Policy Gradient (continue)

Recap: the REINFORCE Algorithm

$$\tau = \{s_0, a_0, s_1, a_1, \ldots, s_{H-1}, a_{H-1}\}$$

$$\rho_\theta(\tau) = \mu(s_0)\pi_\theta(a_0 \mid s_0)P(s_1 \mid s_0, a_0)\pi_\theta(a_1 \mid s_1)\ldots$$

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This formulation has large variance, i.e.,
$\mathbb{E}\left[\|g - \nabla_\theta J(\pi_{\theta_0})\|_2^2\right]$
could be as large as H^3

Today’s Question:

How to Make Policy Gradient really useful in practice

Outline:

1. A $Q(s, a)$ based Policy Gradient and Variance reduction

2. Proximal Policy Optimization (it trains ChatGPT!)

3. Reset to address the exploration challenge

Value / Q function

$$V_h^\pi(s) = \mathbb{E}\left[\sum_{t=h}^{H-1} r(s_t, a_t) \,|\, s_h = s, a_t \sim \pi\right]$$

$$Q_h^\pi(s, a) = \mathbb{E}\left[\sum_{t=h}^{H-1} r(s_t, a_t) \,|\, s_h = s, a_h = a, a_t \sim \pi\right]$$

$$= r(s, a) + \mathbb{E}_{s' \sim \mathcal{T}(s,a)} V_{h+1}^\pi(s')$$

Policy Gradient w/ Q_h^π

$$\nabla_\theta J(\pi_\theta) := \sum_{h=0}^{H-1} \mathbb{E}_{s,a\sim d_h^{\pi_\theta}} \left[\underline{\nabla_\theta \ln \pi_\theta(a_h \mid s_h) \cdot Q_h^{\pi_\theta}(s_h, a_h)} \right]$$

State-action
distribution at h

Adjust θ s.t. policy increases
(decreases) prob of a with high (low)
expected reward-to-go

Variance reduction via a Baseline

$$\nabla_\theta J(\pi_\theta) := \sum_{h=0}^{H-1} \mathbb{E}_{s,a\sim d_h^{\pi_\theta}} \left[\nabla_\theta \ln \pi_\theta(a_h \mid s_h) \cdot \left(Q_h^{\pi_\theta}(s_h, a_h) - b(s_h) \right) \right]$$

Baseline: as long as it is action-indepenent, it does not affect the gradient

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Baseline: as long as it is action-indepenent, it does not affect the gradient

Just need to show $\mathbb{E}_{s,a\sim\pi_\theta(.|s)} \nabla \ln \pi_\theta(a \,|\, s) \cdot b(s) = 0$:

Value function as a baseline

$$\nabla_\theta J(\pi_\theta) := \sum_{h=0}^{H-1} \mathbb{E}_{s,a\sim d_h^{\pi_\theta}}\left[\nabla_\theta \ln \pi_\theta(a_h \mid s_h)\cdot\left(\underline{Q_h^{\pi_\theta}(s_h,a_h) - V_h^{\pi_\theta}(s_h)}\right)\right]$$

This is called **Advantage** function:

$$A_h^{\pi_\theta}(s,a) = Q_h^{\pi_\theta}(s,a) - V_h^{\pi_\theta}(s)$$

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Value function as a baseline

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1. Advantage can be as small as a constant (e.g., this is the condition where DAgger works better than BC)

2. V is not the theoretically optimal baseline, but is used almost in every practical PG alg/implementation

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Example application of PPO:

Train a robot to “run” forward as fast as possible:

State: joint angles, center of mass, velocity, etc

Action: torques on joints

Reward: distance of moving forward between two steps

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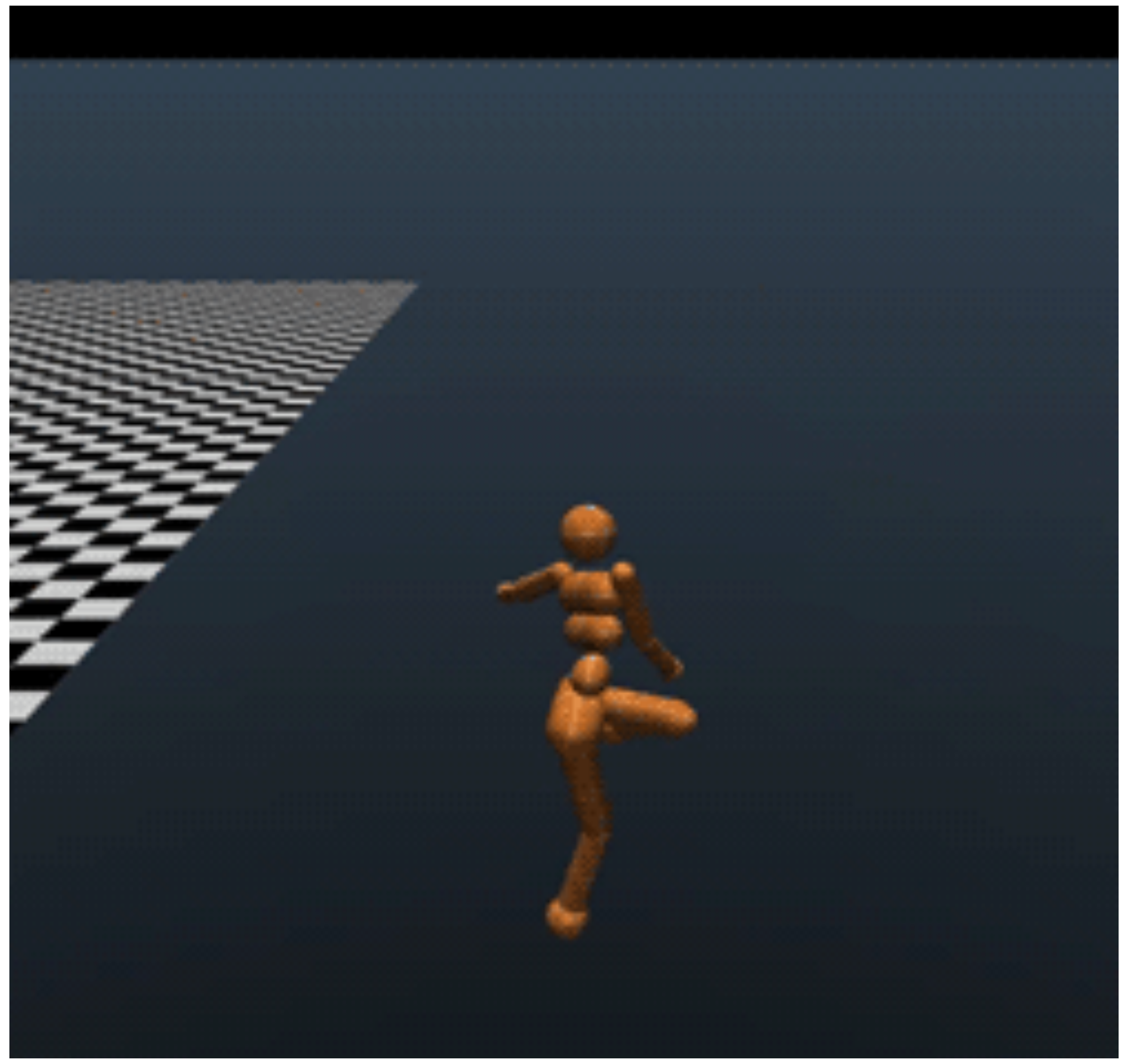

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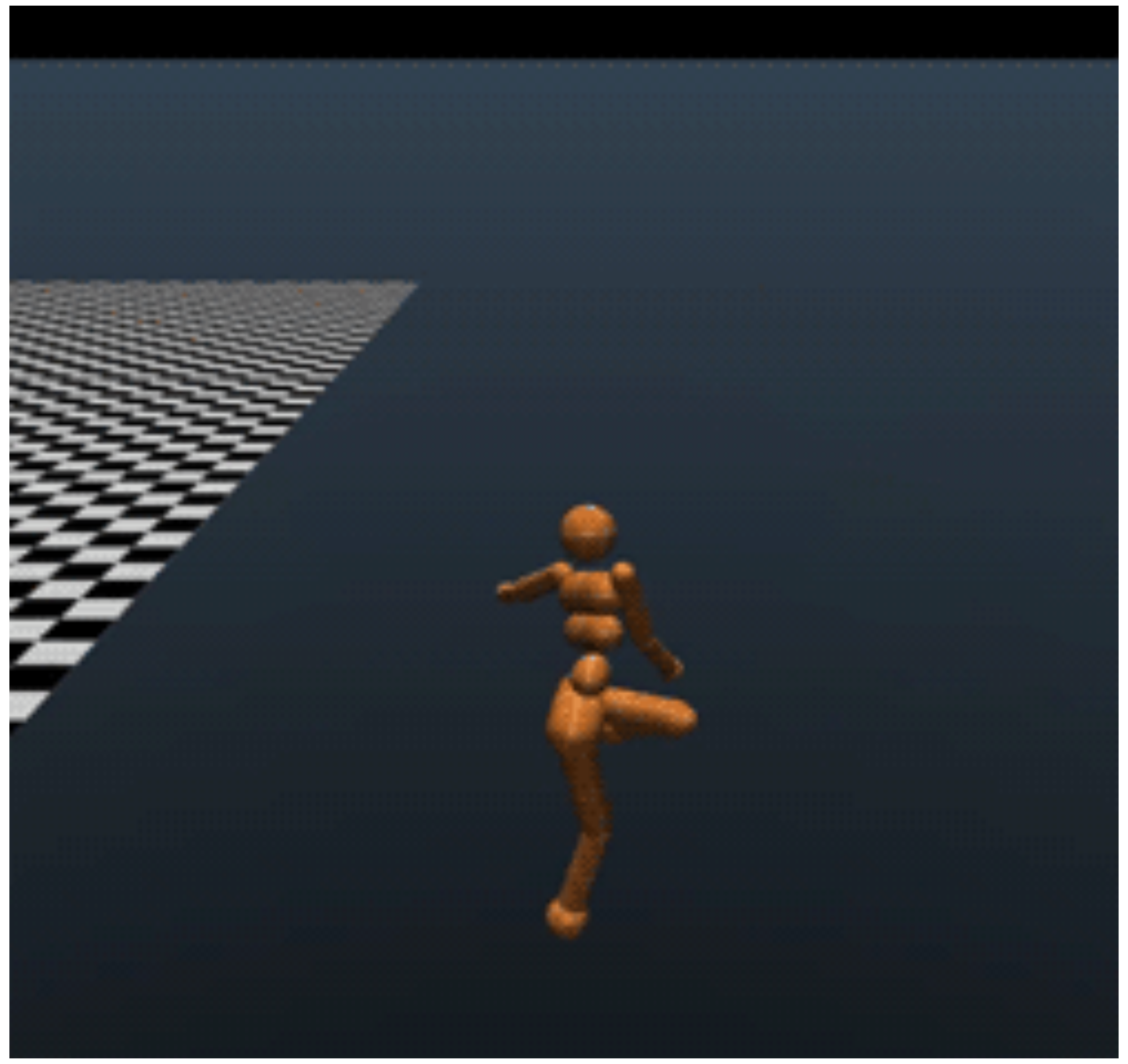

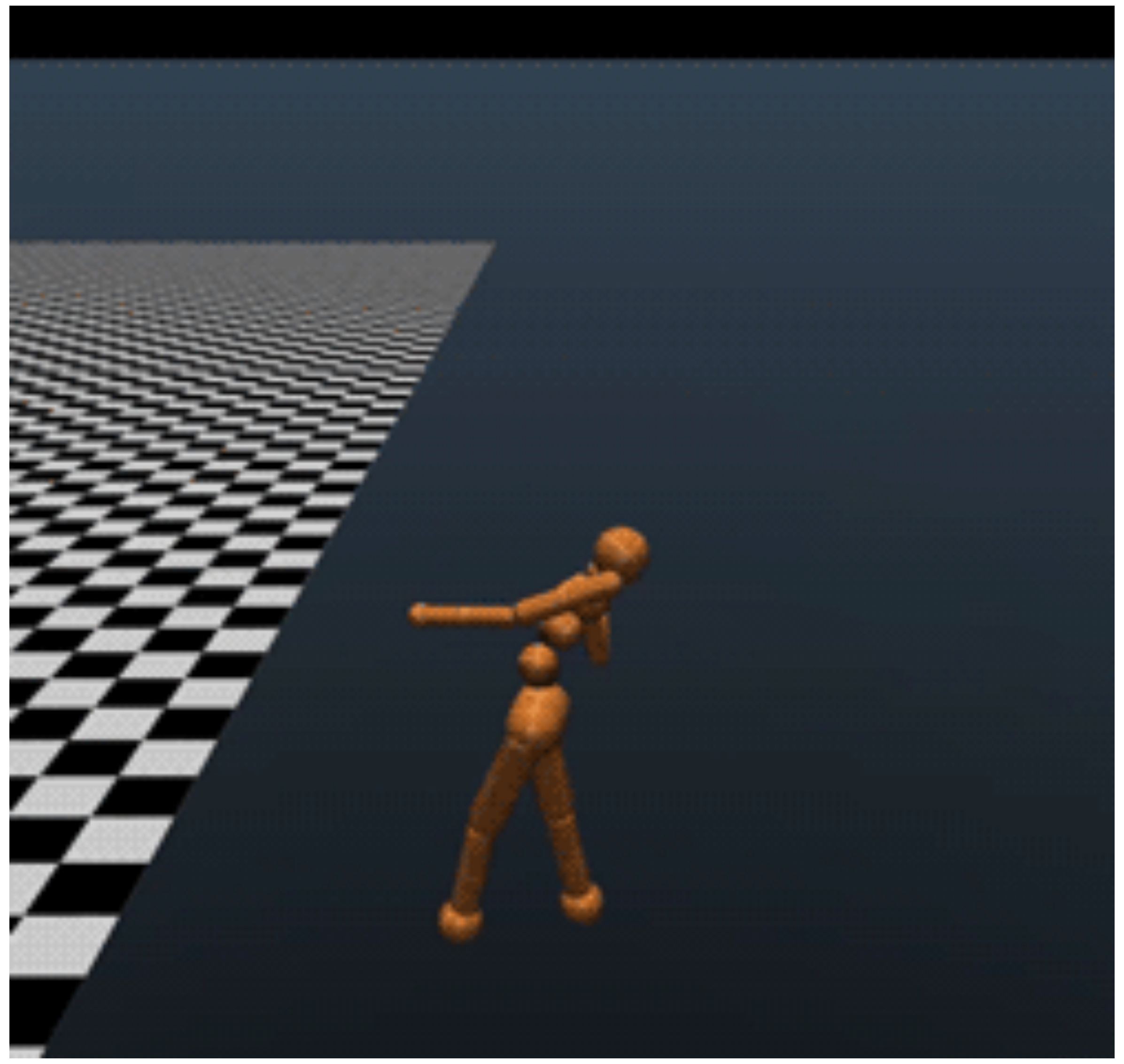

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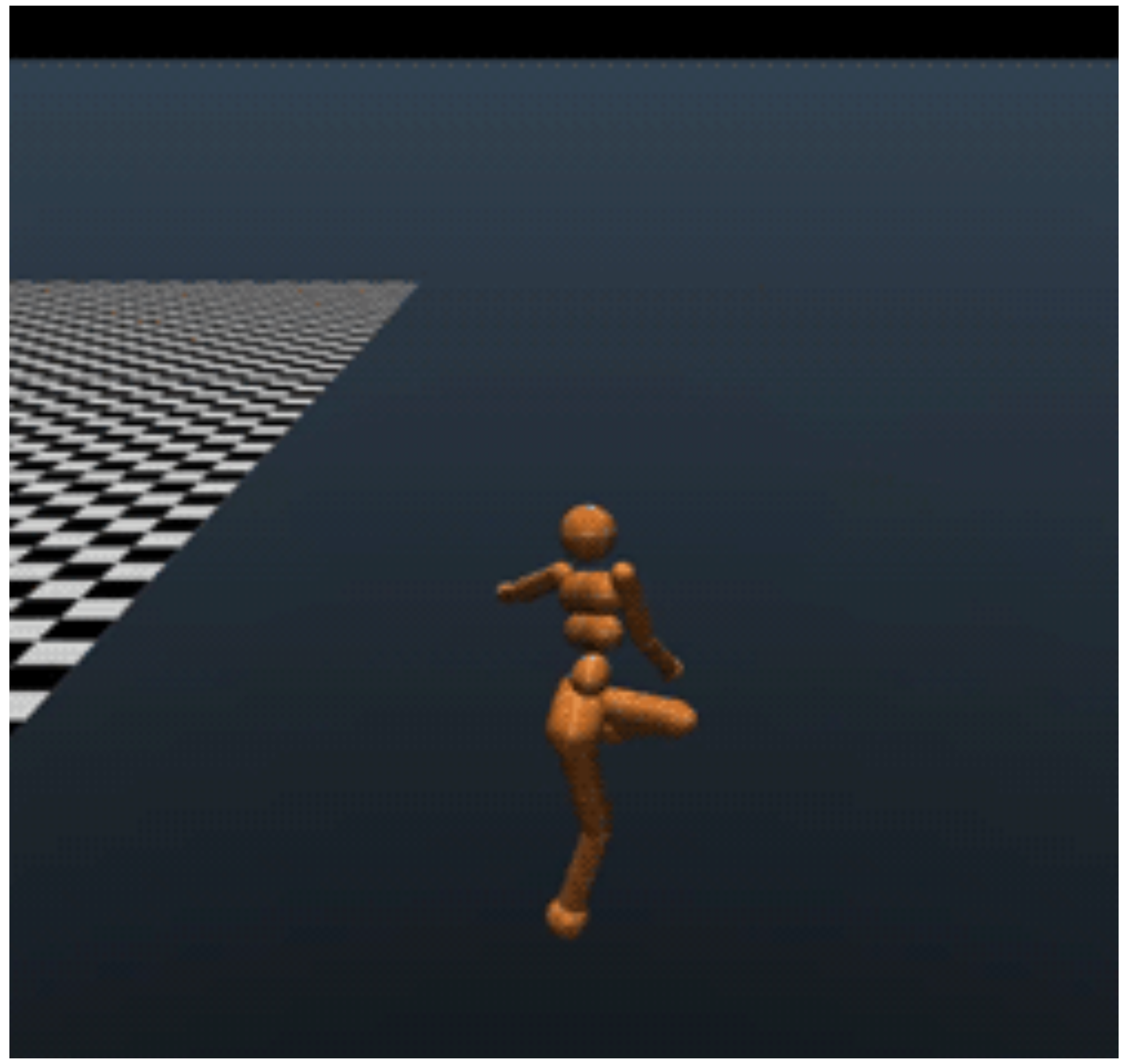
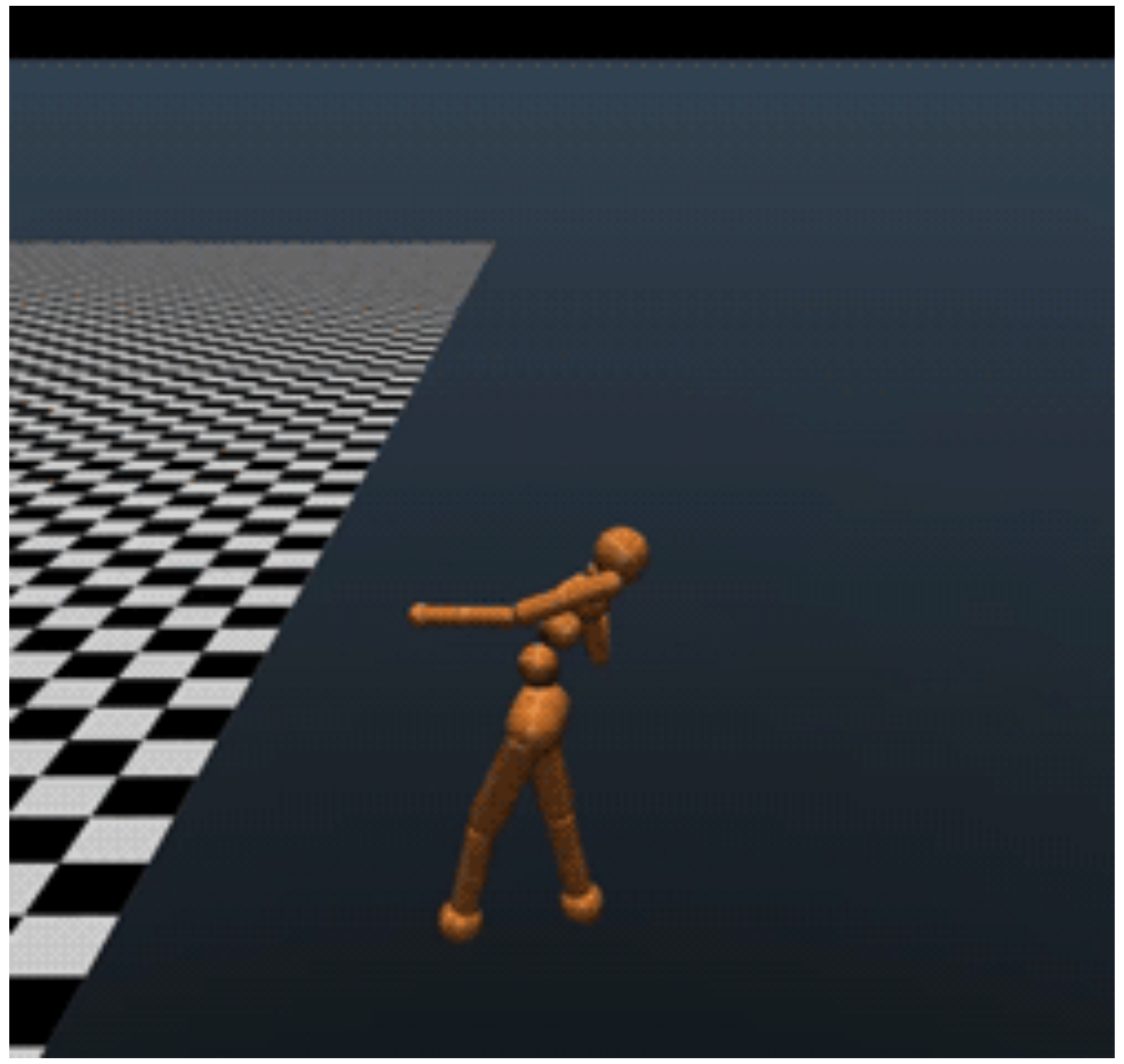
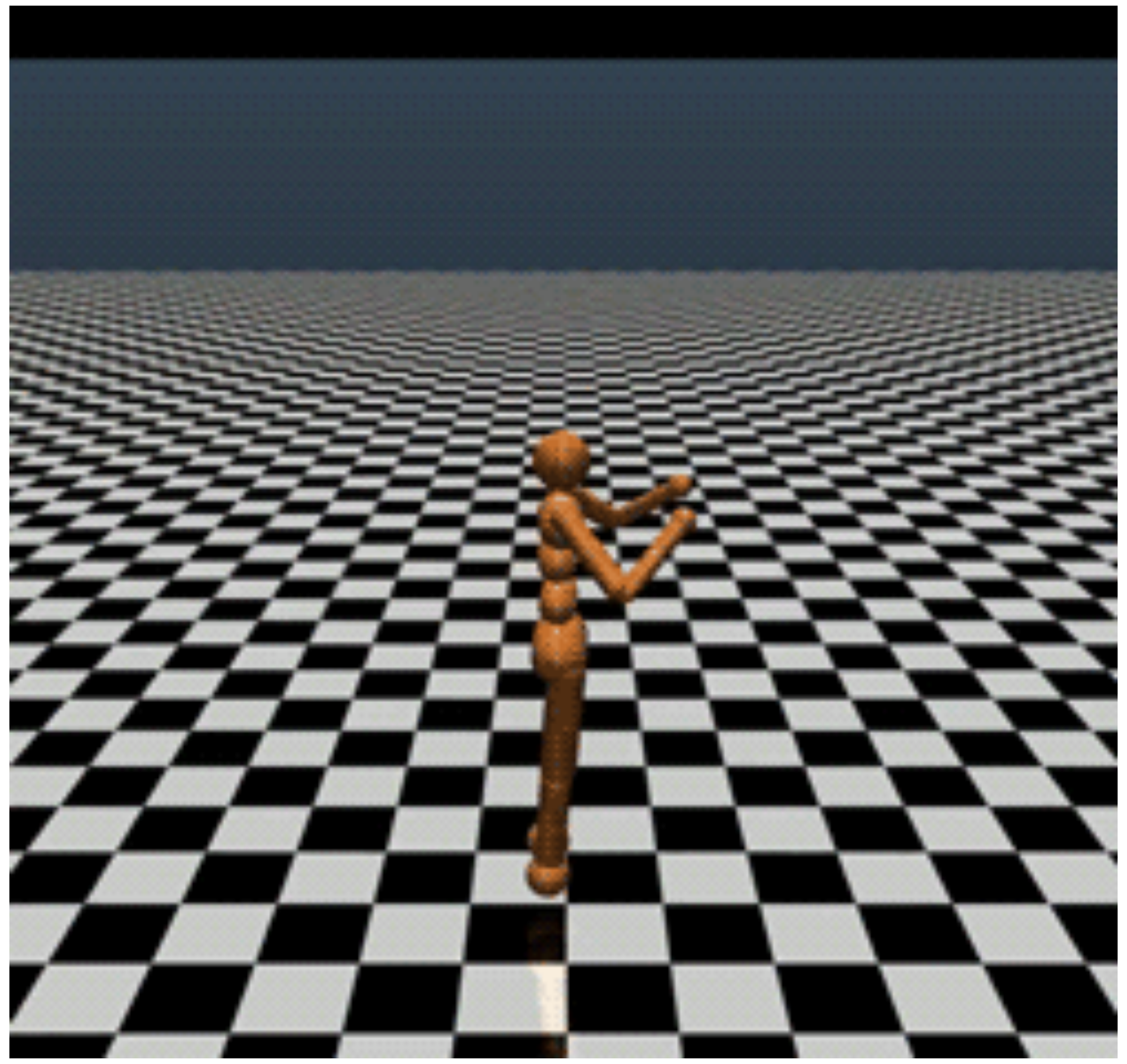

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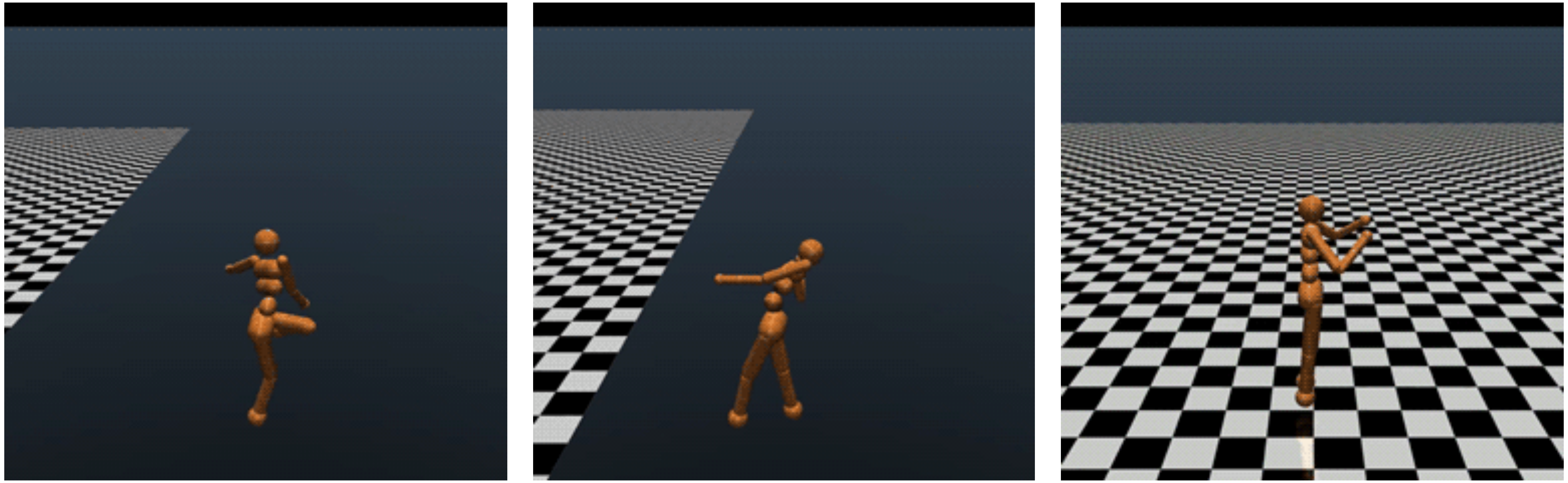
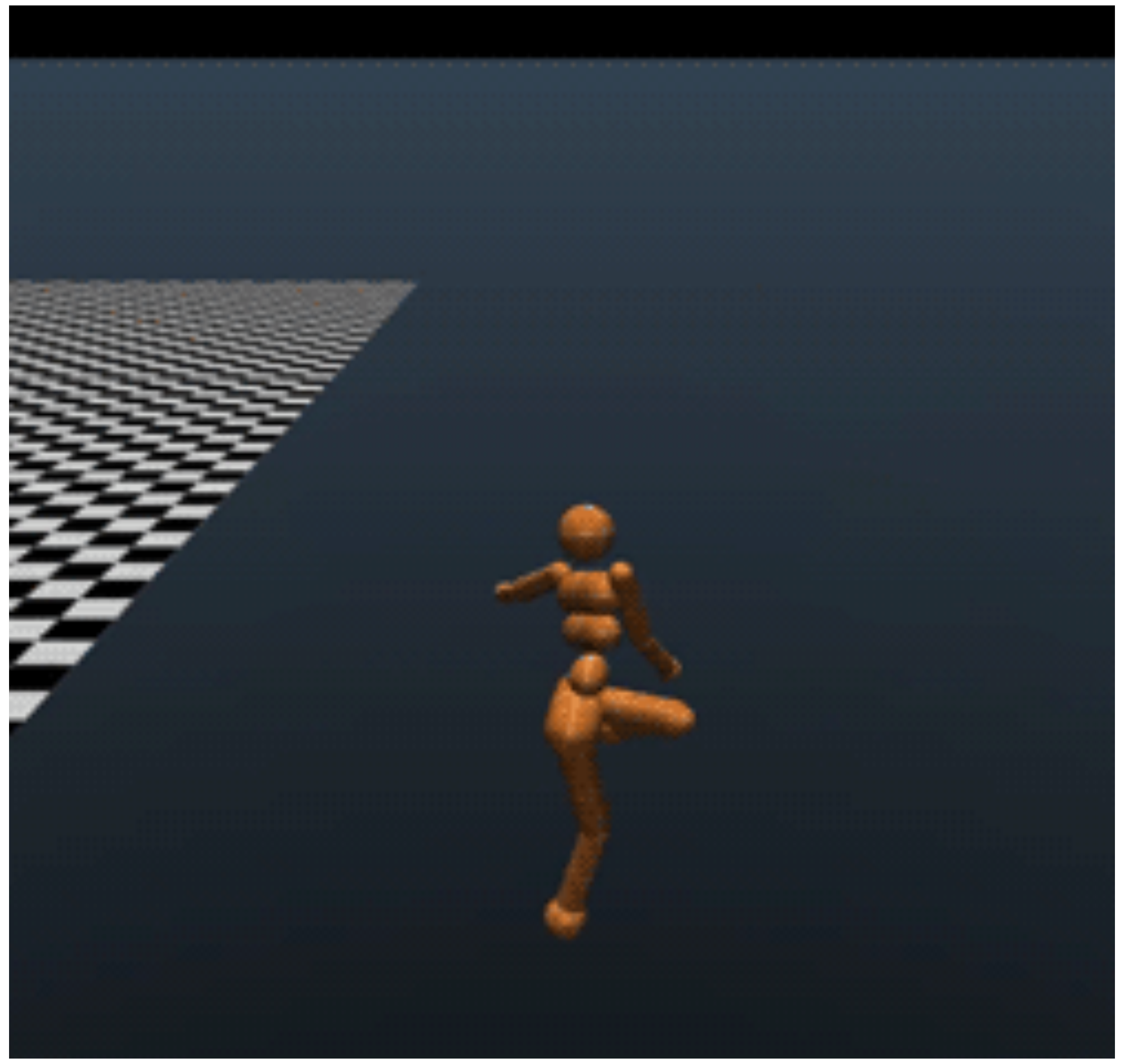


(BTW, This reveals an issue on reward design—you may study it in Inverse RL lectures)

Naive Policy Gradient can unstable and slow

The potential high-variance in PG can make learning very unstable

Naive Policy Gradient can unstable and slow

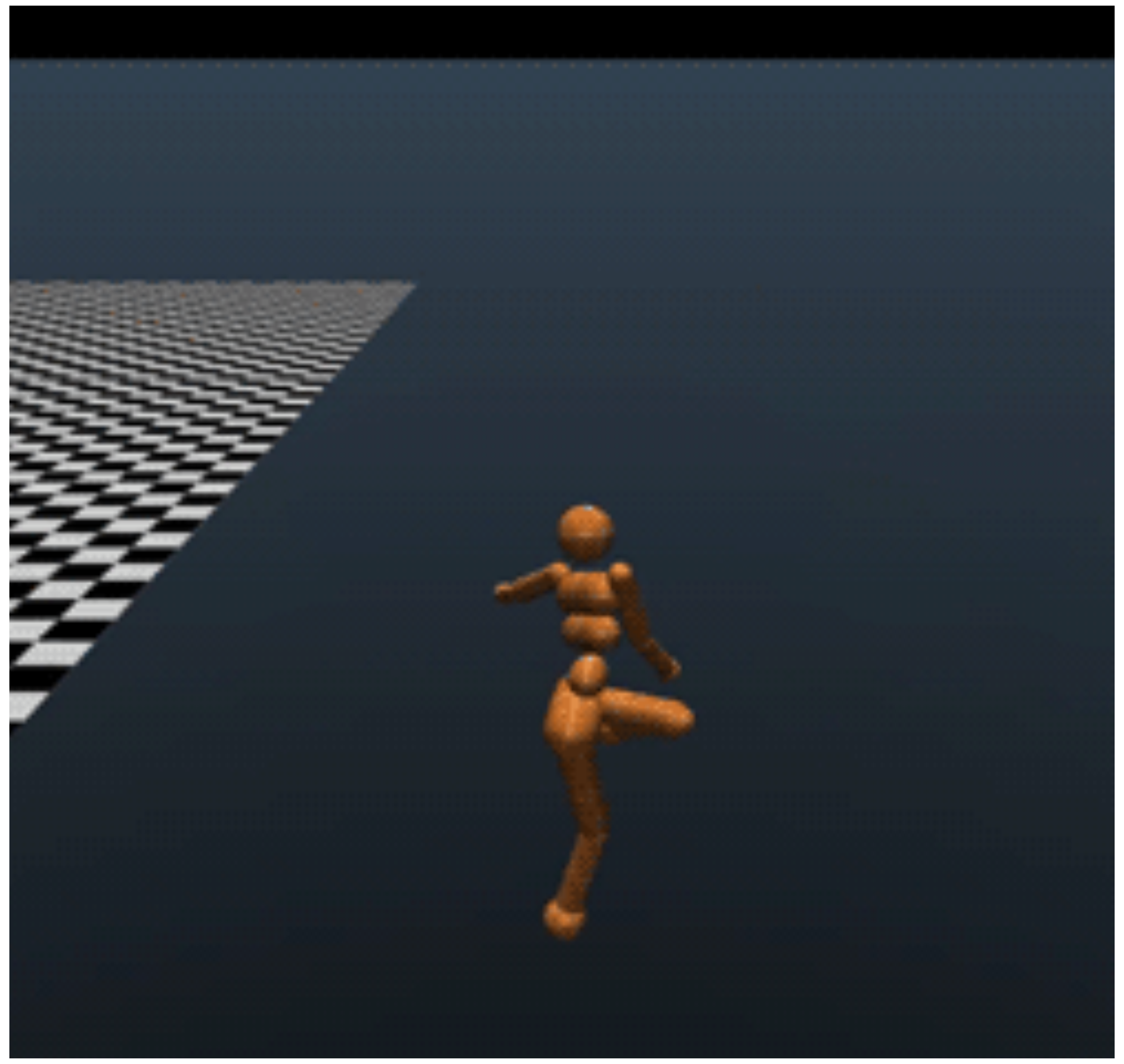


Too frequent!

GPU usage can be very low…

Proximal Policy Optimization (PPO)

π_{θ_t}



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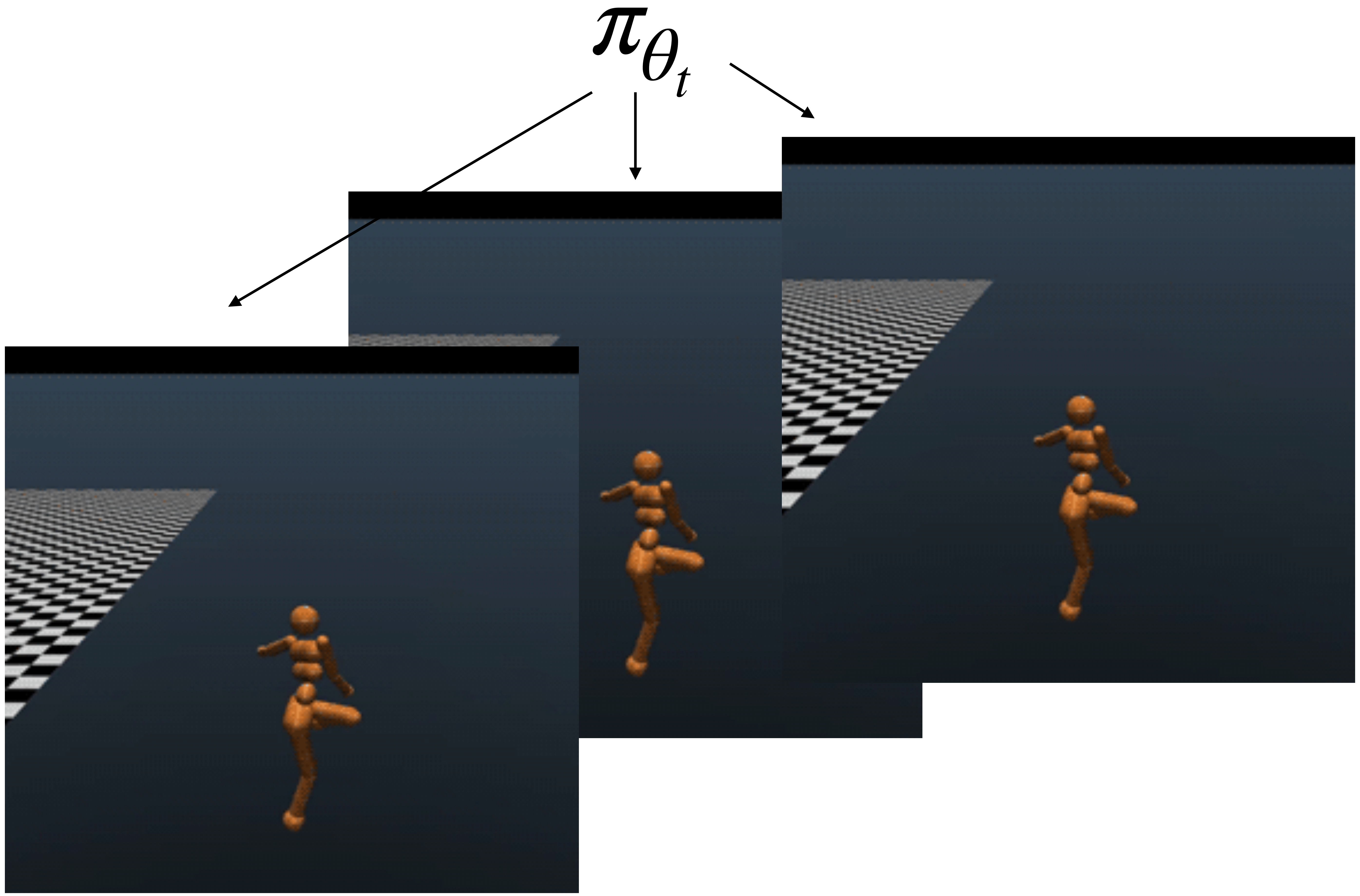

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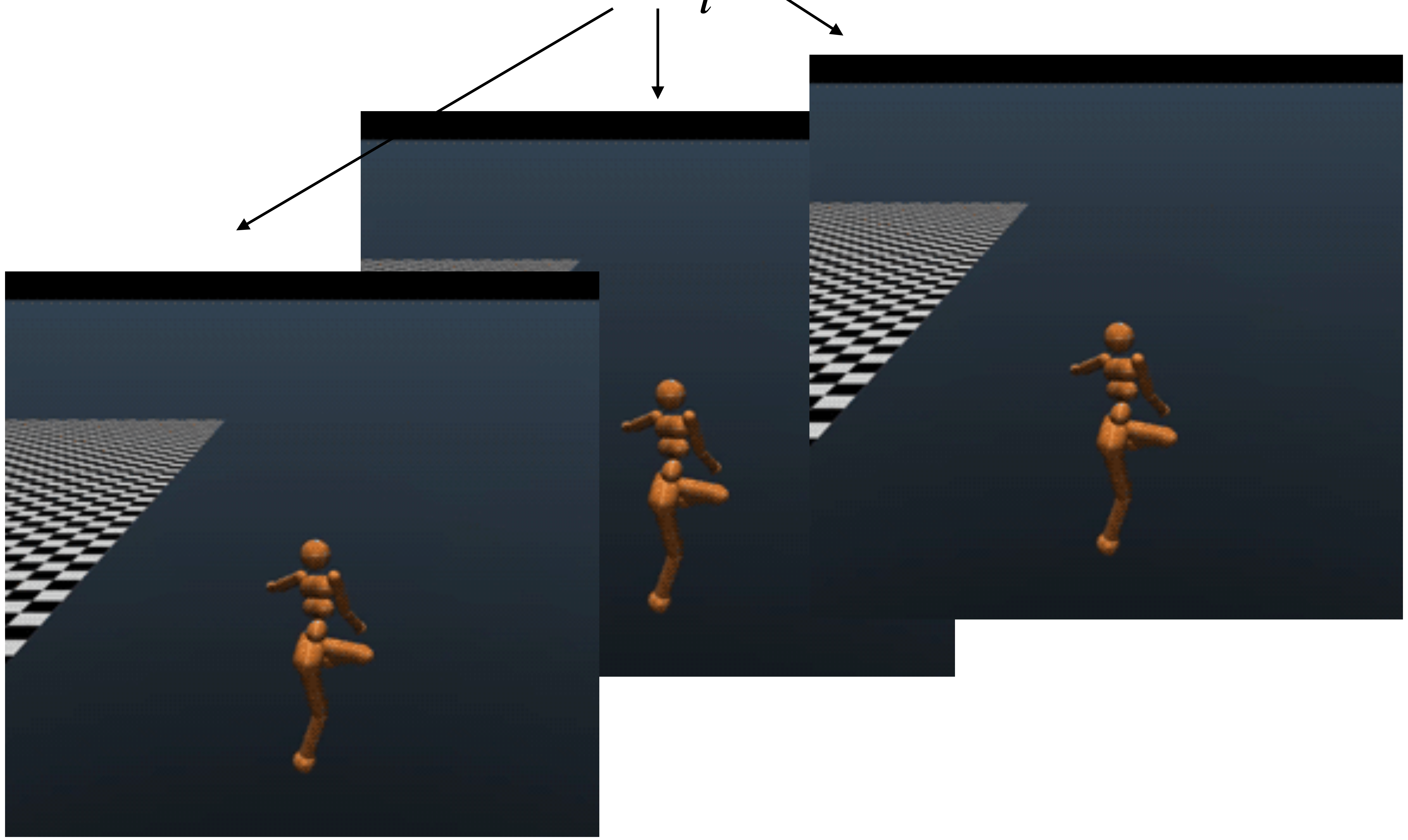

Collect a large dataset

$$\longrightarrow \mathcal{D} = \left\{ s, a, A^{\pi_{\theta_t}}(s, a) \right\}$$

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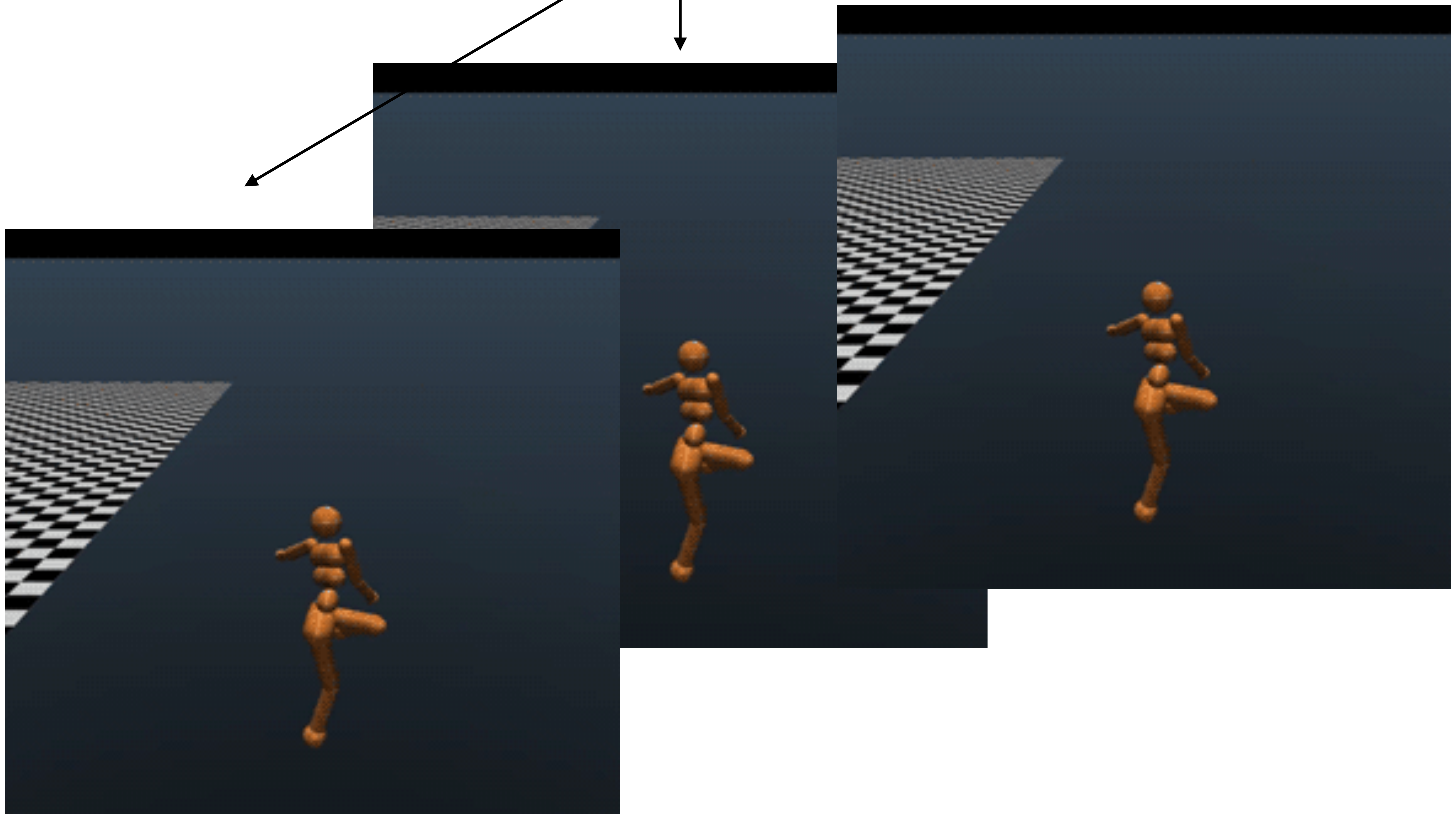

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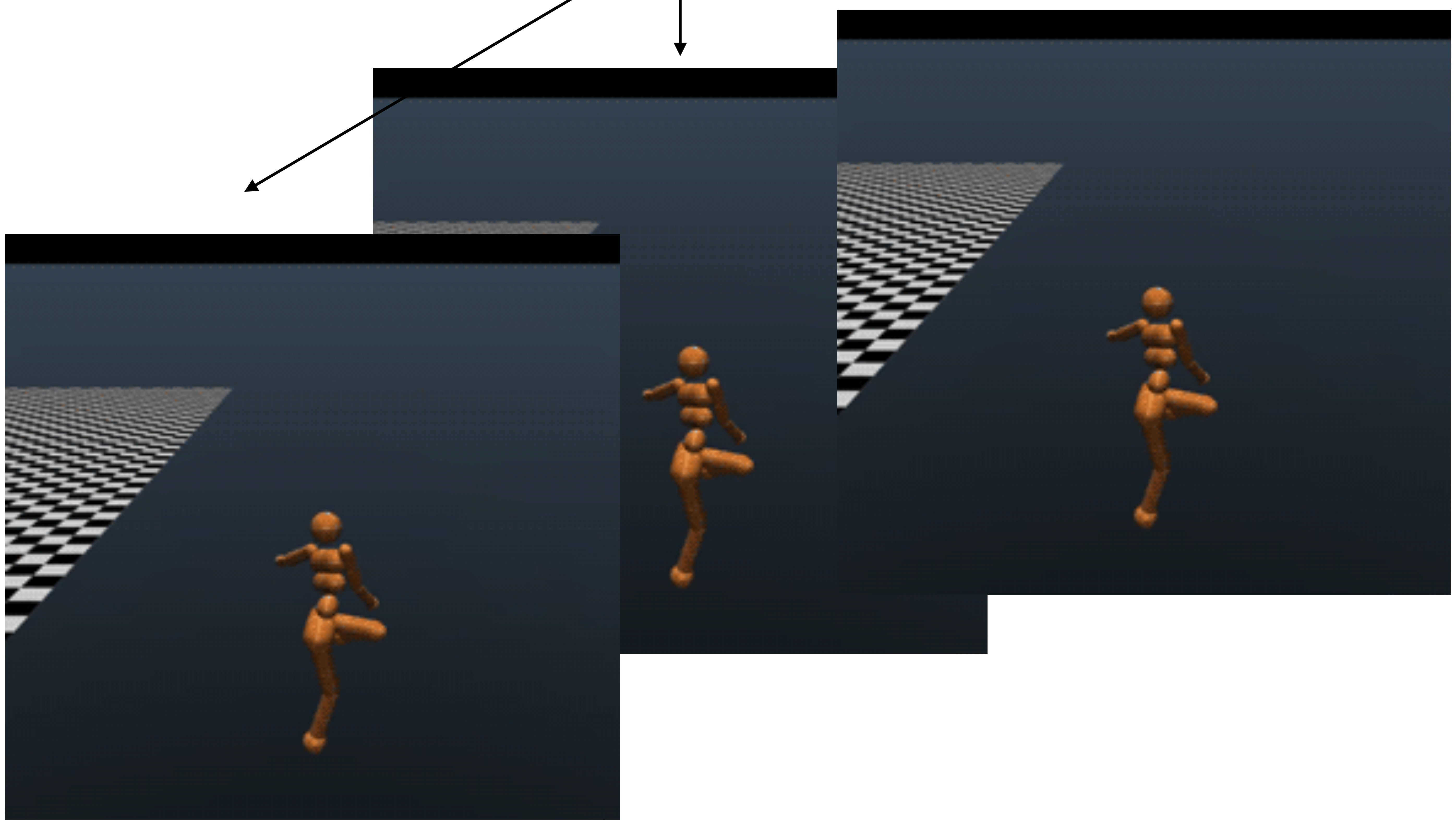

Collect a large dataset

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Now let's do multiple epoches of min-batch gradient update on the dataset

Proximal Policy Optimization (PPO)

Construct a batch Supervised Learning style objective using $\mathscr{D} = \{s, a, A^{\pi_{\theta_t}}(s, a)\}$

$$\max_\theta \ell(\theta) = \max_\theta \mathbb{E}_{s\sim d^{\pi_{\theta_t}}}\mathbb{E}_{a\sim\pi_\theta(\cdot|s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

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IW trick $\rightarrow \mathbb{E}_{s\sim d^{\pi_{\theta_t}}}\mathbb{E}_{a\sim\pi_{\theta_t}(\cdot|s)} \dfrac{\pi_\theta(\,\cdot\,|\,s)}{\pi_{\theta_t}(a\,|\,s)} \cdot A^{\pi_{\theta_t}}(s, a)$

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IW trick $$\to \mathbb{E}_{s\sim d^{\pi_{\theta_t}}} \mathbb{E}_{a\sim\pi_{\theta_t}(\cdot|s)} \frac{\pi_\theta(\,\cdot \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

$$\approx \sum_{s,a} \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

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$$\hat{\ell}_{clip}(\theta) = \sum_{s,a} \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1 - \epsilon, 1 + \epsilon\right) \cdot A^{\pi_{\theta_t}}(s, a)$$

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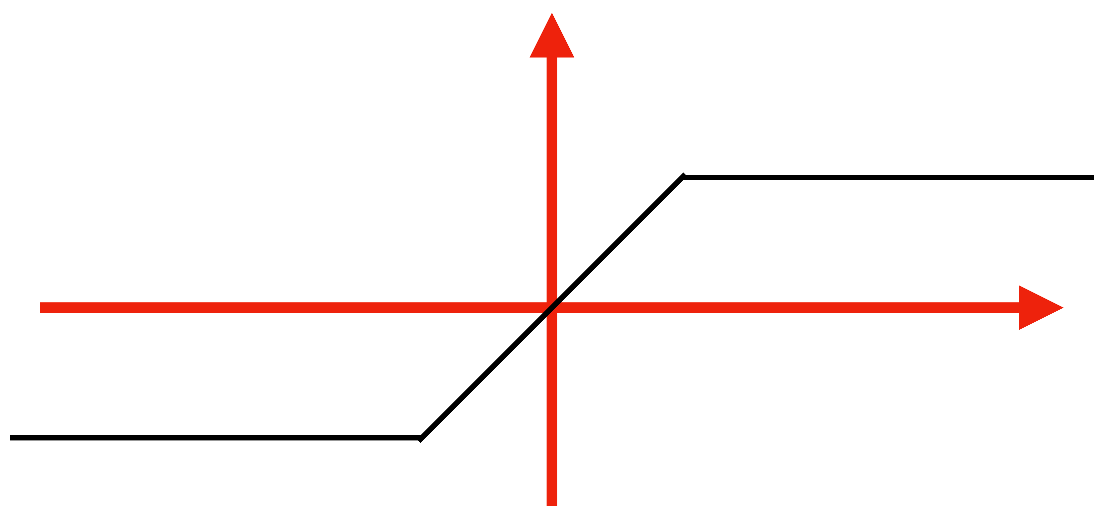

$\text{clip}(x, 1 - \epsilon, 1 + \epsilon)$

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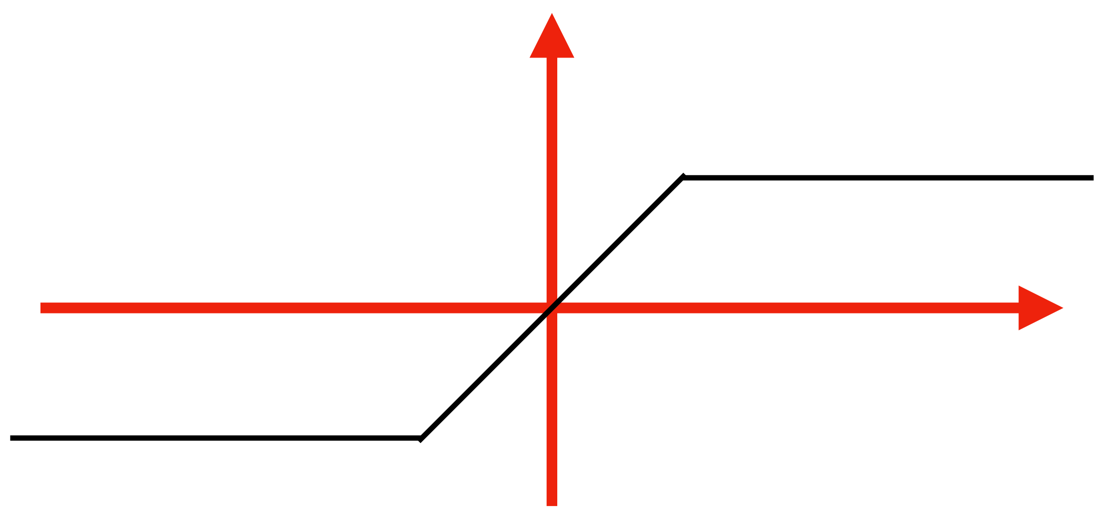

clip$(x, 1-\epsilon, 1+\epsilon)$

Stop updating $\pi_\theta(a \mid s)$ if it is too different from $\pi_{\theta_t}(a \mid s)$

Proximal Policy Optimization (PPO)

Trick 2, take the min of the clipped and uncipped (original) obj

$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s, a), \quad \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1 - \epsilon, 1 + \epsilon \right) \cdot A^{\pi_{\theta_t}}(s, a) \right\}$$

Original obj

clipped obj which ensures no abrupt change in action probabilities

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Original obj

clipped obj which ensures no abrupt change in action probabilities

We compute $\theta_{t+1} \approx \arg\max_\theta \hat{\ell}_{final}(\theta)$, via performing a few epoches of minbatch SG ascent (or Adam/Adagrad) on $\hat{\ell}_{final}$

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Policy gradient cannot do exploration…

Initialization: s_0

Length of chain is H

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Probability of random walk hitting reward 1 is $(1/3)^{-H}$

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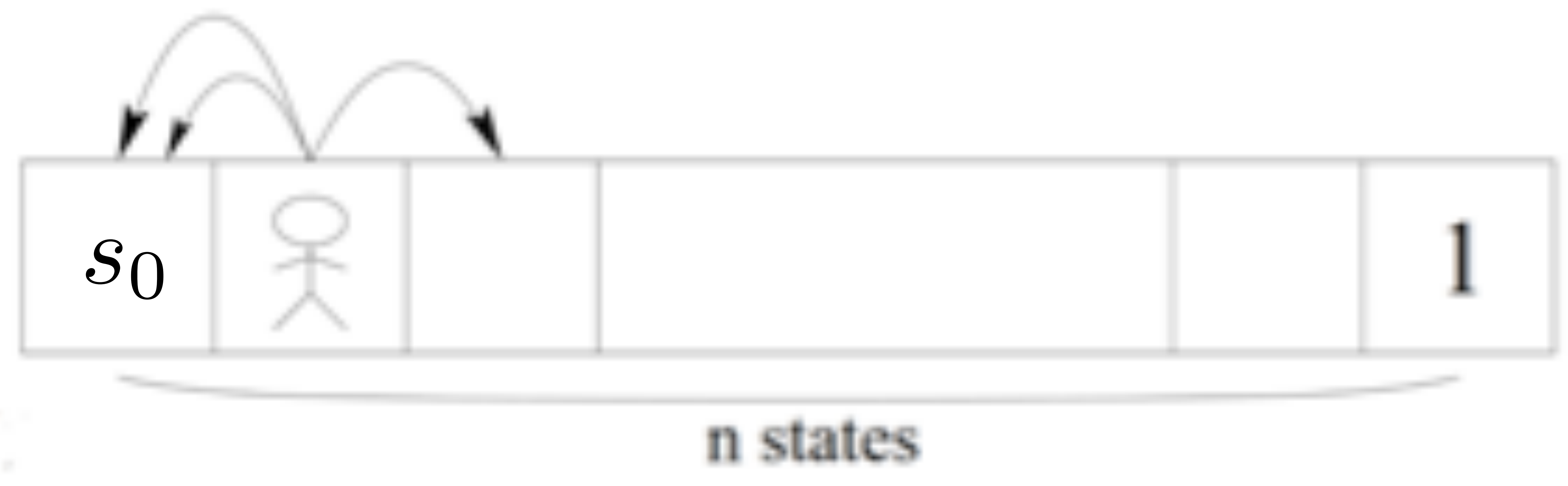

Thrun '92

Length of chain is H

Probability of random walk hitting reward 1 is $(1/3)^{-H}$

Unless we take exponentially many trajs, empirical policy gradient is zero

Common fixes

1. Instead of starting from randomly initialized policy, starting from a good pre-trained policy

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BC -> PG (e.g., GPT3->ChatGPT)

2. Reset based on some informative state distribution (e.g., explorative distribution, expert demos)

e.g., start uniform randomly along the chain instead of just s_0

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Summary

The advantage A^π based policy gradient formulation

PPO: the cliping trick to stabilize training; fast, efficient, and scalable

PG can stuck at local optimal; use BC or reset to help PG succeed