## CS412/413

Introduction to Compilers
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Lecture 28: Loop Optimizations
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## Loop Invariant Code

- Idea: if a computation produces same result in all loop iterations, move it out of the loop
- Example:
```
for (i=0; i<10; i++)
    a[i] = 10*i + x*x;
```
- Expression `x*x` produces the same result in each iteration; move it of the loop:
```
t = x*x;
for (i=0; i<10; i++)
    a[i] = 10*i + t;
```



## Loop Invariant Computation

- An instruction a = b OP c is loop-invariant if each operand is:
  - Constant, or
  - Has all definitions outside the loop, or
  - Has exactly one definition, and that is a loop-invariant computation
- Reaching definitions analysis computes all the definitions of x and y which may reach t = x OP y



## Code Motion

- Suppose a = b OP c is loop-invariant
- We want to hoist it out of the loop
- Code motion of a definition d: a = b OP c in pre-header is valid if:
  1. Definition d dominates the nodes after the loop exit where a is live
  2. There is no other definition of a in loop
  3. All uses of a in the loop can only be reached from d

  otherwise, just move the computation b OP c



## Other Issues

- Preserve dependencies between loop-invariant instructions when hoisting code out of the loop
```
for (i=0; i<N; i++) {
  x = y+z;
  a[i] = 10*i + x*x;
}
```
```
x = y+z;
t = x*x;
for(i=0; i<N; i++)
    a[i] = 10*i + t;
```
- Nested loops: apply loop invariant code motion algorithm multiple times
```
for (i=0; i<N; i++)
  for (j=0; j<M; j++)
    a[i][j] = x*x + 10*i +
              100*j;
```
```
t1 = x*x;
for (i=0; i<N; i++) {
    t2 = t1+ 10*i;
    for (j=0; j<M; j++)
        a[i][j] = t2 + 100*j;
}
```



## Induction Variables

- An induction variable is a variable in a loop, whose value is linear with respect to the loop iteration number

  v = f(i)

  f(i) = c*i + d
- Observation: linear combinations of linear functions are linear functions
  - Consequence: linear combinations of induction variables are induction variables

## Induction Variables

- Two categories of induction variables
- Basic induction variables: only incremented in loop body
  i = i + c
  where c is a constant (positive or negative)
- Derived induction variables: expressed as a linear function of an induction variable
  k = c*j + d
  where:
  - either j is basic induction variable
  - or j is derived induction variable relative to i and:
    1. No definition of j outside the loop reaches definition of k
    2. i is not defined between the definitions of j and k



## Families of Induction Variables

- Each basic induction variable defines a family of induction variables
  – Each variable in the family of i is a linear function of i
- A variable k is in the family of basic variable i if:
  1. k = i ( the basic variable itself)
  2. k is a linear function of other variables in the family of i:
     k = c*j+d, where $j \in$ Family(i)
- A triple <i, a, b> denotes an induction variable k in the family of i such that: k = i*a + b
  – Triple for basic variable i is <i, 1, 0>



## Dataflow Analysis Formulation

- Detection of induction variables: can formulate problem using the dataflow analysis framework
  – Analyze loop body sub-graph, except the back edge
  – Analysis is similar to constant folding
- Dataflow information: a function F that assigns a triple to each variable:
  F(k) = <i,a,b>, if k is an induction variable in family of i
  F(k) = $\perp$ : k is not an induction variable
  F(k) = $\top$ : don't know if k is an induction variable



## Dataflow Analysis Formulation

- Meet operation: if F1 and F2 are two functions, then:

$$(F1 \sqcap F2)(v) = \begin{cases} \langle i,a,b \rangle & \text{if } F1(k)=F2(k)=\langle i,a,b \rangle \\ \perp, & \text{otherwise} \end{cases}$$

  (in other words, use a flat lattice)
- Initialization:
  – Detect all basic induction variables
  – At loop header: F(i) = <i,1,0> for each basic variable i
- Transfer function:
  – consider F is information before node n
  – Compute information F' after n



## Dataflow Analysis Formulation

- For a definition k = j+c, where k is not basic induction variable
  F'(v) = <i, a, b+c>, if v=k and F(j)=<i,a,b>
  F'(v) = F(v), otherwise
- For a definition k = j*c, where k is not basic induction variable
  F'(v) = <i, a*c, b*c>, if v=k and F(j)=<i,a,b>
  F'(v) = F(v), otherwise
- For any other instruction and any variable k in def[n] :
  F'(v) = $\perp$, if F(v) = <k, a, b>
  F'(v) = F(v), otherwise



## Strength Reduction

- Basic idea: replace expensive operations (multiplications) with cheaper ones (additions) in definitions of induction variables

```
while (i<10) {
  j = ...;   // <i,3,1>
  a[j] = a[j] -2;
  i = i+2;
}
```

⇒

```
s = 3*i+1;
while (i<10) {
  j = s;
  a[j] = a[j] -2;
  i = i+2;
  s= s+6;
}
```

- Benefit: cheaper to compute s = s+6 than j = 3*i
  – s = s+6 requires an addition
  – j = 3*i requires a multiplication

## General Algorithm

- Algorithm:

For each induction variable j with triple <i,a,b>
whose definition involves multiplication:

1. create a new variable s
2. replace definition of j with j=s
3. immediately after i=i+c, insert s = s+a*c
   (here a*c is constant)
4. insert s = a*i+b into preheader

- Correctness:
  this transformation maintains the invariant that s = a*i+b



## Strength Reduction

- Gives opportunities for copy propagation, dead code elimination

```
s = 3*i+1;
while (i<10) {
  j = s;
  a[j] = a[j] –2;
  i = i+2;
  s= s+6;
}
```

⇨

```
s = 3*i+1;
while (i<10) {

  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```



## Induction Variable Elimination

- Idea: eliminate each basic induction variable whose only uses are in loop test conditions and in their own definitions i = i+c
  - rewrite loop test to eliminate induction variable

```
s = 3*i+1;
while (i<10) {
  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```

- When are induction variables used only in loop tests?
  – Usually, after strength reduction
  – Use algorithm from strength reduction even if definitions of induction variables don't involve multiplications



## Induction Variable Elimination

- Rewrite test condition using derived induction variables
- Remove definition of basic induction variables (if not used after the loop)

```
s = 3*i+1;
while (i<10) {
  a[s] = a[s] –2;
  i = i+2;
  s= s+6;
}
```

⇨

```
s = 3*i+1;
while (s<31) {
  a[s] = a[s] –2;
  s= s+6;
}
```



## Induction Variable Elimination

For each basic induction variable i whose only uses are
– The test condition i < u
– The definition of i: i = i + c

Take a derived induction variable k in its family, with triple <i,c,d>
Replace test condition i < u with k < c*u+d

Remove definition i = i+c if i is not live on loop exit



## Where We Are

- Defined dataflow analysis framework
- Used it for several analyses
  – Live variables
  – Available expressions
  – Reaching definitions
  – Constant folding
- Loop transformations
  – Loop invariant code motion
  – Induction variables
- Next:
  – Pointer alias analysis

## Pointer Alias Analysis

- Most languages use variables containing addresses
  - E.g. pointers (C,C++), references (Java), call-by-reference parameters (Pascal, C++, Fortran)
- Pointer aliases: multiple names for the same memory location, which occur when dereferencing variables that hold memory addresses
- Problem:
  - Don't know what variables read and written by accesses via pointer aliases (e.g. *p=y, x=*p, p.f=y, x=p.f, etc.)
  - Need to know accessed variables to compute dataflow information after each instruction



## Pointer Alias Analysis

- Worst case scenarios
  - *p = y may write any memory location
  - x = *p may read any memory location
- Such assumptions may affect the precision of other analyses
- Example1: Live variables
  before any instruction x = *p, all the variables may be live
- Example 2: Constant folding
  a = 1; b = 2;*p = 0; c = a+b;
- c = 3 at the end of code only if *p is not an alias for a or b!
- Conclusion: precision of result for all other analyses depends on the amount of alias information available
  - hence, it is a fundamental analysis



## Alias Analysis Problem

- Goal: for each variable v that may hold an address, compute the set Ptr(v) of possible targets of v
  - Ptr(v) is a set of variables (or objects)
  - Ptr(v) includes stack- and heap-allocated variables (objects)
- Is a "may" analysis: if $x \in$ Ptr(v), then v may hold the address of x in some execution of the program
- No alias information: for each variable v, Ptr(v) = V, where V is the set of all variables in the program



## Simple Alias Analyses

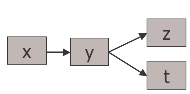
- Address-taken analysis:
  - Consider AT = set of variables whose addresses are taken
  - Then, Ptr(v) = AT, for each pointer variable v
  - Addresses of heap variables are always taken at allocation sites (e.g. x = new int[2], x=malloc(8))
  - Hence AT includes all heap variables
- Type-based alias analysis:
  - If v is a pointer (or reference) to type T, then Ptr(v) is the set of all variables of type T
  - Example: p.f and q.f can be aliases only if p and q are references to objects of the same type
  - Works only for strongly-typed languages



## Dataflow Alias Analysis

- Dataflow analysis: for each variable v, compute points-to set Ptr(v) at each program point
- Dataflow information: set Ptr(v) for each variable v
  - Can be represented as a graph $G \subseteq 2^{V \times V}$
  - Nodes = V (program variables)
  - There is an edge $v \rightarrow u$ if $u \in$ Ptr(v)

Ptr(x) = {y}
Ptr(y) = {z,t}






## Dataflow Alias Analysis

- Dataflow Lattice: $(2^{V \times V}, \supseteq)$
  - V x V is set of all possible points-to relations
  - "may" analysis: top element is $\emptyset$, meet operation is $\cup$
- Transfer functions: use standard dataflow transfer functions:
  out[I] = (in[I]-kill[I]) U gen[I]

| | | |
|---|---|---|
| p = addr q | kill[I]={p} x V | gen[I]={(p,q)} |
| p = q | kill[I]={p} x V | gen[I]={p} x Ptr(q) |
| p = *q | kill[I]={p} x V | gen[I]={p} x Ptr(Ptr(q)) |
| *p = q | kill[I]= … | gen[I]=Ptr(p) x Ptr(q) |

For all other instruction, kill[I] = {}, gen[I] = {}

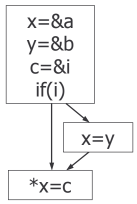
- Transfer functions are monotonic, but not distributive!

# Alias Analysis Example

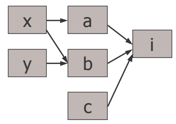
Program

```
x=&a;
y=&b;
c=&i;
if(i) x=y;
*x=c;
```

CFG




Points-to Graph
(at the end of program)






# Alias Analysis Uses

- Once alias information is available, use it in other dataflow analyses
- Example: Live variable analysis
  Use alias information to compute use[I] and def[I] for load and store statements:

| | | |
|---|---|---|
| x = *y | use[n] = {y}∪Ptr(y) | def[n]={x} |
| *x = y | use[n] = {x,y} | def[n]=Ptr(x) |