CS 481 Homework 4
due Friday September 23

Do each problem on a separate set of pages and please remember to write your name, net-id and problem number on the top right hand corner of each page.

1. 4.2.6 from the book using machine constructions.

2. Let $\mathcal{L}$ be a set of languages such that $\mathcal{L}$ is closed under the following operations:

   (a) If L_1, L_2 are in $\mathcal{L}$, so is $L_1 \cdot L_2 = \{xy | x \in L_1, y \in L_2\}$.

   (b) If L is in $\mathcal{L}$, so is $L \cap R$ for any regular language R.

   (c) If L is in $\mathcal{L}$, so are $h(L)$ and $h^{-1}(L)$ where h is a homomorphism and h^{-1} is the inverse homomorphism.

   Show that $\mathcal{L}$ is closed under union i.e. if L_1 and L_2 are in $\mathcal{L}$, then $L_1 \cup L_2$ is too.

3. For A to be a set of natural numbers, define

$$\mathbf{binaryA} = \{\text{binary representation of number of } A\} \subseteq \{0, 1\}^*$$
$$\mathbf{unaryA} = \{\text{unary representation of number of } A\} \subseteq \{0\}^*$$

   For example, if $A = \{2, 3, 5\}$, $\mathbf{binaryA} = \{10, 11, 101\}$ and $\mathbf{unaryA} = \{00, 000, 00000\}$.

   One of the following statements is true and the other is false. State which is which and prove.

   (a) $\forall A$, if **binaryA** is regular, then **unaryA** is regular.

   (b) $\forall A$, if **unaryA** is regular, then **binaryA** is regular.

4. For this question assume that the following two languages L_1 and L_2 are not regular.

$$L_1 = \{0^i 1^i \mid i \geq 0\}$$
$$L_2 = \{0^i 1 0^i \mid i \geq 0\}$$

   Assuming the closure properties of regular languages, under union, intersection, closure and homomorphisms and its inverse, show that none of the following languages are regular. *Hint :* Show that if they were regular, then L_1 or L_2 would be regular.

(a) $\{0^{2i}1^{3i} \mid i \geq 0\}$.

(b) $\{ww \mid w \in \{0,1\}^*\}$.

(c) $\{0^i1^j \mid i = j + 50\}$.

(d) $\{0^i1^j2^k \mid i = j \vee j = k\}$.