Automata and Computability	**481 Homework 5**
CS481 Fall 2005	**September 23**

1. Prove non-regularity of the following languages.
   (a) $L = \{0^{i^2} \mid i \geq 0\}$.
   (b) $L = \{w \mid w \in \{0,1\}^*, w = w^R\}$, where w^R is the reverse of the string w.
   (c) The set PAREN of balanced parenthesis (). For example, (()())(()) belongs to PAREN but ())(does not.
   (d) $\{0^i ww 0^j \mid i, j > 0, w \in \{0,1\}^*\}$.
2. Prove that if A is a regular language over Σ and $\{a, b\} \subseteq \Sigma$, then the following language is also regular.
   $$\{c^n \mid \exists w \in A,\ \#a(w) + \#b(w) = n\}$$
3. 4.2.11.
4. Consider the language $L = \{a^i b^j c^k \mid i, j, k \geq 0 \wedge \text{ if } i = 1 \text{ then } j = k\}$.

   Prove that L satisfies the conditions of the pumping lemma, i.e. show that there is a number p where, if s is a string of length at least p in L, then s may be written as $s = xyz$ such that

   - for each $i \geq 0$, $xy^i z \in L$.
   - $y \neq \varepsilon$.
   - $|xy| \leq p$.

   Prove that L is nonregular. Explain why this fact does not contradict the pumping lemma.