CS412/CS413

Introduction to Compilers

Tim Teitelbaum

Lecture 10: LR Parsing

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LR(0) Parsing Summary

- LR(0) item = a production with a dot in RHS
- LR(0) state = set of LR(0) items valid for viable prefixes
- Compute LR(0) states and build DFA:
  - Start state: $V(\varepsilon) = \{ [S' \to .S] \}\downarrow^*$
  - Other states: $V(\alpha X) = V(\alpha)\to_X\downarrow^*$
- Build the LR(0) parsing table from the DFA
- Use the LR(0) parsing table to determine whether to reduce or to shift

LR(0) Limitations

- An LR(0) machine only works if each state with a reduce action has only one possible reduce action and no shift action
- With some grammars, construction gives states with shift/reduce or reduce/reduce conflicts
- Need to use look-ahead to choose

ok	shift /reduce	reduce / reduce
[L → L,S.]	[L → L,S.] [S → S.,L]	[L → S,L .] [L → S.]

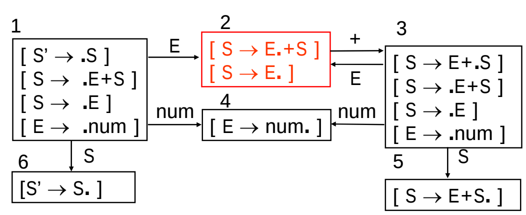
LR(0) Parsing Table

	(	)	id	,	ε	S	L
1	s3		s2			g4	
2	S→id	S→id	S→id	S→id	S→id		
3	s3		s2			g7	g5
4					accept		
5		s6		s8			
6	S→(L)	S→(L)	S→(L)	S→(L)	S→(L)		
7	L→S	L→S	L→S	L→S	L→S		
8	s3		s2			g9	
9	L→L,S	L→L,S	L→L,S	L→L,S	L→L,S		

A Non-LR(0) Grammar

- Grammar for addition of numbers:
  S → S + E | E
  E → num
- Left-associative version is LR(0)
- Right-associative version is not LR(0)
  S → E + S | E
  E → num

LR(0) Parsing Table



What to do in state 2: shift or reduce?

	num	+	ε	E	S
1	s4			g2	g6
2	S→E	s3/S→E	S→E		

SLR(1) Parsing

- SLR Parsing = easy extension of LR(0)
  - For each reduction $A \to \beta$, look at the next symbol c
  - Apply reduction only if c is in FOLLOW(A), or c= ε and $S \Rightarrow^* \gamma A$
- SLR parsing table eliminates some conflicts
  - Same as LR(0) table except reduction rows
  - Adds reductions $A \to \beta$ only in the columns of symbols in FOLLOW(A)
- Example:

FOLLOW(S)={}
but $S \Rightarrow^* \gamma E$

	num	+	ε	E	S
1	s4			g2	g6
2		s3	$S \to E$		

SLR Parsing Table

- Reductions do not fill entire rows
- Otherwise, same as LR(0)

	num	+	ε	E	S
1	s4			g2	g6
2		s3	$S \to E$		
3	s4			g2	g5
4			$S \to E$		
5			$S \to E+S$		
6			s7		
7			accept		

SLR(k)

- Use the LR(0) machine states as rows of table
- Let Q be a state and u be a lookahead string
  - Action(Q,u) = shift Goto(Q,b)

    if Q contains an item of the form $[A \to \beta_1 . b\beta_3]$, with $u \in FIRST_k(b\beta_3\, FOLLOW_k(A))$
  - Action(Q,u) = accept

    if Q = { $[S' \to S.]$ } and $u=\varepsilon$
  - Action(Q,u) = reduce i

    if Q contains the item $[A \to \beta .]$, where $A \to \beta$ is the ith production of G and $u \in FOLLOW_k(A)$, or $u=\varepsilon$ and $S \Rightarrow^* \gamma A$
  - Action(Q,u) = error otherwise
- G is SLR(k) iff the Action function given above is single-valued for all Q and u, i.e, there are no shift-reduce or reduce-reduce conflicts.

LR(1) Parsing

- Get as much power as possible out of 1 look-ahead symbol parsing table
- LR(1) grammar = recognizable by a shift/reduce parser with 1-symbol look-ahead
- LR(1) parsing uses similar concepts as LR(0)
  - Parser states = sets of items
  - LR(1) item = LR(0) item + look-ahead symbol following the production

LR(0) item : [$S \to .S+E$]

LR(1) item : [$S \to .S+E$ +]

LR(1) States

- LR(1) state = set of LR(1) items
- LR(1) item = [$A \to \alpha . \beta$ b], where b in $\Sigma \cup \{\varepsilon\}$
- Meaning: α already matched at top of the stack; next expect to see βb
- Shorthand notation

  [$A \to \alpha . B$ $b_1, \ldots, b_n$]

  means:

  [$A \to \alpha . \beta$ b_1]

  ...

  [$A \to \alpha . \beta$ b_n]

[$S \to S.+E$	+,ε]
[$S \to S+.E$	num]

- Extend closure and goto operations

LR(1) Closure

- LR(1) closure operation on set of items S
  - For each item in S:

    [$A \to \alpha . B\beta$ b]

    and for each production $B \to \gamma$, add the following item to S:

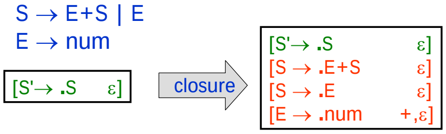
    [$B \to .\gamma$ FIRST(βb)], or

    [$B \to .\gamma$ ε] if FIRST(βb)={}
  - Repeat until nothing changes
- Similar to LR(0) closure, but also keeps track of the look-ahead symbol

LR(1) Start State

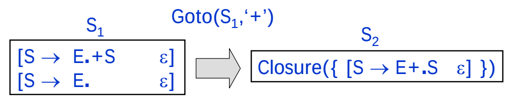
- Initial state: start with [S' → .S ε], then apply the closure operation
- Example: sum grammar

  S' → S
  S → E+S | E
  E → num



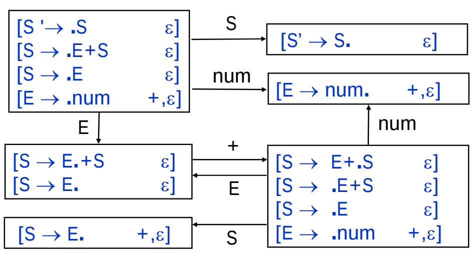
LR(1) Goto Operation

- LR(1) goto operation = describes transitions between LR(1) states
- Algorithm: for a state S and a symbol Y
  – S' = { [A → αY.β b] | [A → α.Yβ b] ∈ S}
  – Goto(S, Y) = Closure(S')



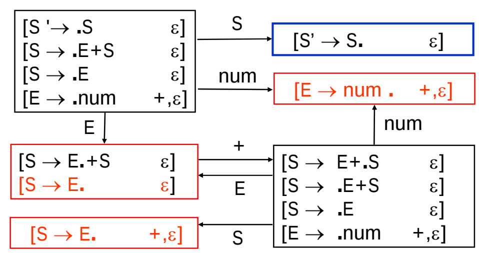
LR(1) DFA Construction

- If S' = Goto(S,X) then add an edge labeled X from S to S'



LR(1) Reductions

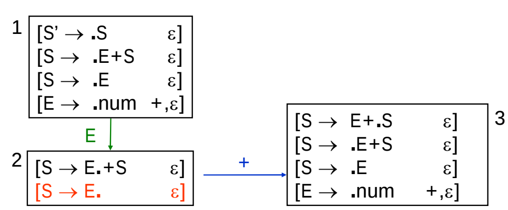
- Reductions correspond to LR(1) items of the form [A → β. x]



LR(1) Parsing Table Construction

- Same as construction of LR(0) parsing table, except for reductions
- If [A → β. b] ∈ state Q, then:
  Action(Q,b) is Reduce(A → β)

LR(1) Parsing Table Example



Fragment of the Parsing table:

	+	ε	E
1			2
2	s3	S→E	

LR(1) but not SLR(1)

- Let G have productions

  S → aAb | Ac

  A → a | ε

- V(a) = {

  [S → a.Ab]

  [A → a.]

  [A → .a]

  [A → .]

  }

FOLLOW(A) = {b,c}

reduce-reduce conflict

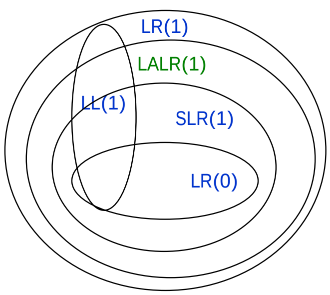
LALR(1) Grammars

- Problem with LR(1): too many states
- LALR(1) Parsing (Look-Ahead LR)
  - Construct LR(1) DFA and then merge any two LR(1) states whose items are identical except look-ahead
  - Results in smaller parser tables
  - Theoretically less powerful than LR(1)

[S → id. +] [S → E. ε]	+	[S → id. ε] [S → E. +]	= ?

- LALR(1) Grammar = a grammar whose LALR(1) parsing table has no conflicts

Classification of Grammars



$LR(k) \subseteq LR(k+1)$

$LL(k) \subseteq LL(k+1)$

$LL(k) \subseteq LR(k)$

$LR(0) \subseteq SLR(1)$

$LALR(1) \subseteq LR(1)$

Automate the Parsing Process

- Can automate:
  - The construction of LR parsing tables
  - The construction of shift-reduce parsers based on these parsing tables
- Automatic parser generators: yacc, bison, CUP
- LALR(1) parser generators
  - Not much difference compared to LR(1) in practice
  - Smaller parsing tables than LR(1)
  - Augment LALR(1) grammar specification with declarations of precedence, associativity
- output: LALR(1) parser program

Associativity

S → S + E | E
E → num

⇒

E → E + E
E → num

What happens if we run this grammar through LALR construction?

Shift/Reduce Conflict

E → E + E
E → num

[E → E+E. +]   [E → E.+E +,ε]

→ +

shift/reduce conflict

shift: 1+(2+3)
reduce: (1+2)+3

1+2+3
  ^

Grammar in CUP

nonterminal E; terminal PLUS, LPAREN...
precedence left PLUS;

"when shifting a '+' conflicts with reducing a production, choose reduce"

```
E ::= E  PLUS  E
    |  LPAREN  E  RPAREN
    |  NUMBER ;
```


Precedence

- CUP can also handle operator precedence

$E \to E + E \mid T$
$T \to T \times T \mid num \mid (E)$

⇩

$E \to E + E \mid E \times E$
$\mid num \mid (E)$

Conflicts without Precedence

$E \to E + E \mid E \times E$
$\mid num \mid (E)$

[E → E.+E ...]
[E → E×E. +]

[E → E+E . ×]
[E → E.×E ...]

Predecence in CUP

precedence left PLUS;
precedence left TIMES; // TIMES > PLUS
E ::= E PLUS E | E TIMES E | ...

RULE: in conflict, choose reduce if last terminal of production has higher precedence than symbol to be shifted; choose shift if vice-versa. In tie, use associativity (left or right) given by precedence rule

[E → E .+E ...]
[E → E×E. +]

reduce E→E×E

[E → E+E. ×]
[E → E.×E ...]

Shift ×

Summary

- Look-ahead information makes SLR(1), LALR(1), LR(1) grammars expressive
- Automatic parser generators support LALR(1) grammars
- Precedence, associativity declarations simplify grammar writing