CS 4120
Introduction to Compilers

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Lecture 24: Data-flow, control-flow analysis

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"Classic" constant propagation

- Idea: propagate and fold integer constants in one pass

```
x = 1;        x = 1;
y = 5+x;      y = 6;
z = y*y;      z = 36;
```

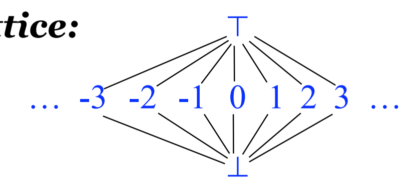
- Information about a single variable:
  - i. Variable never defined
  - ii. Variable has single constant value
  - iii. Variable has multiple values

One-variable Const. Prop.

never defined ⊓ *constant* = *constant*

constant c_1 ⊓ *constant* c_2 = *multiple*

Full lattice:



Rest of definition

- Flow function for $x = x$ OP c_1:

  $F_n(\top) = \top$

  $F_n(\bot) = \bot$

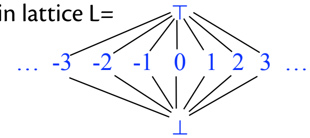
  $F_n(c_2) = c_2 \text{ OP } c_1$

- Flow function is monotonic, distributive: iterative solution works, gives MOP
- What about multiple variables $x_1 \ldots x_n$?

  Want tuple $(v_1, \ldots v_n)$,

Multiple vars

- Dataflow value is a tuple $(v_1,\ldots v_n)$, each v_i in lattice L=



- Set of tuples $(v_1,\ldots,v_n)$ is also a lattice under component-wise ordering:

$$(v_1,\ldots,v_n) \sqsubseteq (v'_1,\ldots,v'_n) \Leftrightarrow \forall_i\, v_i \sqsubseteq v'_i$$

$$(v_1,\ldots,v_n) \sqcap (v'_1,\ldots,v'_n) = (v_1 \sqcap v'_1,\ldots,v_n \sqcap v_n)$$

- For any two lattices L_1, L_2, have *product lattice* $L_1 \times L_2$ $(v_1, v_2) \sqsubseteq (v'_1, v'_2) \Leftrightarrow v_1 \sqsubseteq v'_1$ & $v_2 \sqsubseteq v'_2$

- Tuple dataflow values are in $L \times \ldots \times L = L^n$

Flow functions

- Consider $x_1 = x_2$ OP x_3

  $F(x_1, \top, x_3) = (\top, \top, x_3)$

  $F(x_1, x_2, \top) = (\top, x_2, \top)$

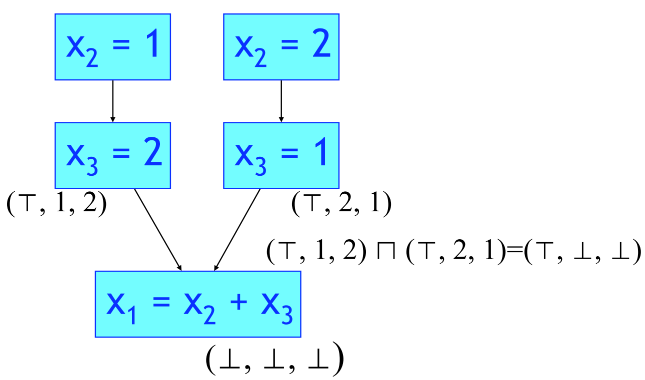
  $F(x_1, \bot, x_3) = (\bot, \bot, x_3)$

  $F(x_1, x_2, \bot) = (\bot, x_2, \bot)$

  $F(x_1, c_2, c_3) = (c_2 \text{ OP } c_3, c_2, c_3)$

- Monotonic? Distributes over $\sqcap$?

Not MOP!



$F((\top, 1, 2) \sqcap (\top, 2, 1)) \neq F(\top, 1, 2) \sqcap F(\top, 2, 1)$

Loops

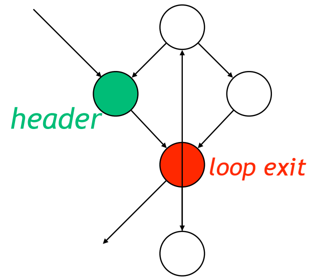
- Most execution time in most programs is spent in loops: 90/10 is typical
- Most important targets of optimization: loops
- Loop optimizations:
  - loop-invariant code motion
  - loop unrolling
  - loop peeling
  - strength reduction of expressions containing induction variables
  - removal of bounds checks
  - loop tiling
- When to apply loop optimizations?

High-level optimization?

- Loops may be hard to recognize in IR or quadruple form -- should we apply loop optimizations to source code or high-level IR?
  - Many kinds of loops: while, do/while, continue
  - loop optimizations benefit from other IR-level optimizations and vice-versa -- want to be able to interleave
- Problem: identifying loops in flowgraph

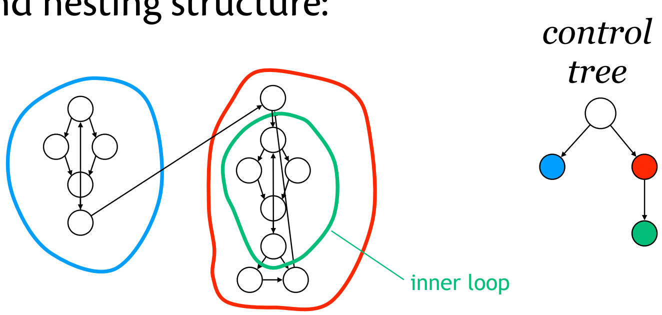
Definition of a loop

- A *loop* is a set of nodes in the control flow graph, with one distinguished node called the *header* (entry point)
- Every node is reachable from header, header reachable from every node: *strongly-connected component*
- No entering edges from outside except to header
- nodes with outgoing edges: *loop exit nodes*



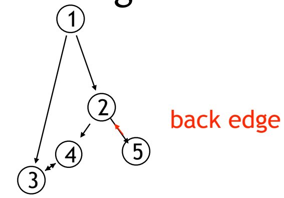
Nested loops

- Control-flow graph may contain many loops, and loops may contain each other
- *Control-flow analysis* : identify the loops and nesting structure:



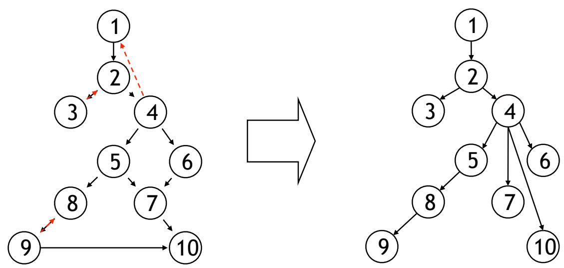
Dominators

- CFA based on idea of *dominators*
- Node A *dominates* node B if the only way to reach B from start node is through A
- Edge in flowgraph is a *back edge* if destination dominates source
- A loop contains at least one back edge



Dominator tree

- Domination is transitive; if A dominates B and B dominates C, then A dominates C
- Domination is anti-symmetric
- Every flowgraph has *dominator tree* (Hasse diagram of domination relation)



Dominator dataflow analysis

- Forward analysis; out[n] is set of nodes dominating n
- "A node $\mathbf{B}$ is dominated by another node $\mathbf{A}$ if $\mathbf{A}$ dominates *all* of the predecessors of $\mathbf{B}$"

$$\mathrm{in}[n] = \bigcap_{n' \in \mathrm{pred}[n]} \mathrm{out}[n']$$

- "Every node dominates itself"

$$\mathrm{out}[n] = \mathrm{in}[n] \cup \{n\}$$

- Formally: L = sets of nodes ordered by $\subseteq$, flow functions $F_n(x) = x \cup \{n\}$, $\sqcap = \cap$, $\top = \{\text{all } n\}$
  $\Rightarrow$ Standard iterative analysis gives best soln

Completing control-flow analysis

- Dominator analysis gives all back edges
- Each back edge $n \rightarrow h$ has an associated *natural loop* with h as its header: all nodes reachable from h that reach n without going through h
- For each back edge, find natural loop
- Nest loops based on subset relationship between natural loops
- Exception: natural loops may share same header; merge them into larger loop.
- Control tree built using nesting relationship