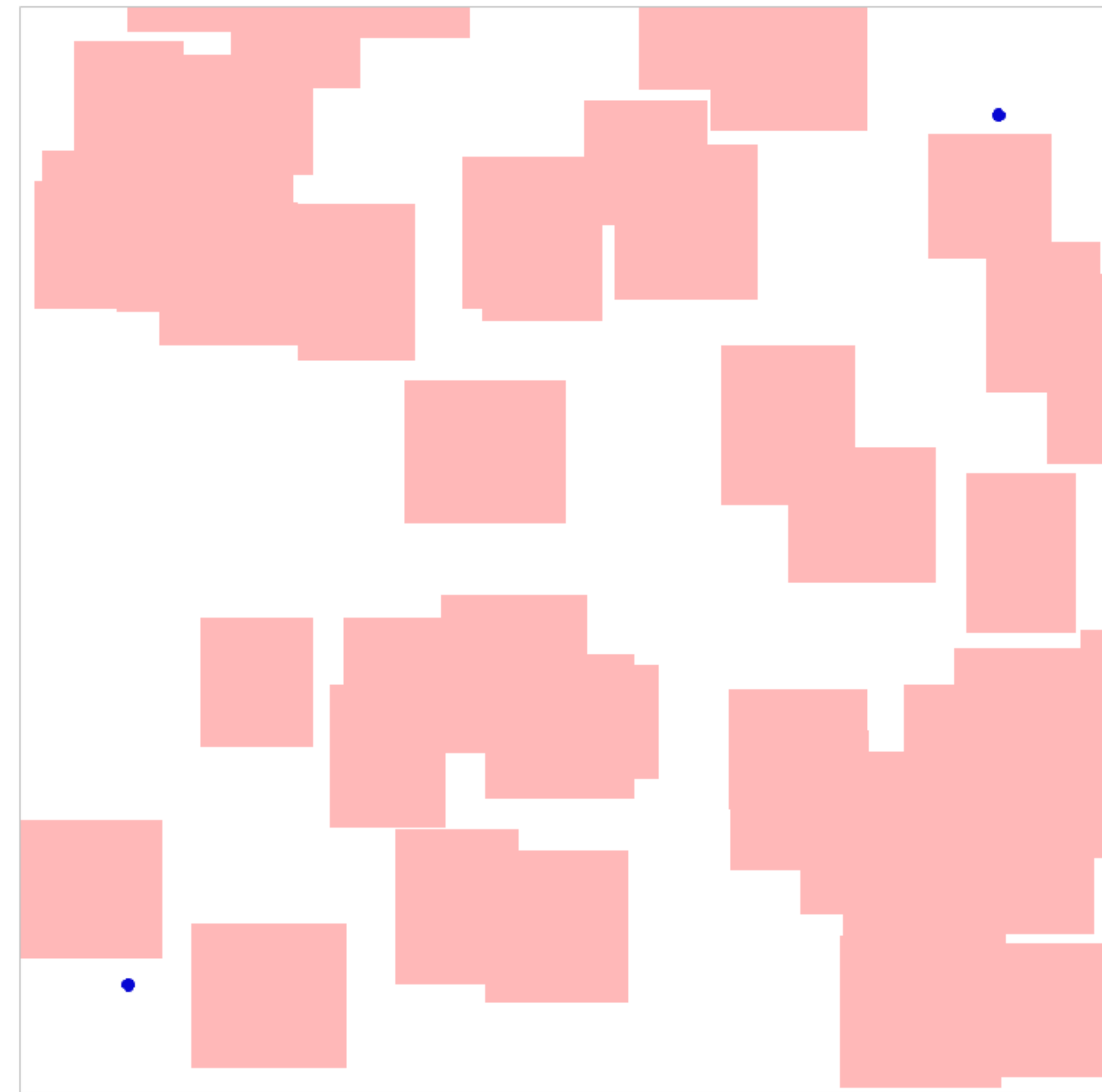
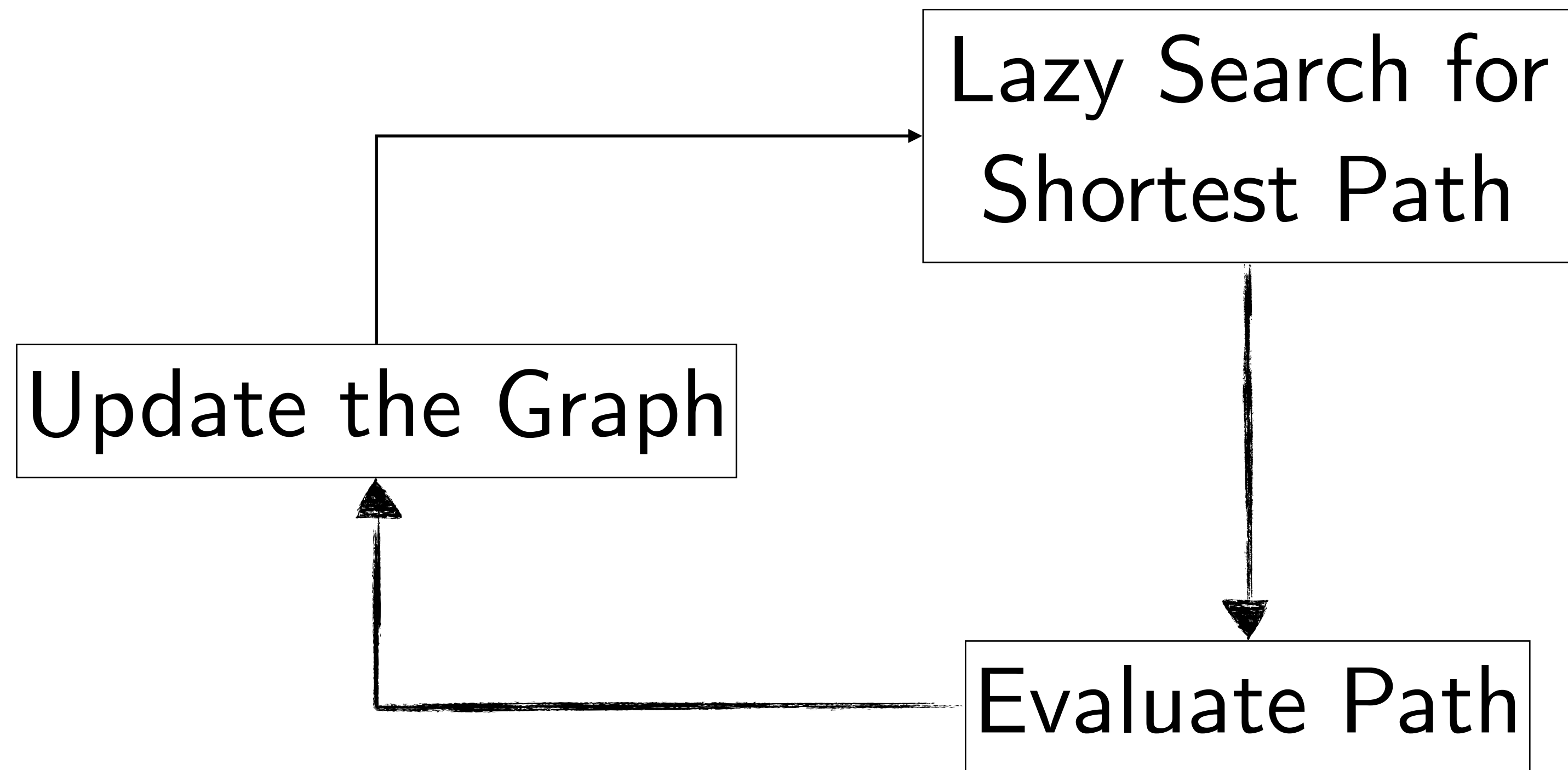
# LazySP

Greedy Best-first Search over **Paths**

To find the shortest path,  
eliminate all shorter paths!

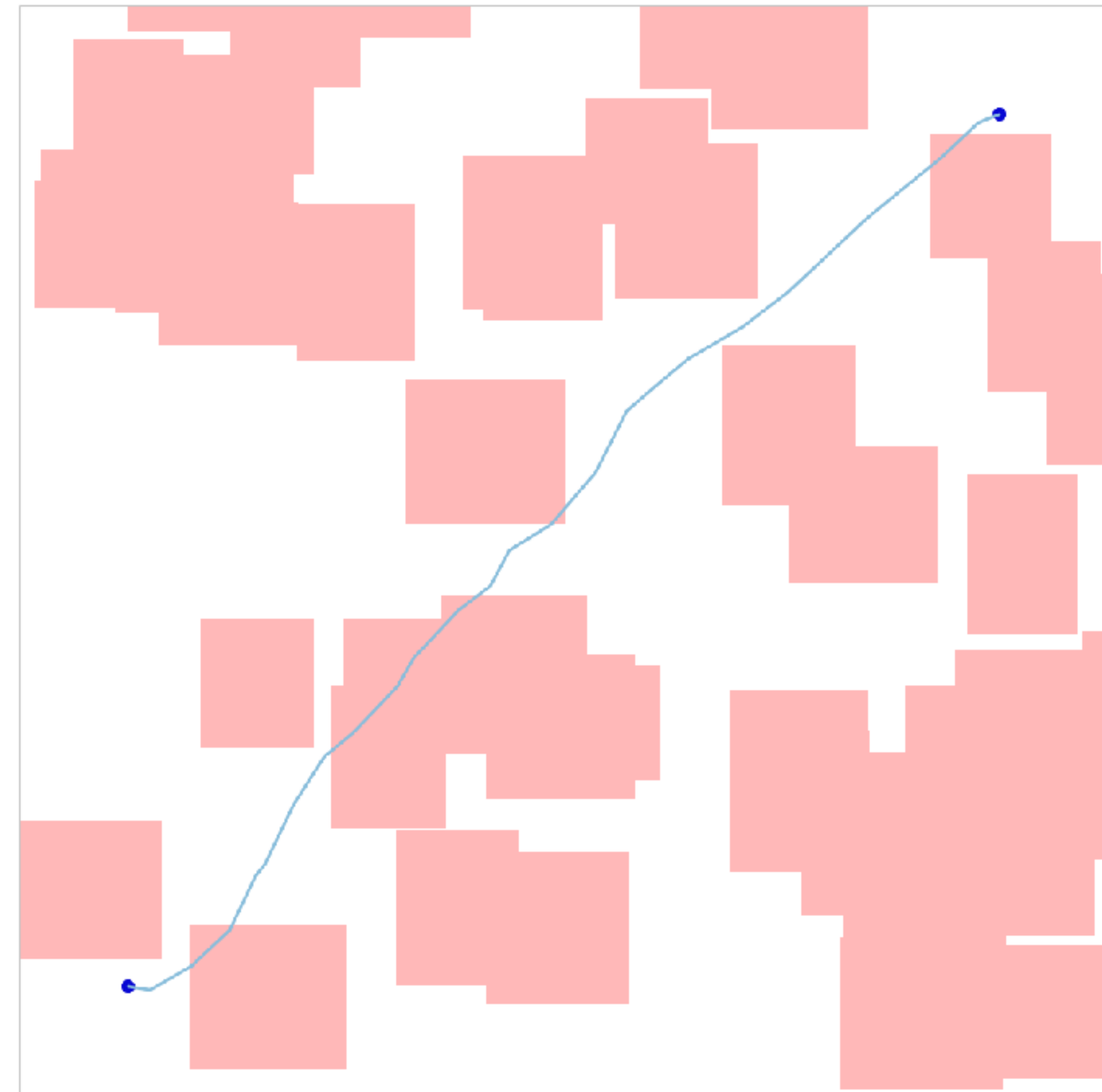
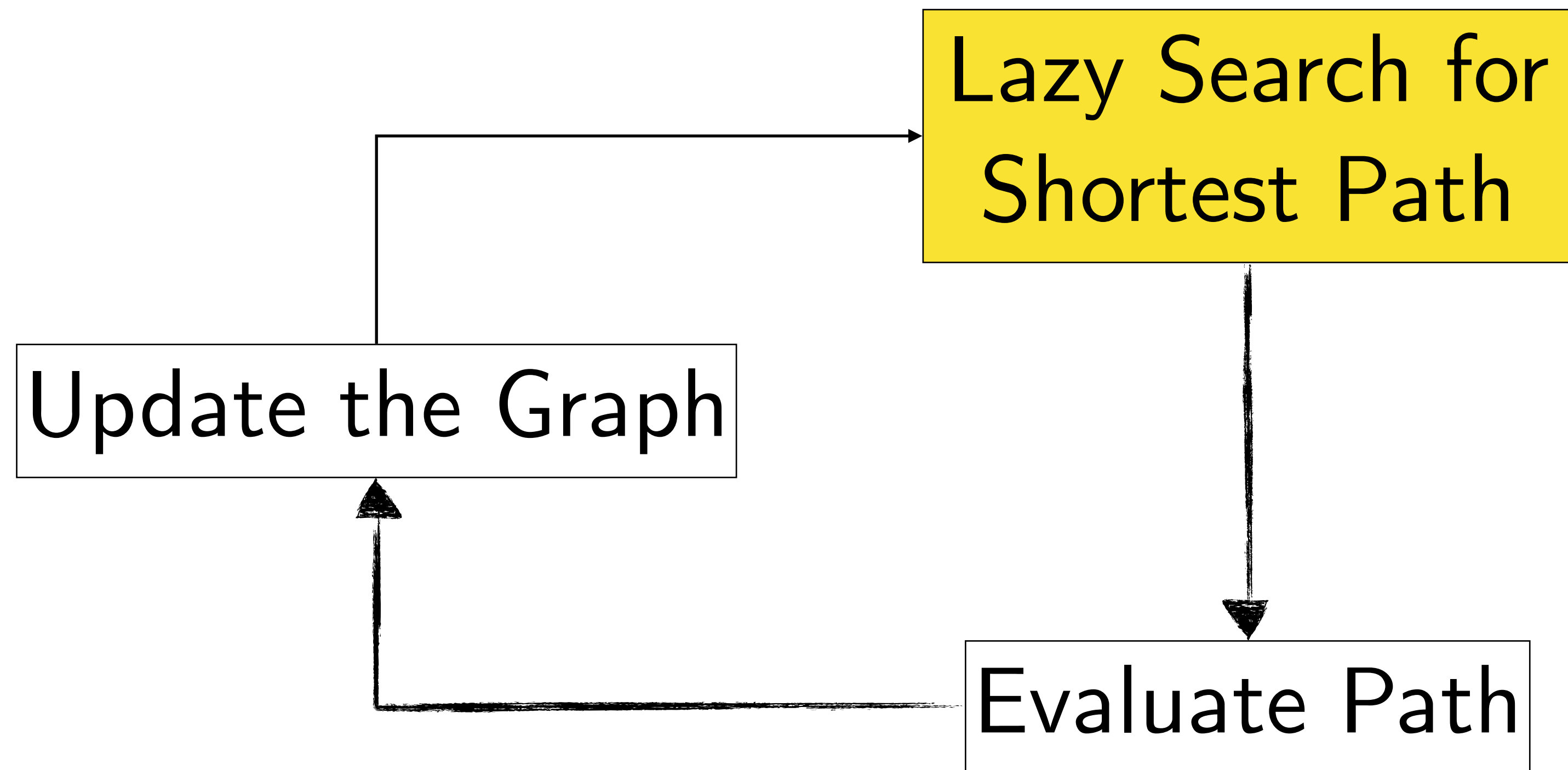
# LazySP

Optimism Under Uncertainty



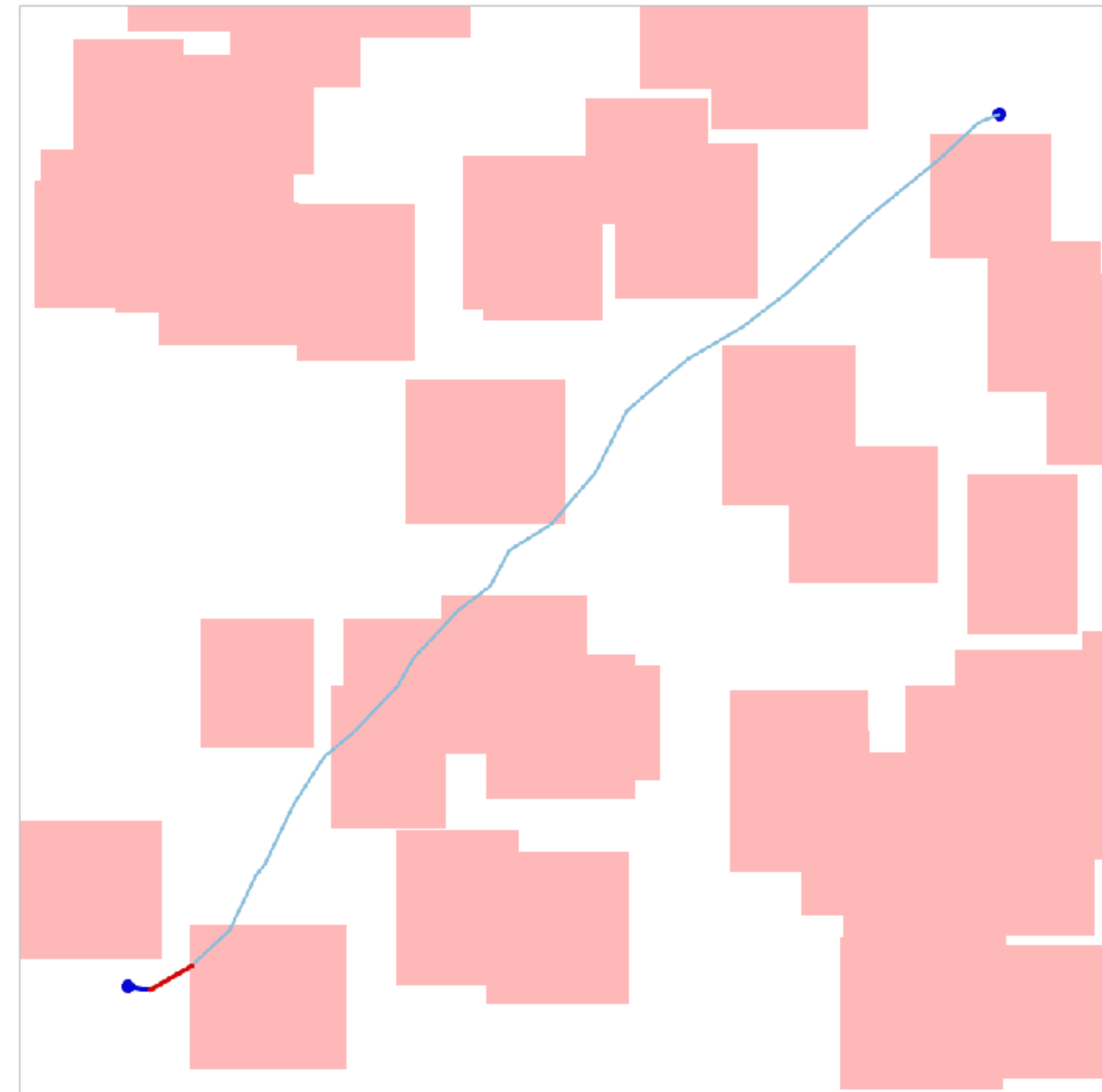
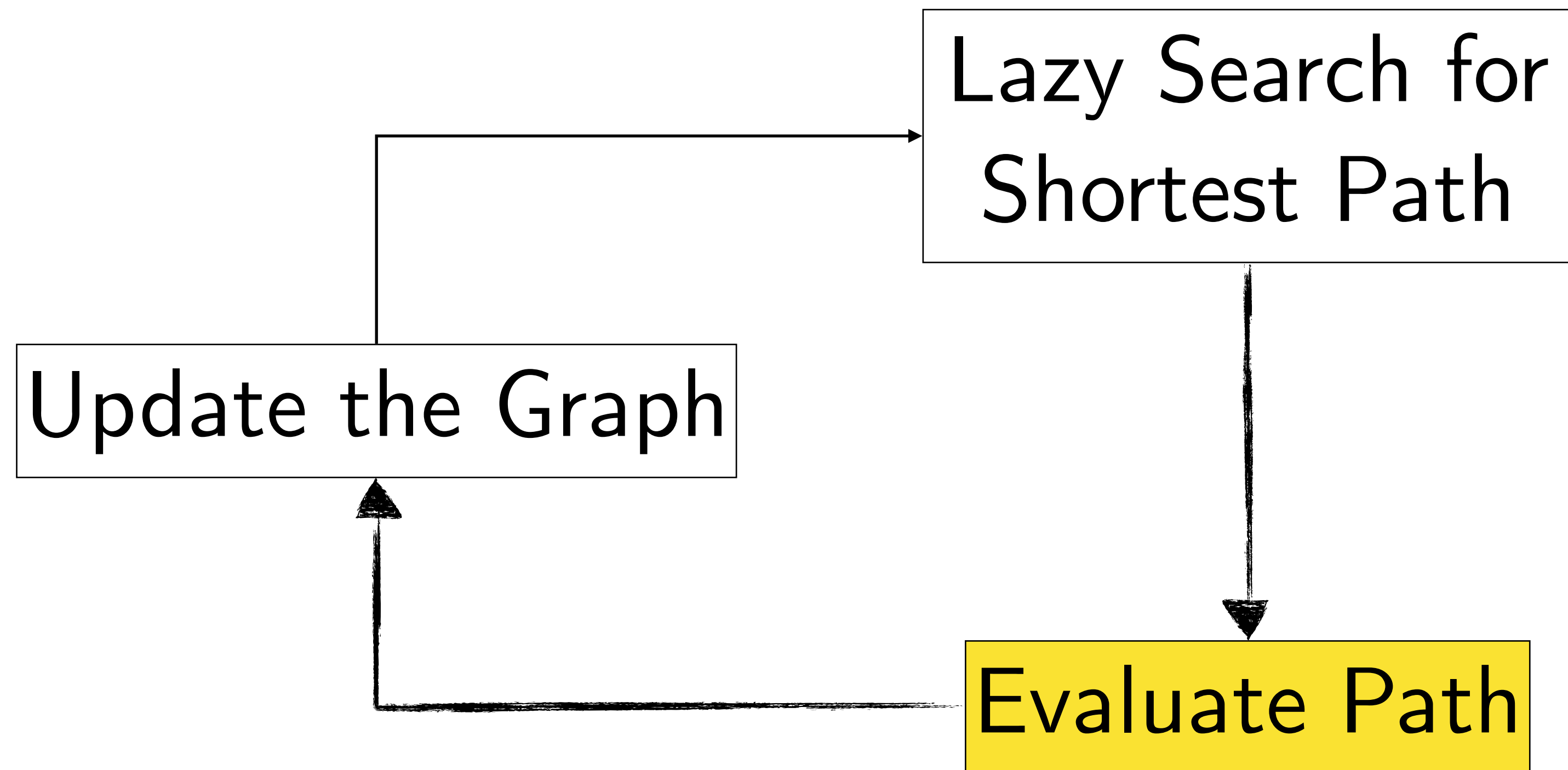
# LazySP

Optimism Under Uncertainty



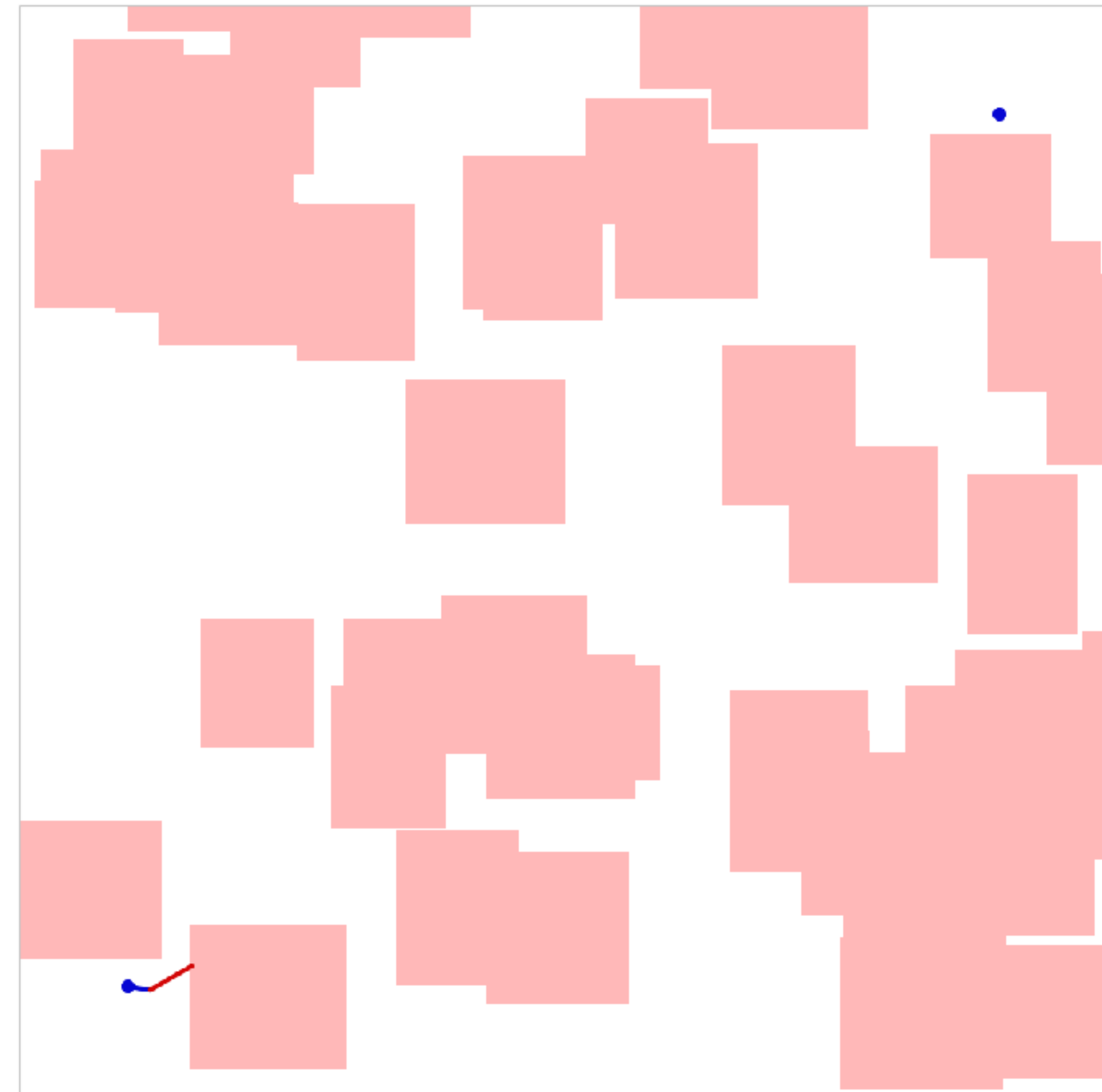
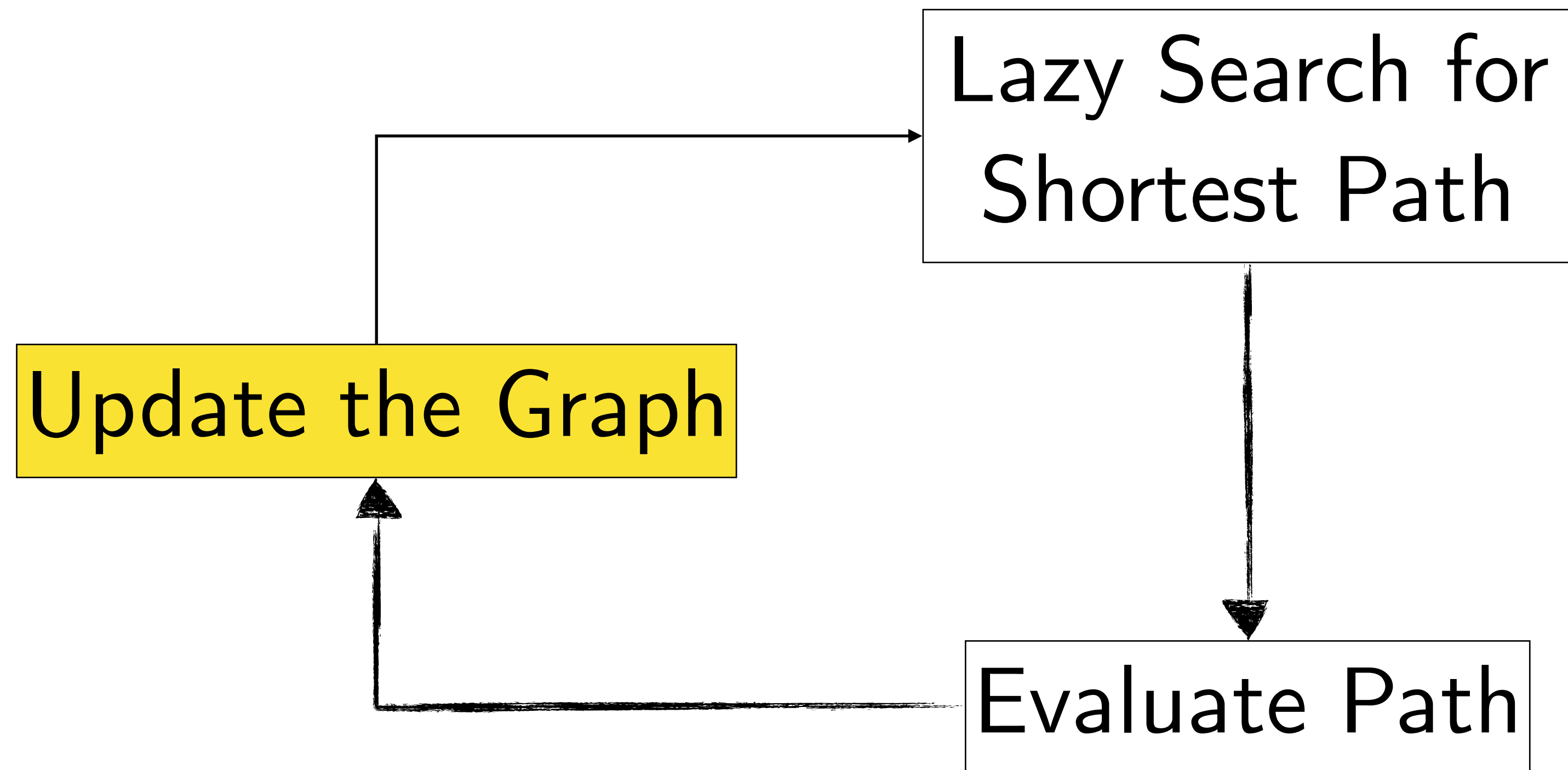
# LazySP

Optimism Under Uncertainty



# LazySP

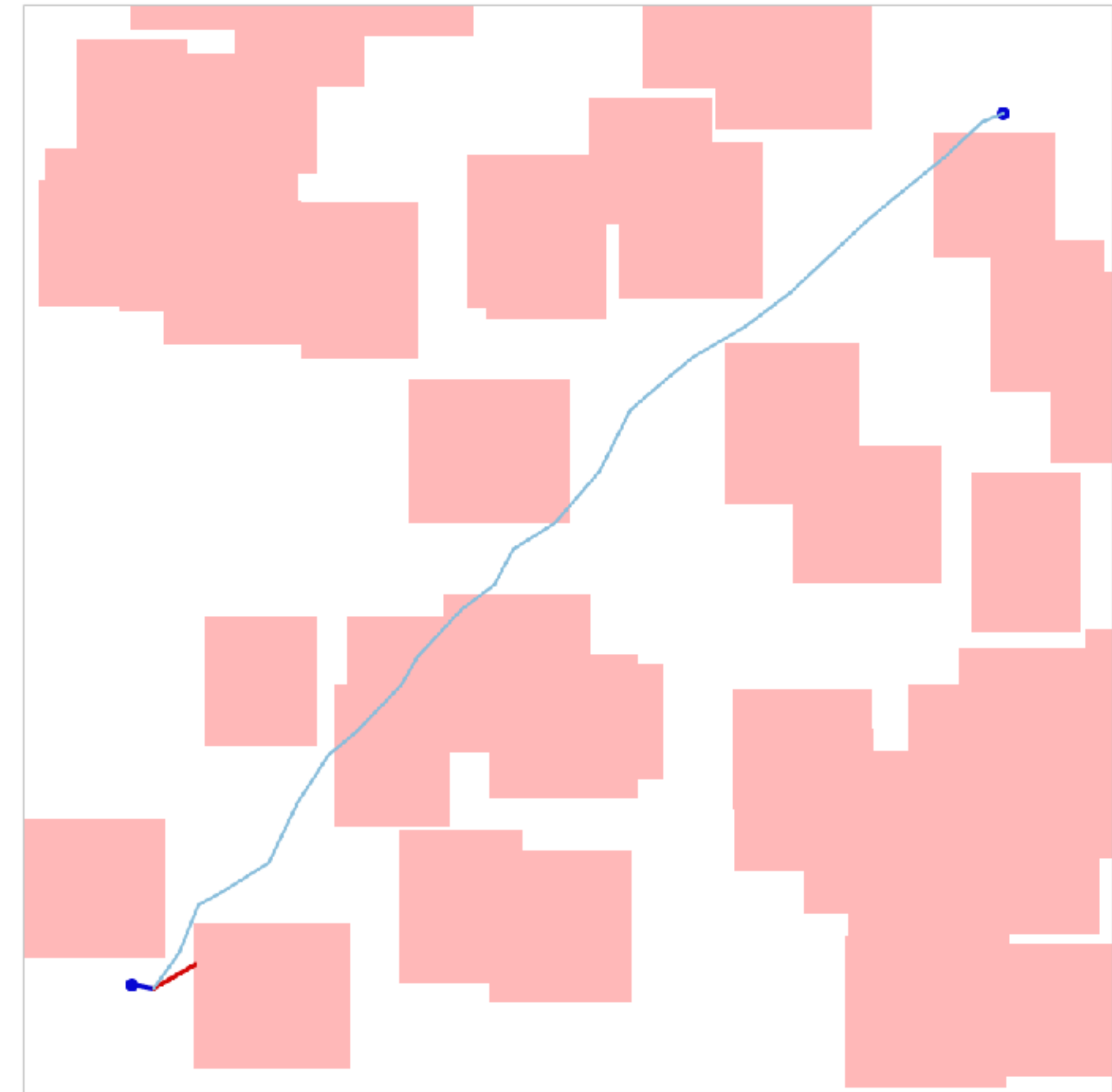
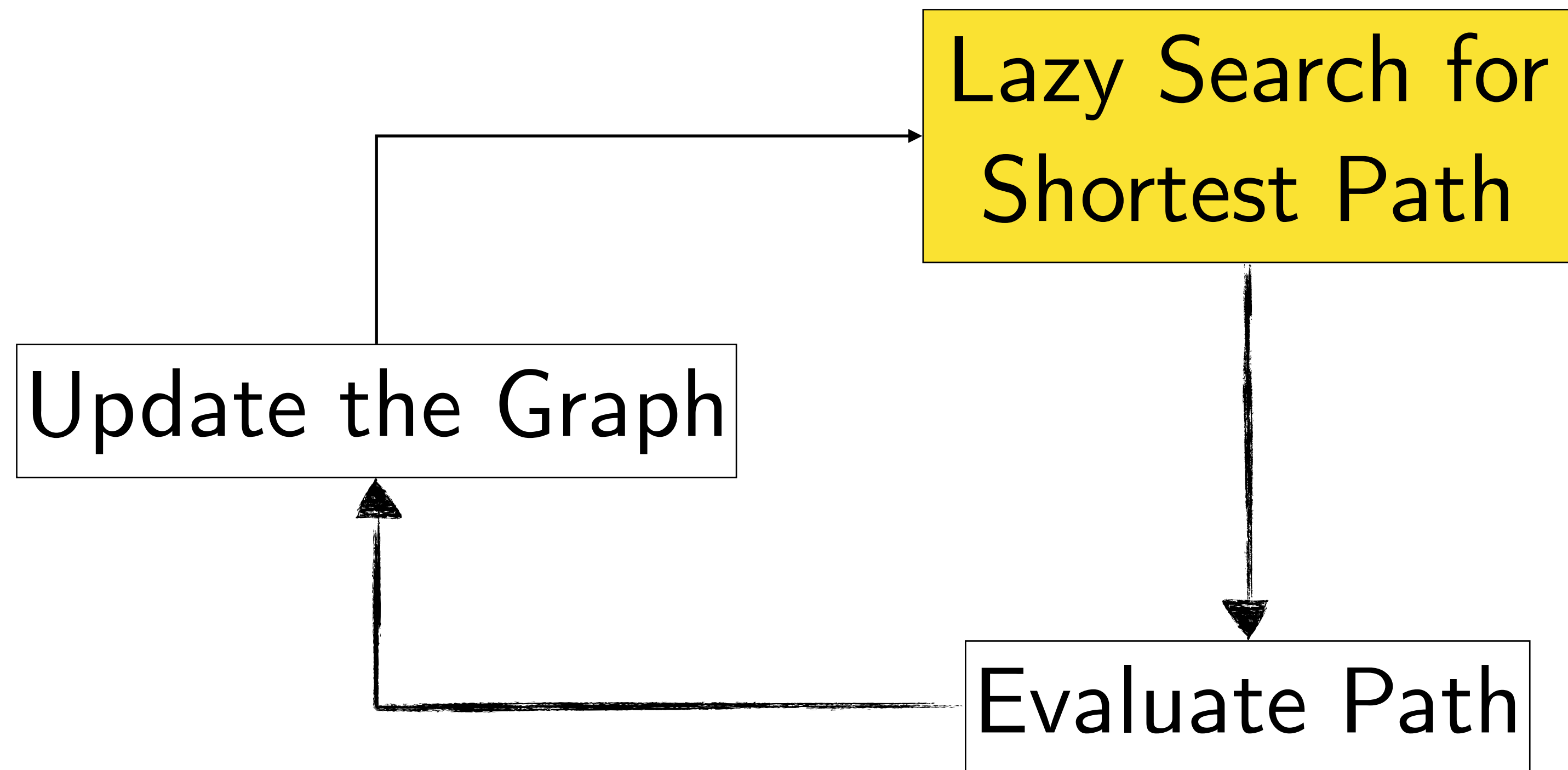
Optimism Under Uncertainty





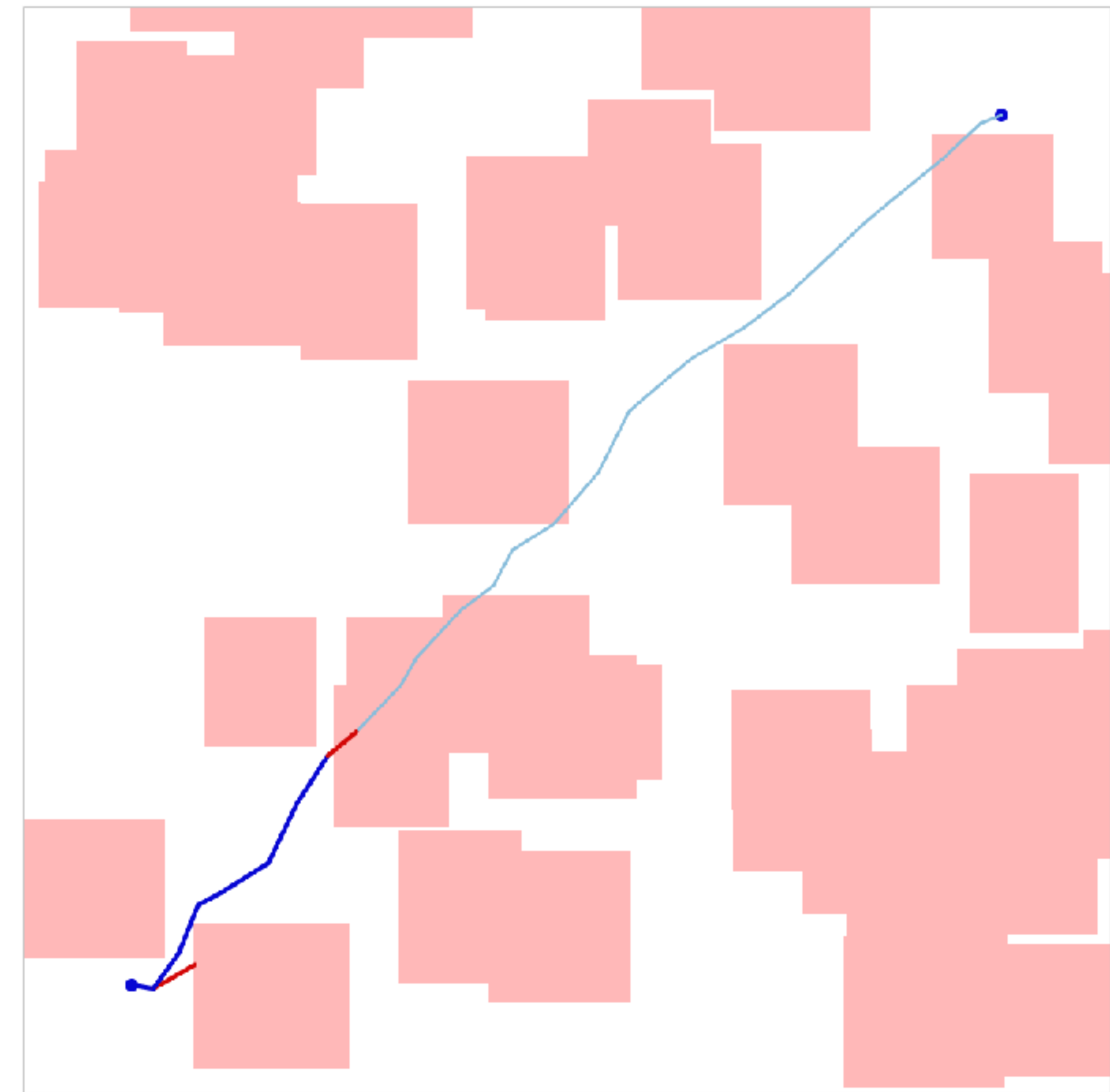
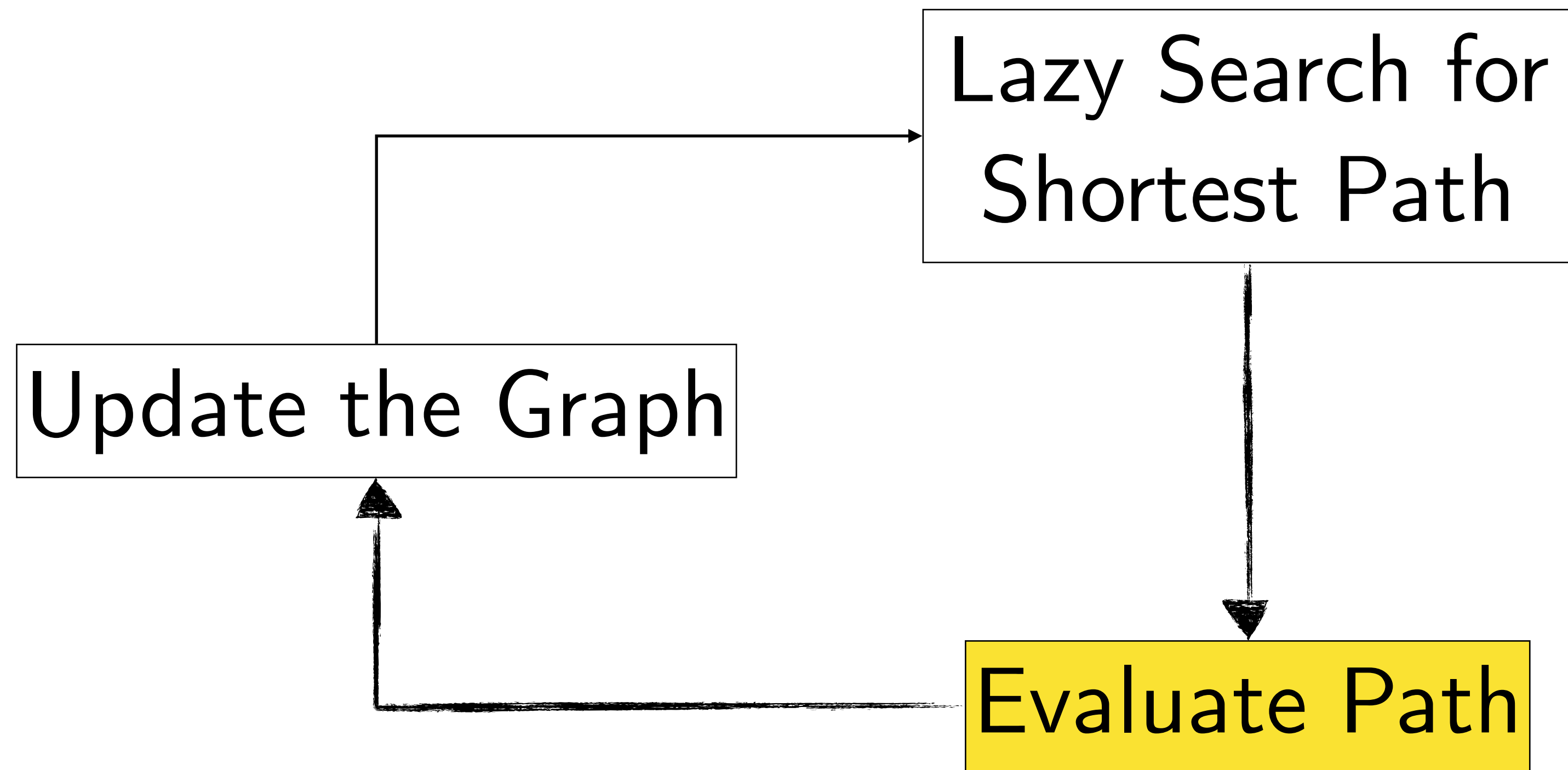
# LazySP

Optimism Under Uncertainty



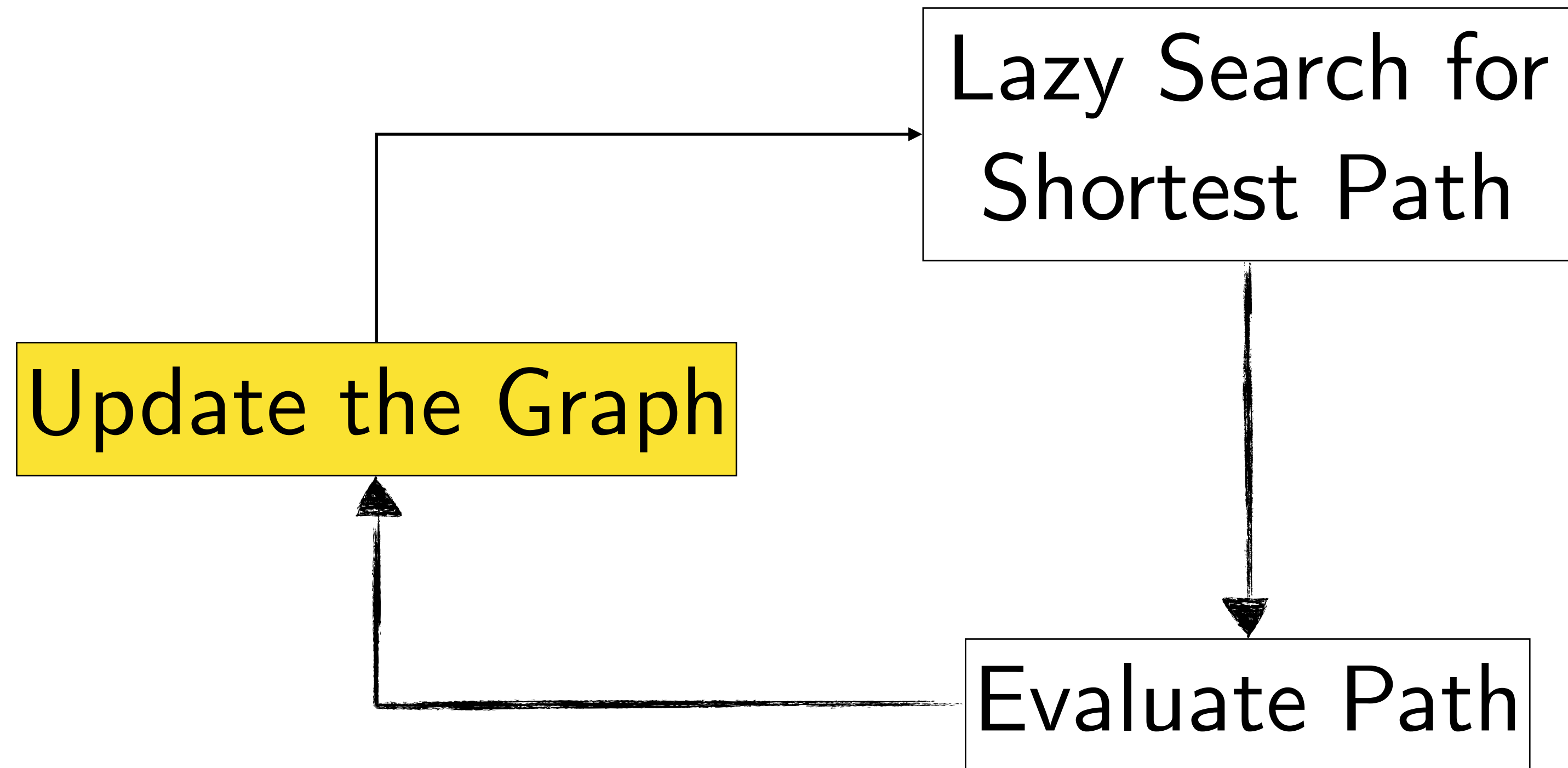
# LazySP

Optimism Under Uncertainty



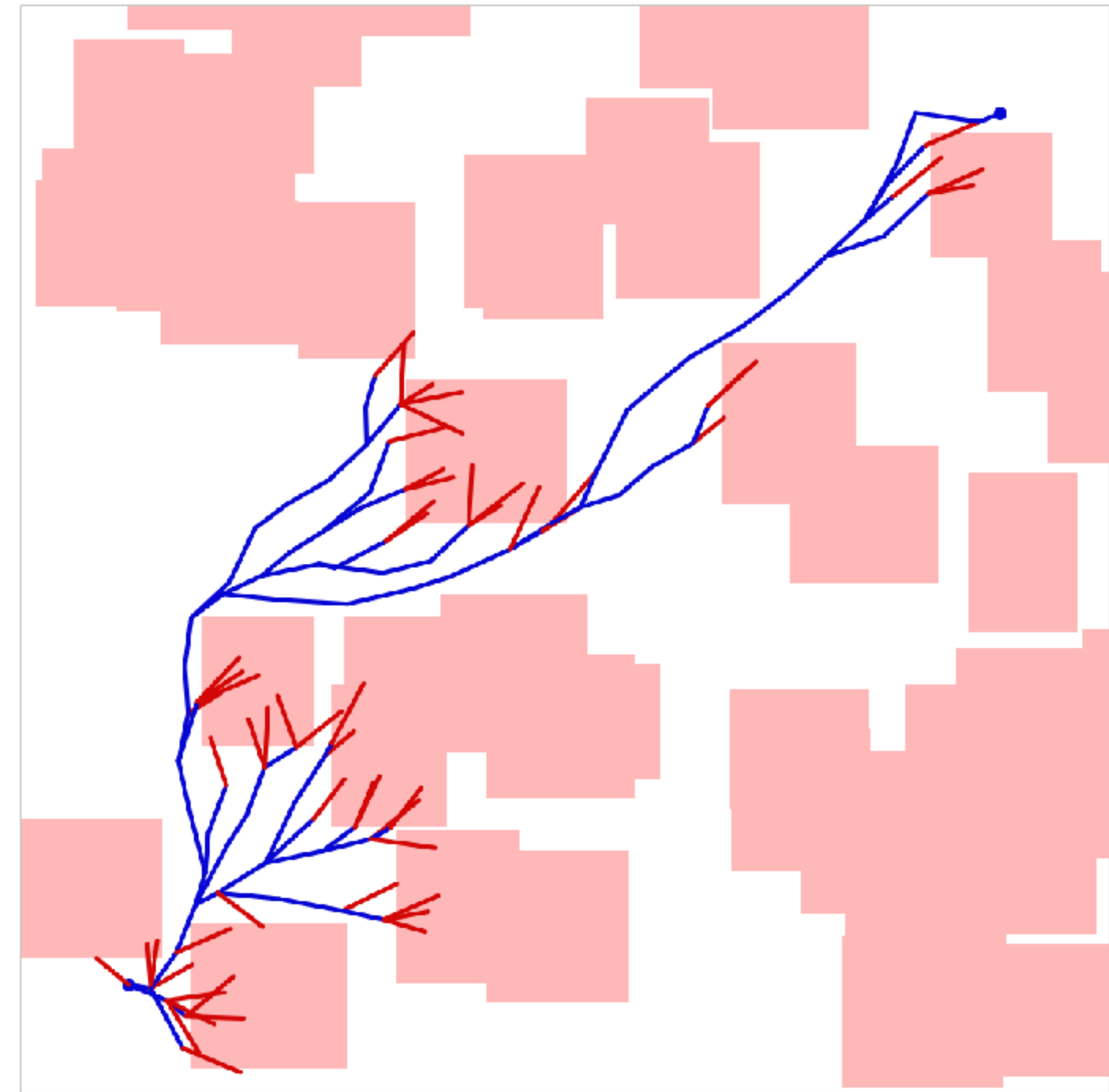
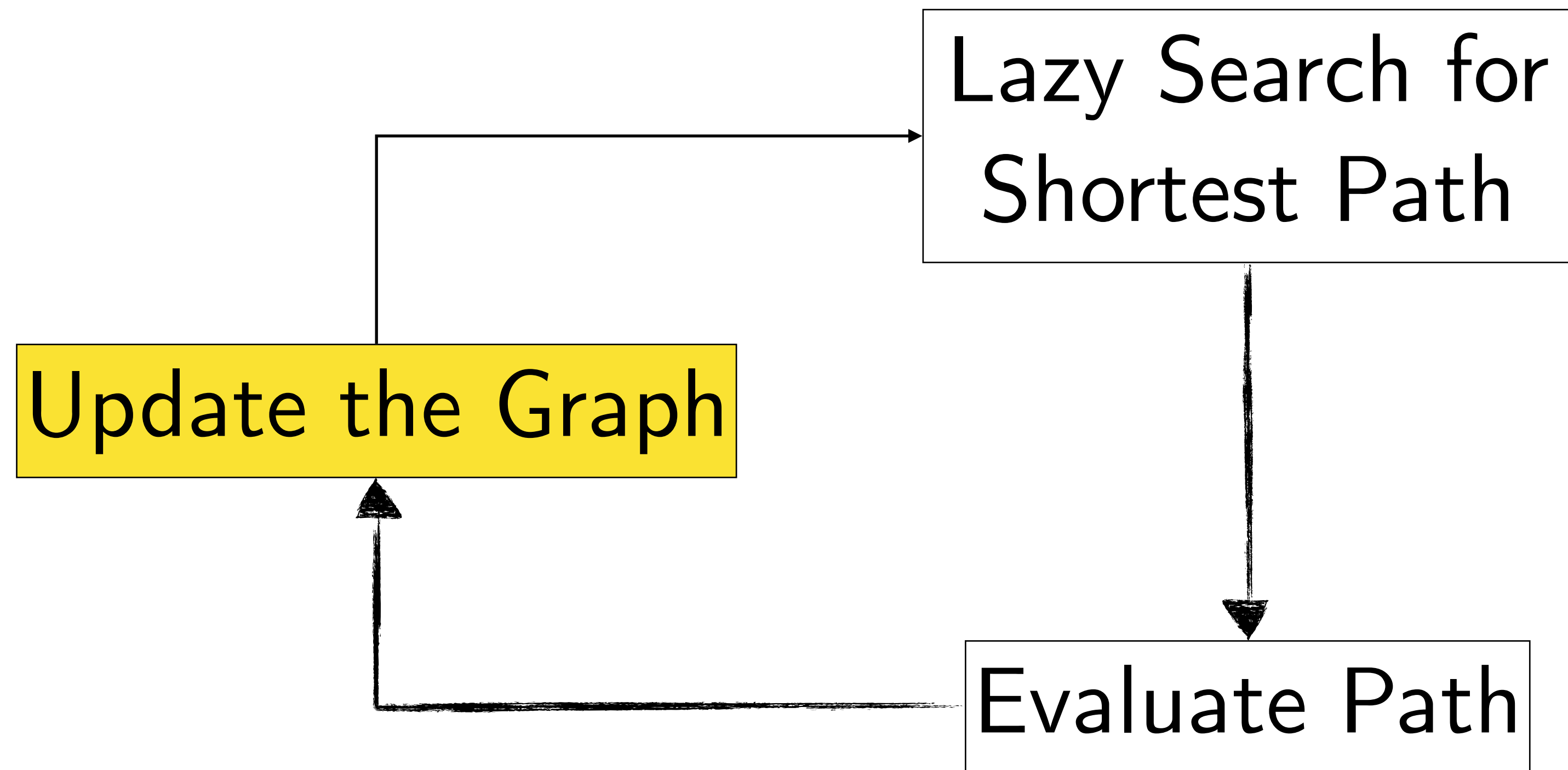
# LazySP

Optimism Under Uncertainty



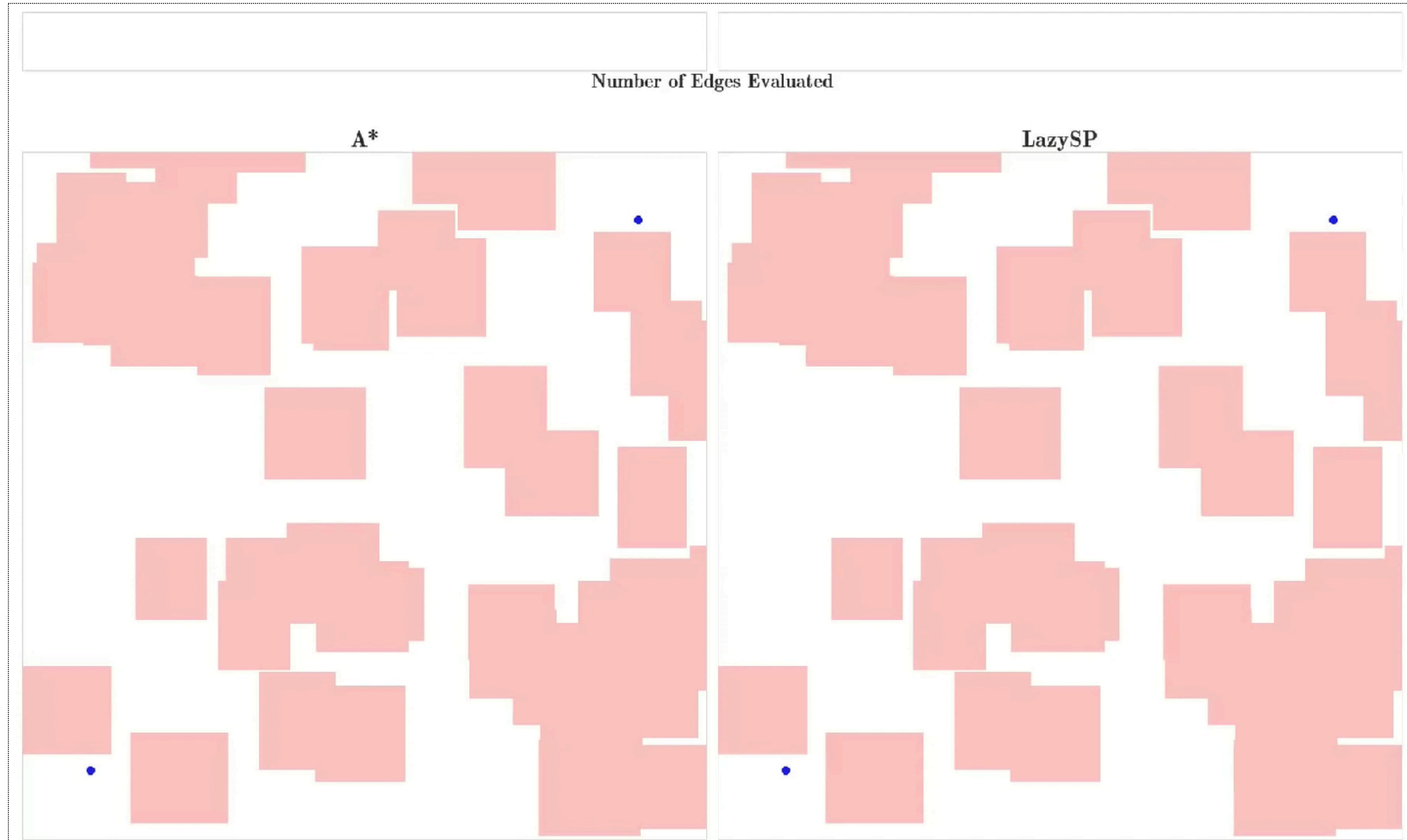
# LazySP

Optimism Under Uncertainty

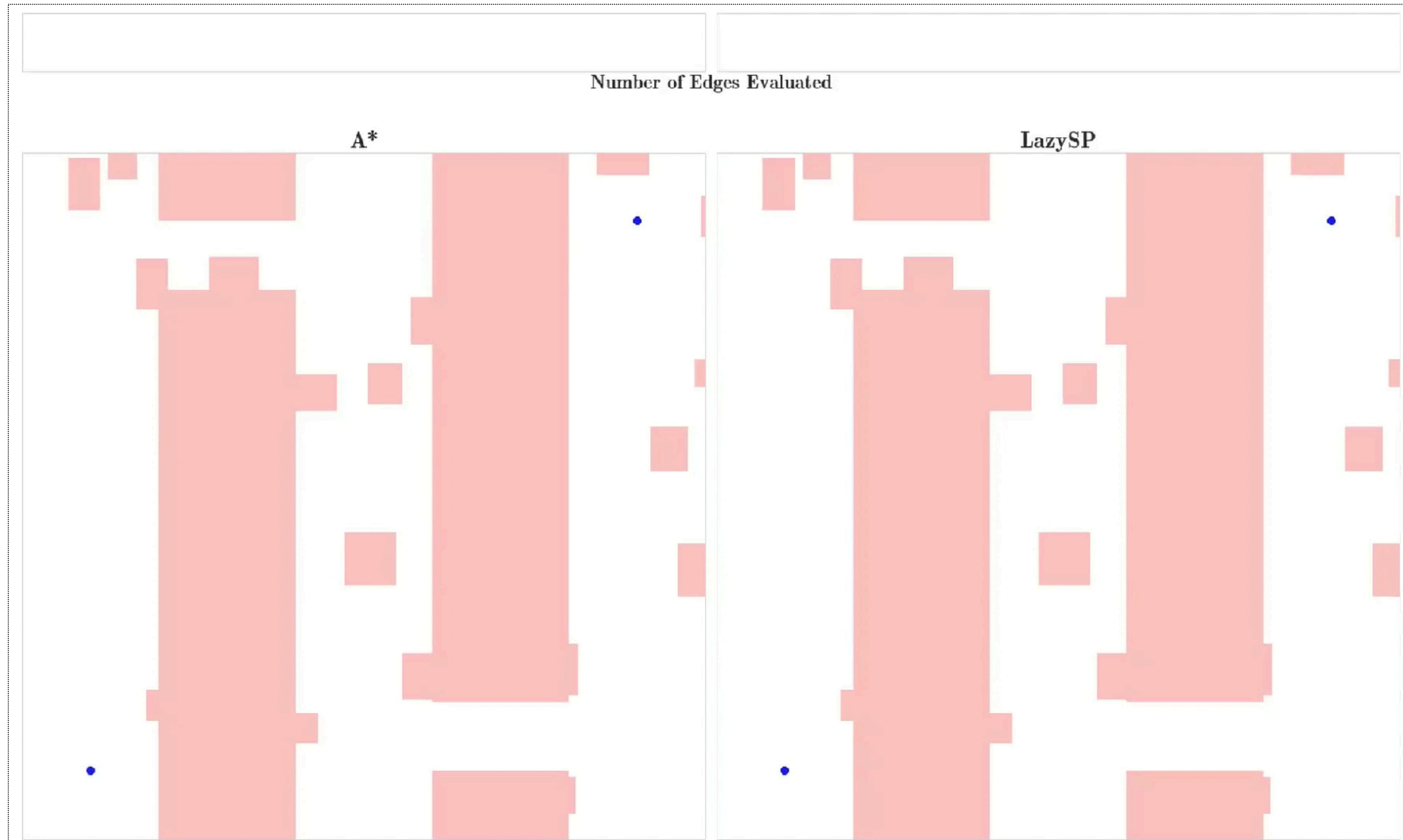




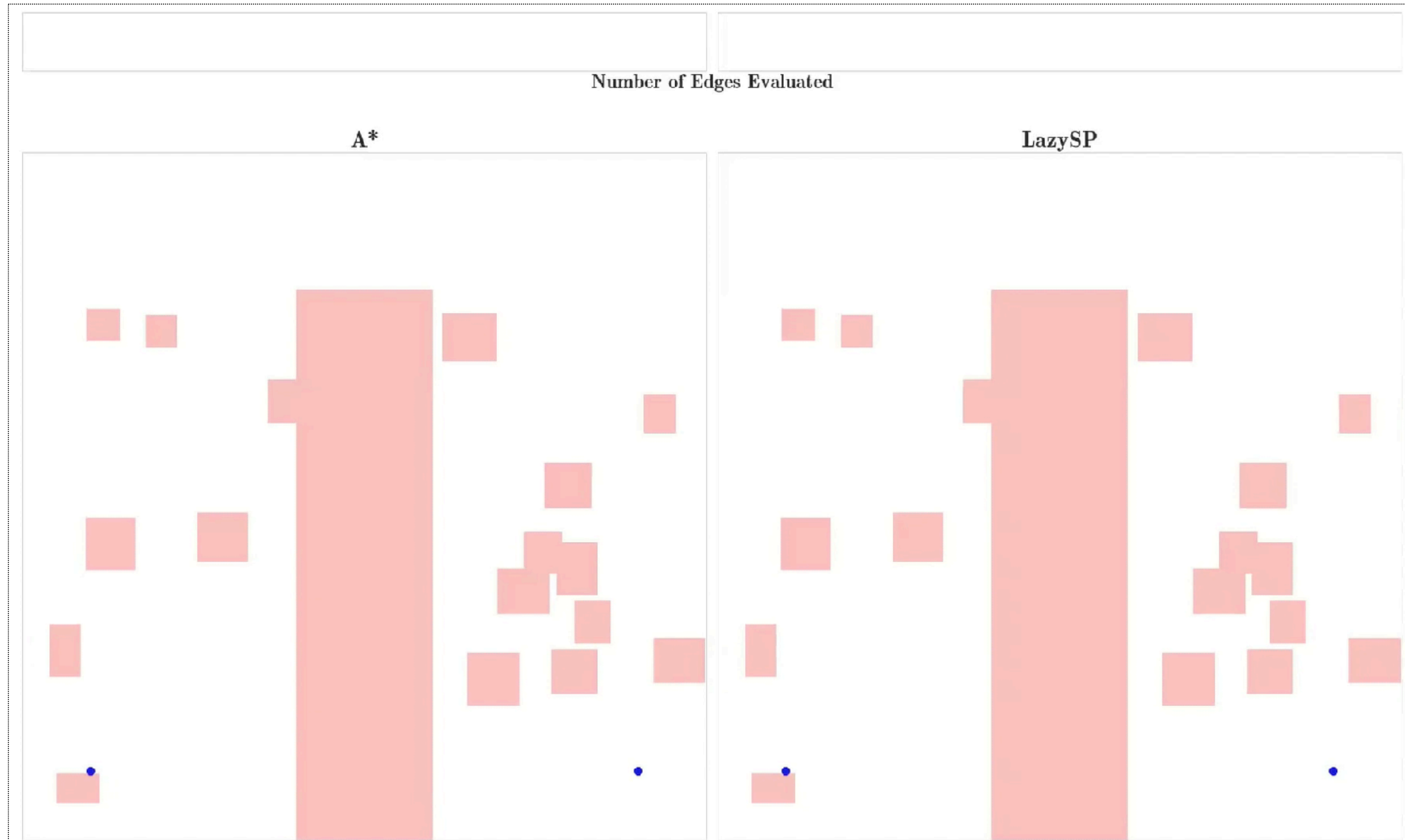
# A\* vs LazySP



# A\* vs LazySP

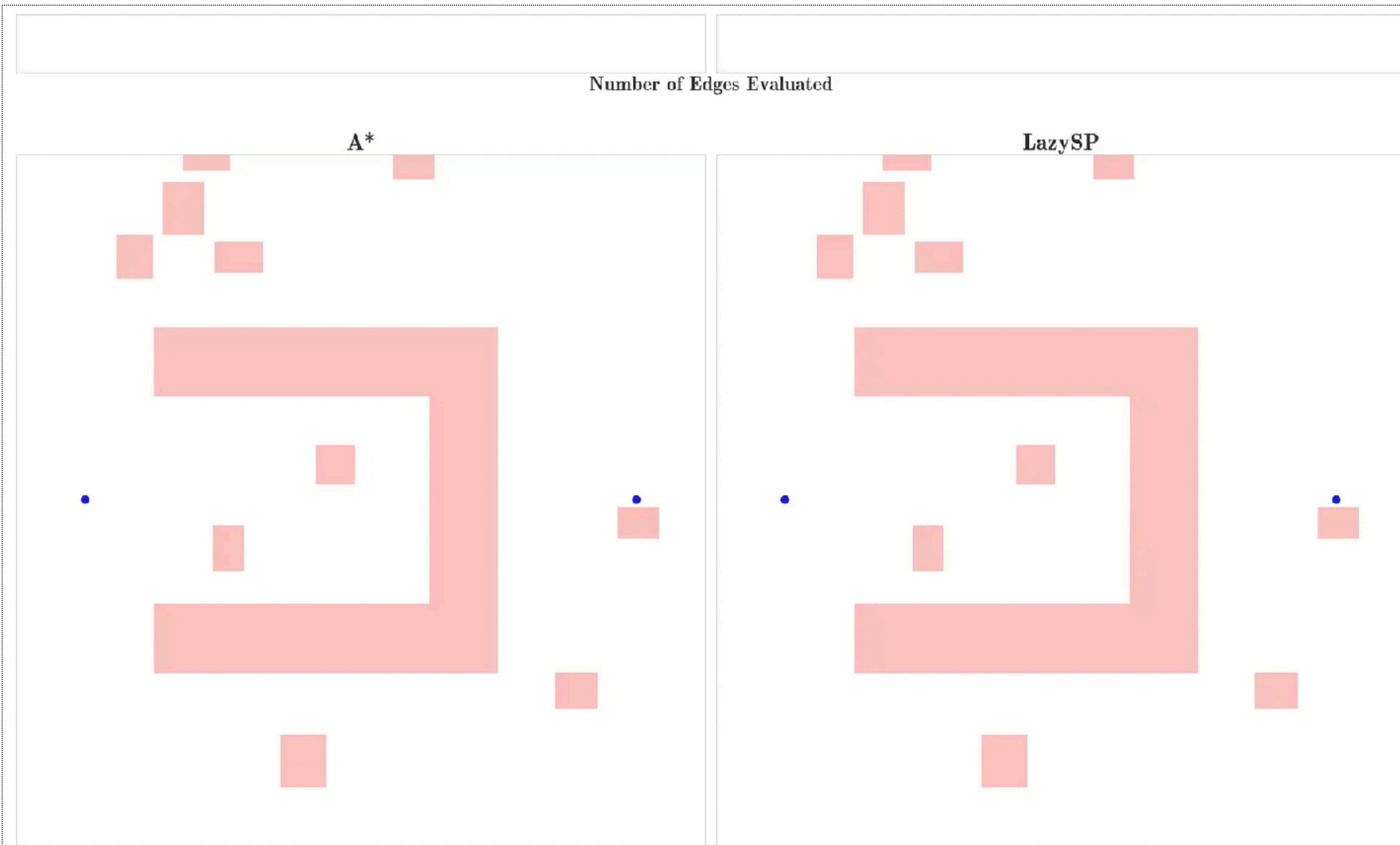


# A\* vs LazySP

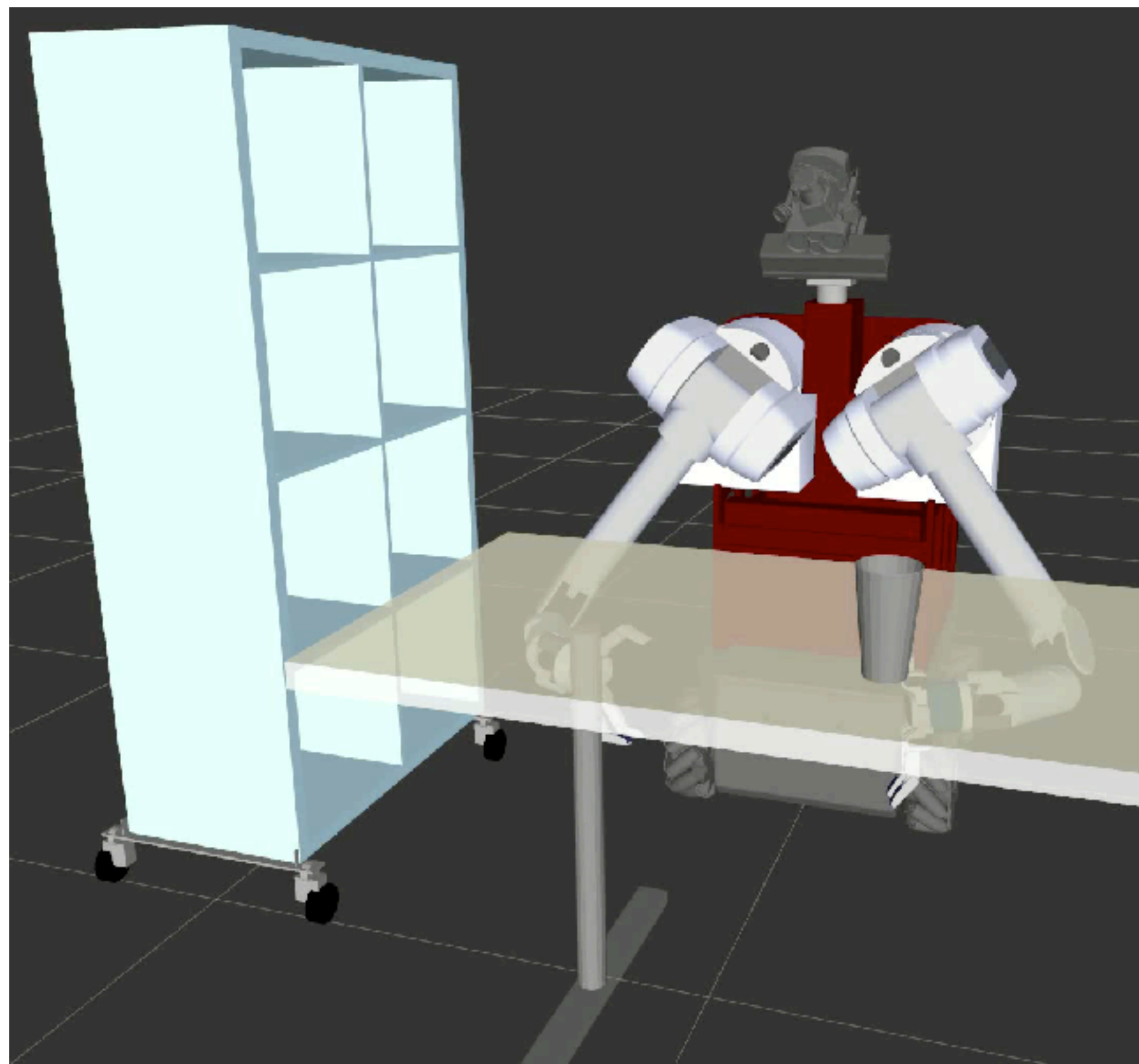




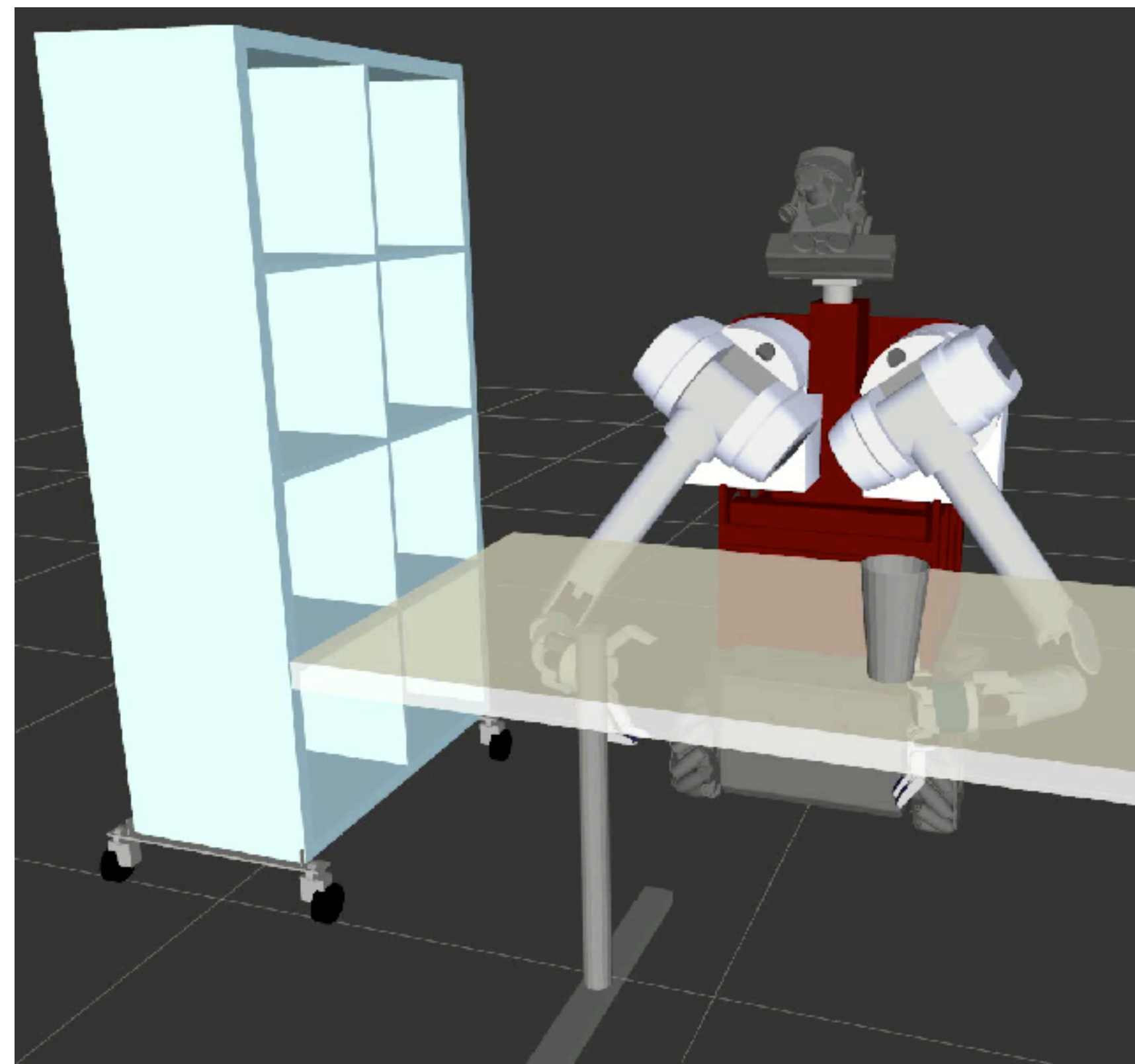
# A\* vs LazySP



# A\* vs LazySP



A\* (191 edges)



LAZYSP (38 edges)

# What can we prove about Lazy SP?

LazySP finds the optimal path

LazySP evaluates the minimal number of edges

*(For a given edge selector policy)*

How can learning help  
make LazySP even lazier?  
(i.e. faster)

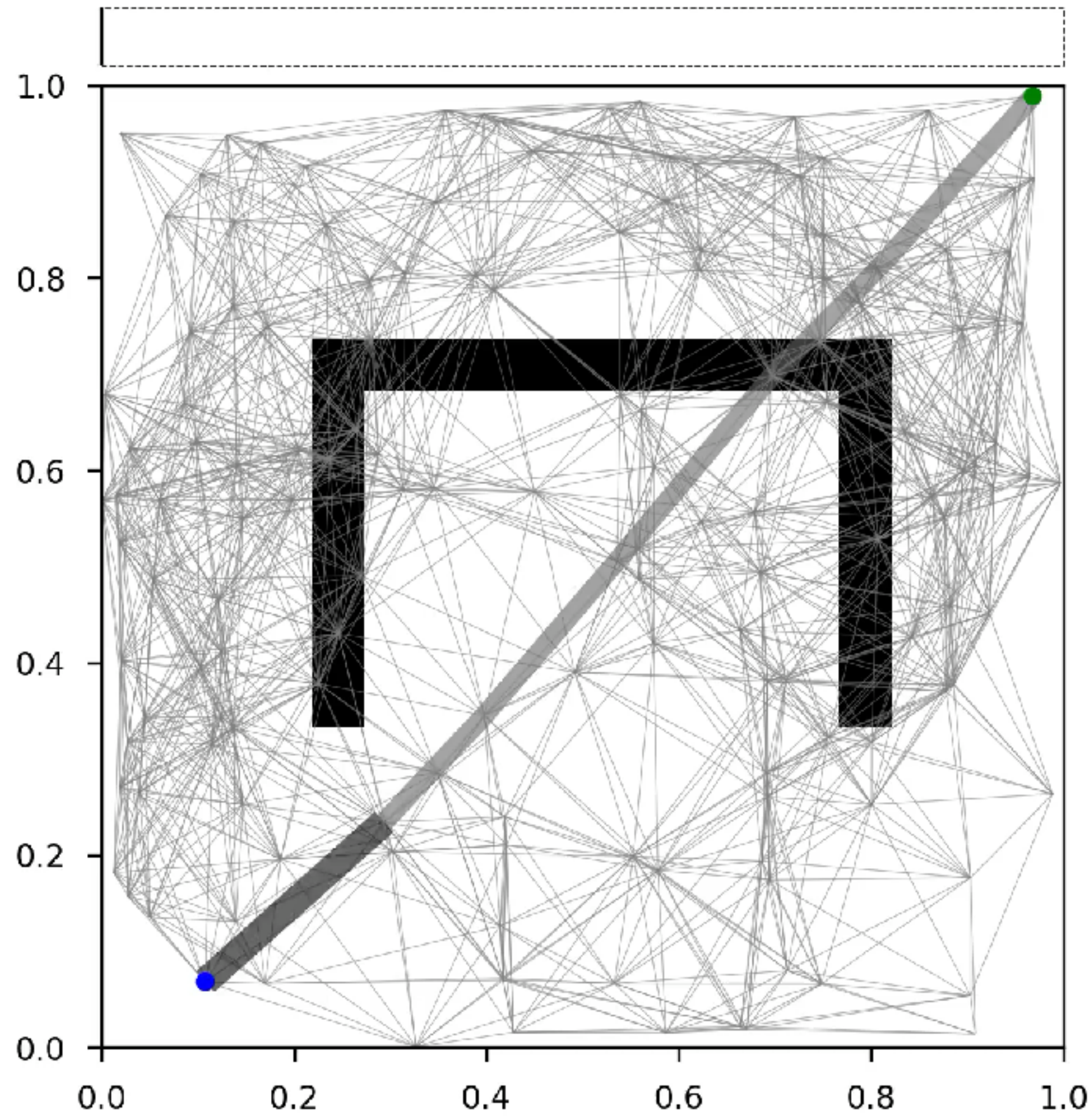
## Leveraging Experience in Lazy Search

Mohak Bhardwaj <sup>\*</sup>, Sanjiban Choudhury <sup>†</sup>, Byron Boots <sup>\*</sup> and Siddhartha Srinivasa <sup>†</sup>  
<sup>\*</sup>Georgia Institute of Technology <sup>†</sup>University of Washington

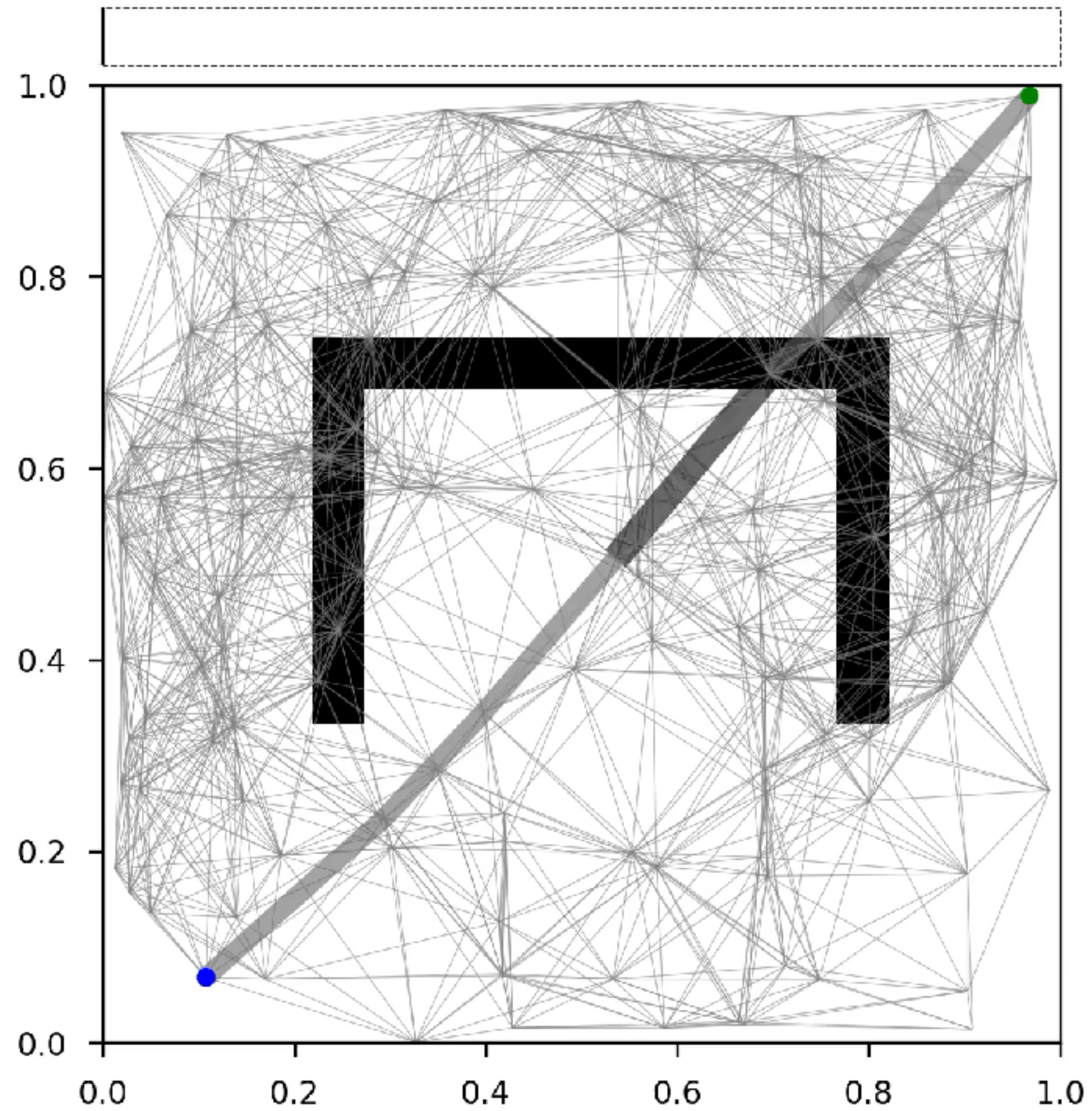




# Learn which edges to evaluate (STROLL)



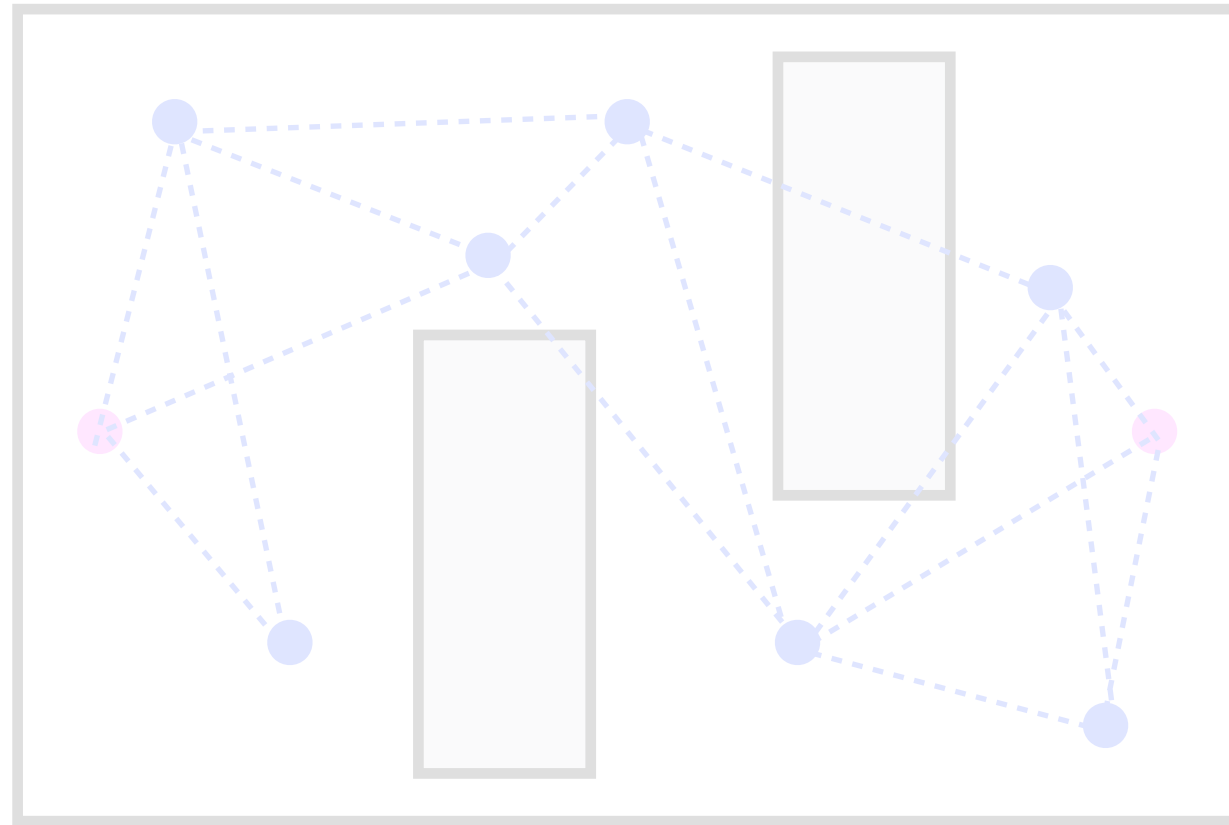
LazySP



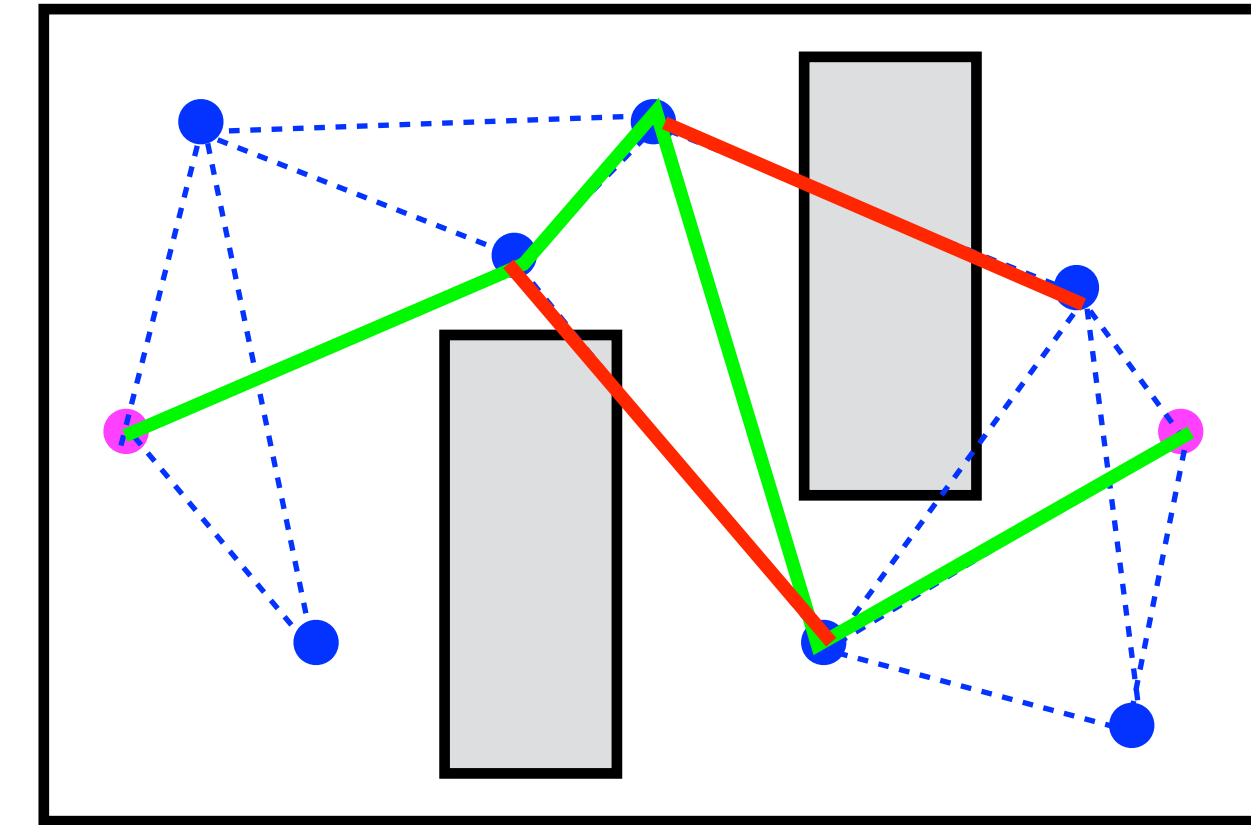
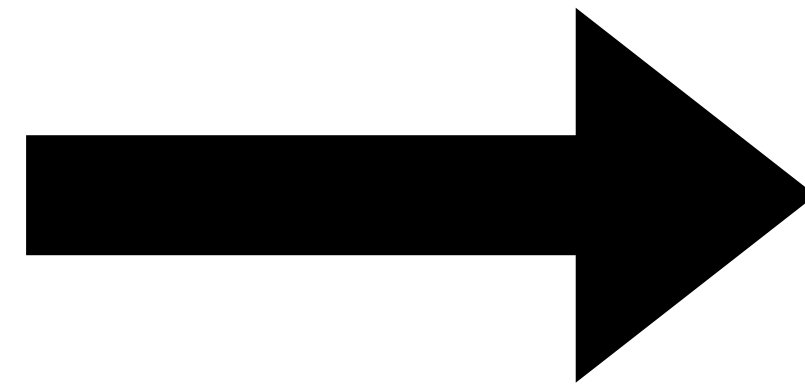
STROLL



# General framework for motion planning



Create a graph

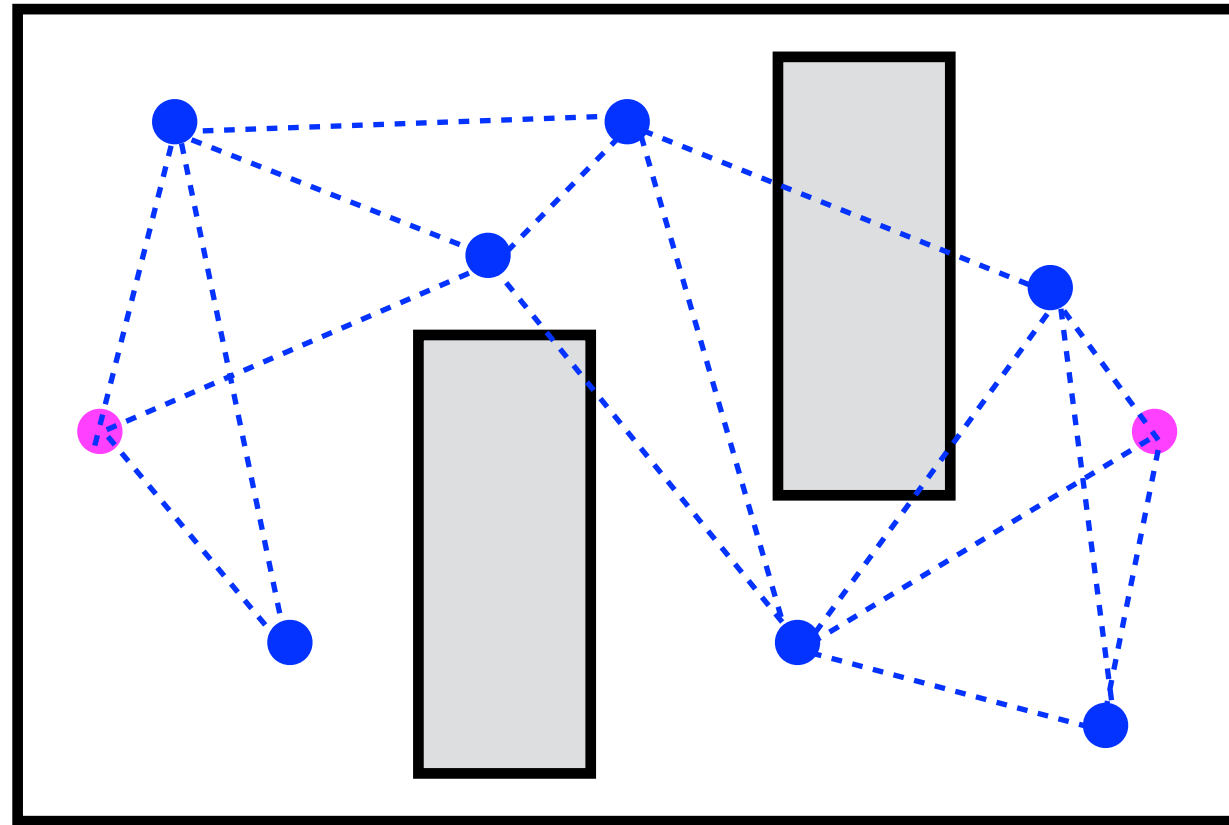


Search the graph

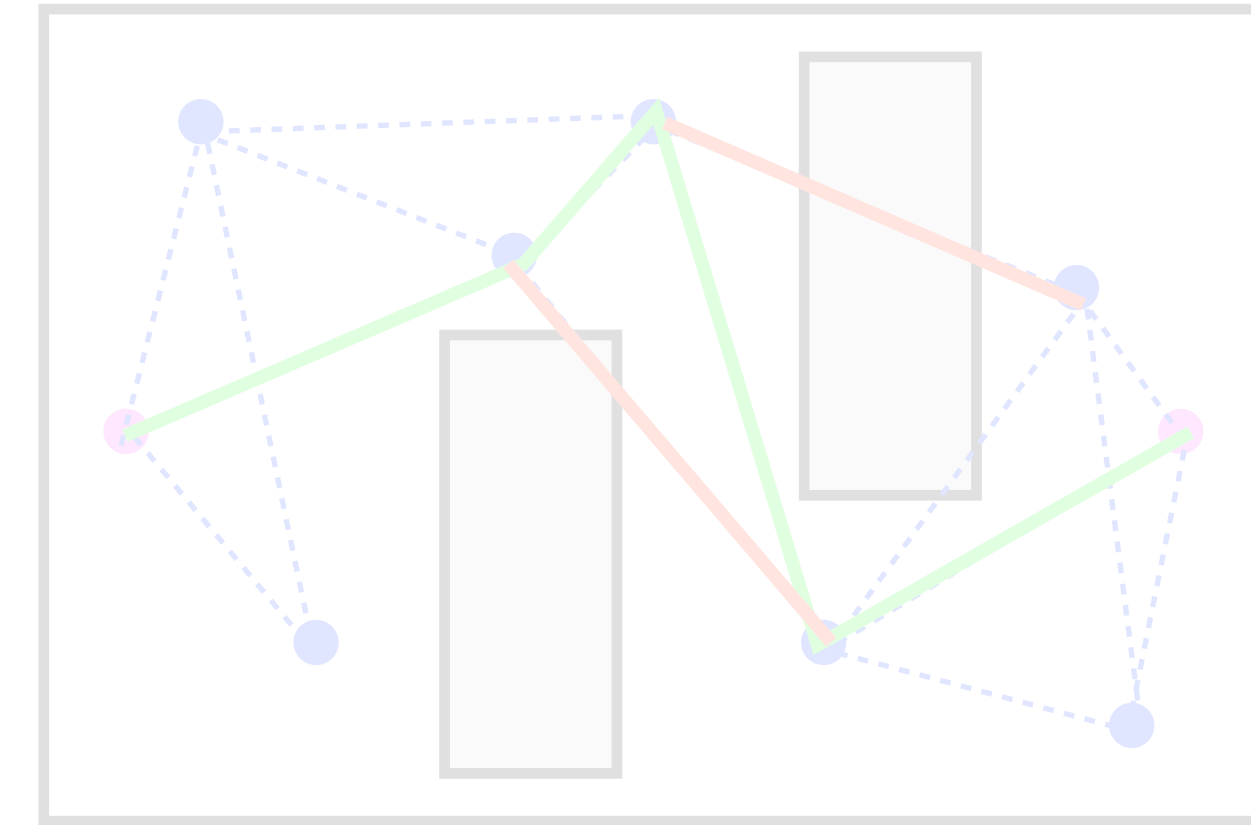
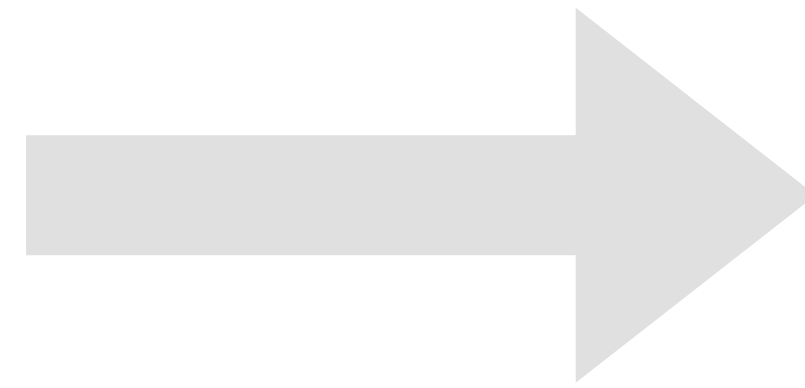


Interleave

# General framework for motion planning



Create a graph



Search the graph



Interleave

# Creating a graph: Abstract algorithm

$$G = (V, E)$$

**Vertices:** set of configurations

**Edges:** paths connecting  
configurations

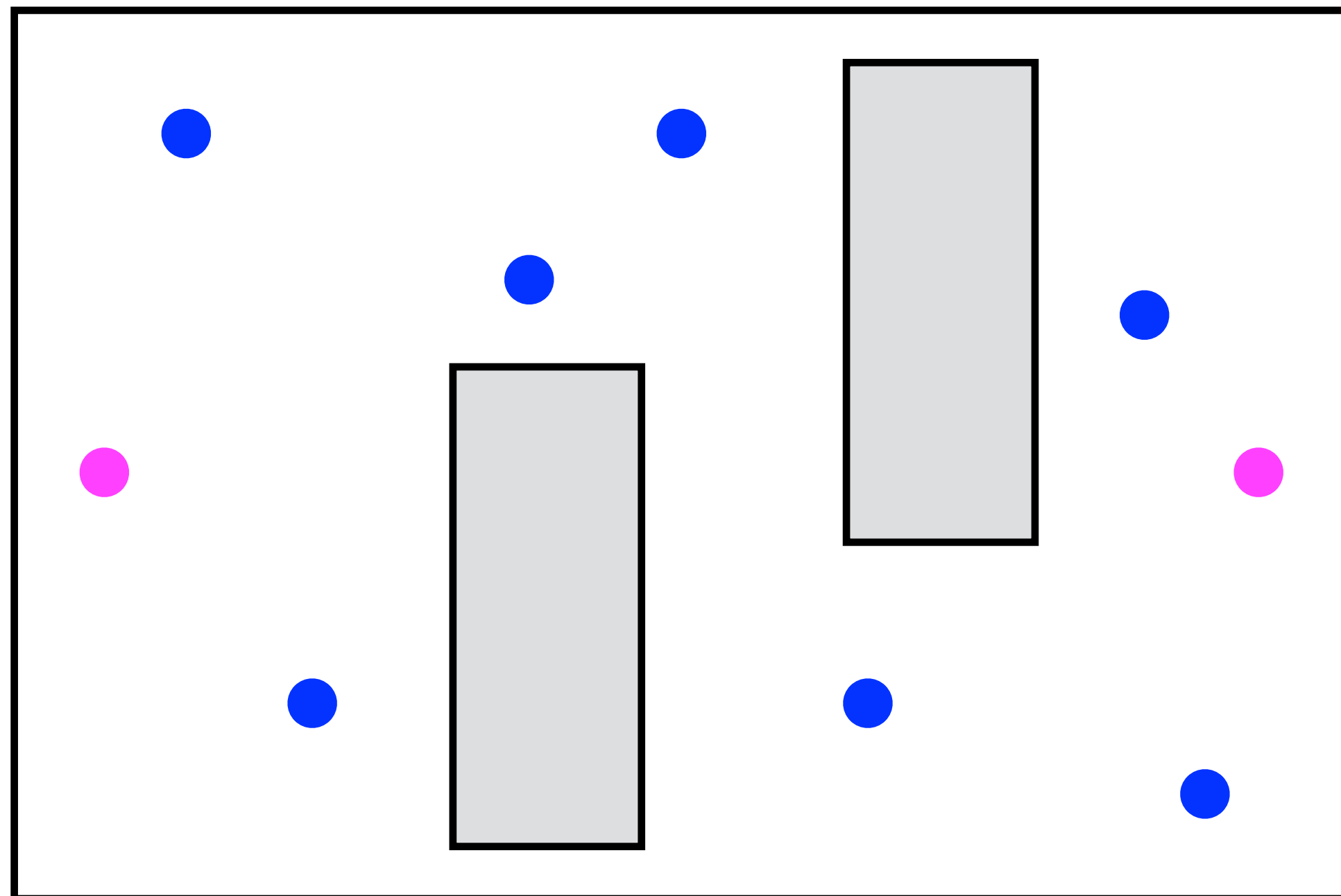


# Creating a graph: Abstract algorithm

$$G = (V, E)$$

**Vertices:** set of configurations

**Edges:** paths connecting configurations



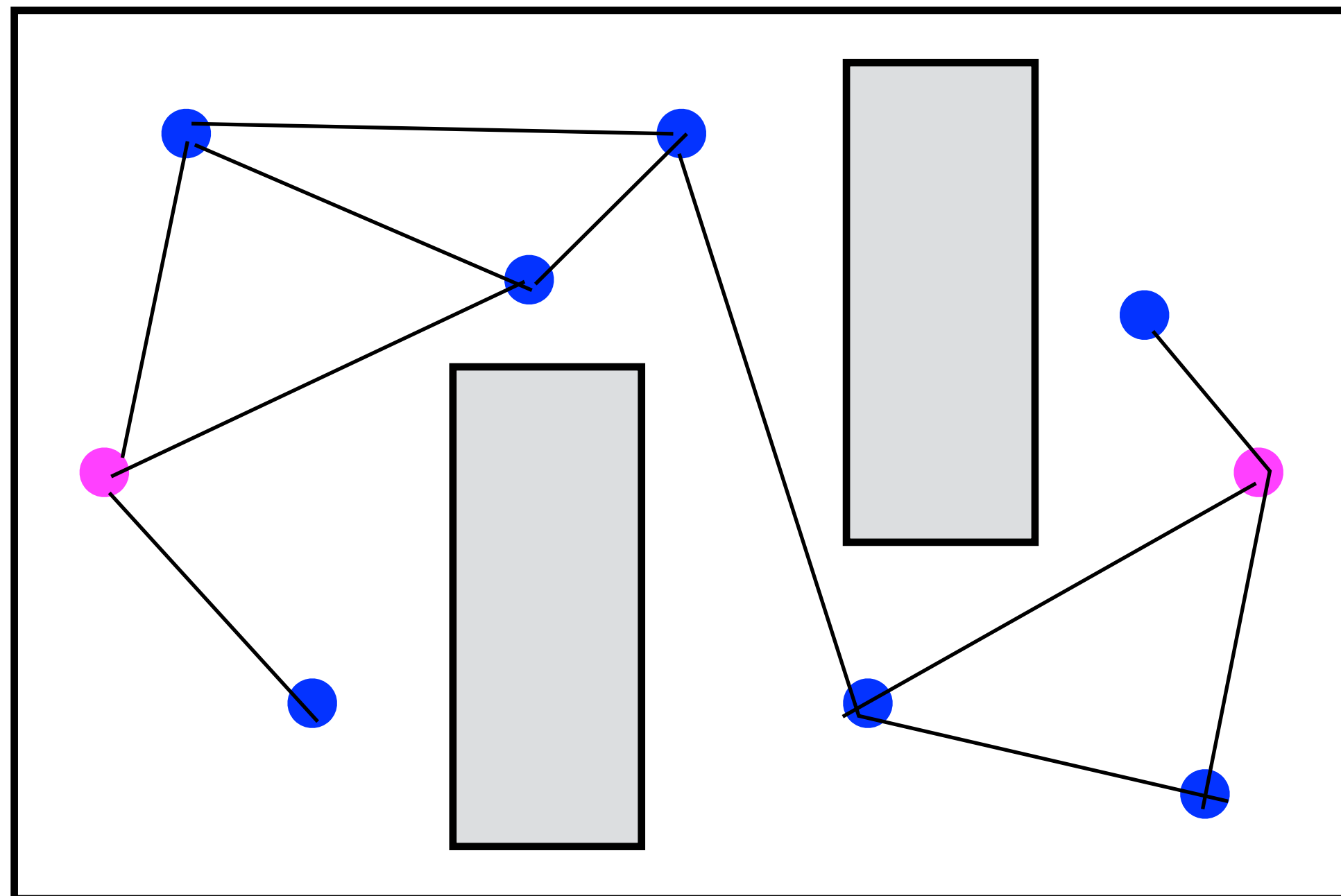
1. Sample a set of collision free vertices  $V$  (add start and goal)

# Creating a graph: Abstract algorithm

$$G = (V, E)$$

**Vertices:** set of configurations

**Edges:** paths connecting configurations

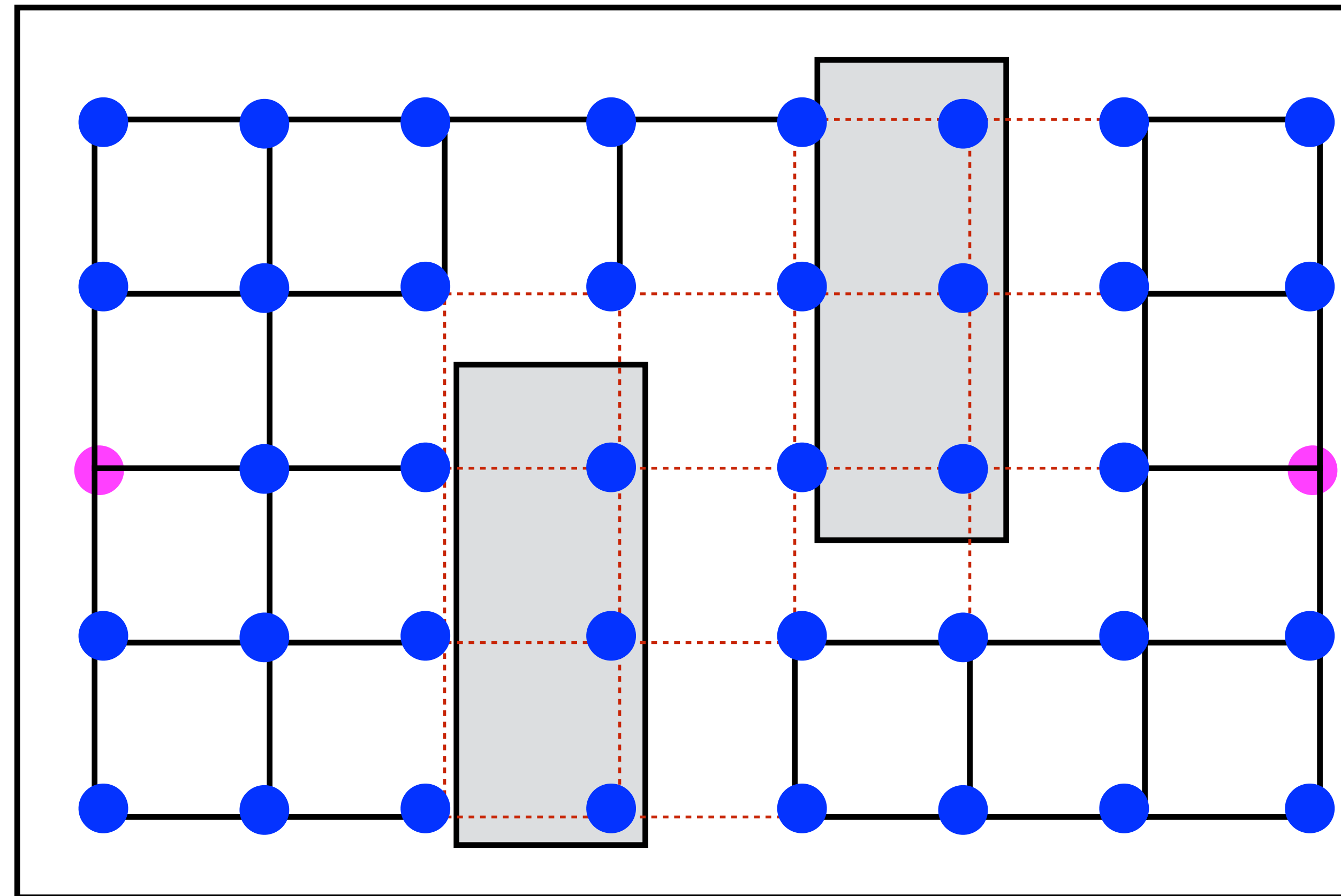


1. Sample a set of collision free vertices  $V$  (add start and goal)

2. Connect “neighboring” vertices to get edges  $E$

# Strategy 1: Discretize configuration space

Create a lattice. Connect neighboring points (4-conn, 8-conn, ...)

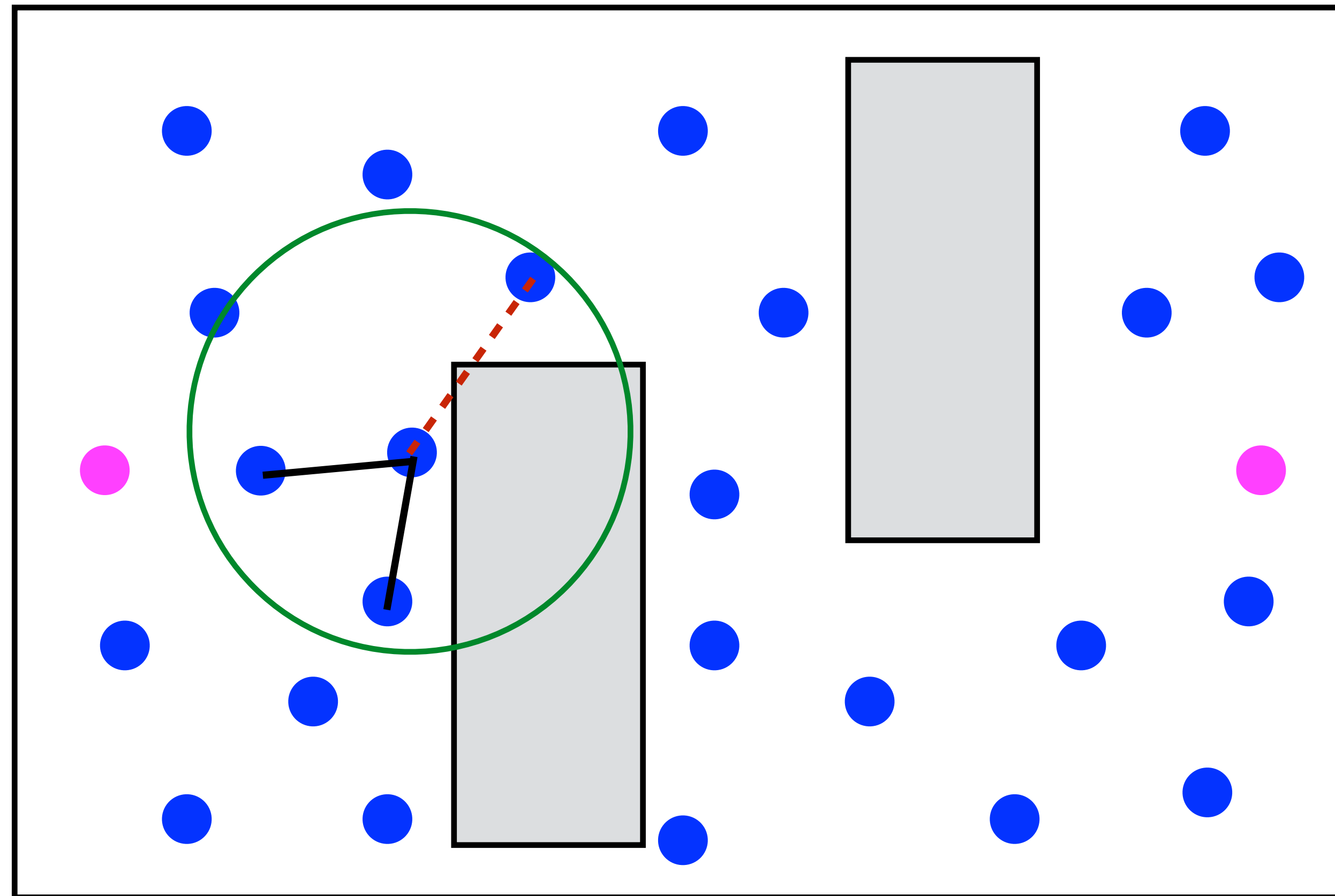


Theoretical guarantees: Resolution complete

What are the pros? What are the cons?

# Strategy 2: Uniformly randomly sample

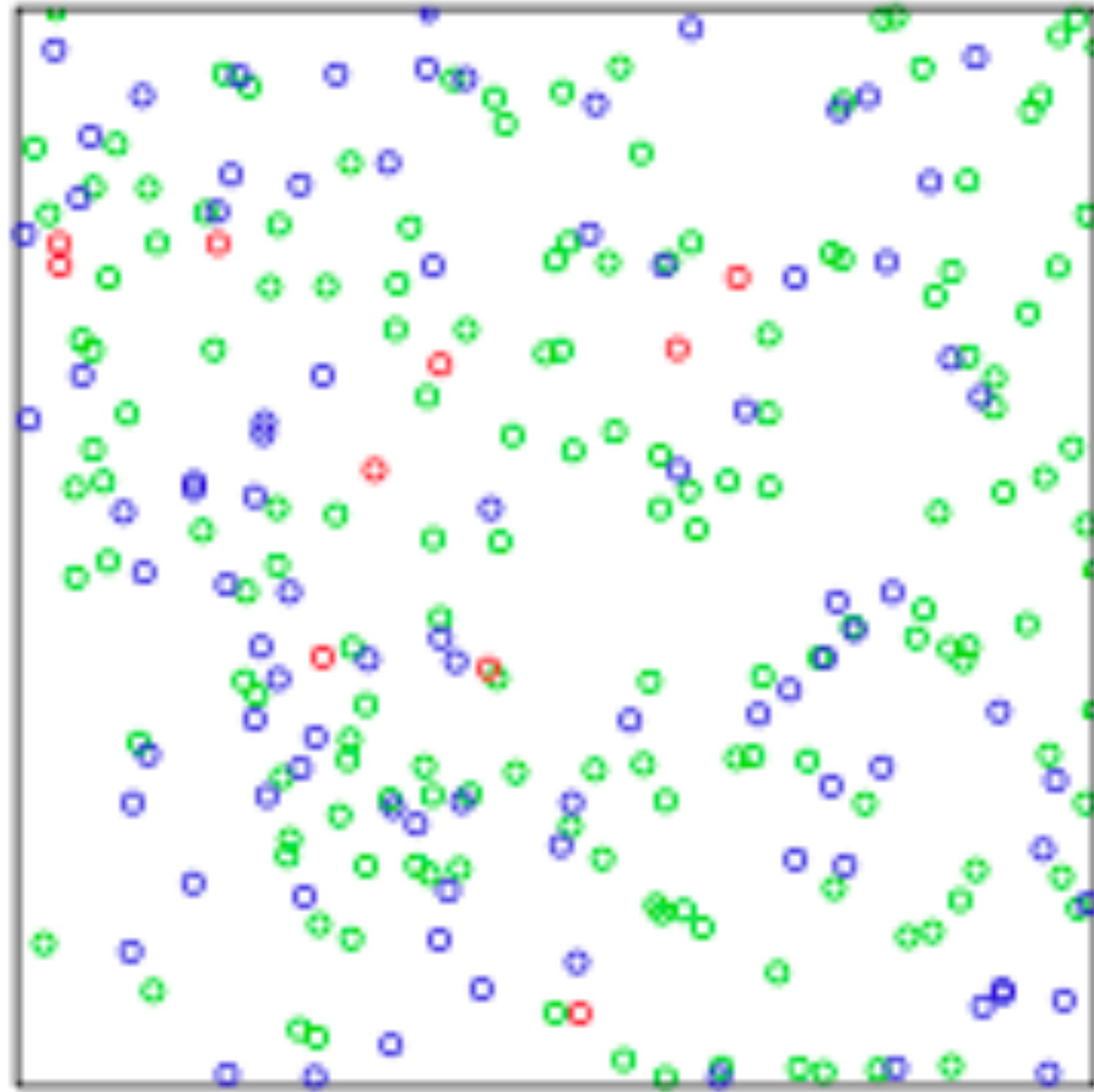
Randomly sample points. Connect all neighbors in a ball!



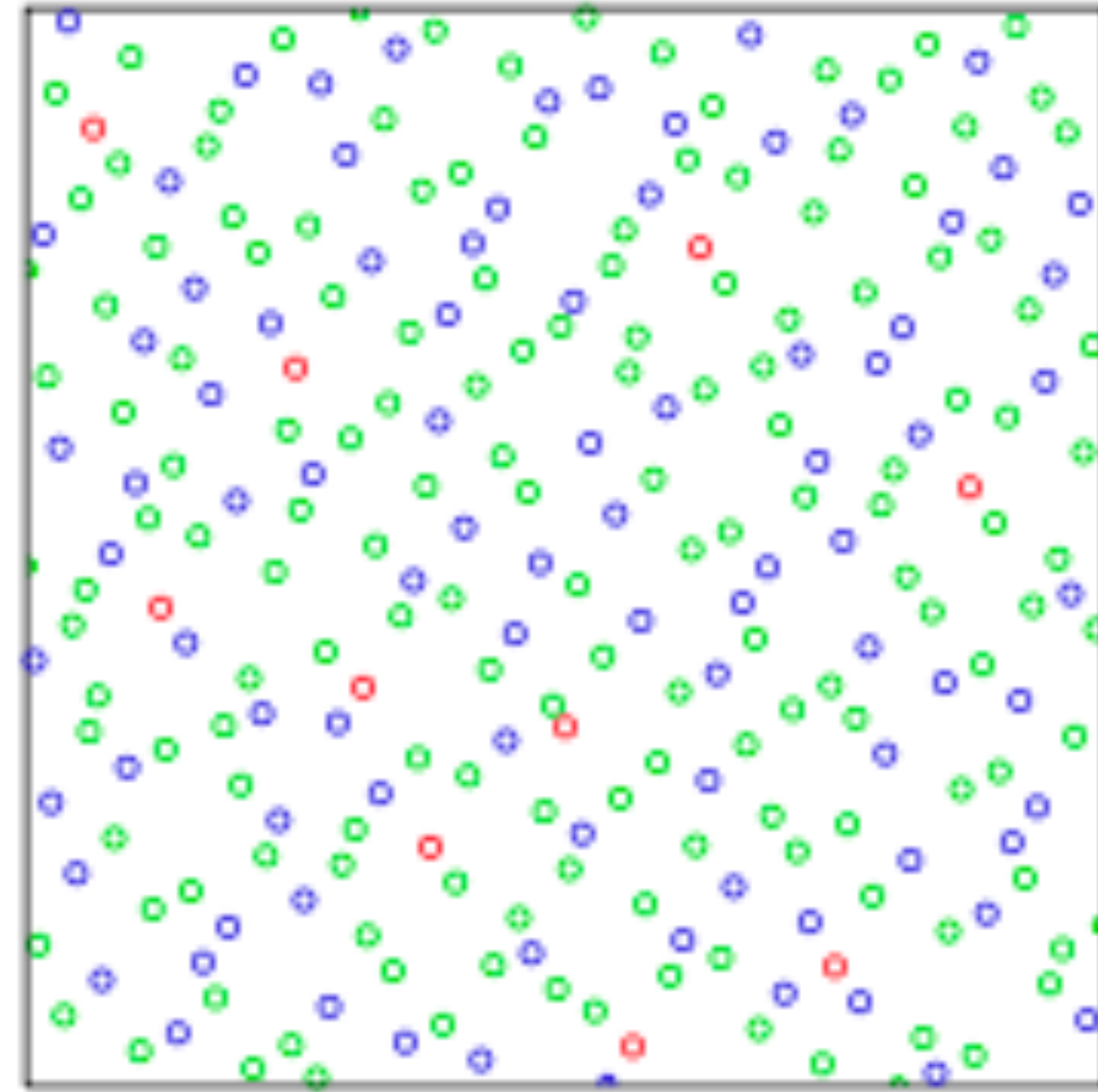
Theoretical guarantees: Probabilistically complete

What are the pros? What are the cons?

# Can we do better than random?



Uniform random sampling tends to clump



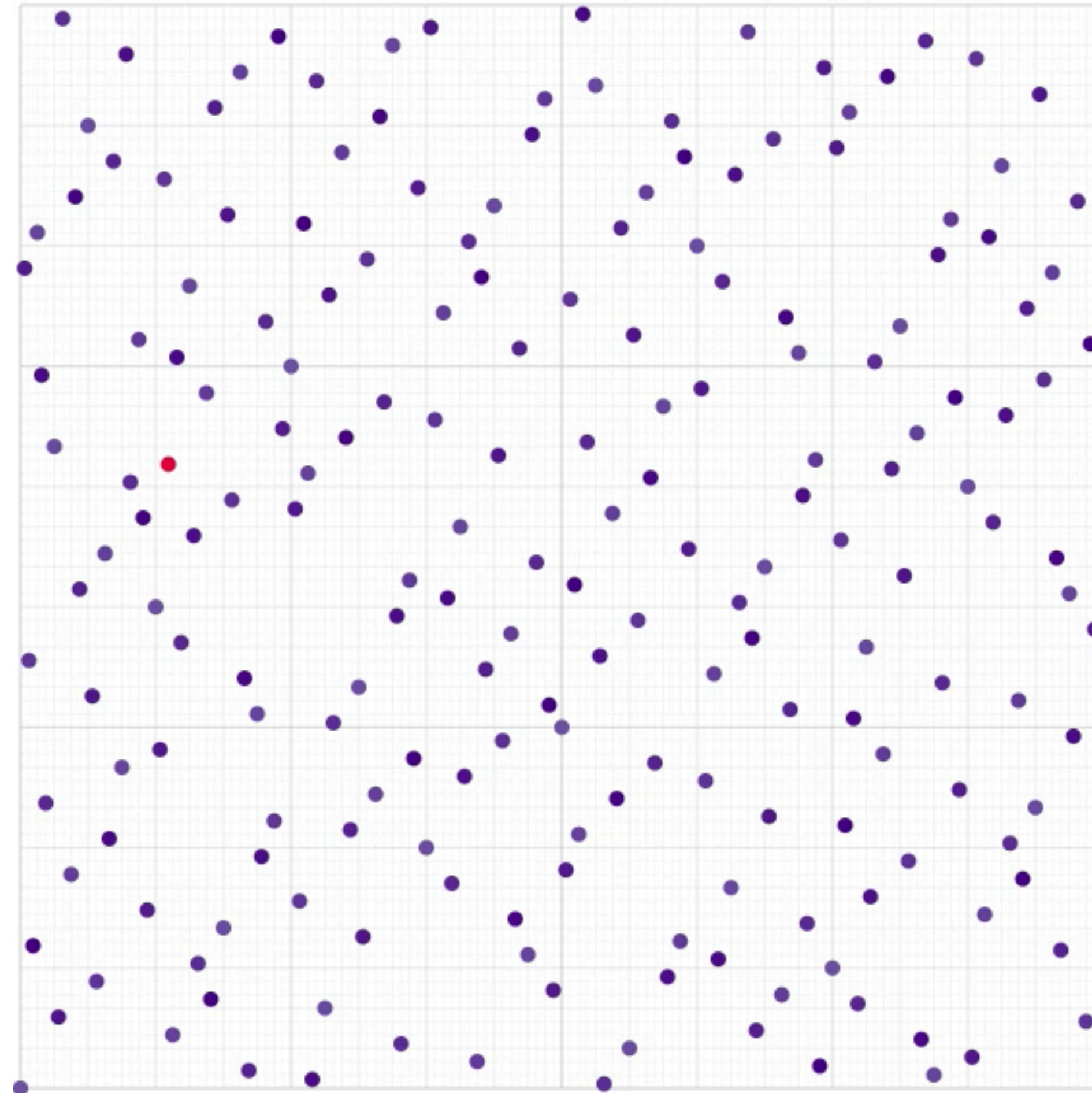
Ideally we would want points to be spread out evenly

**Question:** How do we do this without discretization?



# Halton Sequence

**Intuition:** Create a sequence using prime numbers that uniformly densify space



Link for exact algorithm:

<https://observablehq.com/@jrus/halton>

# How can learning help make better graphs?

**LEGO: Leveraging Experience in Roadmap  
Generation for Sampling-Based Planning**

Rahul Kumar<sup>\*1</sup>, Aditya Mandalika<sup>\*2</sup>, Sanjiban Choudhury<sup>\*2</sup> and Siddhartha S. Srinivasa<sup>\*2</sup>





# Learning a Sampler (LEGO)

