CS 465 Final Exam

Monday 11 December 2006—2.5 hours

Problem 1: Gamma correction (15 pts)

Consider the following formats for storing pixel data in a framebuffer:

a) 8 bit integer, linearly quantized.

b) 8 bit integer, non–linearly quantized with $\gamma = 2$.

c) 16 bit integer, linearly quantized.

1. How well do these formats perform in areas of an image that are near 10% gray (i.e. the output intensity is 10% of the maximum white value)? Rank them in order of quality (that is, list the format with lowest quantization error first).
2. How well do these formats perform in areas of an image that are near 90% gray? Rank them in order of quality (that is, list the format with lowest quantization error first).
3. Which format provides the best overall performance, and why?

Problem 2: Ray Tracing (15 points)

Consider the following pseudocode for intersecting a ray with a surface.

```
function surfaceIntersect(Ray r)
    Point3 p = r.origin;
    Vector3 d = r.direction;
    [tmin,tmax] = quadraticSolve(d.x*d.x + d.y*d.y,
                                 2*(p.x*d.x + p.y*d.y),
                                 p.x*p.x + p.y*p.y - 1);
    if (tmin == null) return INFINITY;
    z1 = p.z + tmin*d.z;
    if (tmin > 0 && z1 < 1 && z1 > -1) return tmin;
    z2 = p.z + tmax*d.z;
    if (tmax > 0 && z2 < 1 && z2 > -1) return tmax;
    return INFINITY;
```

Assume that the function `quadraticSolve(a,b,c)` returns the solutions to the quadratic equation $ax^2 + bx + c = 0$ in ascending order, or `NULL` if no real solutions exist.

Give a complete and precise description of the surface that this function intersects.

Problem 3: Transformation matrices (20 points)

Each of the matrices in this problem represents an affine transformation (in homogeneous coordinates) that can be described as exactly one of the following types.

a) A non–zero translation

b) A rotation around an axis through the origin

c) A reflection across an axis (in 2D) or a plane (in 3D) through the origin

d) A shear along an axis (in 2D) or a plane (in 3D) through the origin

e) A non–uniform scale with a positive scale factor along an axis through the origin

For each matrix below, state which type a) – e) it is, and give the specific quantities of the transformation (for translations, give the translation vector; for rotations, give the axis and angle of rotation; for reflections, give the normal to the axis/plane of reflection; for shears, give the normal to the axis/plane of the shear and the shear factor; and for scales give the axis and the positive scale factor).

Note that this is not a matching exercise; there may be matrix types that occur more than once, or types that do not appear at all. However, each problem matrix has exactly 1 correct matrix type.

$$1.\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad 2.\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3.\ \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$4.\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 5.\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 6.\ \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 4: Curves (15 points)

Consider a Bézier curve with the following control points:

$$\mathbf{p}_0 = (1,0)$$
$$\mathbf{p}_1 = (1,1)$$
$$\mathbf{p}_2 = (-1,1)$$
$$\mathbf{p}_3 = (0,0)$$

Recall that the equation for a Bézier curve is

$$\mathbf{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix}$$

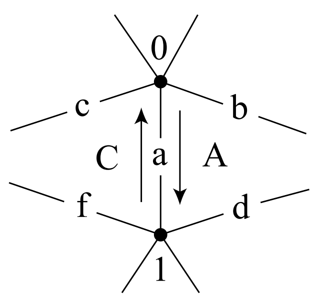
What is the minimal bounding box for this curve? That is, what are the minimum and maximum x and y values that the curve achieves?

Problem 5: Meshes (20 points)

Consider the closed polygonal mesh described by the following edge table of a winged-edge data structure:

edge	start vertex	end vertex	face L	face R	pred L	succ L	pred R	succ R
a	0	1	A	C	b	d	f	c
b	0	2	B	A	c	e	d	a
c	0	3	C	B	a	f	e	b
d	1	2	A	D	a	b	h	g
e	2	3	B	E	b	c	i	h
f	1	3	F	C	g	i	c	a
g	1	4	D	F	d	h	i	f
h	2	4	E	D	e	i	g	d
i	3	4	F	E	f	g	h	e

Note that we are using the conventions from Shirley et al., so that edge a in the table above implies the following mesh structure, viewed from outside the mesh:

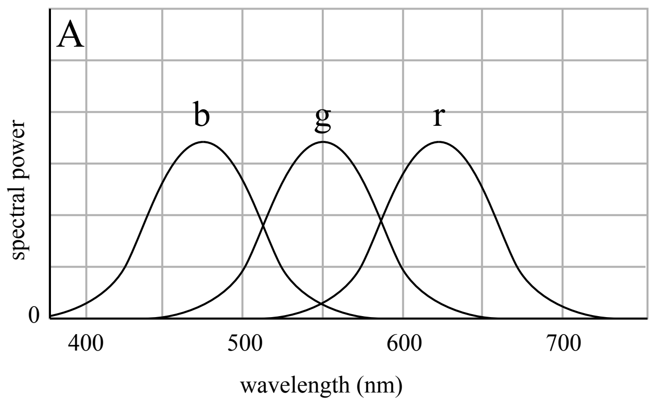


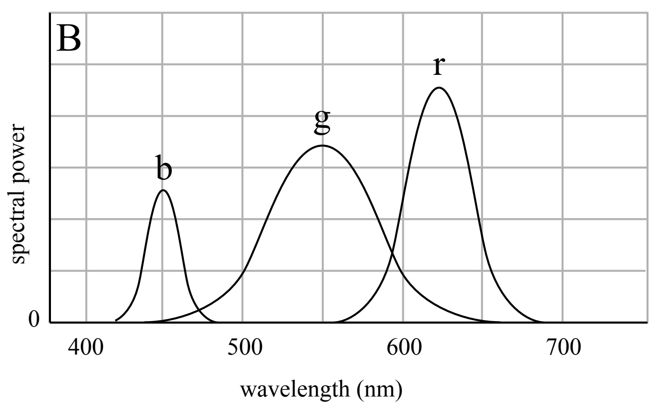
1. List all vertices adjoining edge g.
2. List all faces adjoining edge h.
3. List all edges adjoining face B.
4. List all vertices adjoining face D.
5. List all edges adjoining vertex 2.
6. List all faces adjoining vertex 1.

For questions 3 – 6, list the items in counter–clockwise order when viewed from outside the mesh.

Problem 6: Color (15 points)

Consider the following two sets of spectra, A and B:





1. For each of the three spectra in figure B, is the saturation greater than, less than, or roughly the same as the corresponding spectrum in figure A?
2. For each of the three spectra in figure B, is the hue significantly different from or roughly the same as the corresponding spectrum in figure A?
3. If you were to choose one of the sets of spectra A or B to use as primaries in a display device, which would be the better choice and why?