This is a 50-minute in class closed book exam. All questions are straightforward and you
should have no trouble doing them. Please show all work and write legibly. Thank you.

1. (a) Give a context-free grammar for the language
$$\{x \mid x \in (a+b)* \; and \; x \; has \; an \; equal \; number \; of \; a's \; and \; b's\}$$

(b) For each variable in the grammar describe the strings that can be generated from the
variable by a statement such as

$$S \overset{*}{\Rightarrow} x \; iff \; x \; has \; an \; equal \; number \; of \; a's \; and \; b's$$

2. Use the pumping lemma to prove that the language $\{ww \mid w \in (a+b)*\}$ is not a
context-free language.

3. (a) What is the meaning of a symbol of the form [qAp] in the conversion of a pda to a
cfg?

(b) Suppose $\delta(q, a, A)$ contains $(s, A_1 A_2 \cdots A_k)$. What productions does this give rise to
in the grammar?

(c) Suppose $\delta(q, a, A)$ contains (s, ε). What productions does this give rise to in the
grammar?