This is a 50-minute in class closed book exam. All questions are straightforward and you should have no trouble doing them. Please show all work and write legibly. Thank you.

1. For each of the following languages, state whether it is regular or non-regular. In each case, give a convincing justification in one or two sentences. You can use the fact that $\{0^n 1^n | n \geq 0\}$ is not regular.

   (a) $L = \{0^{i^2} \mid i \geq 0\}^*$.

   (b) $L = \{0^i w 1^i \mid w \in \{0,1\}^*, i \geq 0\}$.

   (c) $L = \{w\$x \mid w, x \in \{0,1\}^*, \#0(w) = \#1(x)\}$ where $\#0(w)$ is the number of zeros in w.

   (d) $L = \{wx \mid w, x \in \{0,1\}^*, \#0(w) = \#1(x)\}$.

   (e) The language that contains as strings all words from this prelim.

2. Prove that the set $\{ww^R \mid w \in \{0,1\}^*\}$ is not regular using the closure properties of regular languages, and the fact that $\{0^n 1^n | n \geq 0\}$ is not regular.

3. Suppose $L, R \subseteq \Sigma^*$ are regular languages. Define the following operation on two strings: given two strings x and y, $Shuffle(x, y)$ is the set of all strings w such that i) $|w| = |x| + |y|$, and ii) symbols from x and y are interspersed to create w. For instance, if $x = abb$ and $y = baccb$, then $w = abaccbbb$ is in $Shuffle(x, y)$ but $abbbbacc$ is not, since the symbols of y are not in order. In other words, both x and y are present as *non-overlapping* substrings of w. Using machine construction, show that the following language is regular :

$$\{w \mid \exists x \in L, y \in R, w \in Shuffle(x, y)\}$$