CS412/413

Introduction to Compilers
Radu Rugina

Lecture 27: Control Flow Analysis
05 Apr 06

Problem 4: Constant Folding

- Compute constant variables at each program point
- Constant variable = variable having a constant value on all program executions

- Dataflow information: sets of constant values
- Example: `{x=2, y=3}` at program point p
- Is a forward analysis

- Let V = set of all variables in the program
- Let N = set of integer numbers
- The lattice is a map from V to N
- Construct the lattice starting from a lattice for N

Constant Folding Lattice

- Second try: lattice $(N \cup \{\top, \bot\}, \leq)$
  - Where $\bot \leq m$, for all $m \in N$
  - And $m \leq \top$, for all $m \in N$
  - Is complete!

- Meaning:
  - $v=\top$: don't know if v is constant
  - $v=\bot$: v is not constant

Constant Folding Lattice

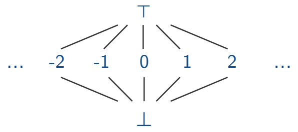
- Second try: lattice $(N \cup \{\top, \bot\}, \leq)$
  - Where $\bot \leq m$, for all $m \in N$
  - And $m \leq \top$, for all $m \in N$
  - Is complete!

- Problem:
  - Is incorrect for constant folding
  - Meet of two constants $c \neq d$ is min(c,d)
  - Meet of different constants should be $\bot$

- Another problem: has infinite height …

Constant Folding Lattice

- Solution: flat lattice $L = (N \cup \{\top, \bot\}, \sqsubseteq)$
  - Where $\bot \sqsubseteq m$, for all $n \in N$
  - And $m \sqsubseteq \top$, for all $n \in N$
  - And distinct integer constants are not comparable



- Note: meet of any two distinct numbers is $\bot$

CF: Transfer Functions

- Transfer function for node n:

  $F_n(X) = (X - \text{kill}[n]) \cup \text{gen}[n]$
- Dataflow information X is a map from V to $N \cup \{\top, \bot\}$
  - Represent it as a set of pairs $(\texttt{var} \mapsto \texttt{m})$
  - Denote by $X[\texttt{var}] = \texttt{m}$ the value of var in this mapping

- If n is `v = c` (constant): $\text{gen}[n] = \{\texttt{v} \mapsto \texttt{c}\}$ $\text{kill}[n] = \{\texttt{v} \mapsto \_\}$
- If n is `v = u+w`: $\text{gen}[n] = \{\texttt{v} \mapsto \texttt{e}\}$ $\text{kill}[n] = \{\texttt{v} \mapsto \_\}$

  where $\texttt{e} = X[\texttt{u}] + X[\texttt{w}]$, if $X[\texttt{u}]$ and $X[\texttt{w}]$ are not $\top, \bot$

  $\texttt{e} = \bot$, if $X[\texttt{u}] = \bot$ or $X[\texttt{w}] = \bot$

  $\texttt{e} = \top$, if $X[\texttt{u}] = \top$ or $X[\texttt{w}] = \top$

CF: Transfer Functions

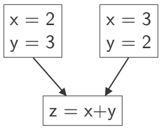
- Transfer function for node n:

  $F_n(X) = (X - \text{kill}[n]) \cup \text{gen}[n]$

- Here gen[n] is not constant, it depends on X
- Exercise: prove that transfer functions are monotonic
- However, transfer functions are not distributive

CF: Distributivity

- Example:



- MFP and MOP yield different solutions

Classification of Analyses

- Forward analyses: information flows from
  - CFG entry block to CFG exit block
  - Input of each block to its output
  - Output of each block to input of its successor blocks
  - Examples: available expressions, reaching definitions, constant folding
- Backward analyses: information flows from
  - CFG exit block to entry block
  - Output of each block to its input
  - Input of each block to output of its predecessor blocks
  - Example: live variable analysis

Another Classification

- “may” analyses:
  - information describes a property that MAY hold in SOME executions of the program
  - Usually: $\sqcap = \cup$, $\top = \emptyset$
  - Hence, initialize info to empty sets
  - Examples: live variable analysis, reaching definitions
- “must” analyses:
  - information describes a property that MUST hold in ALL executions of the program
  - Usually: $\sqcap = \cap$, $\top = S$
  - Hence, initialize info to the whole set
  - Examples: available expressions

Next

- Control flow analysis
  - Detect loops in control flow graphs
  - Dominators
- Loop optimizations
  - Code motion
  - Strength reduction for induction variables
  - Induction variable elimination

Program Loops

- Loop = a computation repeatedly executed until a terminating condition is reached
- High-level loop constructs:
  - While loop: `while(E) S`
  - Do-while loop: `do S while(E)`
  - For loop: `for(i=1, i<=u, i+=c) S`
- Why are loops important:
  - Most of the execution time is spent in loops
  - Typically: 90/10 rule, 10% code is a loop
- Therefore, loops are important targets of optimizations

Detecting Loops

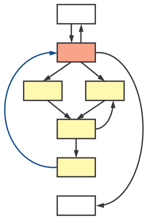
- Need to identify loops in the program
  - Easy to detect loops in high-level constructs
  - Difficult to detect loops in low-level code

- Examples:
  - Languages with unstructured “goto” constructs: structure of high-level loop constructs may be destroyed
  - Optimizing Java bytecodes (without high-level source program): only low-level code is available

Control-Flow Analysis

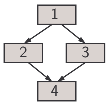
- Goal: identify loops in the control flow graph

- A loop in the CFG:
  - Is a set of CFG nodes (basic blocks)
  - Has a loop header such that control to all nodes in the loop always goes through the header
  - Has a back edge from one of its nodes to the header



Dominators

- Use concept of dominators to identify loops:
  “CFG node d dominates CFG node n if all the paths from entry node to n go through d”



1 dominates 2, 3, 4
2 doesn’t dominate 4
3 doesn’t dominate 4

- Intuition:
  - Header of a loop dominates all nodes in loop body
  - Back edges = edges whose heads dominate their tails
  - Loop identification = back edge identification

Immediate Dominators

- Properties:
  1. CFG entry node n_0 dominates all CFG nodes
  2. If d1 and d2 dominate n, then either
  - d1 dominates d2, or
  - d2 dominates d1

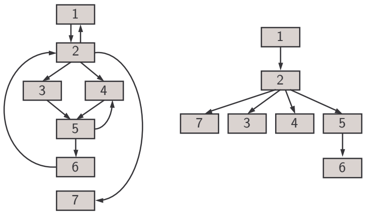
- Immediate dominator idom(n) of node n:
  - idom(n) $\neq$ n
  - idom(n) dominates n
  - If m dominates n, then m dominates idom(n)

- Immediate dominator idom(n) exists and is unique because of properties 1 and 2

Dominator Tree

- Build a dominator tree as follows:
  - Root is CFG entry node n_0
  - m is child of node n iff n=idom(m)

- Example:



Computing Dominators

- Formulate problem as a system of constraints:
  - dom(n) is set of nodes who dominate n
  - dom(n_0)= {n_0}
  - dom(n) = ($\cap$ { dom(p) | p $\in$ pred(n) }) $\cup$ { n }

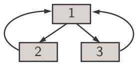
- Can also formulate problem in the dataflow framework
  - What is the dataflow information?
  - What is the lattice?
  - What are the transfer functions?
  - Use dataflow analysis to compute dominators

Natural Loops

- Back edge: edge n→h such that h dominates n
- Natural loop of a back edge n→h:
  - h is loop header
  - Loop nodes is set of all nodes that can reach n without going through h
- Algorithm to identify natural loops in CFG:
  - Compute dominator relation
  - Identify back edges
  - Compute the loop for each back edge

Disjoint and Nested Loops

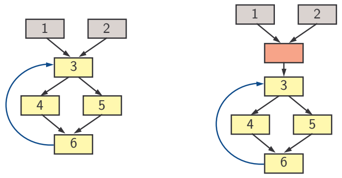
- Property: for any two natural loops in the flow graph, one of the following is true:
  1. They are disjoint
  2. They are nested
  3. They have the same header
- Eliminate alternative 3: if two loops have the same header and none is nested in the other, combine all nodes into a single loop



Two loops: {1,2} and {1,3}
Combine into one loop: {1,2,3}

Loop Preheader

- Several optimizations add code before header
- Insert a new basic block (called preheader) in the CFG to hold this code



Loop Optimizations

- Now we know the loops in the program
- Next: optimize loops
  - Loop invariant code motion
  - Strength reduction of induction variables
  - Induction variable elimination

Loop Invariant Code

- Idea: if a computation produces same result in all loop iterations, move it out of the loop
- Example:

```
for (i=0; i<10; i++)
    a[i] = 10*i + x*x;
```

- Expression `x*x` produces the same result in each iteration; move it of the loop:

```
t = x*x;
for (i=0; i<10; i++)
    a[i] = 10*i + t;
```

Loop Invariant Computation

- An instruction a = b OP c is loop-invariant if each operand is:
  - Constant, or
  - Has all definitions outside the loop, or
  - Has exactly one definition, and that is a loop-invariant computation
- Reaching definitions analysis computes all the definitions of x and y which may reach t = x OP y

Algorithm

```
INV = ∅

Repeat
    for each instruction ∉ INV
       if operands are constants, or
          have definitions outside the loop, or
          have exactly one definition d ∈ INV
       then
          INV = INV ∪ {i}
Until no changes in INV
```

Code Motion

- Next: move loop-invariant code out of the loop
- Suppose a = b OP c is loop-invariant
- We want to hoist it out of the loop

- Code motion of a definition d: a = b OP c in pre-header is valid if:
  1. Definition d dominates all loop exits where a is live
  2. There is no other definition of a in loop
  3. All uses of a in loop can only be reached from definition d

Other Issues

- Preserve dependencies between loop-invariant instructions when hoisting code out of the loop

```
for (i=0; i<N; i++) {
  x = y+z;
  a[i] = 10*i + x*x;
}
```

```
x = y+z;
t = x*x;
for(i=0; i<N; i++)
    a[i] = 10*i + t;
```

- Nested loops: apply loop invariant code motion algorithm multiple times

```
for (i=0; i<N; i++)
  for (j=0; j<M; j++)
    a[i][j] = x*x + 10*i +
                100*j;
```

```
t1 = x*x;
for (i=0; i<N; i++) {
    t2 = t1+ 10*i;
    for (j=0; j<M; j++)
        a[i][j] = t2 + 100*j;
}
```