CS412/CS413

Introduction to Compilers

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Lecture 28: Dataflow Analysis Instances

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Dataflow Analysis

- Dataflow analysis
  - sets up system of equations
  - iteratively computes MFP
  - Terminates because transfer functions are monotonic and lattice has finite height
- Other possible solutions: FP, MOP, IDEAL
- All are safe solutions, but some are more precise:

  FP ⊑ MFP ⊑ MOP ⊑ IDEAL
- MFP = MOP if transfer functions are distributive
- MOP and IDEAL are intractable
- Compilers use dataflow analysis and MFP

Dataflow Analysis Instances

- Apply dataflow framework to several analysis problems:
  - Live variable analysis
  - Available expressions
  - Reaching definitions
  - Constant folding
- Discuss:
  - Implementation issues
  - Classification of dataflow analyses

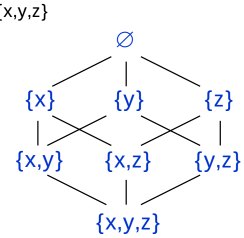
Problem 1: Live Variables

- Compute live variables at each program point
- Live variable = variable whose value may be used later, in some execution of the program
- Dataflow information: sets of live variables
- Example: variables {x,z} may be live at program point p
- Is a backward analysis
- Let V = set of all variables in the program
- Lattice (L, ⊑), where:
  - L = 2^V (power set of V, i.e., set of all subsets of V)
  - Partial order ⊑ is set inclusion: ⊇

    $S_1 \sqsubseteq S_2$ iff $S_1 \supseteq S_2$

LV: The Lattice

- Consider set of variables V = {x,y,z}
- Partial order: ⊇
- Set V is finite implies lattice has finite height
- Meet operator: ∪ (set union: out[B] is union of in[B'], for all B'∈succ(B)
- Top element: ∅ (empty set)



- Smaller sets of live variables = more precise analysis
- All variables may be live = least precise

LV: Dataflow Equations

- Equations:

  in[B] = F_B(out[B]), for all B

  out[B] = ∪{in[B'] | B'∈succ(B)}, for all B

  out[B_e] = X_0
- Meaning of union meet operator:

  "A variable is live at the end of a basic block B if it is live at the beginning of one of its successor blocks"

LV: Transfer Functions

- Transfer functions for basic blocks are composition of transfer functions of instructions in the block
- Define transfer functions for instructions
- General form of transfer functions:
  $F_I(X) = (X - def[I]) \cup use[I]$
  where:
  def[I] = set of variables defined (written) by I
  use[I] = set of variables used (read) by I
- Meaning of transfer functions:
  "Variables live before instruction I include: (1) variables live after I, but not written by I, and (2) variables used by I"

LV: Transfer Functions

- Define def/use for each type of instruction

if I is x = y OP z :	use[I] = {y, z}	def[I] = {x}
if I is x = OP y :	use[I] = {y}	def[I] = {x}
if I is x = y :	use[I] = {y}	def[I] = {x}
if I is x = addr y :	use[I] = {}	def[I] = {x}
if I is if (x) :	use[I] = {x}	def[I] = {}
if I is return x :	use[I] = {x}	def[I] = {}
if I is $x = f(y_1,..., y_n)$:	use[I] = $\{y_1,..., y_n\}$ def[I] = {x}	

- Transfer functions $F_I(X) = (X - def[I]) \cup use[I]$
- For each F_I, def[I] and use[I] are constants: they don't depend on input information X

LV: Monotonicity

- Are transfer functions: $F_I(X) = (X - def[I]) \cup use[I]$ monotonic?
- Because def[I] is constant, X – def[I] is monotonic:
  $X_1 \supseteq X_2$ implies $X_1 - def[I] \supseteq X_2 - def[I]$
- Because use[I] is constant, $Y \cup use[I]$ is monotonic:
  $Y_1 \supseteq Y_2$ implies $Y_1 \cup use[I] \supseteq Y_2 \cup use[I]$
- Put pieces together: $F_I(X)$ is monotonic
  $X_1 \supseteq X_2$ implies
  $(X_1 - def[I]) \cup use[I] \supseteq (X_2 - def[I]) \cup use[I]$

LV: Distributivity

- Are transfer functions: $F_I(X) = (X - def[I]) \cup use[I]$ distributive?
- Since def[I] is constant: X – def[I] is distributive:
  $(X_1 \cup X_2) - def[I] = (X_1 - def[I]) \cup (X_2 - def[I])$
  because: $(a \cup b) - c = (a - c) \cup (b - c)$
- Since use[I] is constant: $Y \cup use[I]$ is distributive:
  $(Y_1 \cup Y_2) \cup use[I] = (Y_1 \cup use[I]) \cup (Y_2 \sqcap use[I])$
  because: $(a \cup b) \cup c = (a \cup c) \cup (b \cup c)$
- Put pieces together: $F_I(X)$ is distributive
  $F_I(X_1 \cup X_2) = F_I(X_1) \cup F_I(X_2)$

Live Variables: Summary

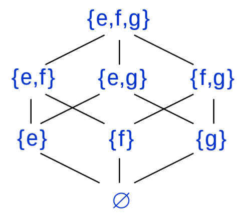
- Lattice: $(2^V, \supseteq)$; has finite height
- Meet is set union, top is empty set
- Is a backward dataflow analysis
- Dataflow equations:
  $in[B] = F_B(out[B])$, for all B
  $out[B] = \cup\{in[B'] \mid B' \in succ(B)\}$, for all B
  $out[B_e] = X_0$
- Transfer functions: $F_I(X) = (X - def[I]) \cup use[I]$
  - are monotonic and distributive
- Iterative solving of dataflow equation:
  - terminates
  - computes MOP solution

Problem 2: Available Expressions

- Compute available expressions at each program point
- Available expression = expression evaluated in all program executions, and its value would be the same if re-evaluated
- Is similar to available copies for constant propagation
- Dataflow information: sets of available expressions
- Example: expressions {x+y, y-z} are available at point p
- Is a forward analysis
- Let E = set of all expressions in the program
- Lattice $(L, \sqsubseteq)$, where:
  - $L = 2^E$ (power set of E, i.e., set of all subsets of E)
  - Partial order $\sqsubseteq$ is set inclusion: $\supseteq$
    $S_1 \sqsubseteq S_2$ iff $S_1 \supseteq S_2$

AE: The Lattice

- Consider set of expressions = {x*z, x+y, y-z}
- Denote e = x*z, f=x+y, g=y-z
- Partial order: ⊆
- Set E is finite implies lattice has finite height
- Meet operator: ∩ (set intersection)
- Top element: {e,f,g} (set of all expressions)



- Larger sets of available expressions = more precise analysis
- No available expressions = least precise

AE: Dataflow Equations

- Equations:

  out[B] = F_B(in[B]), for all B

  in[B] = ∩ {out[B'] | B'∈pred(B)}, for all B

  in[B_s] = X_0

- Meaning of intersection meet operator:
  "An expression is available at entry of block B if it is available at exit of all predecessor nodes"

AE: Transfer Functions

- Define transfer functions for instructions
- General form of transfer functions:

  F_I(X) = (X – kill[I]) ∪ gen[I]

  where:

  kill[I] = expressions "killed" by I

  gen[I] = new expressions "generated" by I

- Note: this kind of transfer function is typical for many dataflow analyses!
- Meaning of transfer functions: "Expressions available after instruction I include: (1) expressions available before I, but not killed by I, and (2) expressions generated by I"

AE: Transfer Functions

- Define kill/gen for each type of instruction

if I is x = y OP z :	gen[I] = {y OP z}	kill[I] = {E | x∈E}
if I is x = OP y :	gen[I] = {OP z}	kill[I] = {E | x∈E}
if I is x = y :	gen[I] = {}	kill[I] = {E | x∈E}
if I is x = addr y :	gen[I] = {}	kill[I] = {E | x∈E}
if I is if (x) :	gen[I] = {}	kill[I] = {}
if I is return x :	gen[I] = {}	kill[I] = {}
if I is x = f(y_1,..., y_n) :	gen[I] = {}	kill[I] = {E | x∈E}

- Transfer functions F_I(X) = (X – kill[I]) ∪ gen[I]
- ... how about x = x OP y?

Available Expressions: Summary

- Lattice: (2^E, ⊆); has finite height
- Meet is set intersection, top element is E
- Is a forward dataflow analysis
- Dataflow equations:

  out[B] = F_B(in[B]), for all B

  in[B] = ∩ {out[B'] | B'∈pred(B)}, for all B

  in[B_s] = X_0

- Transfer functions: F_I(X) = (X – kill[I]) ∪ gen[I]
  - are monotonic and distributive
- Iterative solving of dataflow equation:
  - terminates
  - computes MOP solution

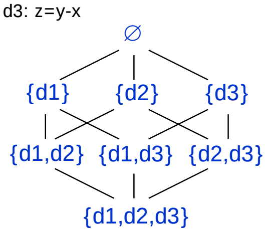
Problem 3: Reaching Definitions

- Compute reaching definitions for each program point
- Reaching definition = definition of a variable whose assigned value may be observed at current program point in some execution of the program
- Dataflow information: sets of reaching definitions
- Example: definitions {d2, d7} may reach program point p
- Is a forward analysis
- Let D = set of all definitions (assignments) in the program
- Lattice (D, ⊑), where:
  – L = 2^D (power set of D)
  – Partial order ⊑ is set inclusion: ⊇

    $S_1 \sqsubseteq S_2$ iff $S_1 \supseteq S_2$

RD: The Lattice

- Consider set of expressions = {d1, d2, d3}
  where d1: x = y, d2: x=x+1, d3: z=y-x
- Partial order: ⊇
- Set D is finite implies lattice has finite height
- Meet operator: ∪ (set union)
- Top element: ∅ (empty set)



- Smaller sets of reaching definitions = more precise analysis
- All definitions may reach current point = least precise

RD: Dataflow Equations

- Equations:
  - $out[B] = F_B(in[B])$, for all B
  - $in[B] = \cup\{out[B'] \mid B' \in pred(B)\}$, for all B
  - $in[B_s] = X_0$
- Meaning of intersection meet operator:
  "A definition reaches the entry of block B if it reaches the exit of at least one of its predecessor nodes"

RD: Transfer Functions

- Define transfer functions for instructions
- General form of transfer functions:
  $F_I(X) = (X – kill[I]) \cup gen[I]$
  where:
  kill[I] = definitions "killed" by I
  gen[I] = definitions "generated" by I
- Meaning of transfer functions: "Reaching definitions after instruction I include: (1) reaching definitions before I, but not killed by I, and (2) reaching definitions generated by I"

RD: Transfer Functions

- Define kill/gen for each type of instruction
- If I is a definition d that defines x:
  gen[I] = {d}   kill[I] = {d' | d' defines x}
- If I is not a definition:
  gen[I] = {}   kill[I] = {}
- Transfer functions $F_I(X) = (X – kill[I]) \cup gen[I]$
- They are monotonic and distributive
  – For each F_I, kill[I] and gen[I] are constants: they don't depend on input information X

Reaching Definitions: Summary

- Lattice: $(2^D, \supseteq)$; has finite height
- Meet is set union, top element is ∅
- Is a forward dataflow analysis
- Dataflow equations:
  - $out[B] = F_B(in[B])$, for all B
  - $in[B] = \cup\{out[B'] \mid B' \in pred(B)\}$, for all B
  - $in[B_s] = X_0$
- Transfer functions: $F_I(X) = (X – kill[I]) \cup gen[I]$
  - are monotonic and distributive
- Iterative solving of dataflow equation:
  - terminates
  - computes MOP solution

Implementation

- Lattices in these analyses = power sets
- Information in these analyses = subsets of a set
- How to implement subsets?

1. Set implementation
   - Data structure with as many elements as the subset has
   - Usually list implementation
2. Bitvectors:
   - Use a bit for each element in the overall set
   - Bit for element x is: 1 if x is in subset, 0 otherwise
   - Example: S = {a,b,c}, use 3 bits
   - Subset {a,c} is 101, subset {b} is 010, etc.

Implementation Tradeoffs

- Advantages of bitvectors:
  - Efficient implementation of set union/intersection:
    set union is bitwise “or” of bitvectors
    set intersection is bitwise “and” of bitvectors
  - Drawback: inefficient for subsets with few elements
- Advantage of list implementation:
  - Efficient for sparse representation
  - Drawback: inefficient for set union or intersection
- In general, bitvectors work well if the size of the (original) set is linear in the program size

Problem 4: Constant Folding

- Compute constant variables at each program point
- Constant variable = variable having a constant value on all program executions
- Dataflow information: sets of constant values
- Example: {x=2, y=3} at program point p
- Is a forward analysis
- Let V = set of all variables in the program, nvar = |V|
- Let N = set of integer numbers
- Use a lattice over the set V x N
- Construct the lattice starting from a lattice for N
- Problem: (N, ≤) is not a complete lattice!
  ... why?

Constant Folding Lattice

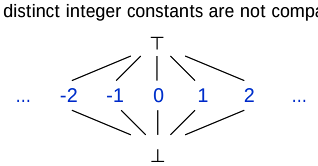
- Second try: lattice (N ∪ {⊤,⊥}, ≤)
  - Where ⊥ ≤ n, for all n∈N
  - And n ≤ ⊤, for all n∈N
  - Is complete!
- Meaning:
  - v= ⊤ : don’t know if v is constant
  - v= ⊥ : v is not constant

Constant Folding Lattice

- Second try: lattice (N ∪ {⊤,⊥}, ≤)
  - Where ⊥ ≤ n, for all n∈N
  - And n ≤ ⊤, for all n∈N
  - Is complete!
- Problem:
  - Is incorrect for constant folding
  - Meet of two constants c≠d is min(c,d)
  - Meet of different constants should be ⊥
- Another problem: has infinite height ...

Constant Folding Lattice

- Solution: flat lattice L = (N ∪ {⊤,⊥}, ⊑)
  - Where ⊥ ⊑ n, for all n∈N
  - And n ⊑ ⊤ , for all n∈N
  - And distinct integer constants are not comparable



- Note: meet of any two distinct numbers is ⊥!

Constant Folding Lattice

- Denote N*=N ∪ {⊤, ⊥}
- Use flat lattice L=(N*, ⊑)
- Constant folding lattice: L’=(V → N*, $\sqsubseteq_C$)
- Where partial order on V → N* is defined as:
  X $\sqsubseteq_C$ Y iff for each variable v: X(v) ⊑ Y(v)
- Can represent a function in V → N* as a set of assignments: { $\{v_1=c_1\}$, $\{v_2=c_2\}$, ..., $\{v_n=c_n\}$ }

CF: Transfer Functions

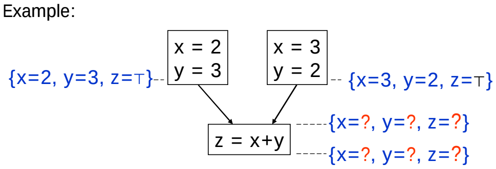
- Transfer function for instruction I:
  $F_I(X) = (X – kill[I]) \cup gen[I]$
  where:
  kill[I] = constants “killed” by I
  gen[I] = constants “generated” by I
- X[v] = c $\in$ N* if {v=c} $\in$ X
- If I is v = c (constant): gen[I] = {v=c} kill[I] = {v} x N*
- If I is v = u+w: gen[I] = {v=e} kill[I] = {v} x N*
  where e = X[u] + X[w], if X[u] and X[w] are not $\top$, $\bot$
  e = $\bot$, if X[u] = $\bot$ or X[w] = $\bot$
  e = $\top$, if X[u] = $\top$ or X[w] = $\top$

CF: Transfer Functions

- Transfer function for instruction I:
  $F_I(X) = (X – kill[I]) \cup gen[I]$
- Here gen[I] is not constant, it depends on X
- However transfer functions are monotonic (easy to prove)
- … but are transfer functions distributive?

CF: Distributivity

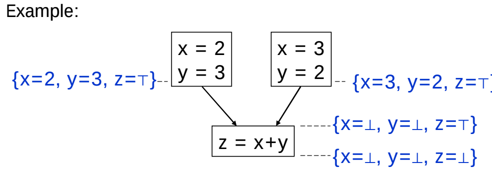
- Example:



- At join point, apply meet operator
- Then use transfer function for z=x+y

CF: Distributivity

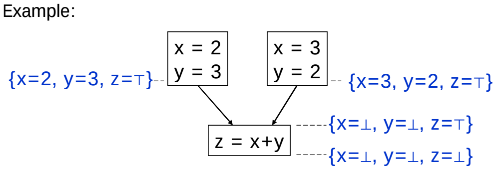
- Example:



- Dataflow result (MFP) at the end: {x=$\bot$, y= $\bot$, z=$\bot$}
- MOP solution at the end?

CF: Distributivity

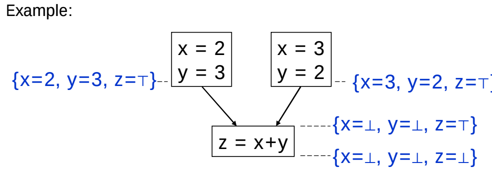
- Example:



- Dataflow result (MFP) at the end: {x=$\bot$, y=$\bot$, z=$\bot$}
- MOP solution at the end: {x=$\bot$, y=$\bot$, z=5} !

CF: Distributivity

- Example:



- Reason for MOP $\neq$ MFP:
  transfer function F of z=x+y is not distributive!
  $F(X_1 \sqcap X_2) \neq F(X_1) \sqcap F(X_2)$
  where X_1 = {x=2, y=3, z=$\top$} and X_2 = {x=3, y=2, z=$\top$}

Classification of Analyses

- Forward analyses: information flows from
  - CFG entry block to CFG exit block
  - Input of each block to its output
  - Output of each block to input of its successor blocks
  - Examples: available expressions, reaching definitions, constant folding
- Backward analyses: information flows from
  - CFG exit block to entry block
  - Output of each block to its input
  - Input of each block to output of its predecessor blocks
  - Example: live variable analysis

Another Classification

- "may" analyses:
  - information describes a property that MAY hold in SOME executions of the program
  - Usually: $\sqcap = \cup$, $\top = \emptyset$
  - Hence, initialize info to empty sets
  - Examples: live variable analysis, reaching definitions
- "must" analyses:
  - information describes a property that MUST hold in ALL executions of the program
  - Usually: $\sqcap = \cap$, $\top = S$
  - Hence, initialize info to the whole set
  - Examples: available expressions