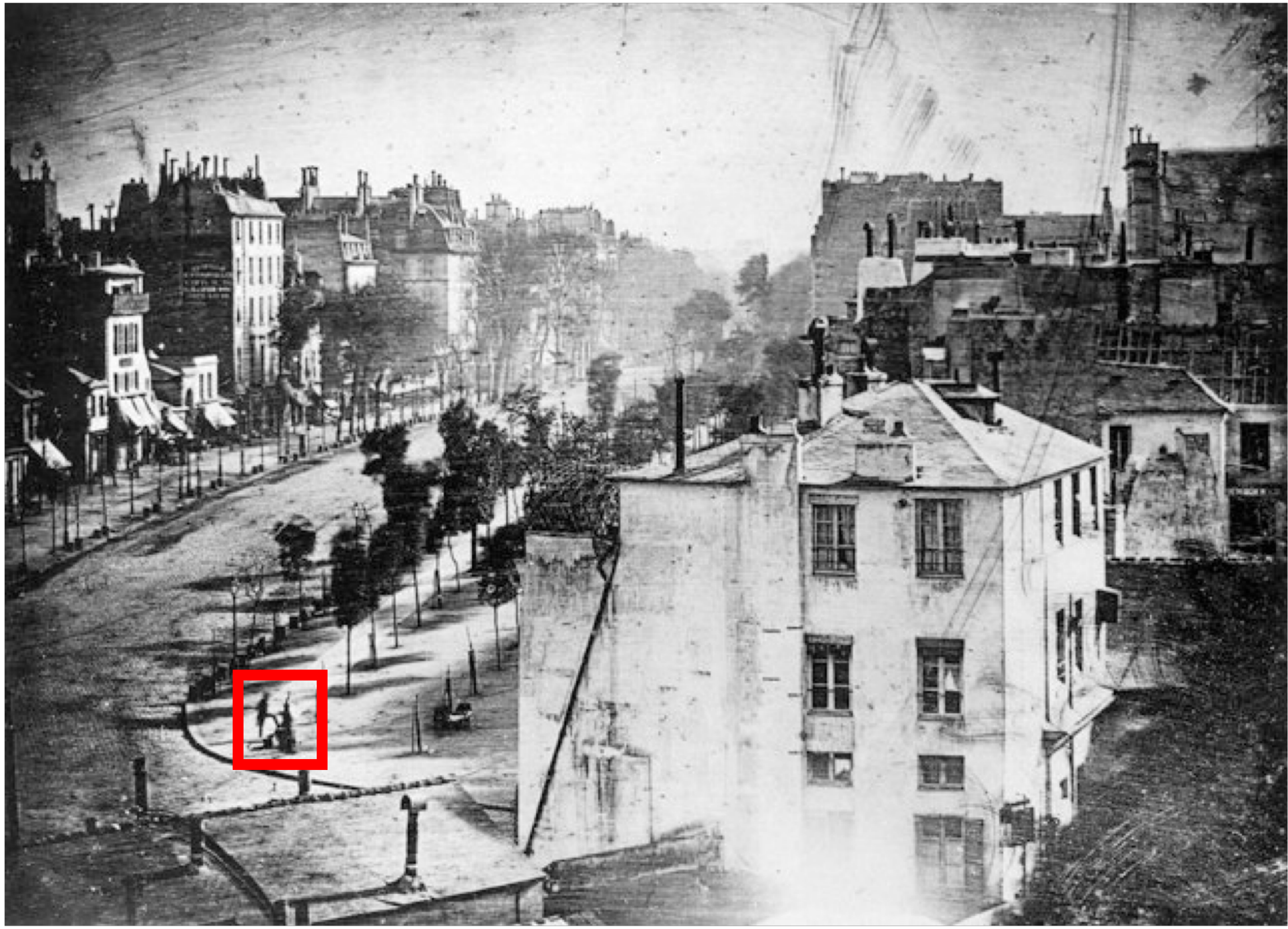
Till now

- Convolution
- Fourier transforms

Image resizing

Why do we need to talk about resizing?

- Need to zoom in to a region to get more details
- Can we get more details?



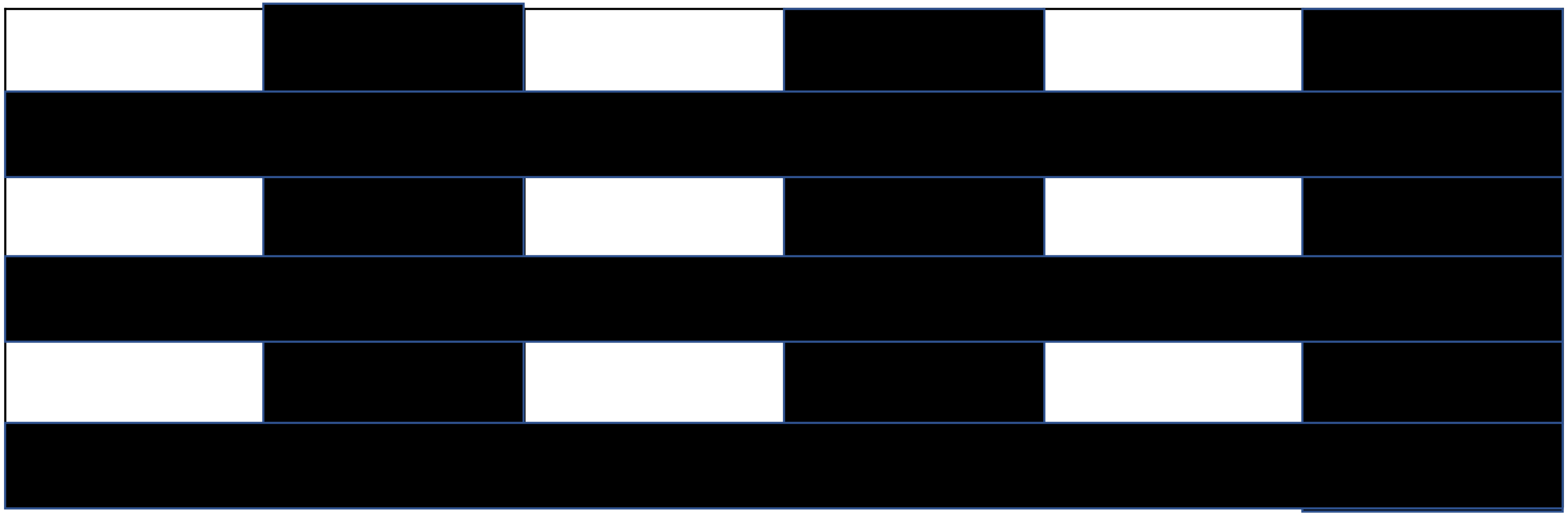
Louis Daguerre, 1838

Why do we need to talk about resizing?

- Far away objects appear small, nearby objects appear larger
- Need to recognize objects at multiple scales
- Resizing images to same size helps recognition

Why is resizing hard?

- E.g, consider reducing size by a factor of 2
- Simple solution: subsampling
- Example: subsampling by a factor of 2



Why is resizing hard?

- Dropping pixels causes problems

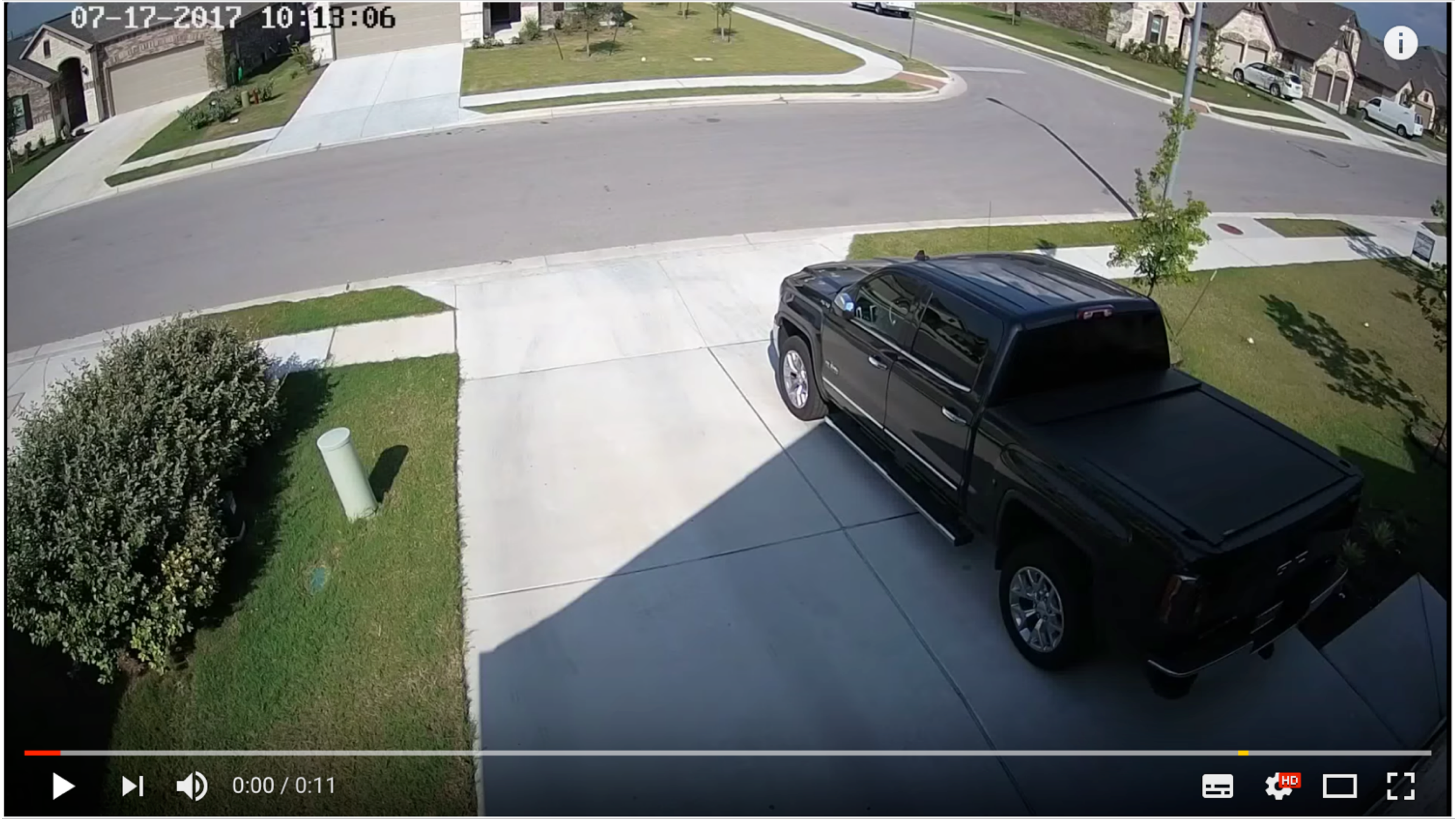
Before

After subsampling

Aliasing!

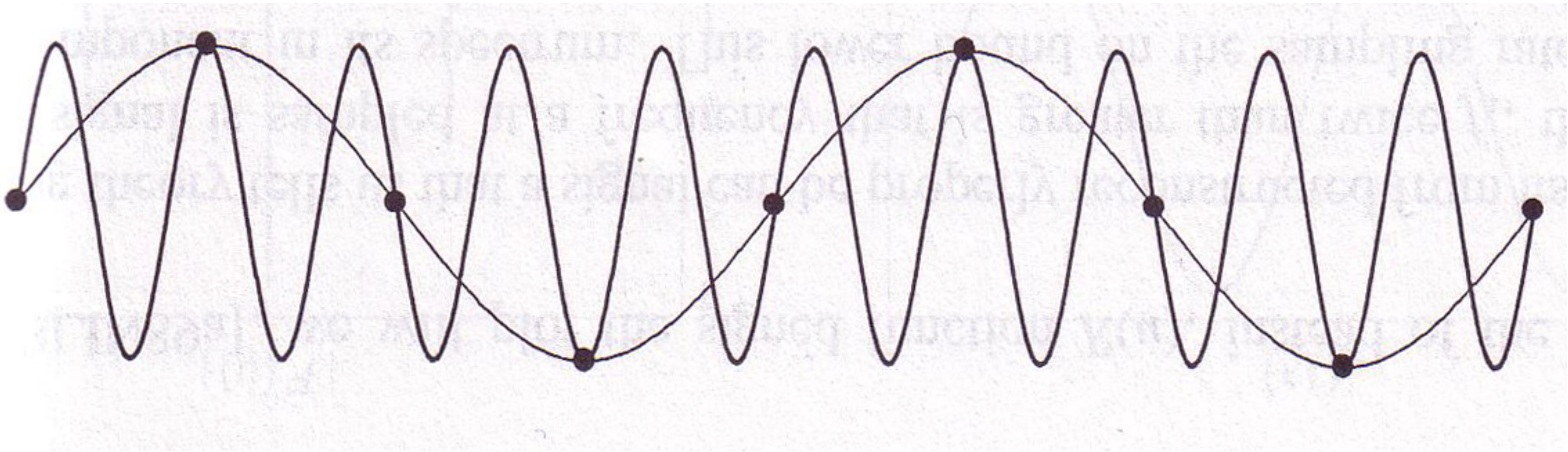
Aliasing in time

Aliasing in time



Why does aliasing happen?

- We ”miss” things between samples
- High frequency signals might appear as low frequency signals
- Called “aliasing”



Let’s look at it mathematically

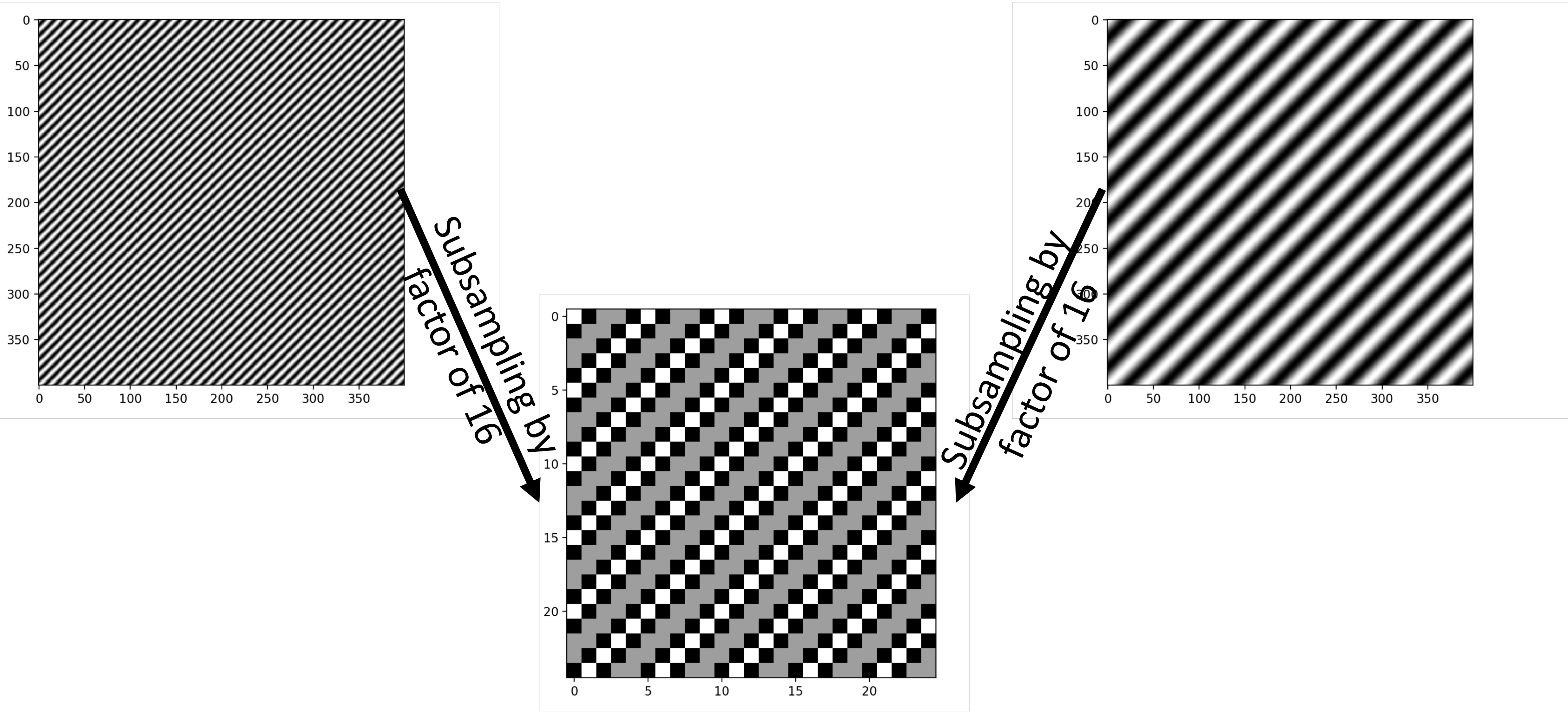
- Fourier basis signals
  - $B_k(n) = e^{\frac{i2\pi kn}{N}}$, time period N/k, frequency k/N
- Suppose we sample every P pixels. Then we are only looking at the values
  - $[B_k(P), B_k(2P), \dots, B_k(nP), \dots]$
- Consider two basis vectors B_k and $B_{k+N/P}$
- $B_{k+N/P}\ (nP) = e^{\frac{i2\pi\left(k+\frac{N}{P}\right)Pn}{N}} = e^{\frac{i2\pi kPn}{T}+i2\pi n(\frac{N}{P})(\frac{P}{N})} = \ e^{\frac{i2\pi kPn}{T}+i2\pi n}$

$$= e^{\frac{i2\pi kPn}{N}} = B_k(nP)$$

Let's look at it mathematically

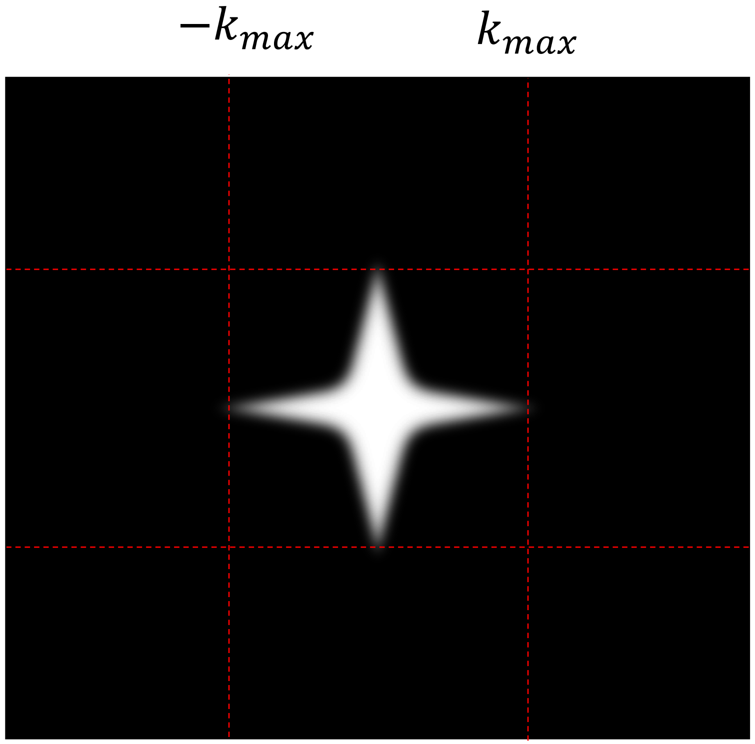
- $B_k(nP) = B_{k+\frac{N}{P}}(nP)$
- The two basis vectors look identical if we sample every P pixels
- Our eyes see the lower frequency: aliasing
- How do we avoid aliasing?
  - *Remove high frequencies that may be aliased*
  - *Sample at a high enough rate (lower P)*

Subsampling Fourier basis



Avoiding aliasing

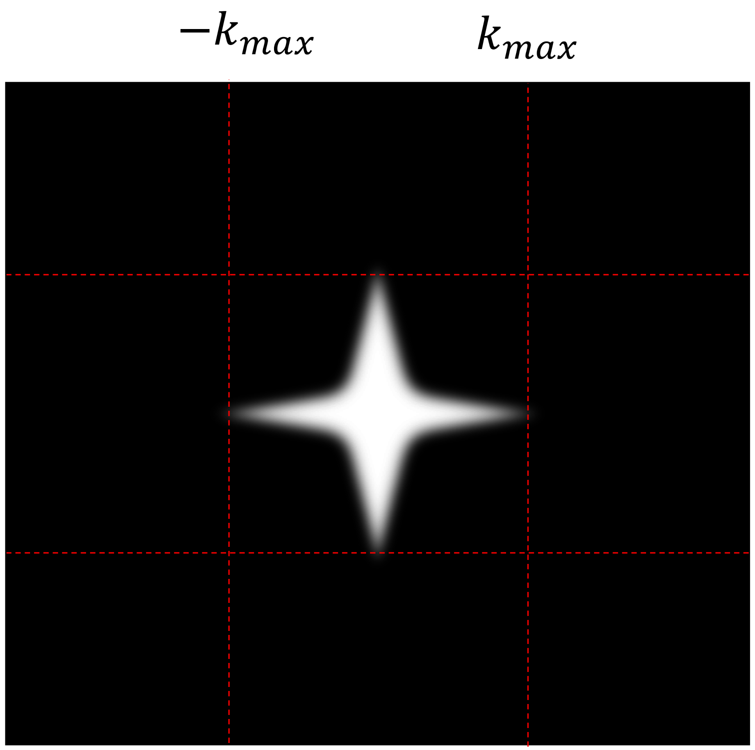
- Suppose image has limited high frequency components
- Fourier transform of x is X
- Suppose X is non-zero only for $-k_{max}$ to k_{max}



Avoiding aliasing

- When sampling once every P pixels, $-k_{max}$ is indistinguishable from $-k_{max} + N/P$
- If image has both coefficients, they will get mixed
- So we want that :

$$k_{max} < -k_{max} + \frac{N}{P} \Rightarrow 2k_{max} < \frac{N}{P}$$



Nyquist Sampling Theorem

- $2k_{max} < \frac{N}{P}$
- Basis B_k has a time period of $\frac{N}{k}$ or frequency of $\frac{k}{N}$
-

$$
\begin{aligned}
2k_{max} &< \frac{N}{P} \\
\Rightarrow \frac{2k_{max}}{N} &< \frac{1}{\mathrm{P}} \\
\Rightarrow 2\nu_{max} &< \nu_{sample}
\end{aligned}
$$

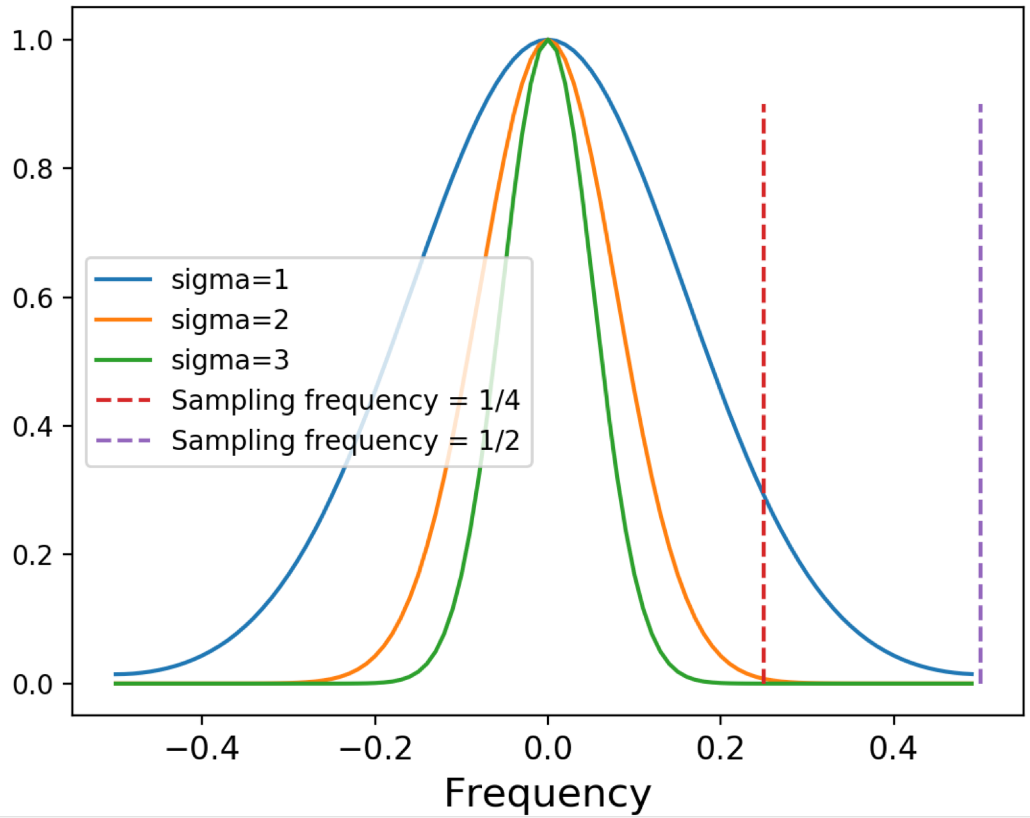
- **Need to sample at at least twice the rate of the highest frequency component**

Avoiding aliasing

- So how to avoid aliasing?
- Make sure we don't subsample too much
  - Sample at at least twice maximum frequency
- *Or remove high frequency components before subsampling*
  - We lose details, but acceptable consequence of reducing image size
- How do we remove high frequency components?
  - *Low-pass filtering: blur !*
- ***Blur with Gaussian, then subsample!***

Subsampling the image

- First smooth the image to remove high frequency components
- How should we smooth?
- One simple answer: blur with Gaussian
  - Recall that Fourier transform of Gaussian is Gaussian
  - Choose sigma so that frequencies above $\frac{\nu_{sample}}{2}$ get killed

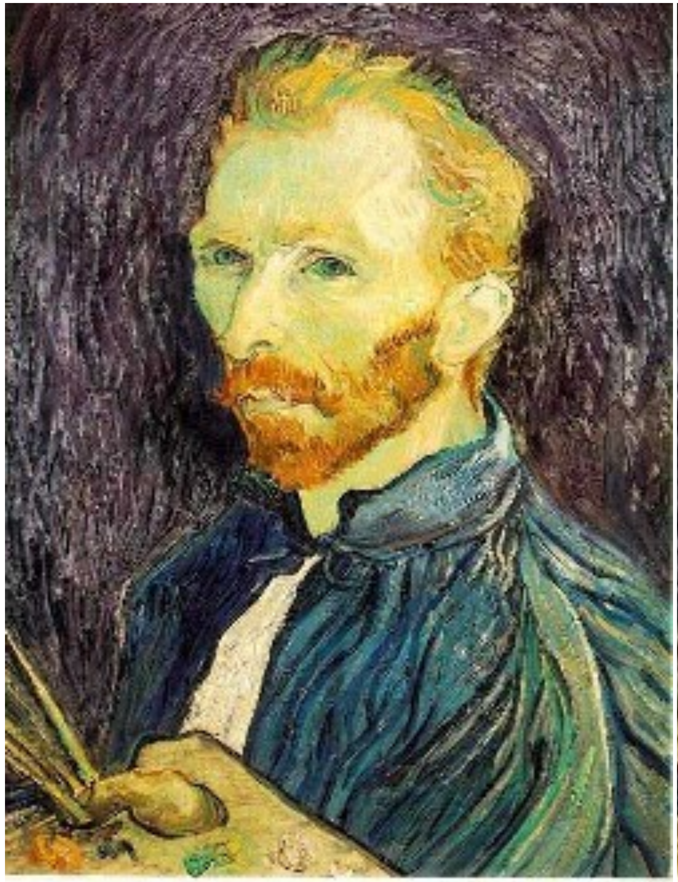


Subsampling before and after smoothing

Before

After

Another example



1/2

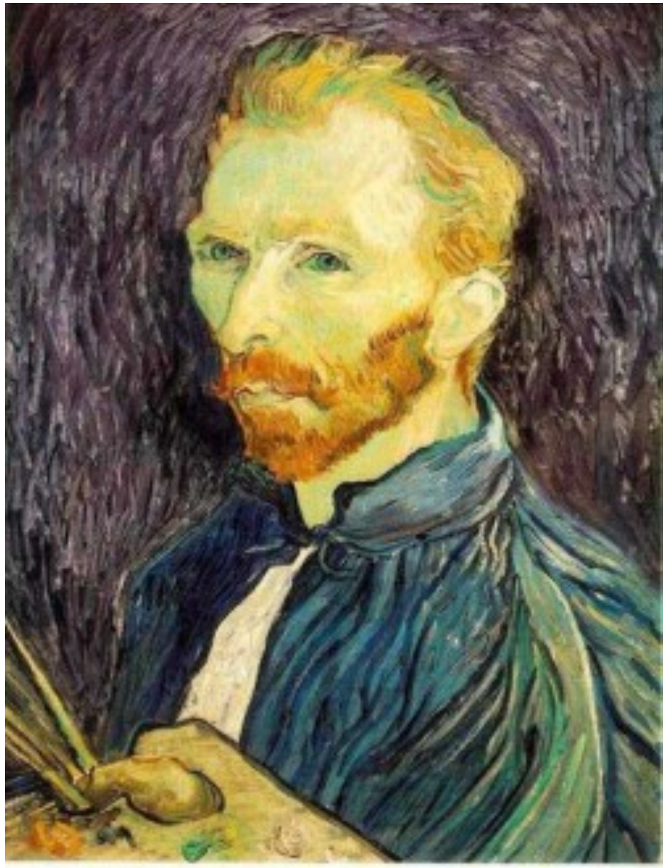
1/4 (2x zoom)

1/16 (4x zoom)

Why does this look so crufty? Aliasing!

Source: S. Seitz

Subsampling images correctly



Gaussian 1/2

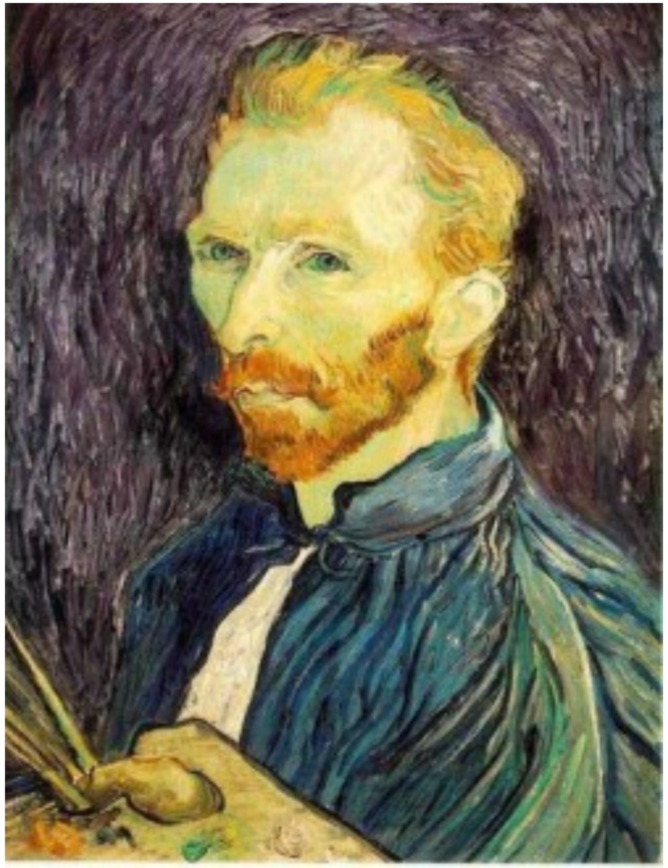
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

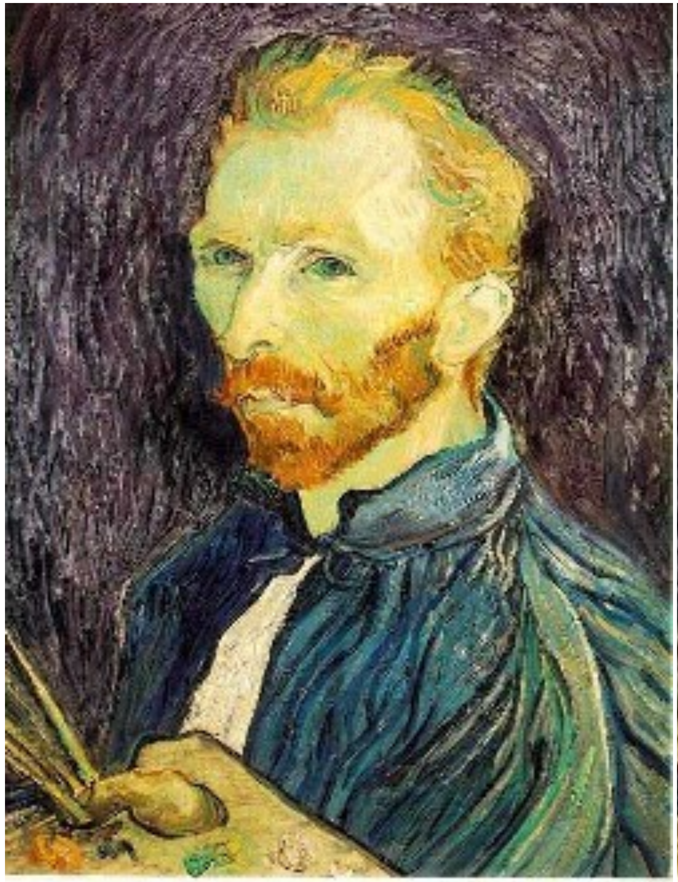
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Compare with...



1/2

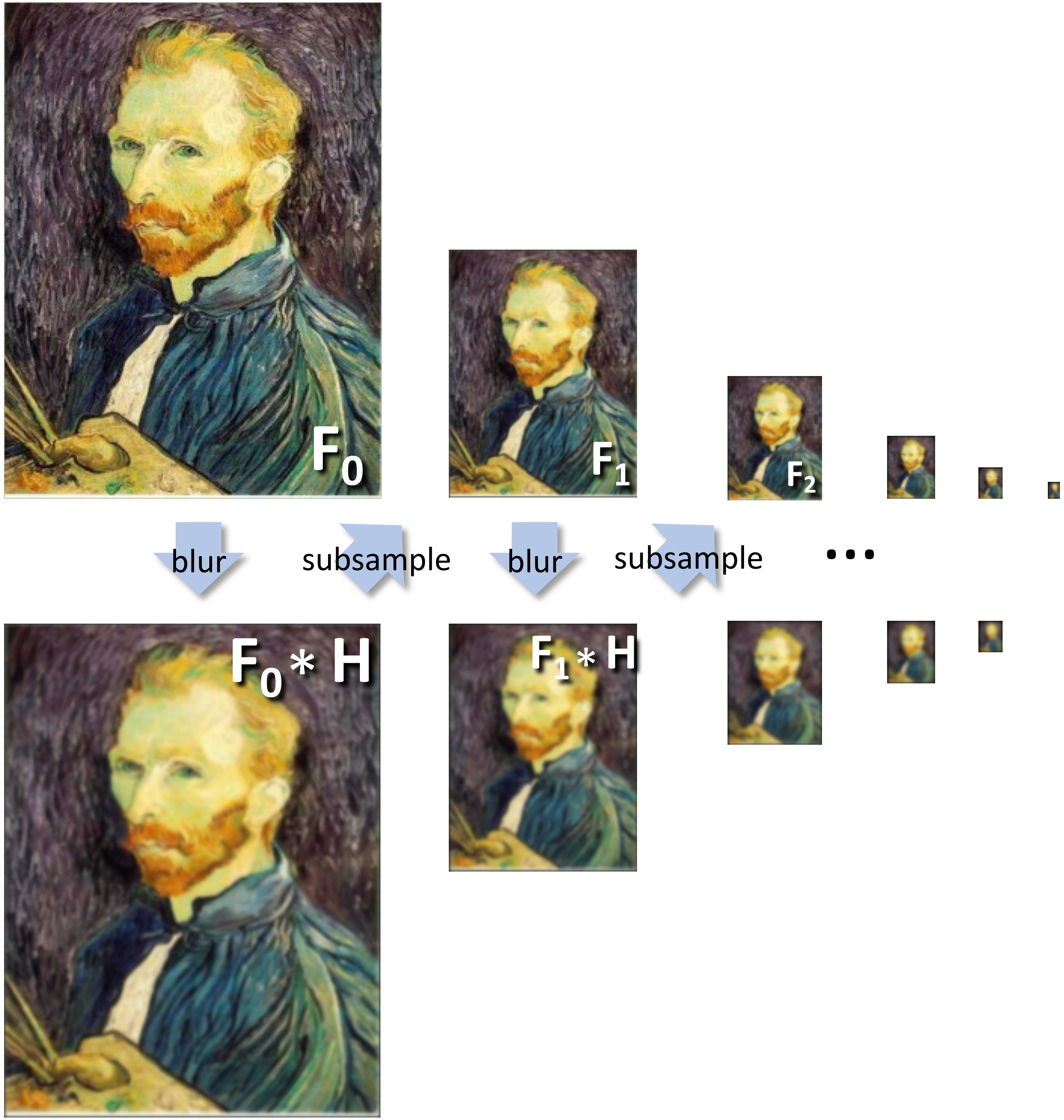
1/4 (2x zoom)

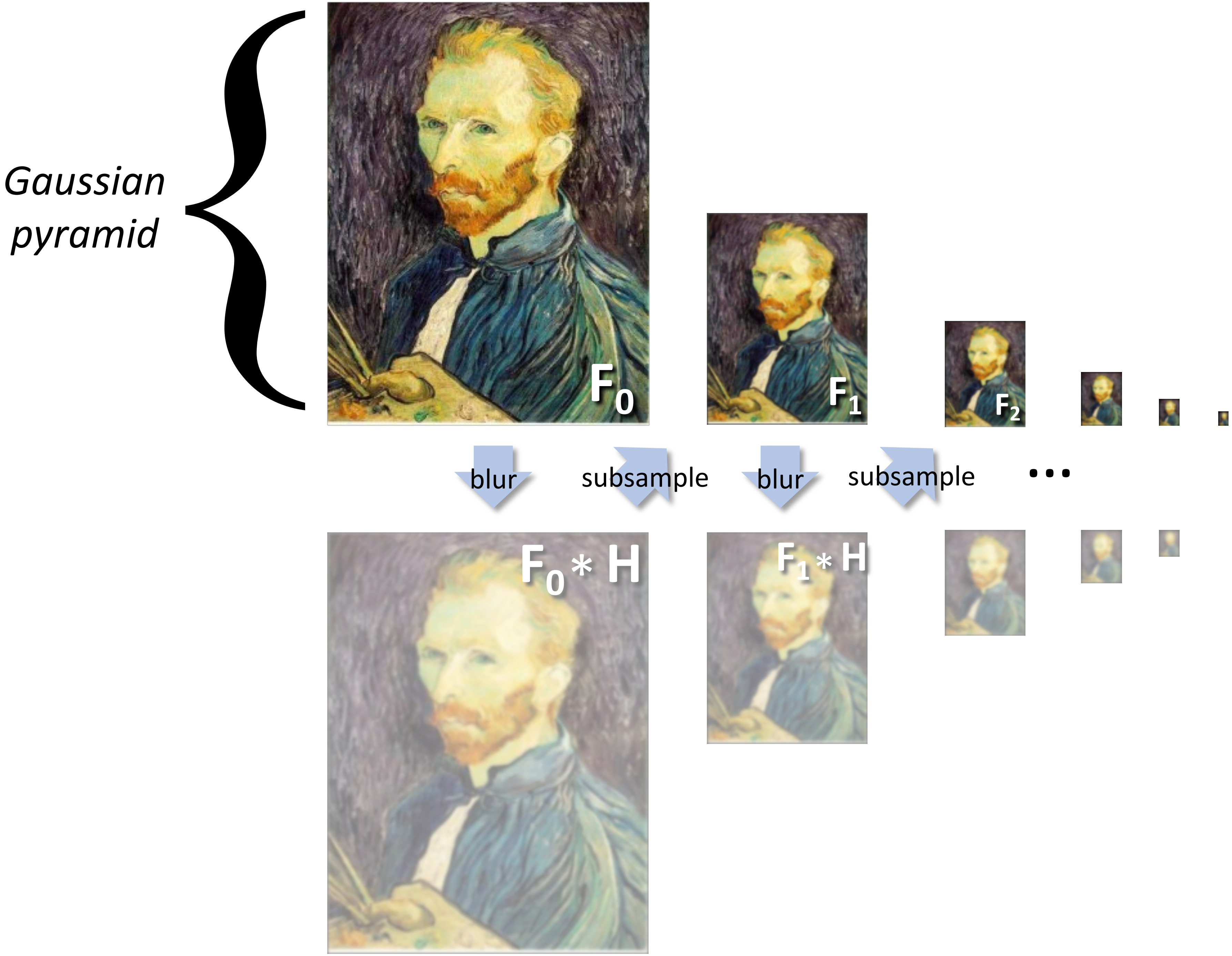
1/8 (4x zoom)

Source: S. Seitz

Gaussian pre-filtering

- Solution: filter the image, *then* subsample

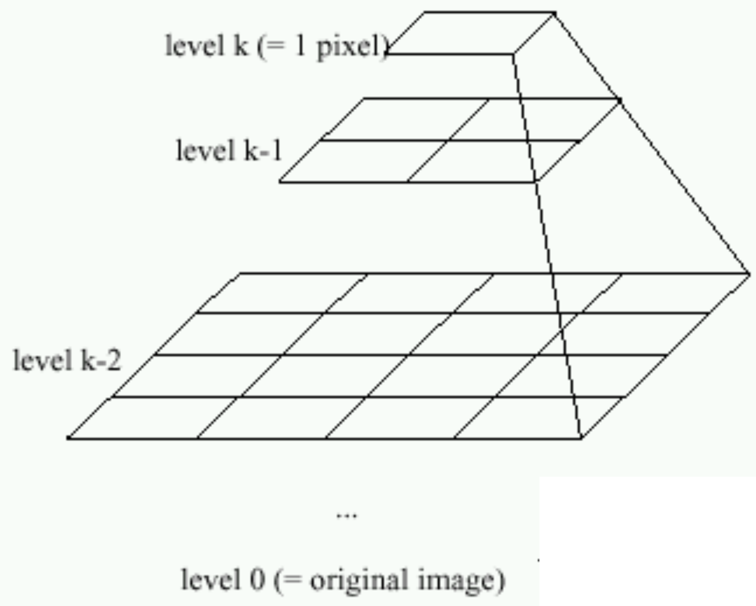




Gaussian pyramid
F0
F1
F2
blur
subsample
blur
subsample
...
F0 * H
F1 * H

Gaussian pyramids
[Burt and Adelson, 1983]

Idea: Represent NxN image as a “pyramid” of $1 \times 1, 2 \times 2, 4 \times 4, \ldots, 2^k \times 2^k$ images (assuming $N=2^k$)

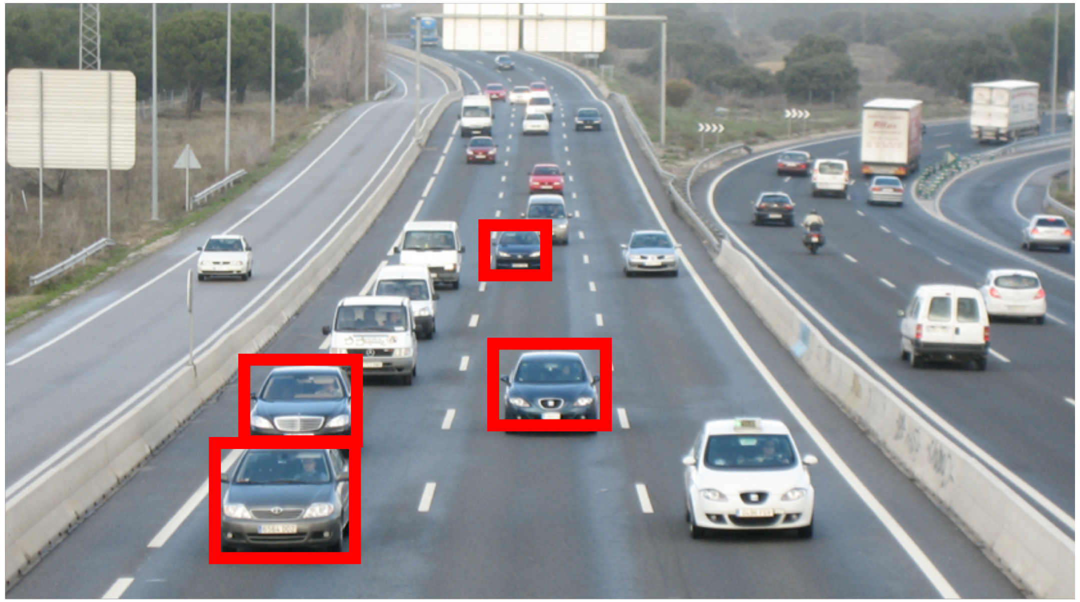


- In computer graphics, a *mip map* [Williams, 1983]

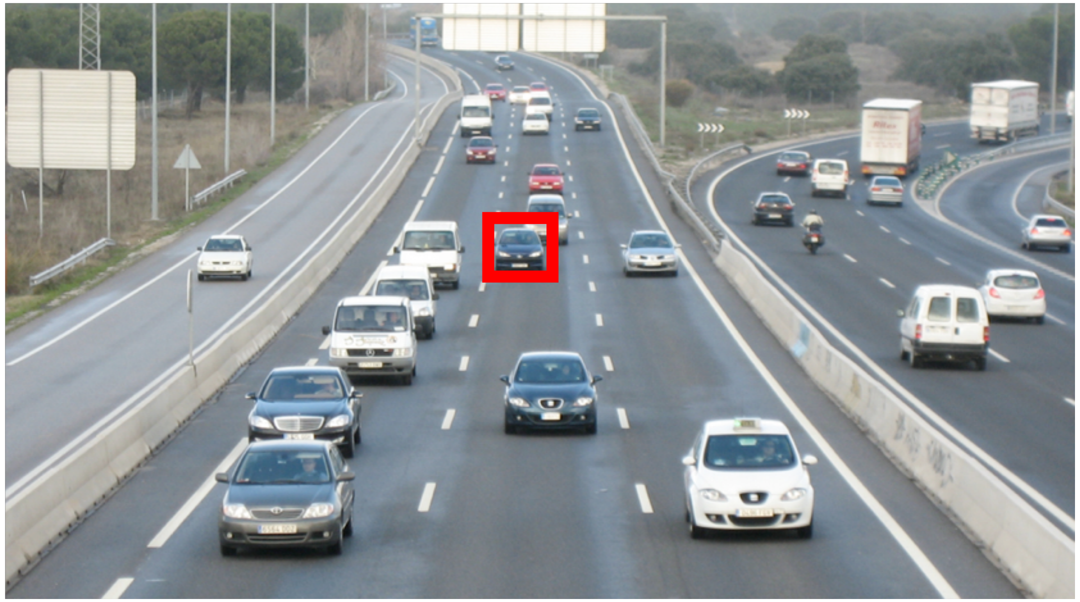
Gaussian Pyramids have all sorts of applications in computer vision

Source: S. Seitz

Gaussian pyramids - Searching over scales

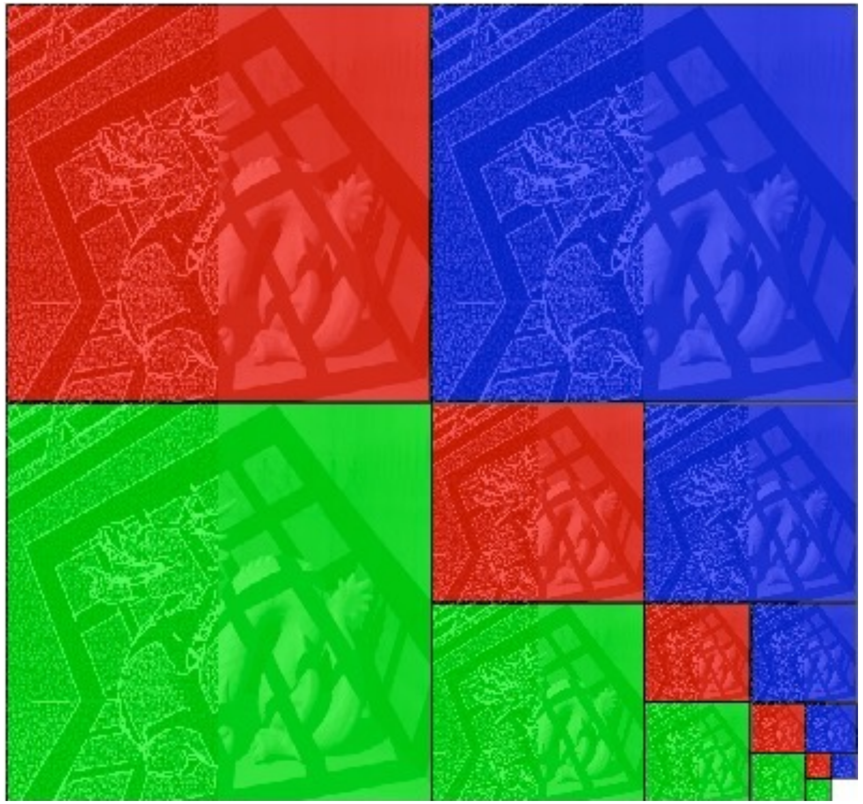


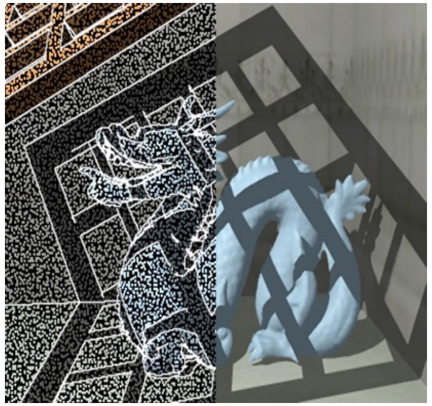
Gaussian pyramids - Searching over scales



Memory Usage

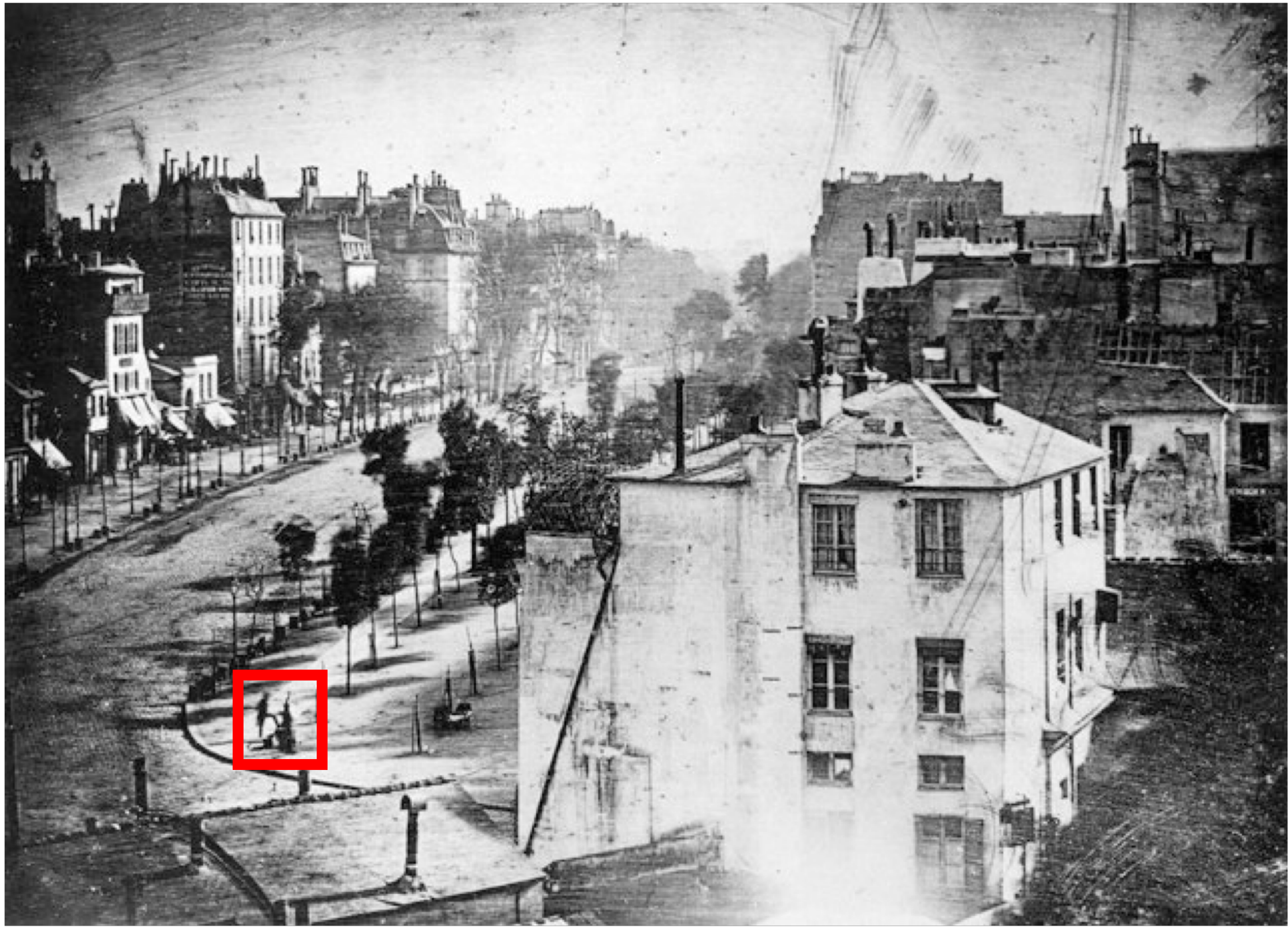
- What is the size of the pyramid?
  - Each color below is one pyramid
  - Together they are twice the original image in each dimension





Going the other way

- Need to zoom in to a region to get more details
- Can we get more details?
- When we reduced size, we had to remove high frequency components
- Unlikely we can get those back



Louis Daguerre, 1838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