CS4670 / 5670: Computer Vision

KavitaBala

Lecture 15: Projection

“The School of Athens,” Raphael

Announcements

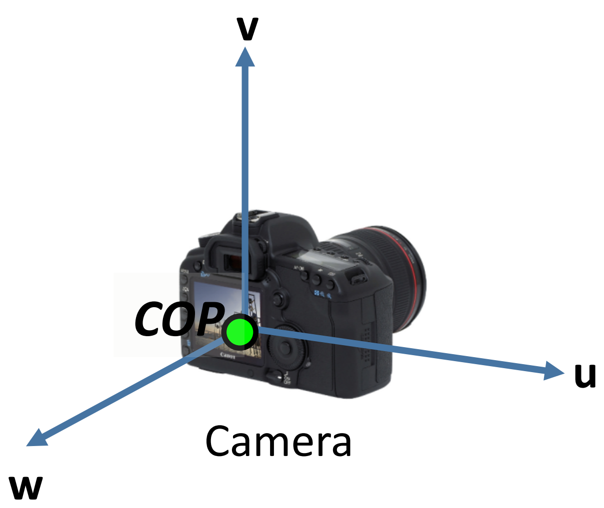
- Prelim on Thu
  - Everything before this slide
  - Bring your calculator
  - **7:30 pm, Location: Call Auditorium, Kennedy Hall**

Camera parameters

- How many numbers do we need to describe a camera?

- We need to describe its *pose* in the world
- We need to describe its internal parameters

A Tale of Two Coordinate Systems



Camera

"The World"

Two important coordinate systems:

1. *World* coordinate system
2. *Camera* coordinate system

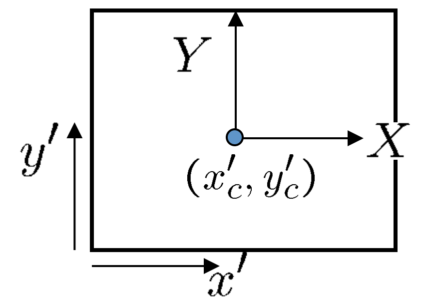
Camera parameters

- To project a point (*x*,*y*,*z*) in *world* coordinates into a camera
- First transform (*x*,*y*,*z*) into *camera* coordinates
- Need to know
  - Camera position (in world coordinates)
  - Camera orientation (in world coordinates)
- Then project into the image plane
  - Need to know camera *intrinsics*
- These can all be described with matrices

Camera parameters

Projection equation

$$\mathbf{x} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$



- The projection matrix models the cumulative effect of all parameters

Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principal point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called “extrinsics,” red are “intrinsics”

Projection equation

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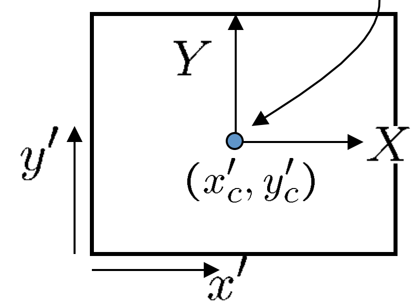

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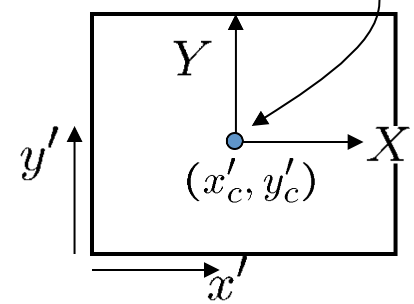


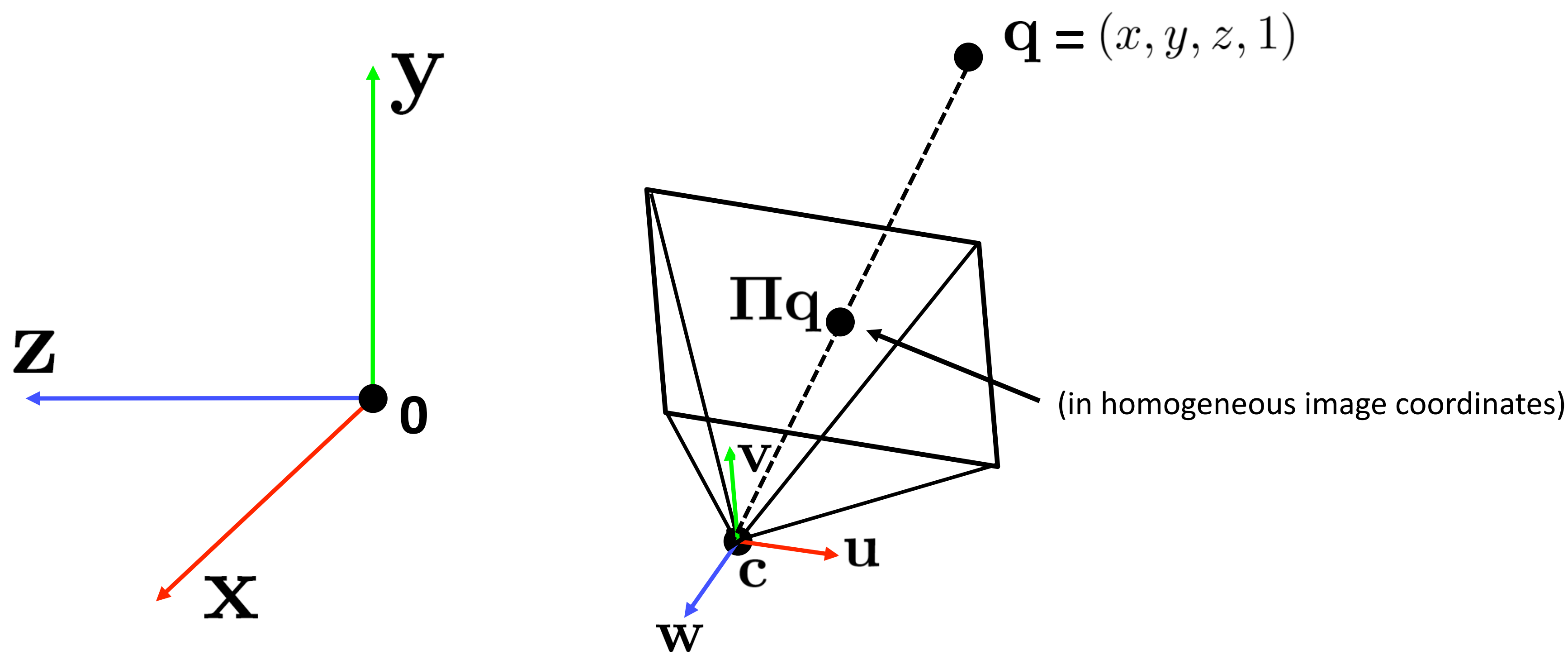
- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

identity matrix

$$\mathbf{\Pi} = \underbrace{\begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{translation}}$$

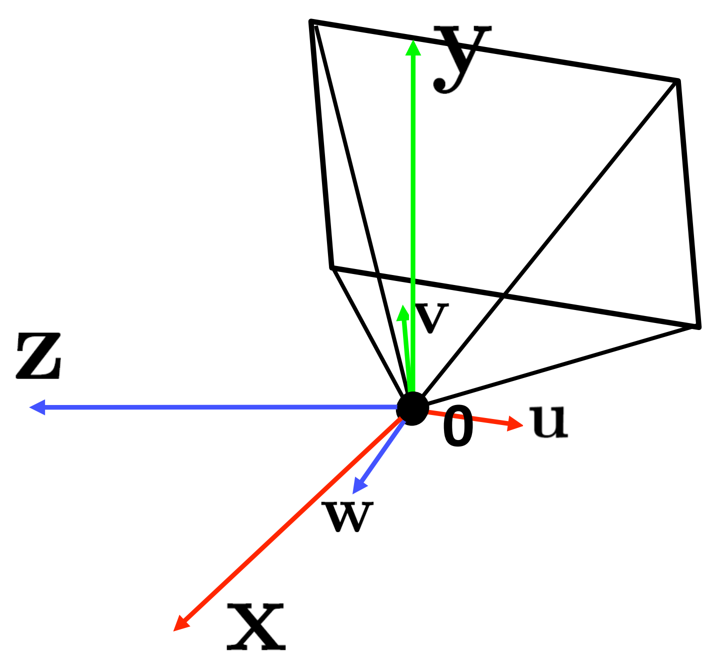
- The definitions of these parameters are **not** completely standardized
  - especially intrinsics—varies from one book to another

Projection matrix

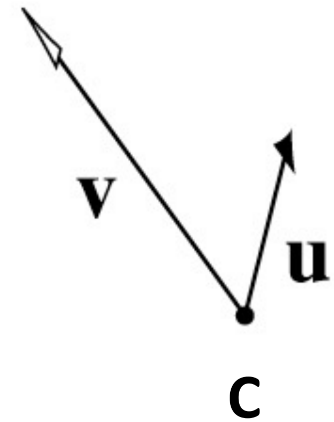


Extrinsics

- How do we get the camera to “canonical form”?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Affine change of coordinates



- Coordinate frame: point plus basis
- Need to change representation of point from one basis to another
- Canonical: origin (0,0) w/ axes e1, e2

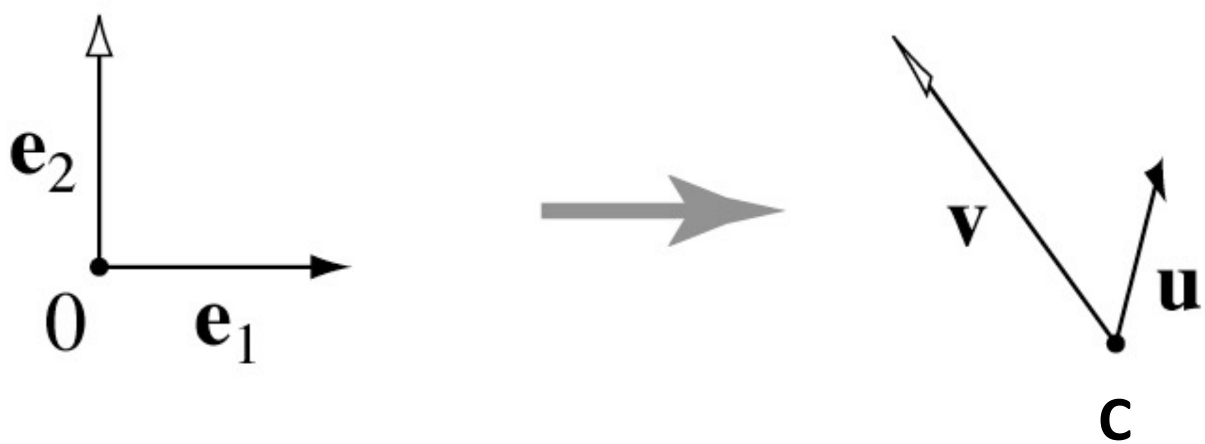
- "Frame to canonical" matrix has frame in columns
  – takes points represented in frame
  – represents them in canonical basis

$$\begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{c} \\ 0 & 0 & 1 \end{bmatrix}$$

Recap from 4620

Another way of thinking about this

- Change of coordinates



Recap from 4620

On the Board

Recap from 4620

Coordinate frame summary

- Frame = point plus basis
- Frame matrix (frame-to-canonical) is

$$F = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{c} \\ 0 & 0 & 1 \end{bmatrix}$$

- Move points to and from frame by multiplying with *F*

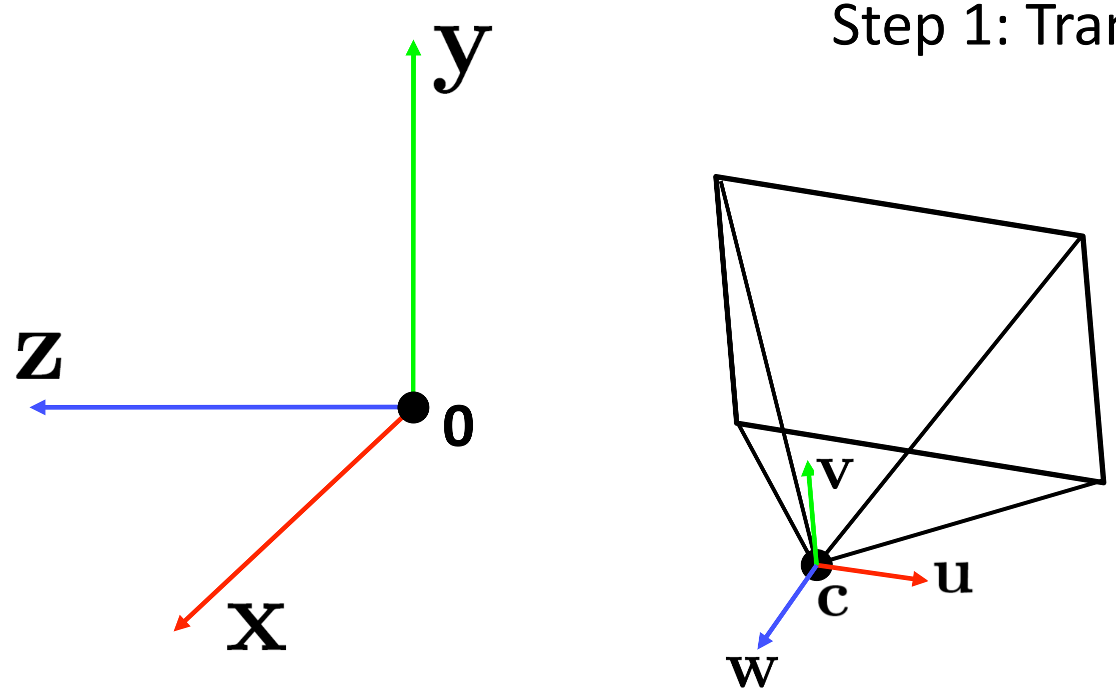
$$p_e = F p_F \quad p_F = F^{-1} p_e$$

Recap from 4620

Extrinsics

- How do we get the camera to “canonical form”?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

Step 1: Translate by -**c**



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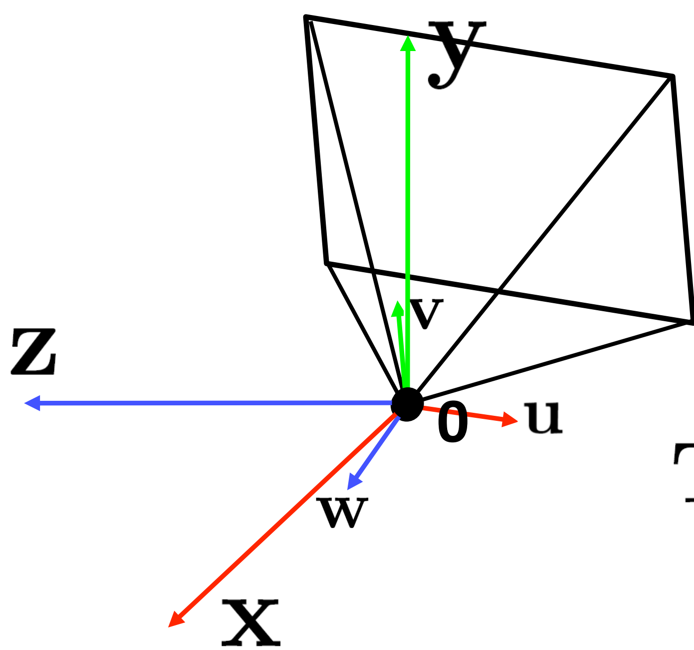

Step 1: Translate by -**c**

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

Extrinsics

- How do we get the camera to "canonical form"?
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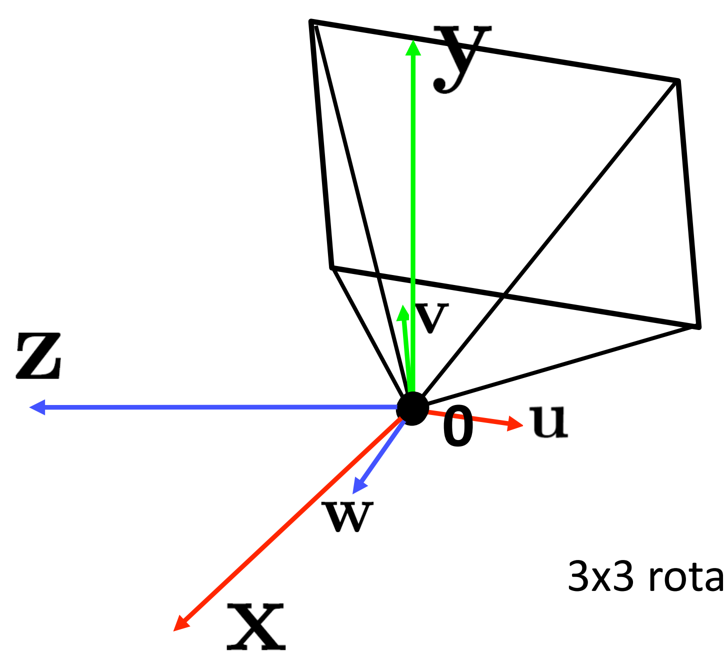


Step 1: Translate by -**c**
Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

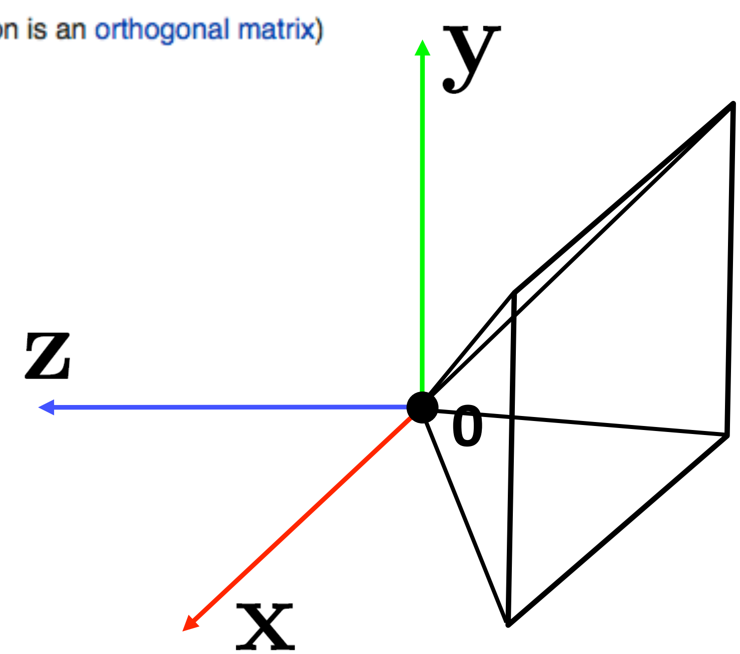
3x3 rotation matrix

Extrinsics

- How do we get the camera to “canonical form”?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

For any rotation matrix R acting on $\mathbb{R}^n$,

- $R^T = R^{-1}$ (The rotation is an orthogonal matrix)



Step 1: Translate by -**c**

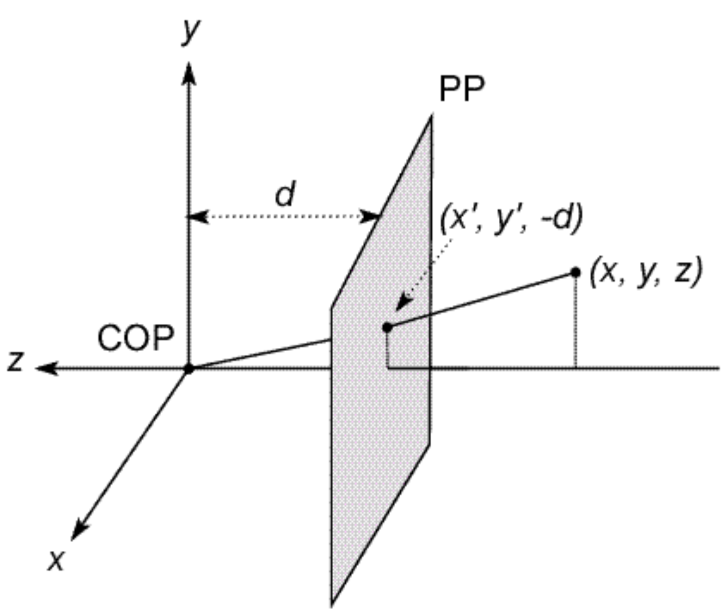
Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

Perspective projection

$$\underbrace{\begin{bmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsics) (converts from 3D rays in camera coordinate system to pixel coordinates)



$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z},\ -d)$$

Perspective projection

$$\underbrace{\begin{bmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$\mathbf{K}$ (intrinsics) (converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} -f & s & c_x \\ 0 & -\alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$ (upper triangular matrix)

α : **aspect ratio** (1 unless pixels are not square)

s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

(c_x, c_y) : **principal point** ((0,0) unless optical axis doesn't intersect projection plane at origin)

Projection matrix

$$\mathbf{\Pi} = \underset{\text{intrinsics}}{\mathbf{K}} \underbrace{\underset{\text{projection}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \underset{\text{rotation}}{\begin{bmatrix} \mathbf{R} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}} \underset{\text{translation}}{\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}}}$$

$$\left[\ \mathbf{R}\ \middle|\ \underbrace{-\mathbf{Rc}}_{\text{(}\mathbf{t}\text{ in book's notation)}}\ \right]$$

$$\mathbf{\Pi} = \mathbf{K} \left[\ \mathbf{R}\ \middle|\ -\mathbf{Rc}\ \right]$$

Focal length

- Can think of as "zoom"

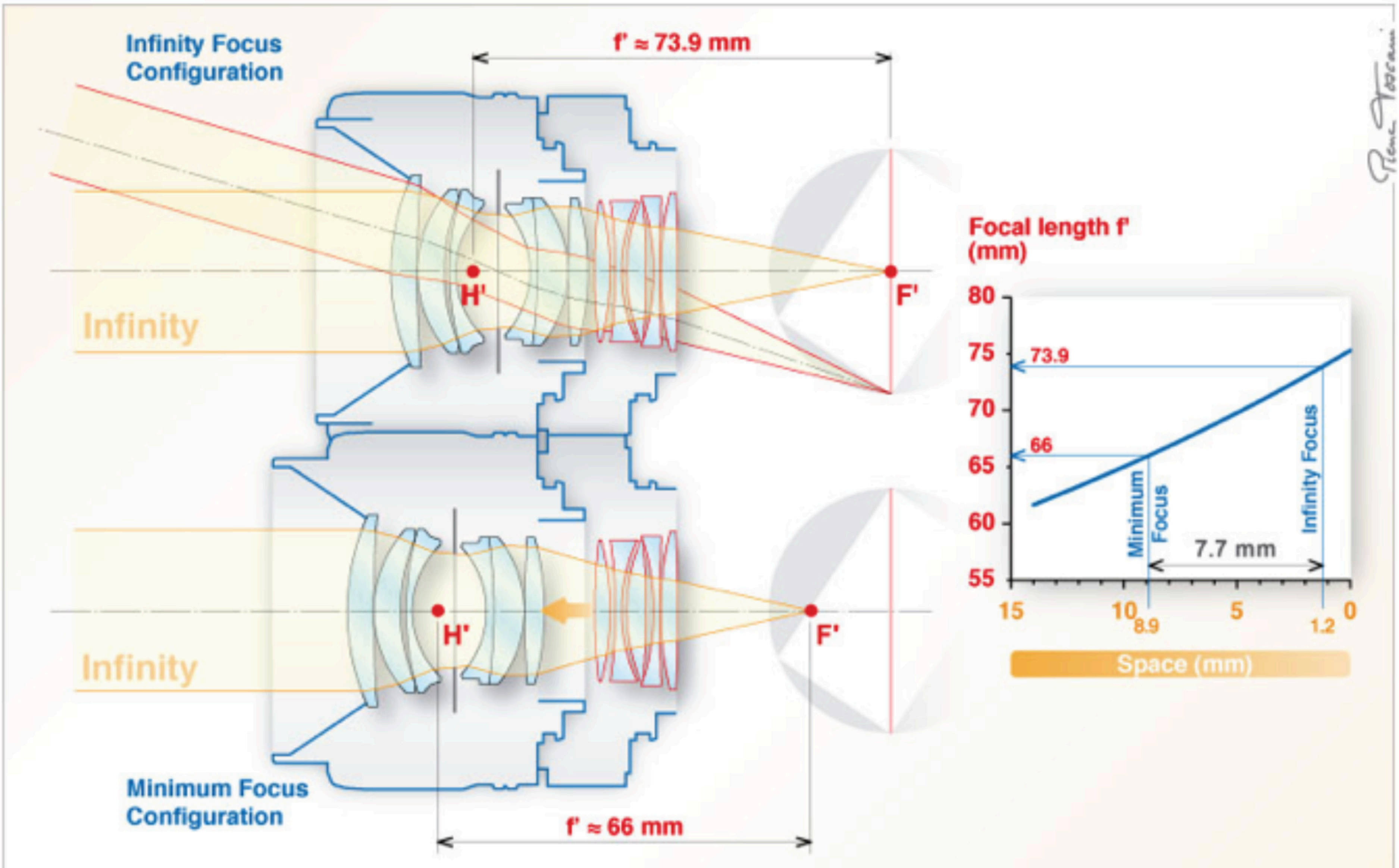
24mm

50mm

200mm

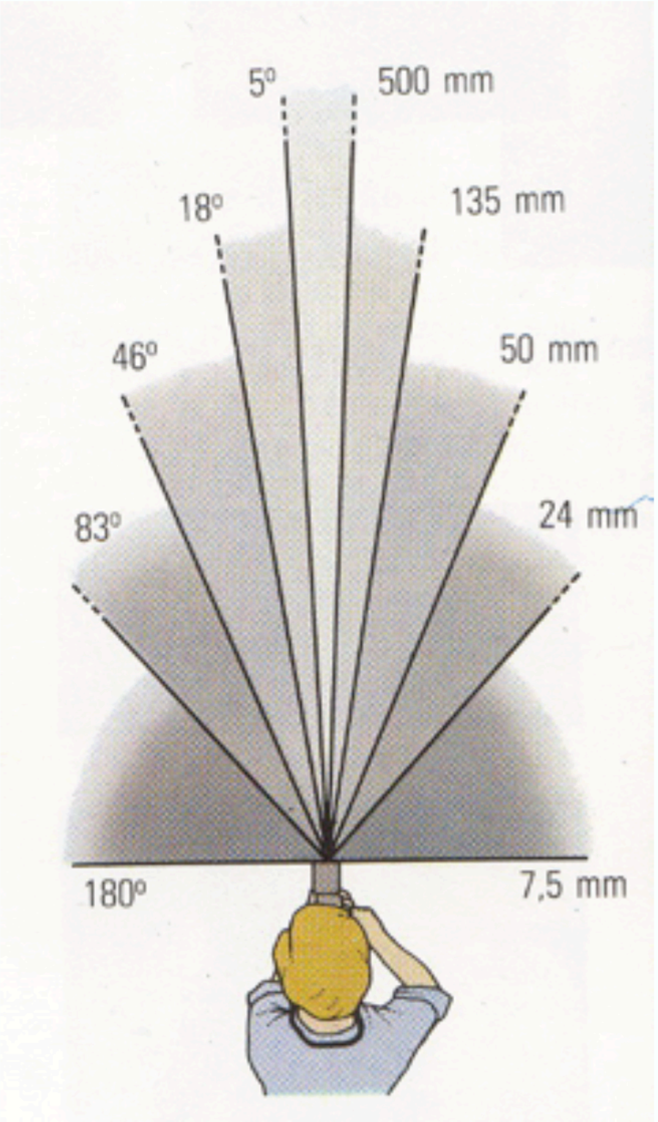
800mm

- Related to *field of view*



http://www.pierretoscani.com/echo_focal_length.html

Focal length in practice



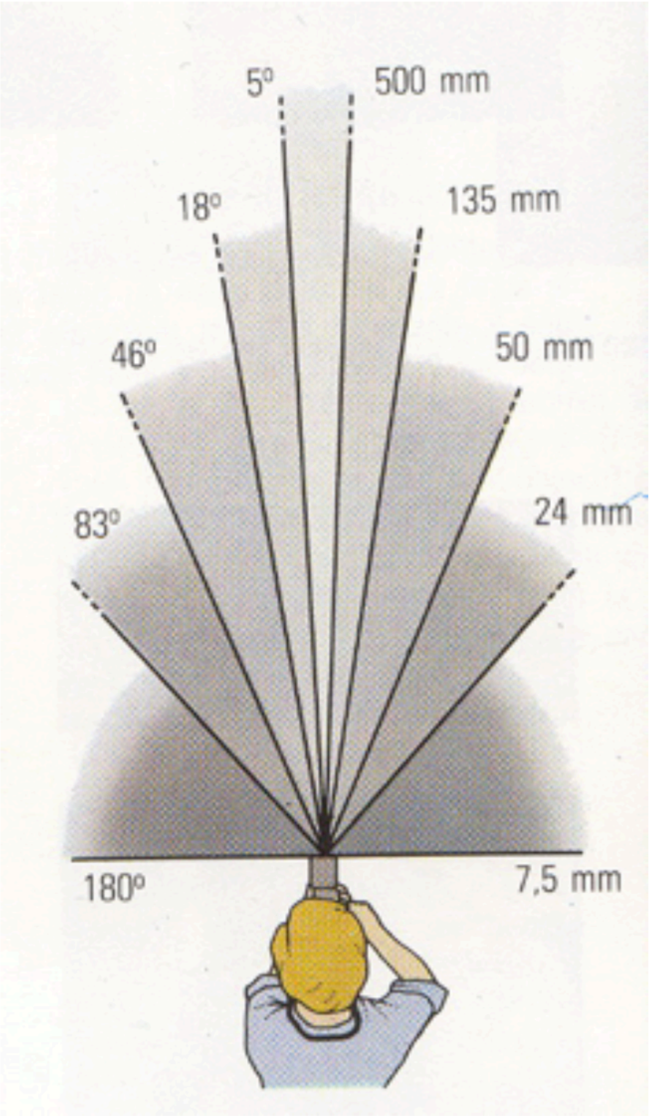
24mm

50mm

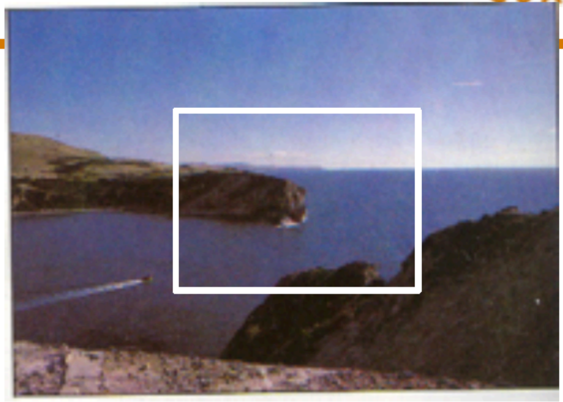
135mm

Fredo Durand

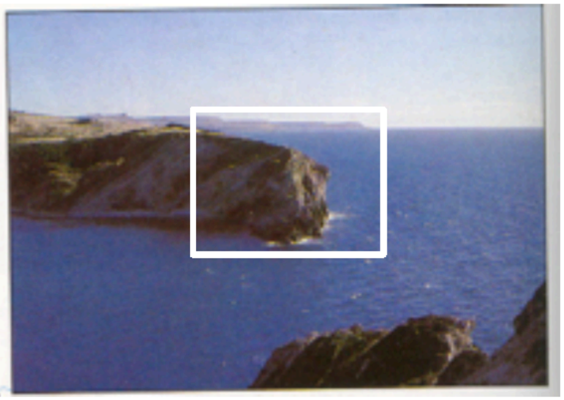
Focal length = cropping



24mm



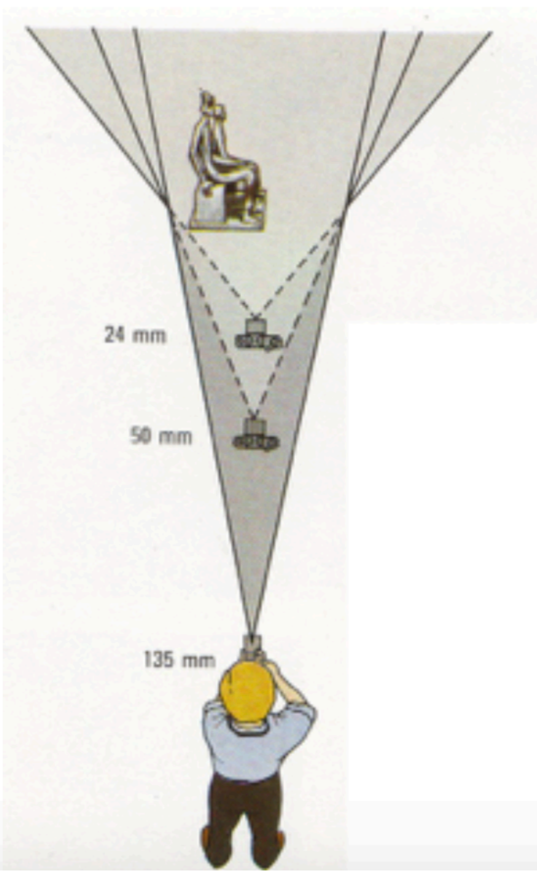
50mm



135mm

Focal length vs. viewpoint

- **Telephoto makes it easier to select background (a small change in viewpoint is a big change in background.**




Grand-angulaire 24 mm


Normal 50 mm


Longue focale 135 mm

Fredo Durand

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- http://www.slate.com/blogs/browbeat/2014/01/21/dolly_zoom_supercut_video_shows_the_vertigo_effect_in_jaws_goodfellas_raging.html

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