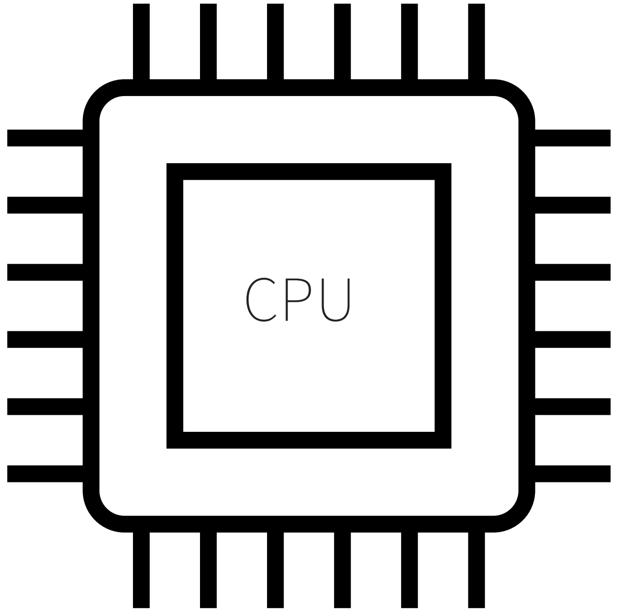
# Gates and Logic

CS 3410: Computer System Organization and Programming

## Simplified Computer Architecture

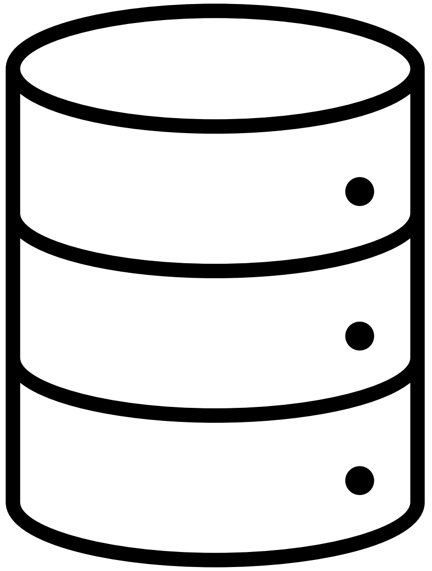
Processor

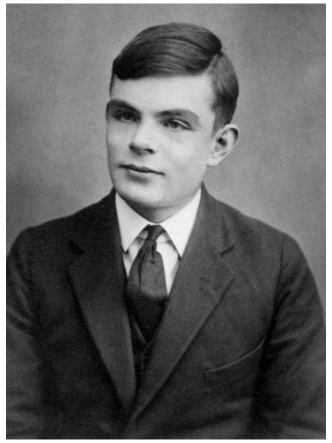


✓ Runs code; does computations

✗ Doesn't remember anything

Memory





✗ Can't compute anything

✓ Stores data

C

```
int x = 10;
x = 2 * x + 15;
```

compiler

How does add work?

x0 = 0

RISC-V assembly language

```
addi x5, x0, 10
muli x5, x5, 2
addi x5, x5, 15
```

x5 = x0 + 10

x5 = x5 * 2

x5 = x5 + 15

assembler

RISC-V machine language

| 10 | x0 | | x5 | op = addi |
|---|---|---|---|---|
| 000000001010 | 00000 | 000 | 00101 | 0010011 |

```
00000000001000101001001010011111
```

| 000000001111 | 00101 | 000 | 00101 | 0010011 |
|---|---|---|---|---|
| 15 | x5 | | x5 | op = addi |

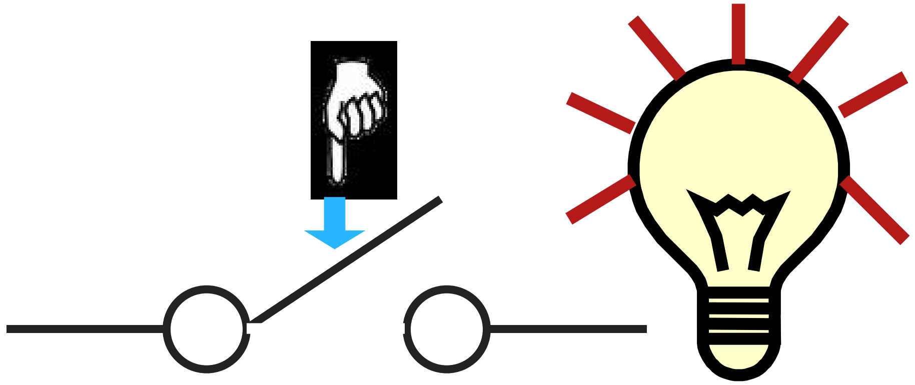
EVERYTHING IS A NUMBER!

## Goals for Today: Bottom Up!

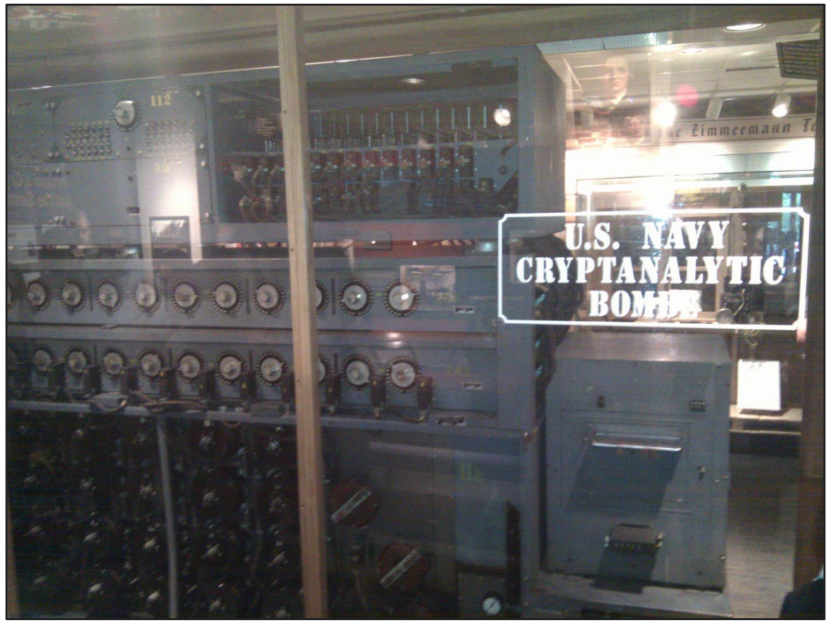
- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Binary Operations
  - One- and four-bit adders
  - Addition (two’s complement)
- Transistors (electronic switch)

# A switch

Acts as a *conductor* or *insulator*.

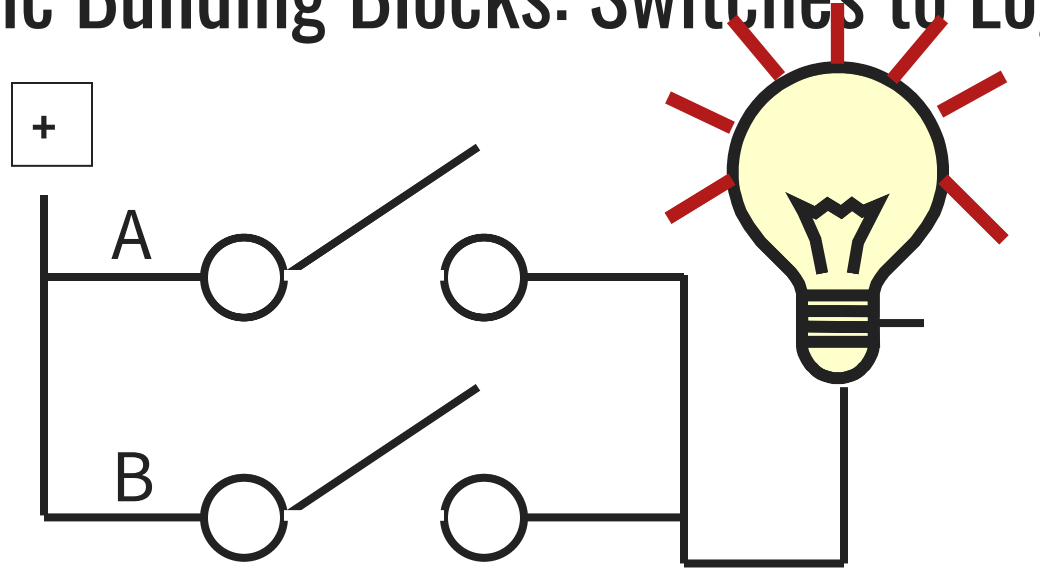


Can be used to build amazing things…



The Bombe used to break the German Enigma machine during World War II

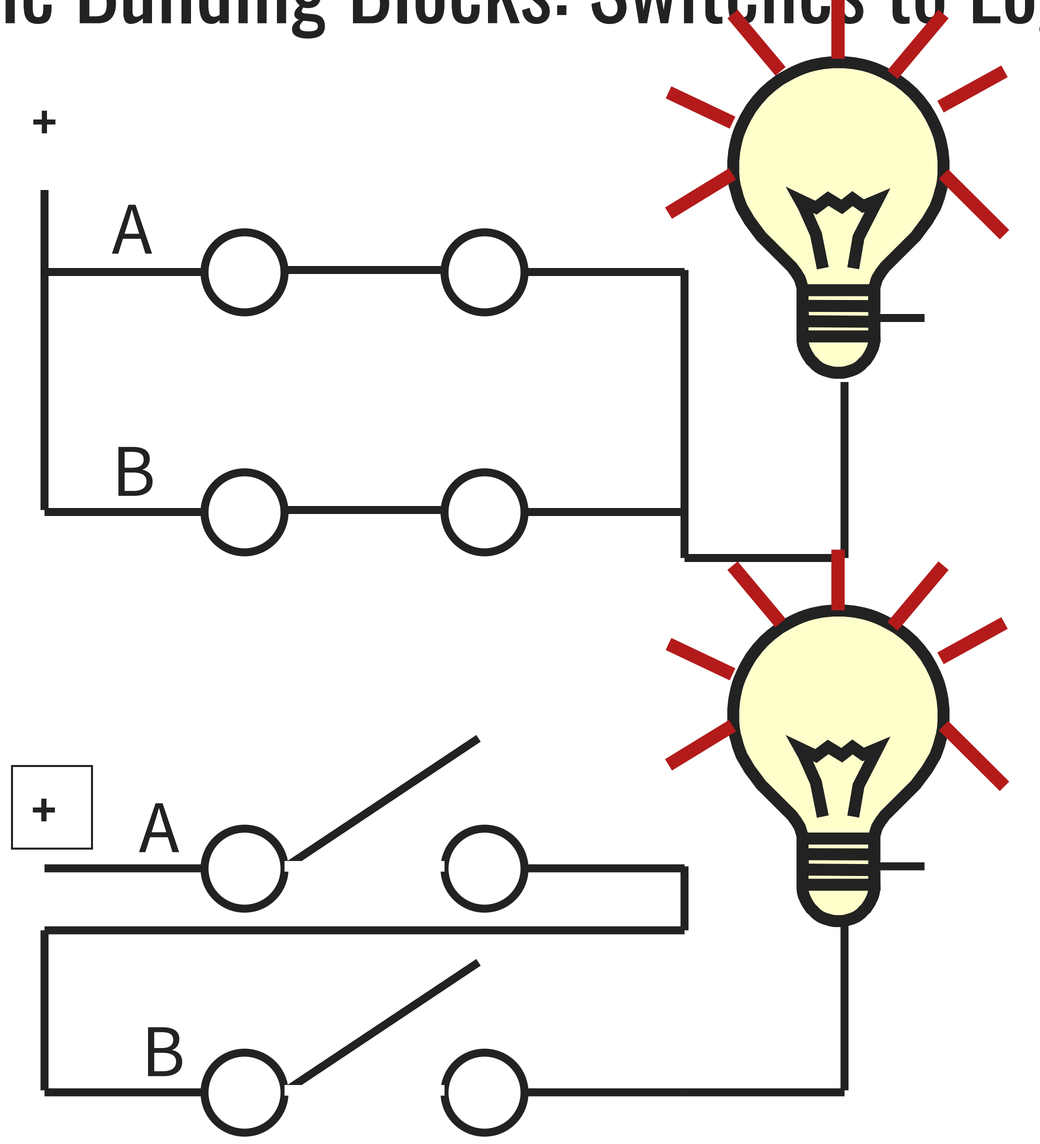
# Basic Building Blocks: Switches to Logic Gates



Truth Table

| A | B | Light |
|---|---|---|
| OFF | OFF | |
| OFF | ON | |
| ON | OFF | |
| ON | ON | |

## Basic Building Blocks: Switches to Logic Gates



Truth Table

| A | B | Light |
|---|---|---|
| OFF | OFF | OFF |
| OFF | ON | ON |
| ON | OFF | ON |
| ON | ON | ON |

| A | B | Light |
|---|---|---|
| OFF | OFF | |
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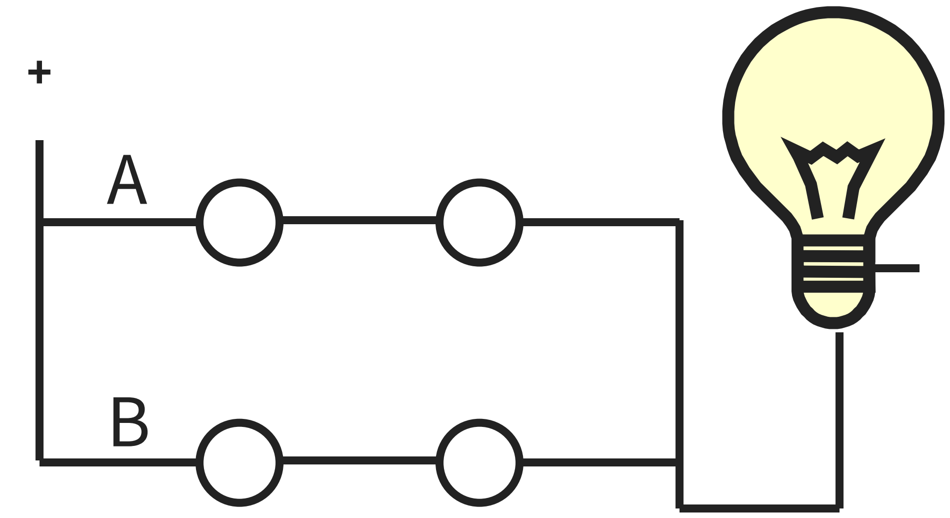
| ON | ON | |

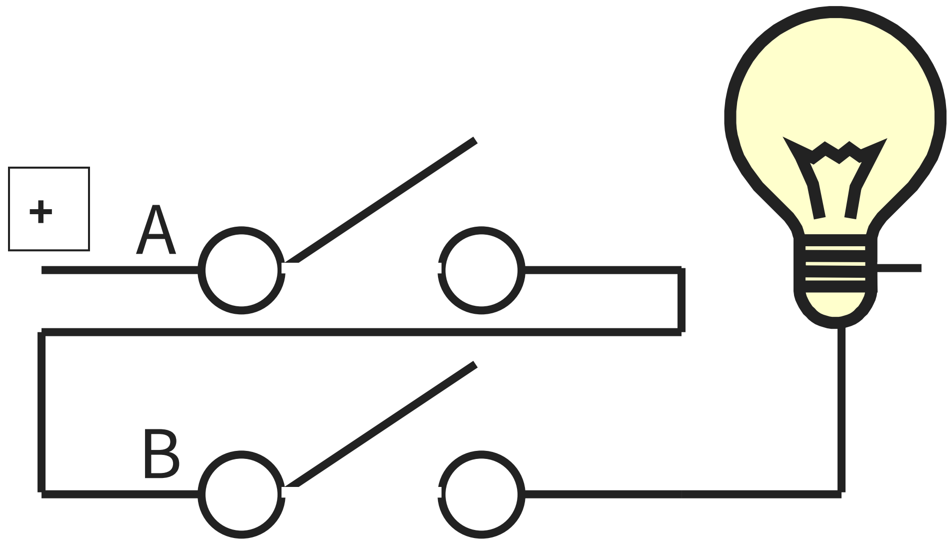
## Basic Building Blocks: Switches to Logic Gates



- Either (OR)

Truth Table

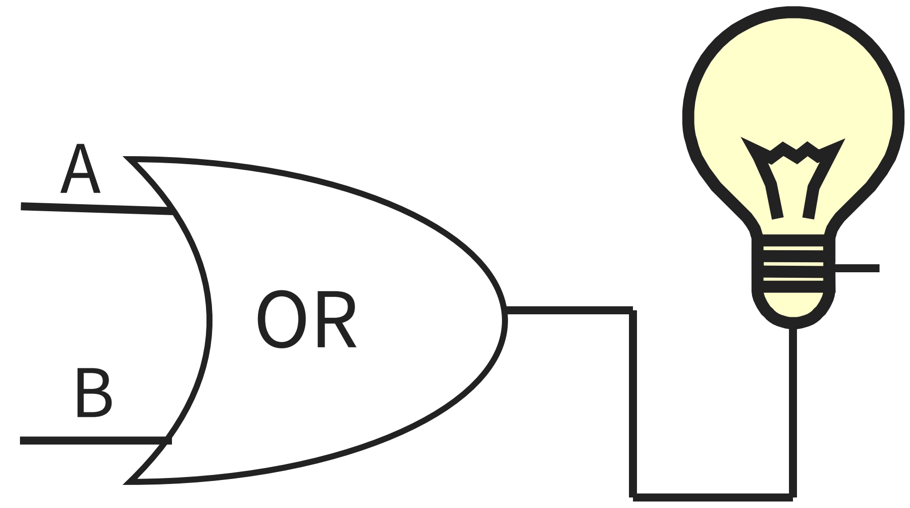
| A | B | Light |
|---|---|---|
| OFF | OFF | OFF |
| OFF | ON | ON |
| ON | OFF | ON |
| ON | ON | ON |



- Both (AND)

| A | B | Light |
|---|---|---|
| OFF | OFF | OFF |
| OFF | ON | OFF |
| ON | OFF | OFF |
| ON | ON | ON |

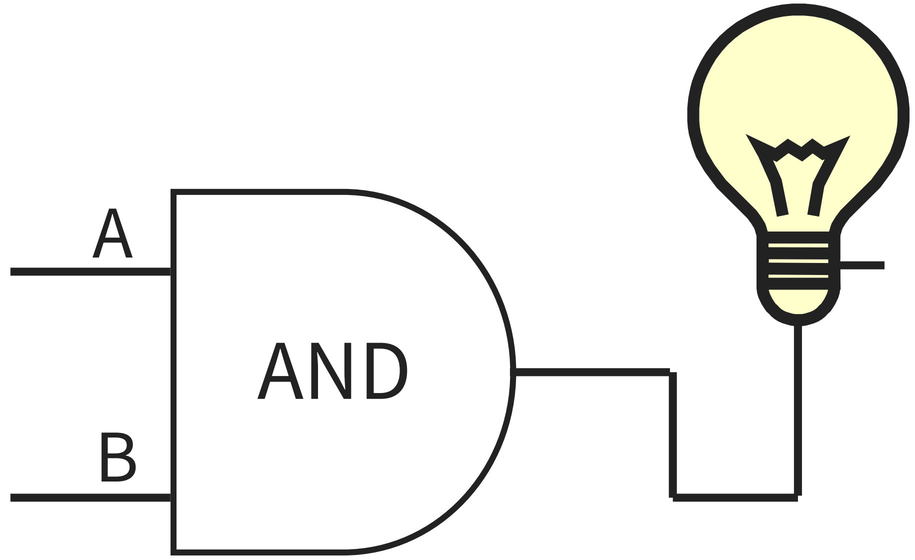
## Basic Building Blocks: Switches to Logic Gates



- Either (OR)

Truth Table

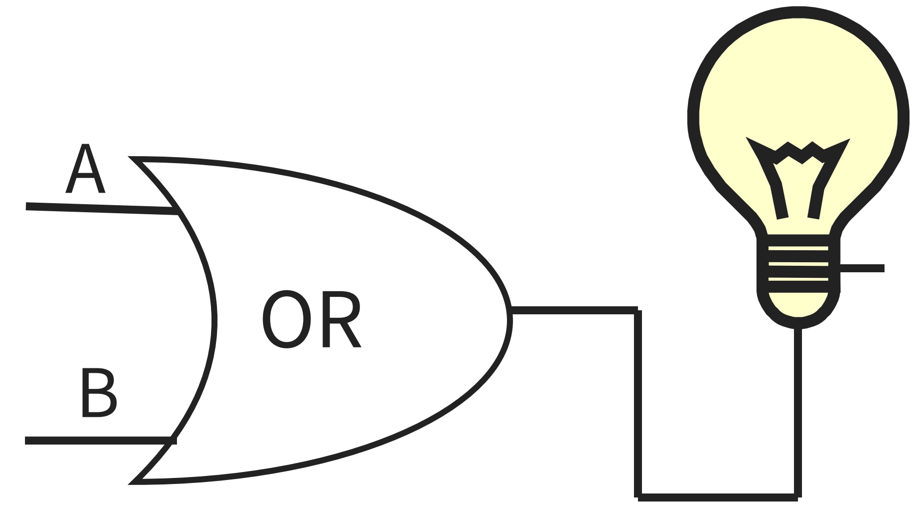
| A | B | Light |
|---|---|---|
| OFF | OFF | OFF |
| OFF | ON | ON |
| ON | OFF | ON |
| ON | ON | ON |



- Both (AND)

| A | B | Light |
|---|---|---|
| OFF | OFF | OFF |
| OFF | ON | OFF |
| ON | OFF | OFF |
| ON | ON | ON |

## Basic Building Blocks: Switches to Logic Gates

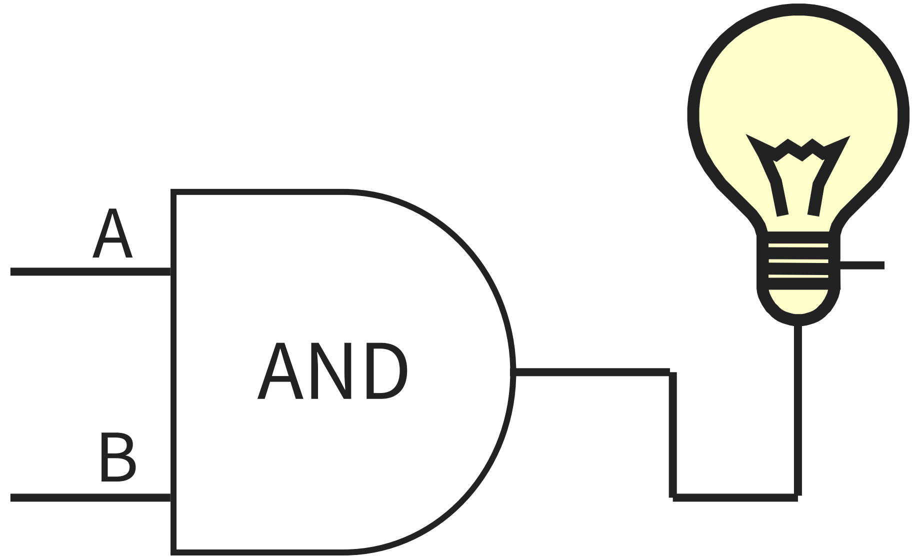


- Either (OR)

Truth Table

| A | B | Light |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

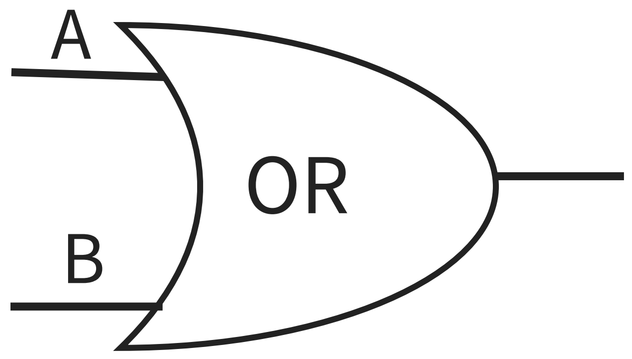
0 = OFF
1 = ON



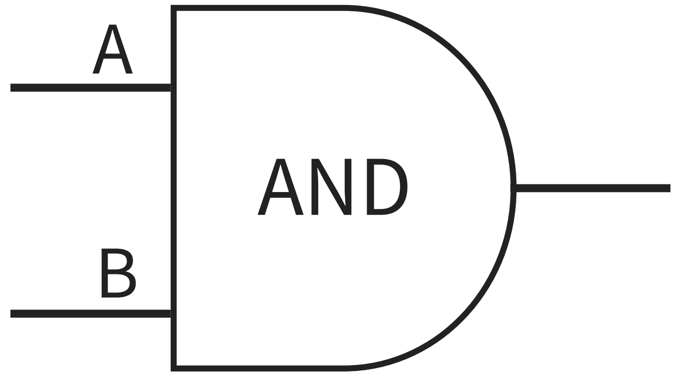
- Both (AND)

| A | B | Light |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## Basic Building Blocks: Switches to Logic Gates



**George Boole (1815-1864)**



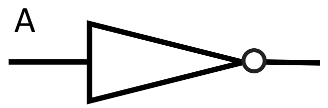
- Did you know?
- George Boole: Inventor of the idea of logic gates. He was born in Lincoln, England and he was the son of a shoemaker in a low class family.

## Takeaway

- Binary (two symbols: true and false) is the basis of Logic Design

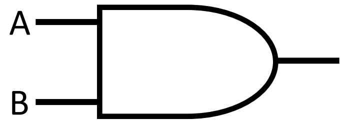
## Building Functions: Logic Gates

- NOT:



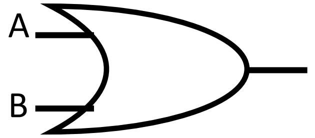
| A | Out |
|---|---|
| | |
| | |

- AND:



| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

- OR:



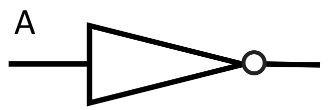
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

- Logic Gates
  - digital circuit that either allows a signal to pass through it or not.
  - Used to build logic functions
  - There are seven basic logic gates:

    AND, OR, **NOT**,

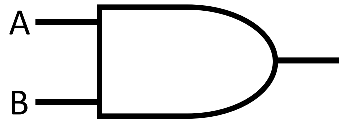
# Building Functions: Logic Gates

- NOT:



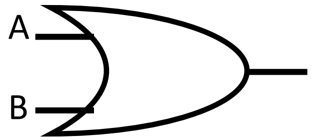
| A | Out |
|---|---|
| 0 | 1 |
| 1 | 0 |

- AND:



| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

- OR:



| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

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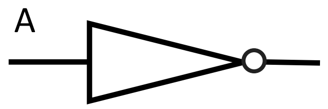
    NAND (not AND), NOR (not OR), XOR, and XNOR (not XOR) [later]

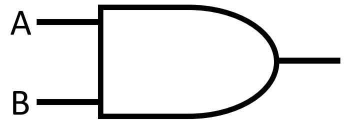
## Building Functions: Logic Gates

- NOT:



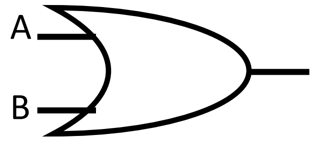
| A | Out |
|---|---|
| 0 | 1 |
| 1 | 0 |

- AND:



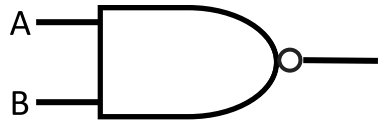
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

- OR:



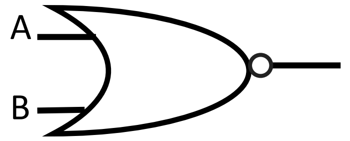
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

NAND:



| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

NOR:



| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

- Logic Gates
  - digital circuit that either allows a signal to pass through it or not.
  - Used to build logic functions
  - There are seven basic logic gates:

    AND, OR, **NOT**,

    NAND (not AND), NOR (not OR), XOR, and XNOR (not XOR) [later]

## Which Gate is this?

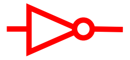
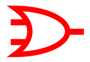
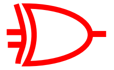
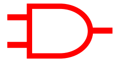
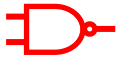
## PollEV Question #1

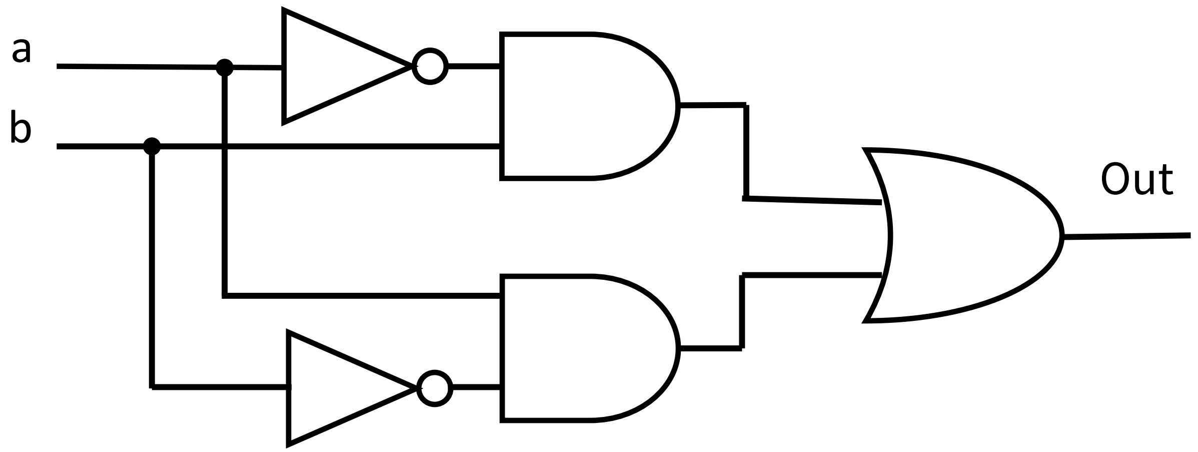
Function:

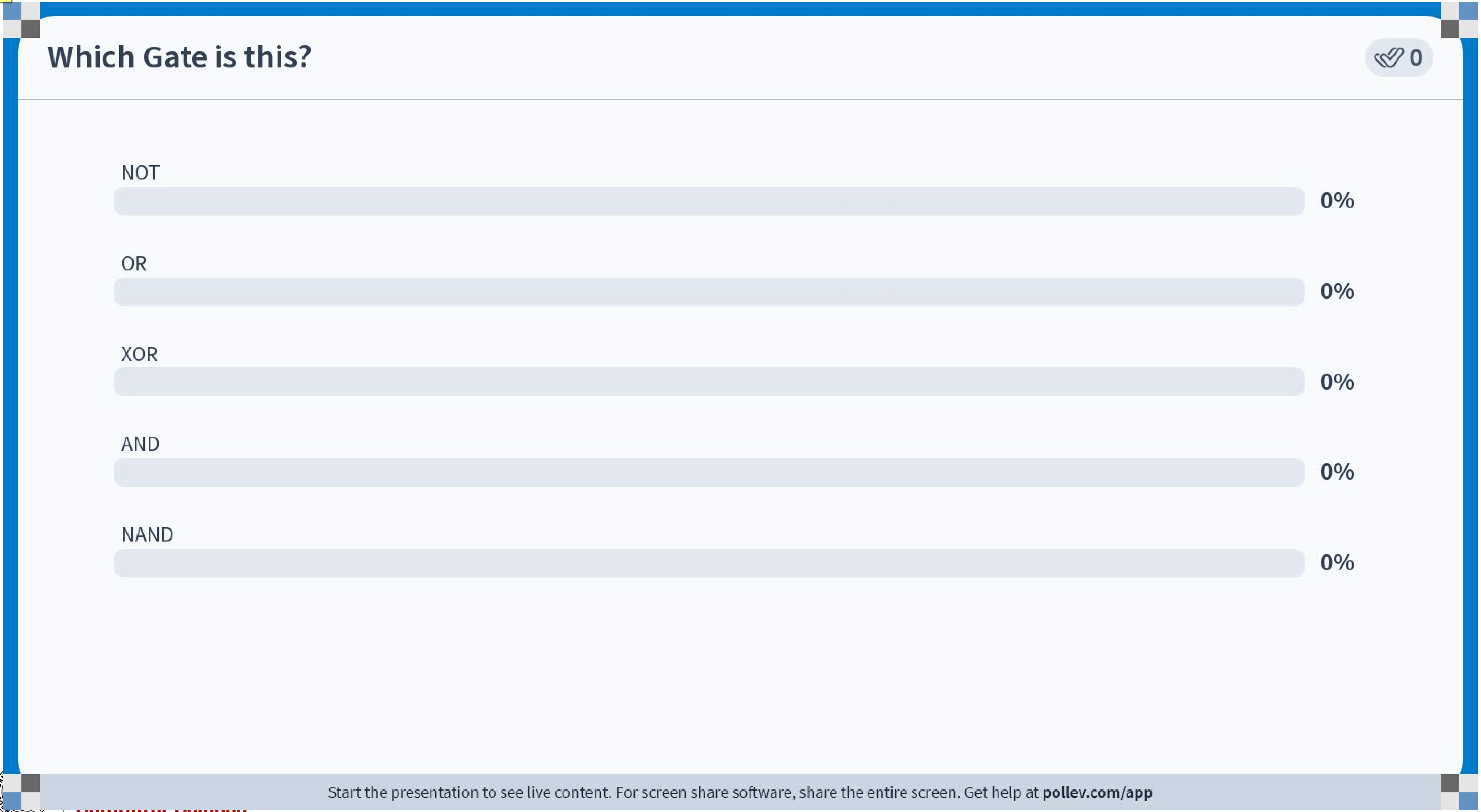
Symbol:

Truth Table:

| a | b | Out |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

(A) NOT 
(B) OR 
(C) XOR 
(D) AND 
(E) NAND 





## Which Gate is this?

0

| | |
|---|---|
| NOT | 0% |
| OR | 0% |
| XOR | 0% |
| AND | 0% |
| NAND | 0% |

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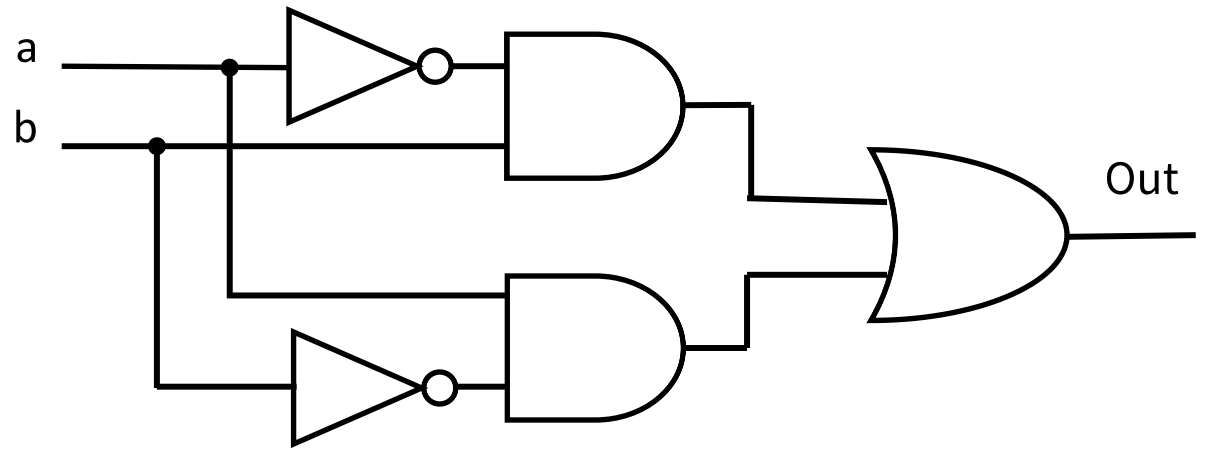
## Which Gate is this?

PollEV Question #1

- XOR: out = 1 if a or b is 1, but not both;
- out = 0 otherwise.
- out = 1, only if a = 1 AND b = 0
- OR a = 0 AND b = 1

| a | b | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(A) NOT
(B) OR
(C) XOR
(D) AND
(E) NAND



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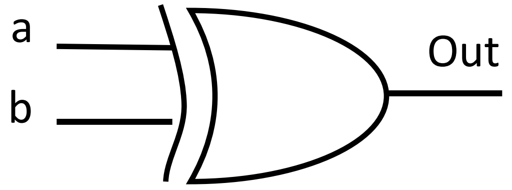
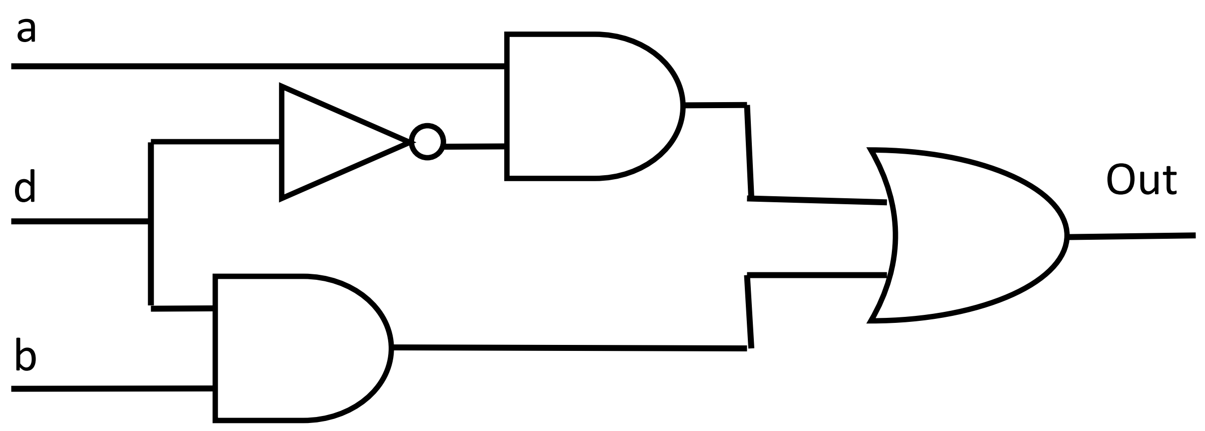
(A) NOT
(B) OR
[✓ (C) XOR]
(D) AND
(E) NAND



# Activity: Logic Gates

- Fill in the truth table, given the following Logic Circuit made from Logic AND, OR, and NOT gates.
- What does the logic circuit do?

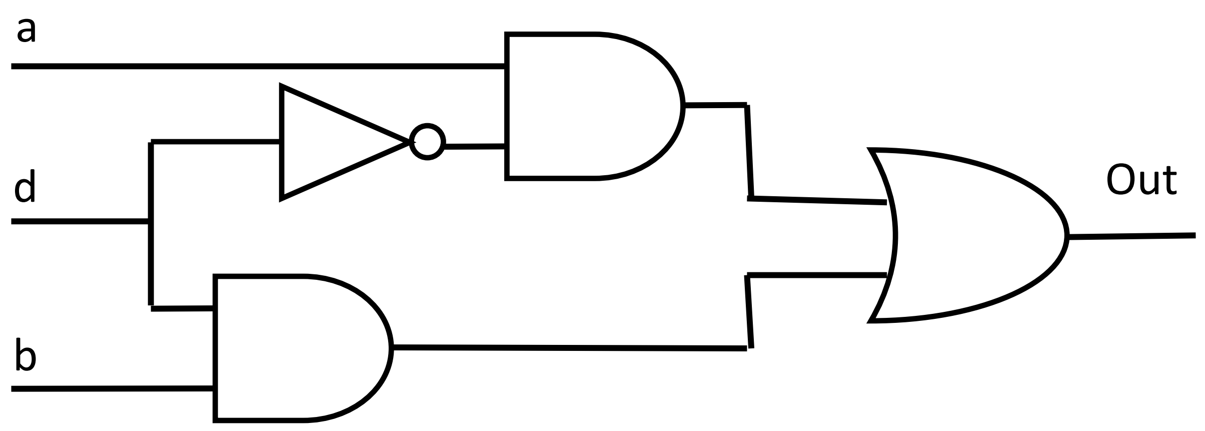
| **a** | **b** | **d** | **Out** |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |



# Activity: Logic Gates

- Multiplexor: select (d) between two inputs (a and b) and set one as the output (out)?
- out = a, if d = 0
- out = b, if d = 1

| a | b | d | Out |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |



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- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Binary Operations
  - One- and four-bit adders
  - Addition (two’s complement)
- Transistors (electronic switch)

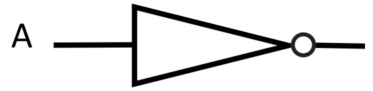
## Next Goal

- Given a Logic function, create a Logic Circuit that implements the Logic Function…
- …and, *with the minimum number of logic gates*

- Fewer gates: A cheaper ($$$) circuit!

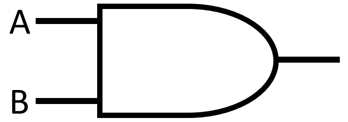
## Logic Gates

NOT:



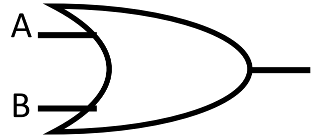
| A | Out |
|---|---|
| 0 | 1 |
| 1 | 0 |

AND:



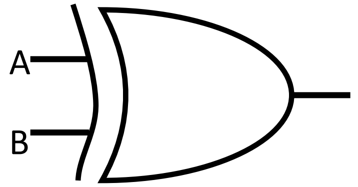
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

OR:



| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

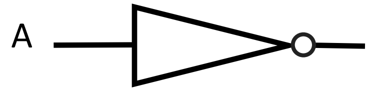
XOR:



| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

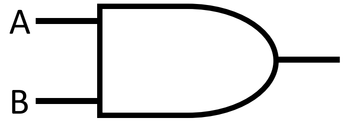
# Logic Gates

NOT:



| A | Out |
|---|---|
| 0 | 1 |
| 1 | 0 |

AND:



| A | B | Out |
|---|---|---|
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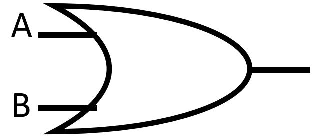
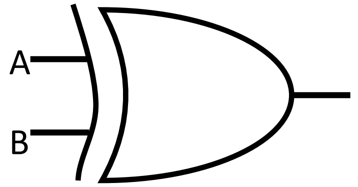
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

OR:



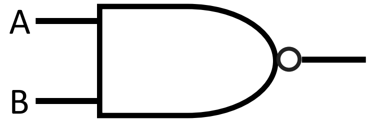
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

XOR:



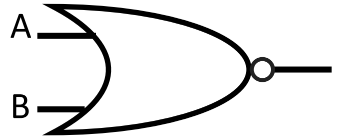
| A | B | Out |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

NAND:



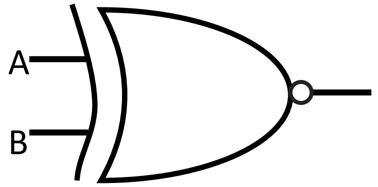
| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

NOR:



| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

XNOR:



| A | B | Out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## Logic Implementation

- How to implement a desired logic function?

| a | b | c | out |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

# Logic Implementation

- How to implement a desired logic function?

| a | b | c | out | minterm |
|---|---|---|---|---|
| **0** | **0** | **0** | **0** | $\bar{a}\,\bar{b}\,\bar{c}$ |
| **0** | **0** | **1** | **1** | $\bar{a}\,\bar{b}\,c$ |
| **0** | **1** | **0** | **0** | $\bar{a}\,b\,\bar{c}$ |
| **0** | **1** | **1** | **1** | $\bar{a}\,b\,c$ |
| **1** | **0** | **0** | **0** | $a\,\bar{b}\,\bar{c}$ |
| **1** | **0** | **1** | **1** | $a\,\bar{b}\,c$ |
| **1** | **1** | **0** | **0** | $a\,b\,\bar{c}$ |
| **1** | **1** | **1** | **0** | $a\,b\,c$ |

1) Write minterms
2) sum of products:
   - OR of all minterms where out=1

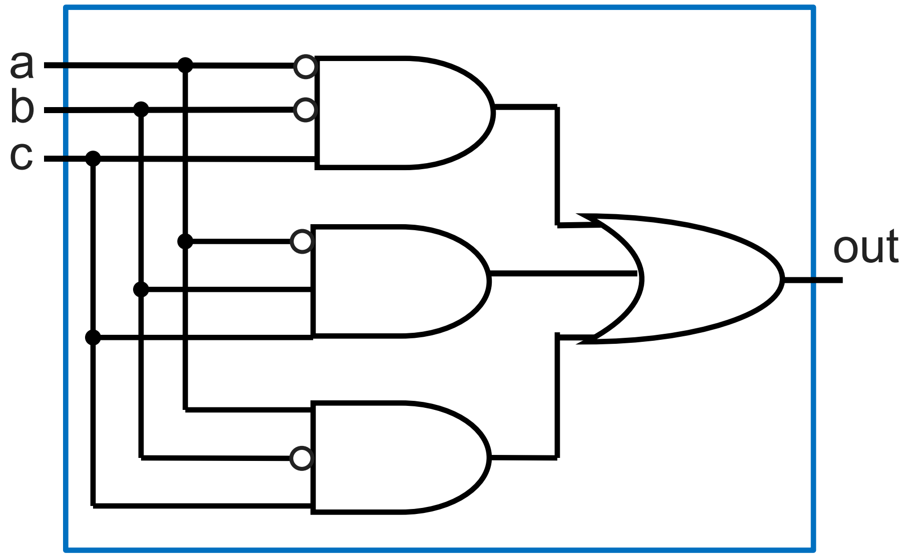
## Logic Implementation

- How to implement a desired logic function?

| a | b | c | out | minterm |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\bar{a}\,\bar{b}\,\bar{c}$ |
| 0 | 0 | 1 | 1 | $\bar{a}\,\bar{b}\,c$ |
| 0 | 1 | 0 | 0 | $\bar{a}\,b\,\bar{c}$ |
| 0 | 1 | 1 | 1 | $\bar{a}\,b\,c$ |
| 1 | 0 | 0 | 0 | $a\,\bar{b}\,\bar{c}$ |
| 1 | 0 | 1 | 1 | $a\,\bar{b}\,c$ |
| 1 | 1 | 0 | 0 | $a\,b\,\bar{c}$ |
| 1 | 1 | 1 | 0 | $a\,b\,c$ |

1) Write minterms
2) sum of products:

- OR of all minterms where out=1
  - E.g. out = $\overline{ab}c + \bar{a}bc + a\bar{b}c$



corollary: *any* combinational circuit *can be* implemented in two levels of logic (ignoring inverters)

# Logic Equations

- NOT:
  - out = $\bar{a}$ = !a = $\neg$a
- AND:
  - out = $a \cdot b$ = a & b = $a \wedge b$
- OR:
  - out = a + b = a | b = $a \vee b$
- XOR:
  - out = $a \oplus b = a\bar{b} + \bar{a}b$
- Logic Equations
  - Constants: true = 1, false = 0
  - Variables: a, b, out, …
  - Operators (above): AND, OR, NOT, etc.

# Logic Equations

- NOT:
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- Logic Equations
  - Constants: true = 1, false = 0
  - Variables: a, b, out, …
  - Operators (above): AND, OR, NOT, etc.

NAND:
- out = $\overline{a \cdot b}$ = !(a & b) = $\neg(a \wedge b)$

NOR:
- out = $\overline{a + b}$ = !(a | b) = $\neg(a \vee b)$

XNOR:
- out = $\overline{a \oplus b} = ab + \overline{ab}$

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## Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$a + 0 =$

$a + 1 =$

$a + \bar{a} =$

$a \cdot 0 =$

$a \cdot 1 =$

$a \cdot \bar{a} =$

## Identities

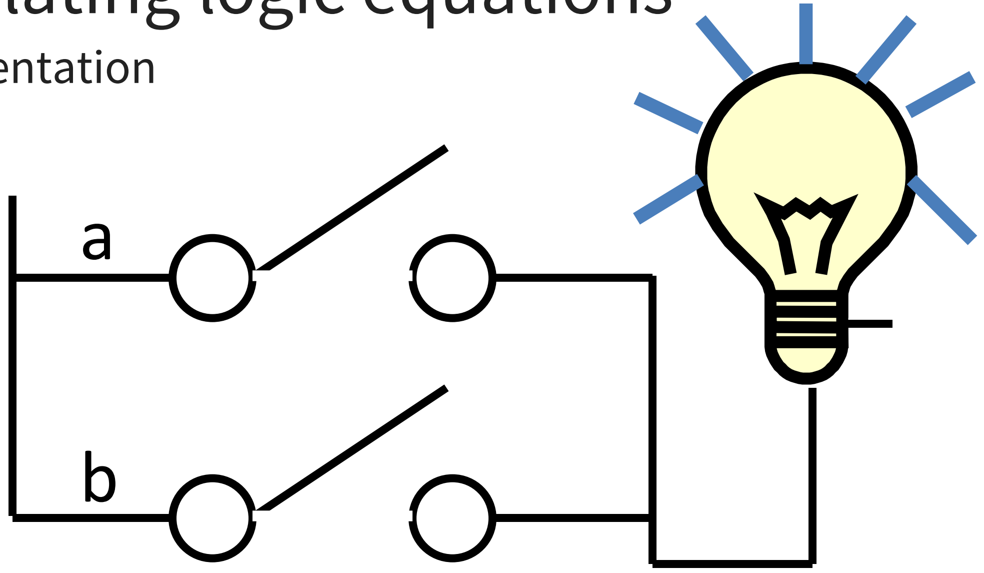
Identities useful for manipulating logic equations

- For optimization & ease of implementation

$a + 0 = a$

$a + 1 = 1$

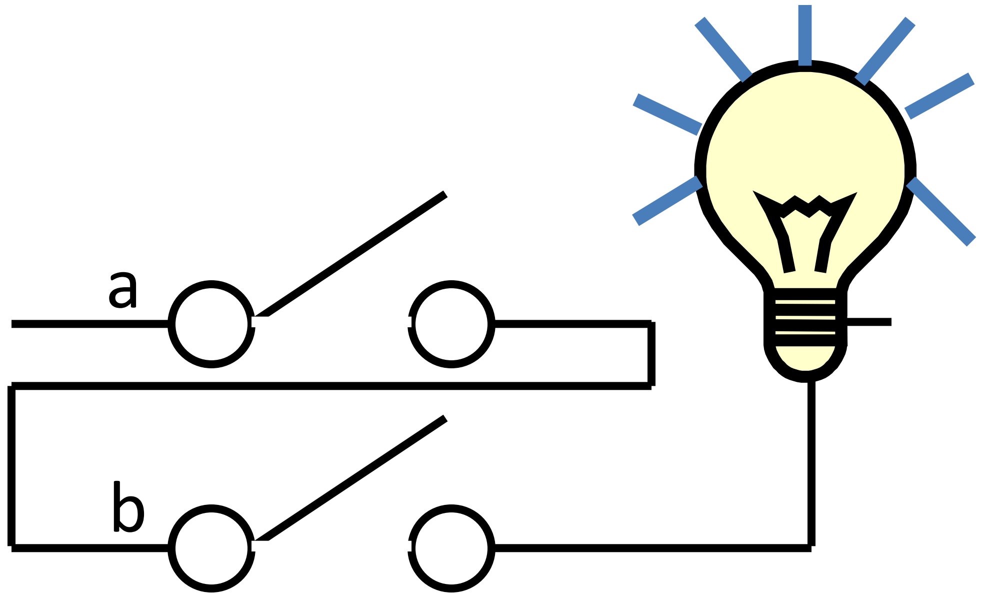
$a + \bar{a} = 1$



$a \cdot 0 = 0$

$a \cdot 1 = a$

$a \cdot \bar{a} = 0$



## Identities

Identities useful for manipulating logic equations

- For optimization & ease of implementation

$\overline{(a+b)}$ =

$\overline{(a \cdot b)}$ =

$a + a\,b$ =

$a(b+c)$ =

$\overline{a(b+c)}$ =

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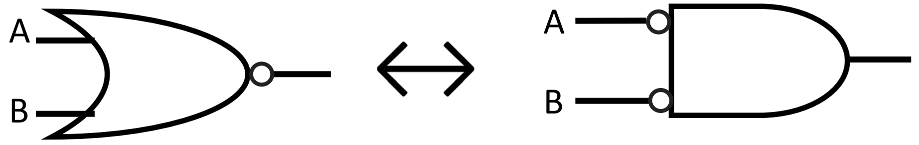
# Identities

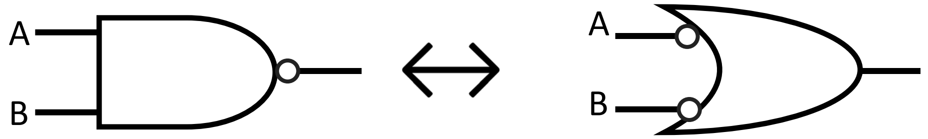
Identities useful for manipulating logic equations

- For optimization & ease of implementation

$\overline{(a + b)} = \bar{a} \cdot \bar{b}$



$\overline{(a \cdot b)} = \bar{a} + \bar{b}$



$a + a\,b = a$

$a(b+c) = ab + ac$

$\overline{a(b + c)} = \bar{a} + \bar{b} \cdot \bar{c}$

## Minimization Example

$a + 0 = a$
$a + 1 = 1$
$a + \bar{a} = 1$
$a \cdot 0 = 0$
$a \cdot 1 = a$
$a \cdot \bar{a} = 0$

$a + ab = a$
$a(b+c) = ab + ac$

$$\overline{(a+b)} = \overline{a} \bullet \overline{b}$$
$$\overline{(ab)} = \overline{a} + \overline{b}$$
$$\overline{a(b+c)} = \overline{a} + \overline{b} \bullet \overline{c}$$

Minimize this logic equation:

$$\begin{aligned}(a+b)(a+c) &= (a+b)a + (a+b)c \\ &= aa + ba + ac + bc \\ &= a + a(b+c) + bc \\ &= a + bc\end{aligned}$$

# Minimization Example

PollEV Question #2

a + 0 = a

a + 1 = 1

a + ā = 1

a · 0 = 0

a · 1 = a

a · ā = 0

a + a b = a

a (b+c) = ab + ac

$$\overline{(a+b)} = \overline{a} \bullet \overline{b}$$

$$\overline{(ab)} = \overline{a} + \overline{b}$$

$$\overline{a(b+c)} = \overline{a} + \overline{b} \bullet \overline{c}$$

(a+b)(a+c) → a + bc

How many gates were required before and after?

| | BEFORE | AFTER |
|---|---|---|
| (A) | 2 OR | 1 OR |
| (B) | 2 OR, 1 AND | 2 OR |
| (C) | 2 OR, 1 AND | 1 OR, 1 AND |
| (D) | 2 OR, 2 AND | 2 OR |
| (E) | 2 OR, 2 AND | 2 OR, 1 AND |

## How many gates were required before and after?

✓ 0

| | |
|---|---|
| 2x OR -> 1x OR | 0% |
| 2x OR, 1x AND -> 2x OR | 0% |
| 2x OR, 1x AND -> 1x OR, 1x AND | 0% |
| 2x OR, 2x AND -> 2x OR | 0% |
| 2x OR, 2x AND -> 2x OR, 1x AND | 0% |

## Minimization Example

PollEV Question #2

$a + 0 = a$

$a + 1 = 1$

$a + \bar{a} = 1$

$a \cdot 0 = 0$

$a \cdot 1 = a$

$a \cdot \bar{a} = 0$

$a + ab = a$

$a(b+c) = ab + ac$

$$\overline{(a+b)} = \overline{a} \bullet \overline{b}$$

$$\overline{(ab)} = \overline{a} + \overline{b}$$

$$\overline{a(b+c)} = \overline{a} + \overline{b} \bullet \overline{c}$$

$(a+b)(a+c) \rightarrow a + bc$

How many gates were required before and after?

| | BEFORE | AFTER |
|---|---|---|
| (A) | 2 OR | 1 OR |
| (B) | 2 OR, 1 AND | 2 OR |
| [✓ (C)] | [✓ 2 OR, 1 AND] | [✓ 1 OR, 1 AND] |
| (D) | 2 OR, 2 AND | 2 OR |
| (E) | 2 OR, 2 AND | 2 OR, 1 AND |

# Checking Equality w/Truth Tables

circuits ↔ truth tables ↔ equations

Example: $(a+b)(a+c) = a + bc$

| a | b | c | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | | | | | |
| 0 | 0 | 1 | | | | | |
| 0 | 1 | 0 | | | | | |
| 0 | 1 | 1 | | | | | |
| 1 | 0 | 0 | | | | | |
| 1 | 0 | 1 | | | | | |
| 1 | 1 | 0 | | | | | |
| 1 | 1 | 1 | | | | | |

## Checking Equality w/Truth Tables

circuits ↔ truth tables ↔ equations

Example: $(a+b)(a+c) = a + bc$

| a | b | c | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | | | | | |
| 0 | 0 | 1 | | | | | |
| 0 | 1 | 0 | | | | | |
| 0 | 1 | 1 | | | | | |
| 1 | 0 | 0 | | | | | |
| 1 | 0 | 1 | | | | | |
| 1 | 1 | 0 | | | | | |
| 1 | 1 | 1 | | | | | |

# Checking Equality w/Truth Tables

circuits ↔ truth tables ↔ equations

Example: (a+b)(a+c) = a + bc

| a | b | c | a+b | a+c | LHS | bc | RHS |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

## Goals for Today: Bottom Up!

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Binary Operations
  - One- and four-bit adders
  - Addition (two's complement)
- Transistors (electronic switch)

## Next Goal

Binary Arithmetic: Add and Subtract two binary numbers

# Binary Addition (Revisited)

Addition works the same for all bases

- Add the digits in each position
- Propagate the carry

Binary addition is pretty easy

- Combine two bits at a time
- Along with a carry

$$\begin{array}{r}
1\phantom{00} \\
183 \\
+\,254 \\
\hline
437
\end{array}$$

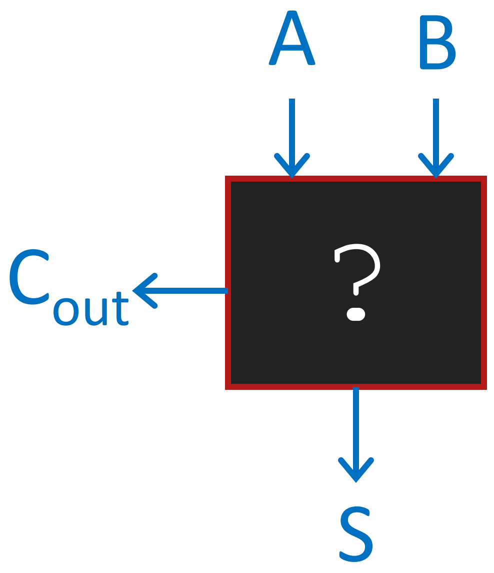
Carry-in

$$\begin{array}{r}
11\textcircled{1}\phantom{000}\ \text{Carry-out} \\
001110 \\
+\,011100 \\
\hline
101010
\end{array}$$

## Binary Addition

- Binary addition requires
  - Add of two bits PLUS carry-in
  - Also, carry-out if necessary

# 1-bit Half Adder



- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

| A | B | $C_{out}$ | S |
|---|---|---|---|
| 0 | 0 | | |
| 0 | 1 | | |
| 1 | 0 | | |
| 1 | 1 | | |

*PollEV Question #3*

What is the equation for $C_{out}$?

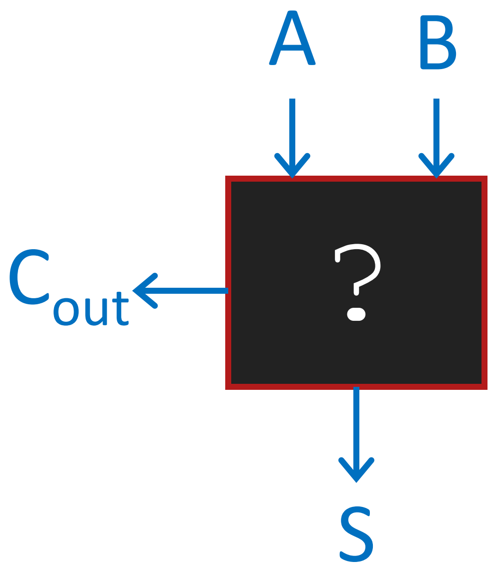
a) $A + B$
b) $AB$
c) $A \oplus B$
d) $A + !B$
e) $!A!B$

## What is the equation for Cout?

0

| | |
|---|---|
| A + B | 0% |
| AB | 0% |
| A ⊠ B | 0% |
| A + !B | 0% |
| !A!B | 0% |

## 1-bit Half Adder



- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

| A | B | $C_{out}$ | S |
|---|---|---|---|
| 0 | 0 | | |
| 0 | 1 | | |
| 1 | 0 | | |
| 1 | 1 | | |

*PollEV Question #3*

What is the equation for $C_{out}$?

a) $A + B$

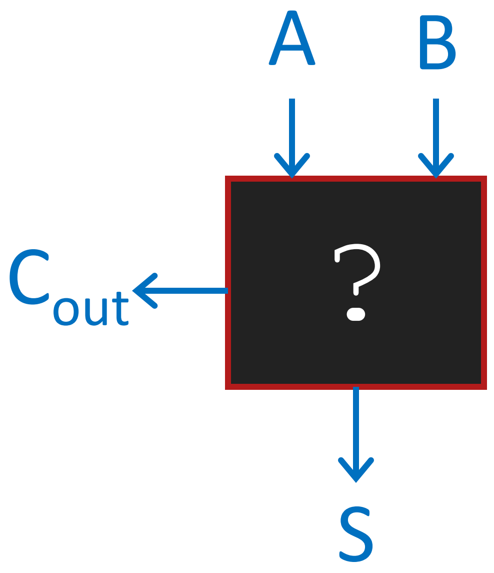
[✓ b) $AB$]

c) $A \oplus B$

d) $A + !B$

e) $!A!B$

# 1-bit Half Adder



- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

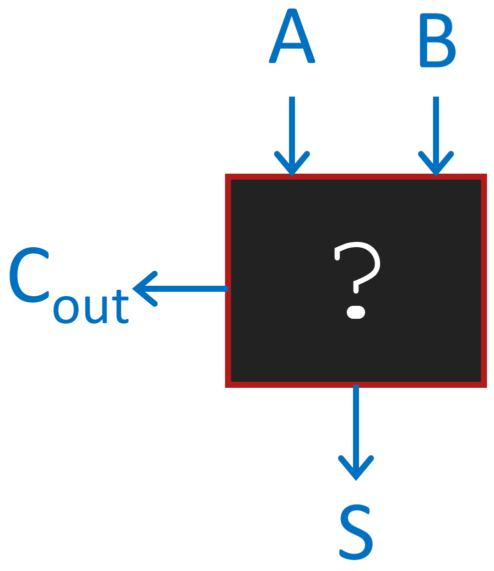
| A | B | $C_{out}$ | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$$\begin{array}{cccc} 0 & 0 & 0 & 1 \\ \phantom{+}0 & \phantom{+}0 & \phantom{+}1 & \phantom{+}1 \\ \underline{+\,0} & \underline{+\,1} & \underline{+\,0} & \underline{+\,1} \\ 0 & 1 & 1 & 0 \end{array}$$

S = one input equals 1

$C_{out}$ = two inputs equal 1

# 1-bit Half Adder

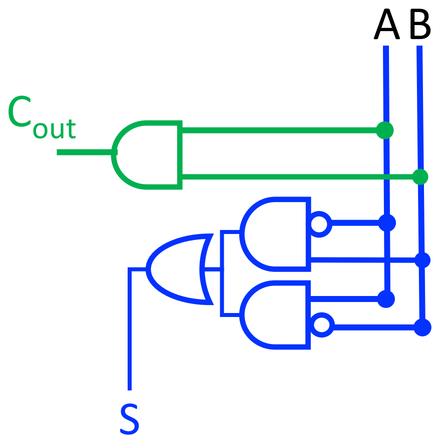


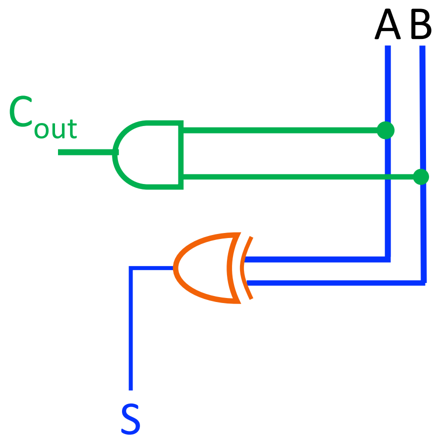
- Adds two 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- No carry-in

| A | B | $C_{out}$ | S |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

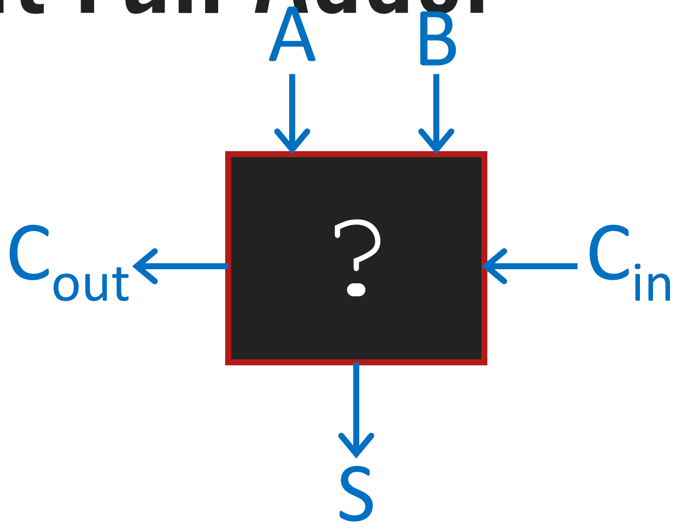
$$S = \overline{A}B + A\overline{B} = A \oplus B$$

$$C_{out} = AB$$





# 1-bit Full Adder

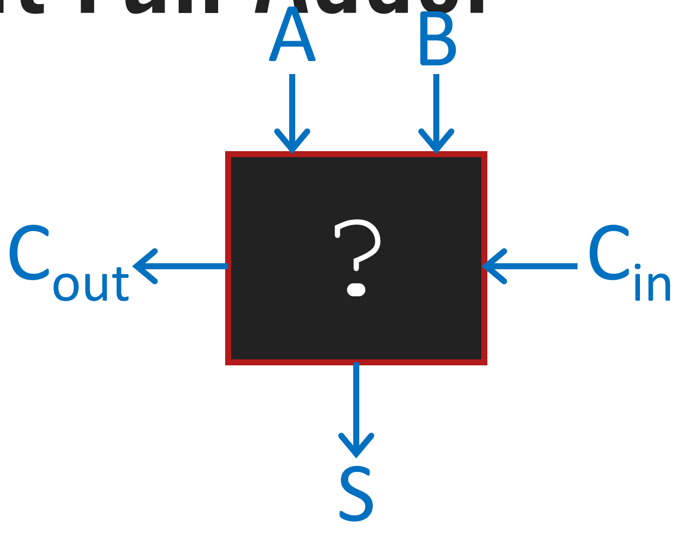


| A | B | $C_{in}$ | $C_{out}$ | S |
|---|---|---|---|---|
| 0 | 0 | 0 | | |
| 0 | 0 | 1 | | |
| 0 | 1 | 0 | | |
| 0 | 1 | 1 | | |
| 1 | 0 | 0 | | |
| 1 | 0 | 1 | | |
| 1 | 1 | 0 | | |
| 1 | 1 | 1 | | |

- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded

- Fill in Truth Table
- Create Sum-of-Product Form
- Draw the Circuits

# 1-bit Full Adder



- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded

| A | B | $C_{in}$ | $C_{out}$ | S |
|---|---|---|---|---|
| 0 | 0 | 0 | | |
| 0 | 0 | 1 | | |
| 0 | 1 | 0 | | |
| 0 | 1 | 1 | | |
| 1 | 0 | 0 | | |
| 1 | 0 | 1 | | |
| 1 | 1 | 0 | | |
| 1 | 1 | 1 | | |

*PollEV Question #4*

What is the equation for $C_{out}$?

a) $A + B + C_{in}$

b) $!A + !B + !\,C_{in}$

c) $A \oplus B \oplus C_{in}$

*d)* $\overline{AB}C + \overline{A}B\overline{C} + A\overline{BC} + ABC$

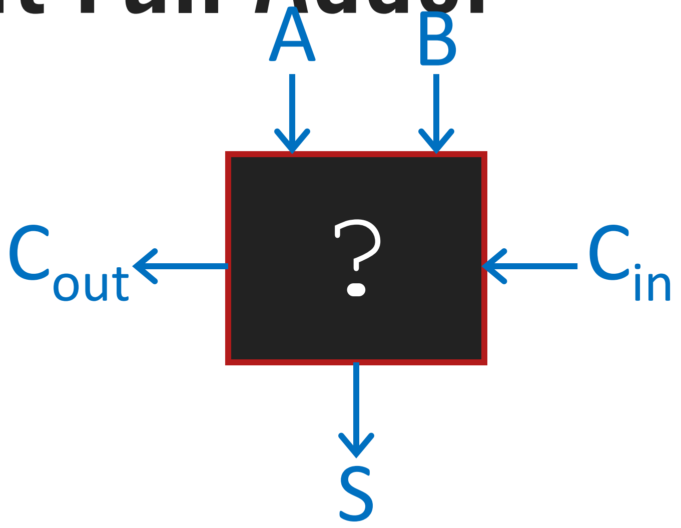
*e)* $\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$

## What is the equation for Cout? (take 2)

0

| | |
|---|---|
| A + B + Cin | 0% |
| !A + !B + ! Cin | 0% |
| A ⊠ B ⊠ Cin | 0% |
| !(AB)Cin + !AB!Cin + A!(BC) + ABC | 0% |
| !ABCin + A!BCin + AB!Cin + ABC | 0% |

# 1-bit Full Adder



- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded

| A | B | $C_{in}$ | $C_{out}$ | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

*PollEV Question #4*

What is the equation for $C_{out}$?

a) $A + B + C_{in}$

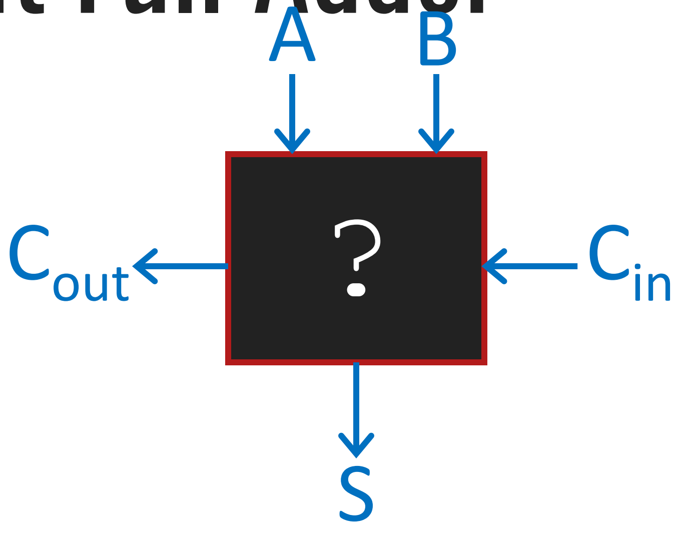
b) $!A + !B + !\ C_{in}$

c) $A \oplus B \oplus C_{in}$

*d)* $\overline{AB}C + \overline{A}B\overline{C} + A\overline{BC} + ABC$

[✓ *e)* $\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$]

# 1-bit Full Adder

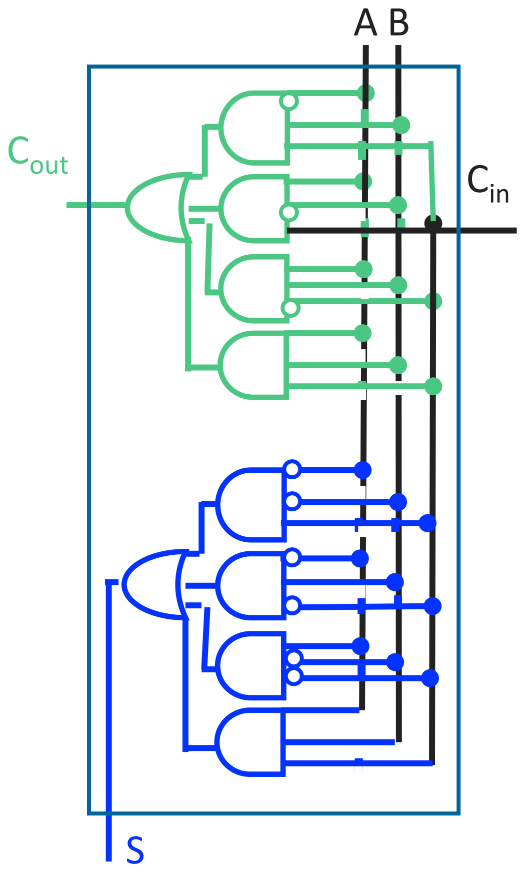


| A | B | $C_{in}$ | $C_{out}$ | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

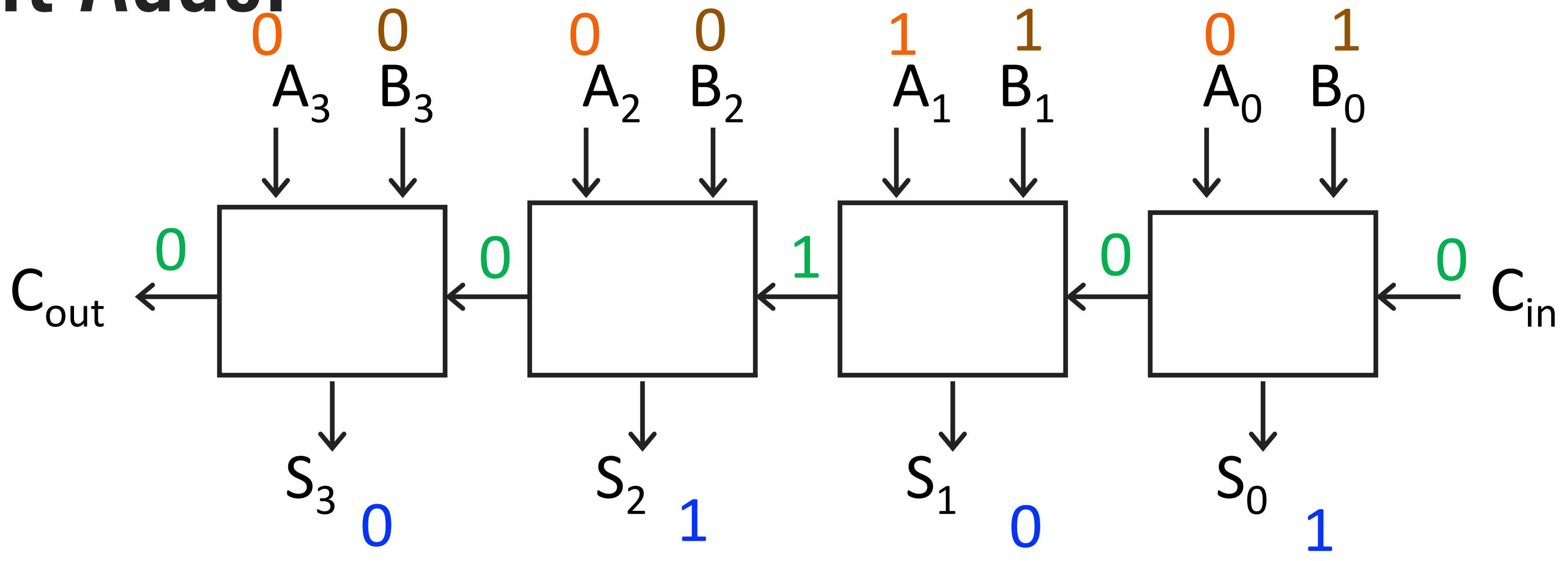
- Adds three 1-bit numbers
- Computes 1-bit result and 1-bit carry-out
- Can be cascaded

$S = \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$

$C_{out} = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$



# 4-bit Adder



- Adds two 4-bit numbers, along with carry-in
- Computes 4-bit result and carry out
- 3 + 2 = 5
- Carry-out → result > 4 bits

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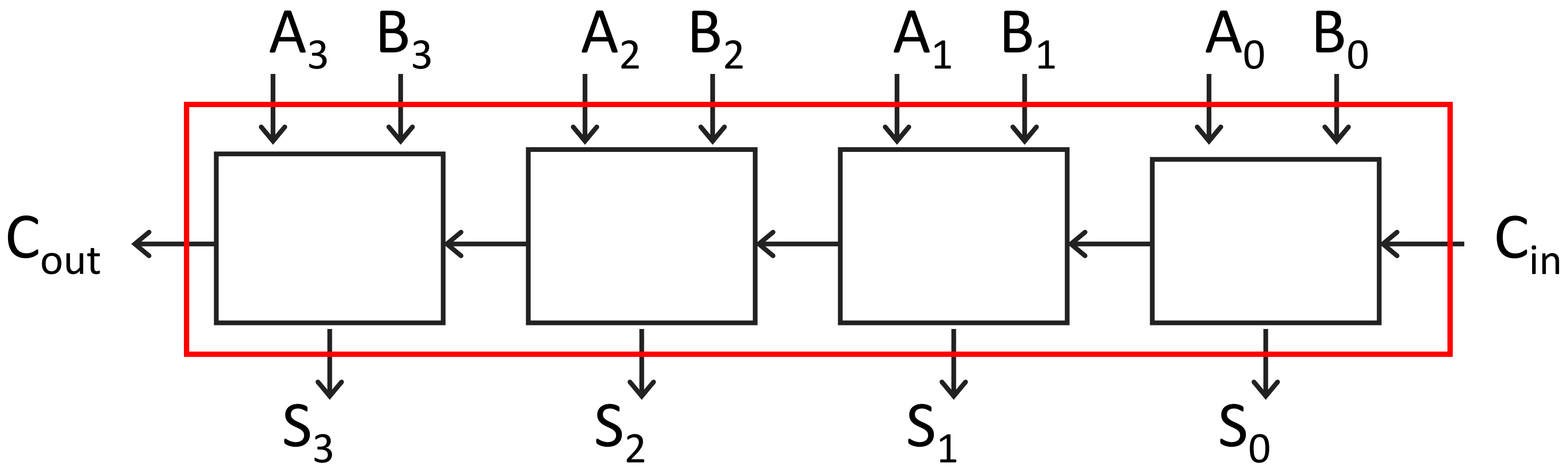
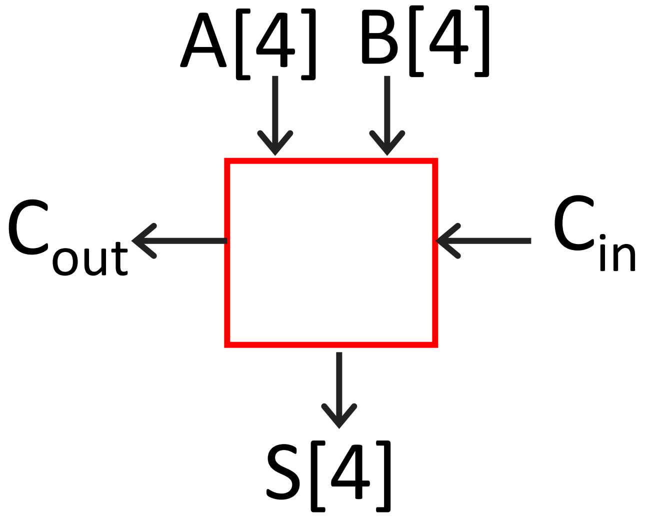
# 4-bit Adder



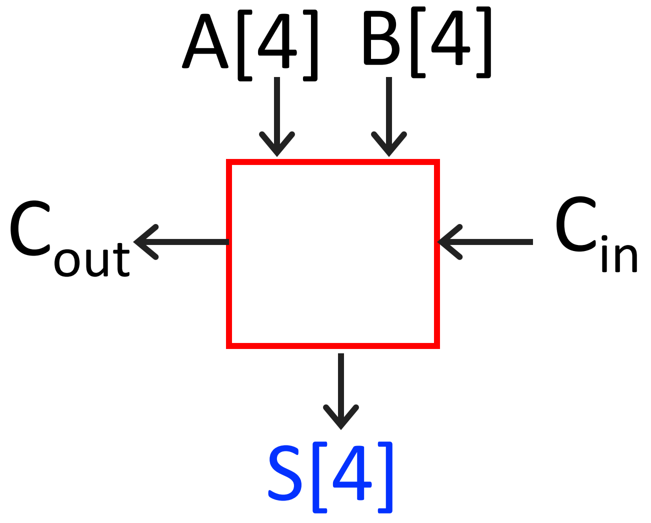


Build it
and
box it!
*(Theme of 3410)*

## PollEV Question #5

What's the largest sum you can calculate with a 4 bit adder?
(Give your answer in base 10. Assume unsigned numbers)

a) 4
b) 1,111
c) 15
d) 16
e) 4000



## What's the largest sum you can calculate with a 4 bit adder? (Give your answer in base 10. Assume unsigned numbers)

0

| | |
|---|---|
| 4 | 0% |
| 1,111 | 0% |
| 15 | 0% |
| 16 | 0% |
| 4000 | 0% |

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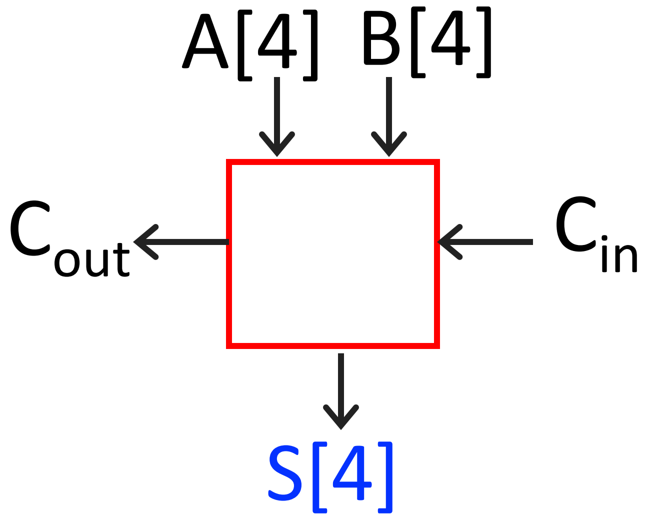
## PollEV Question #5

What's the largest sum you can calculate with a 4 bit adder?
(Give your answer in base 10. Assume unsigned numbers)

a) 4
b) 1,111
[✓ c) 15]
d) 16
e) 4000



# Binary Subtraction

Why create a new circuit?

Just use addition using two's complement math

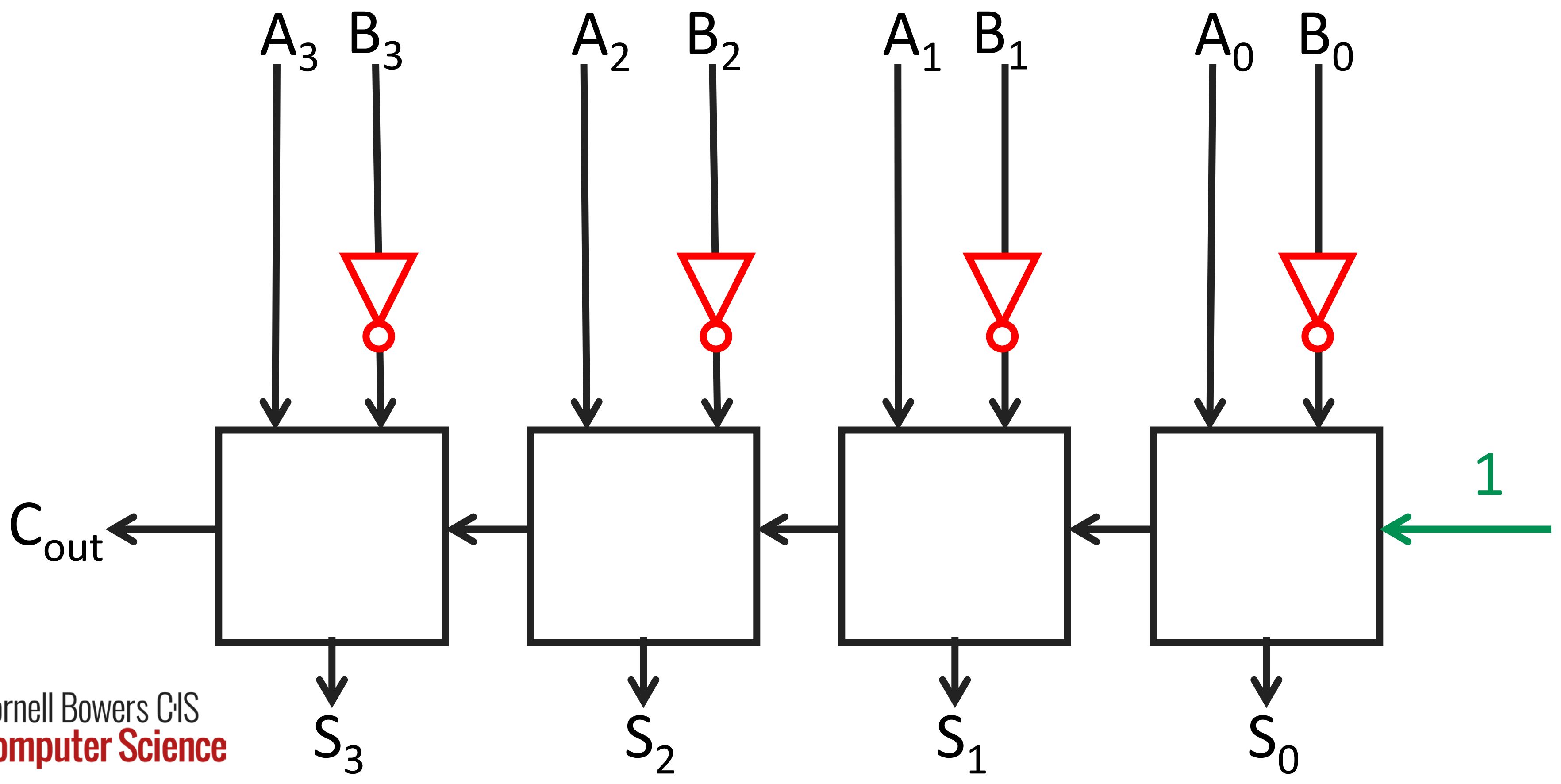
- How?

# Binary Subtraction

## Two's Complement Subtraction

- Subtraction is addition with a negated operand
  - Negation is done by inverting all bits and adding one

$A - B \quad = \quad A + (-B) \quad = \quad A + (\overline{B} + 1)$



## Binary Subtraction

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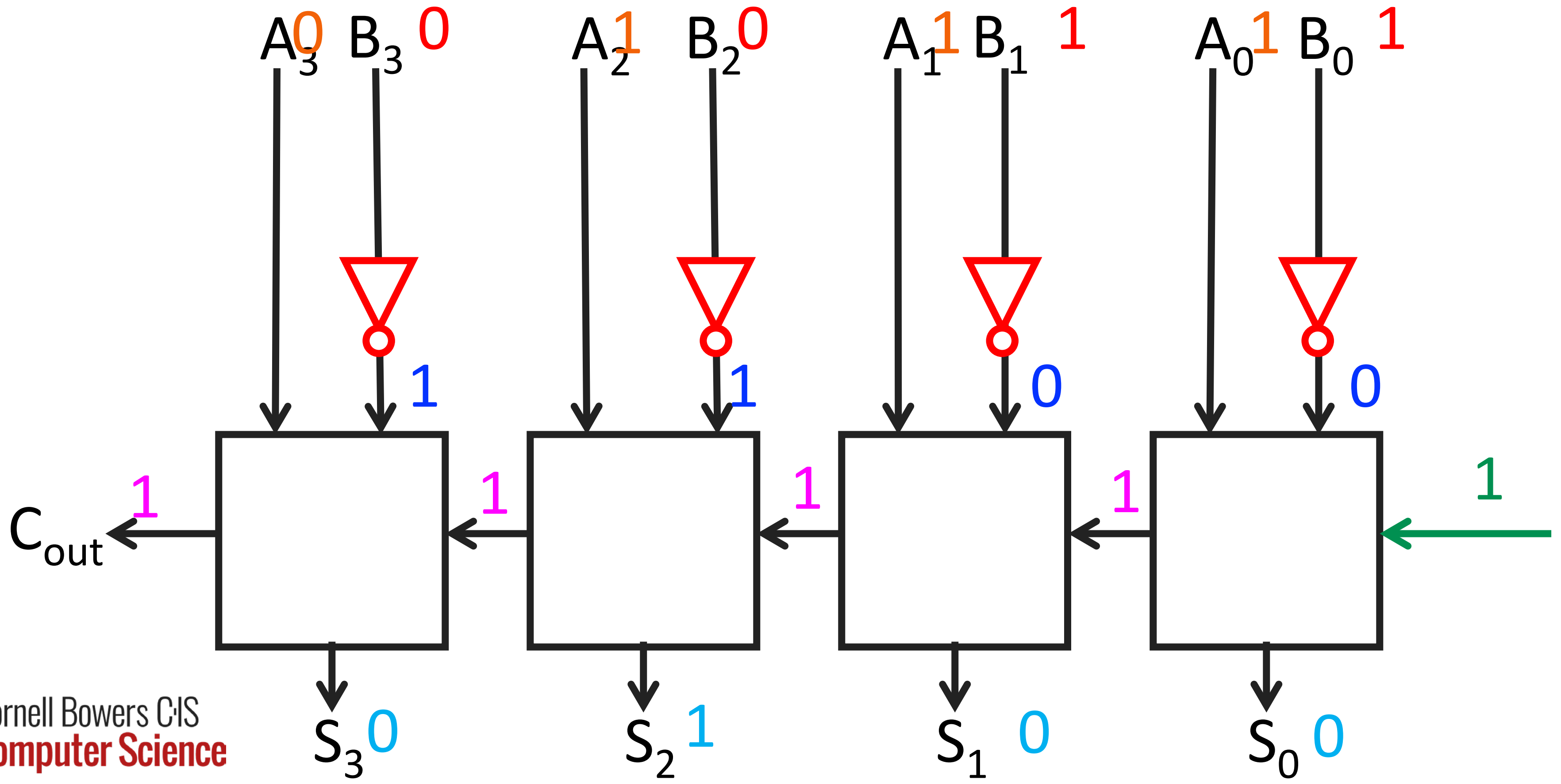

# Binary Subtraction

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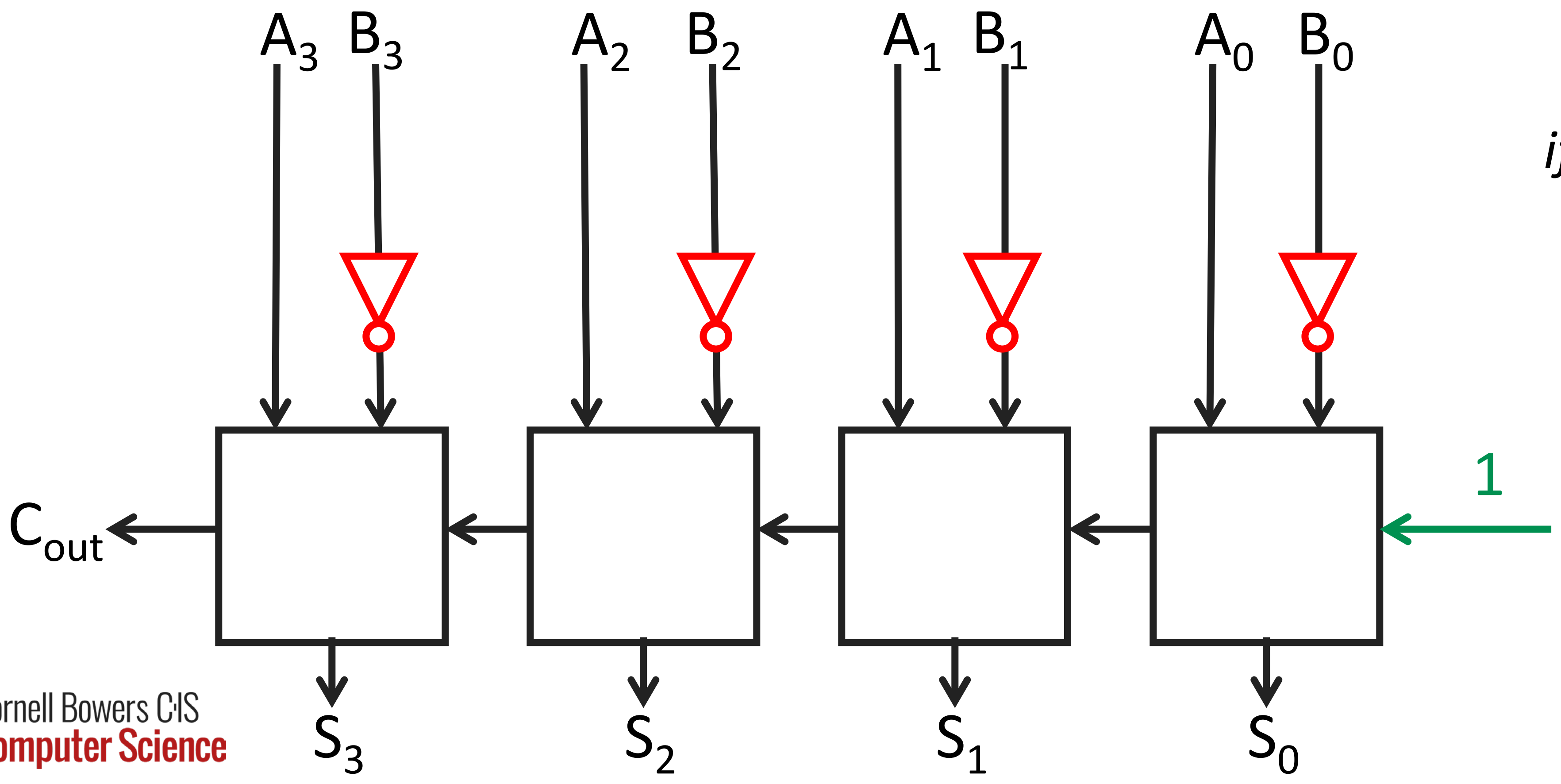

**Add or subtract with XOR gate**

| sub? | $B_0$ | $newB_0$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

*if subtracting, invert $B_0$*

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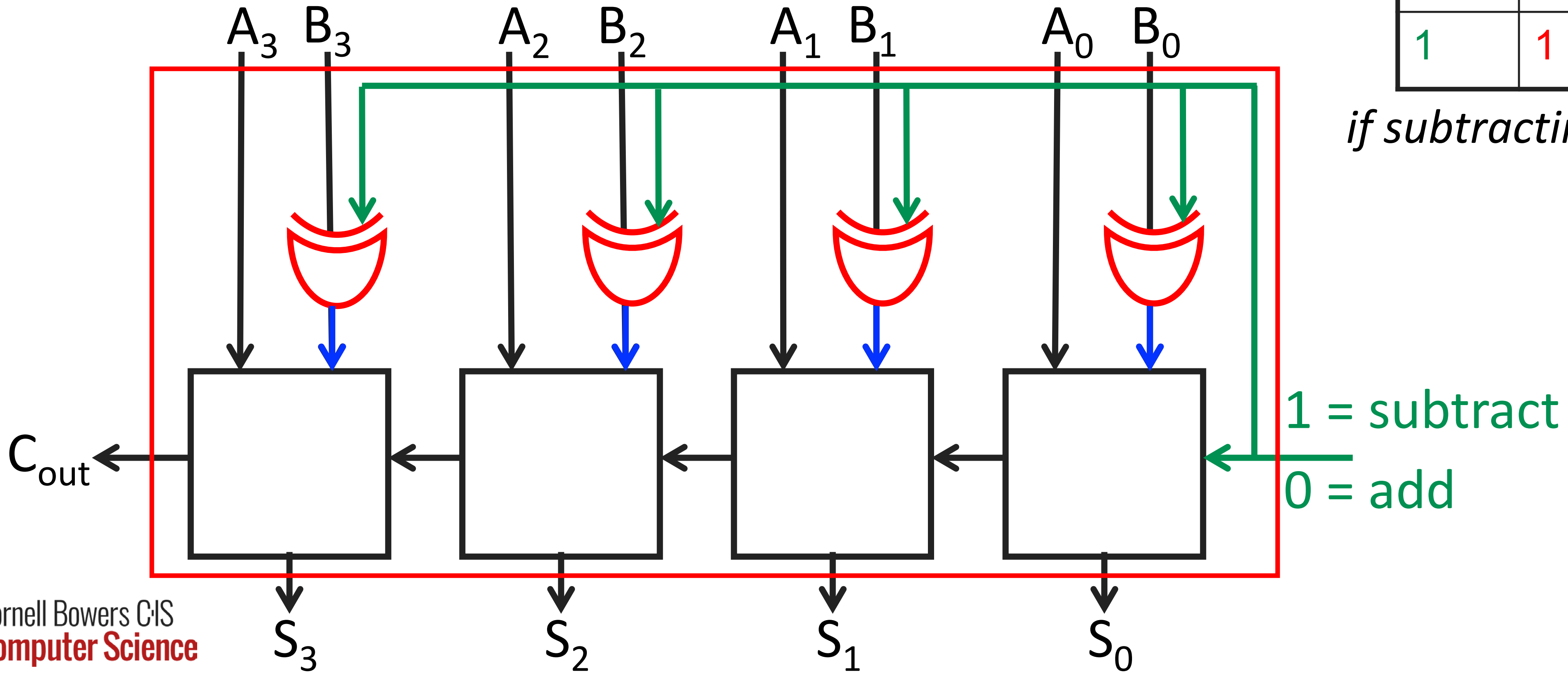

# Binary Subtraction

## Two's Complement Subtraction

- Subtraction is addition with a negated operand
  - E.g. 6 – 7 = -1 → 6 + (-7) = -1

**Add or subtract with XOR gate**

| sub? | $B_0$ | $newB_0$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

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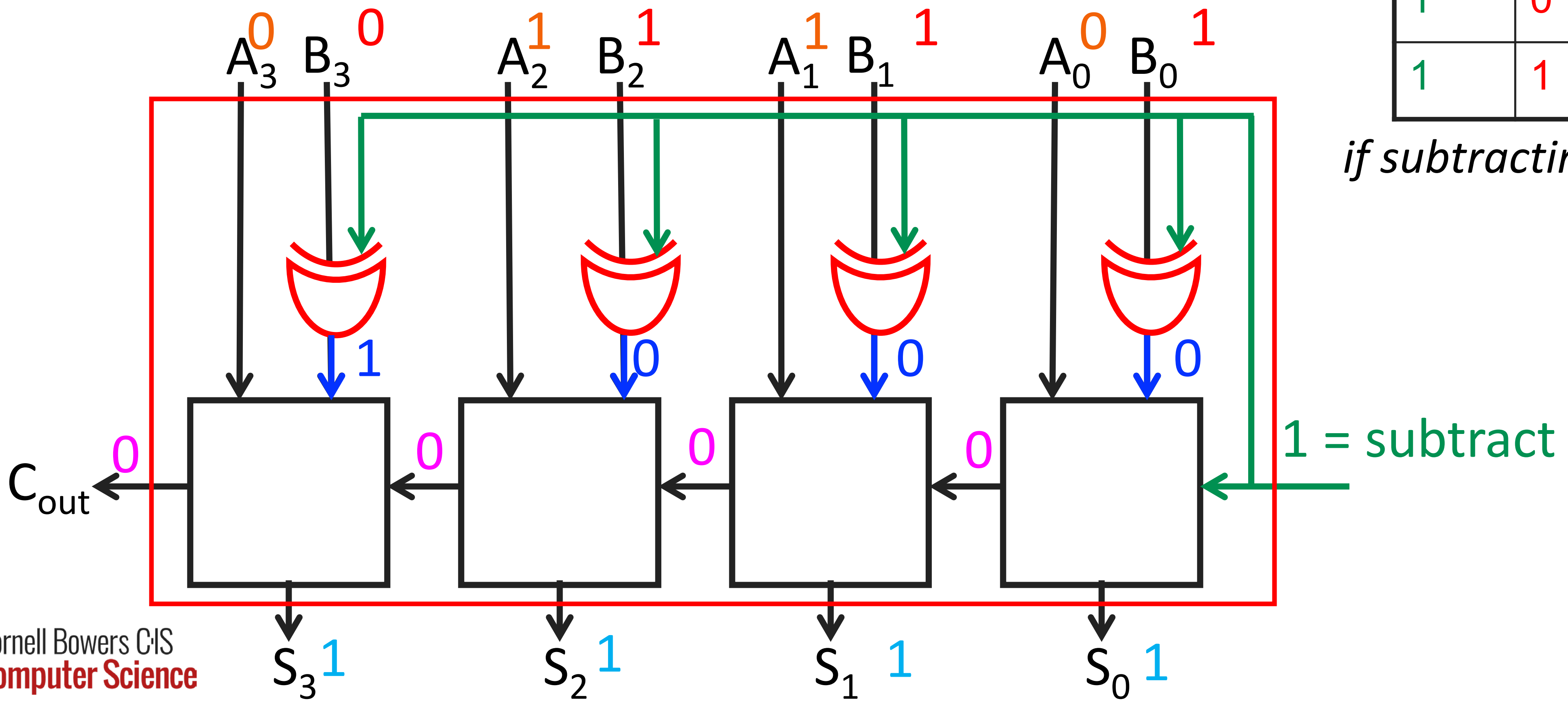


# Binary Subtraction

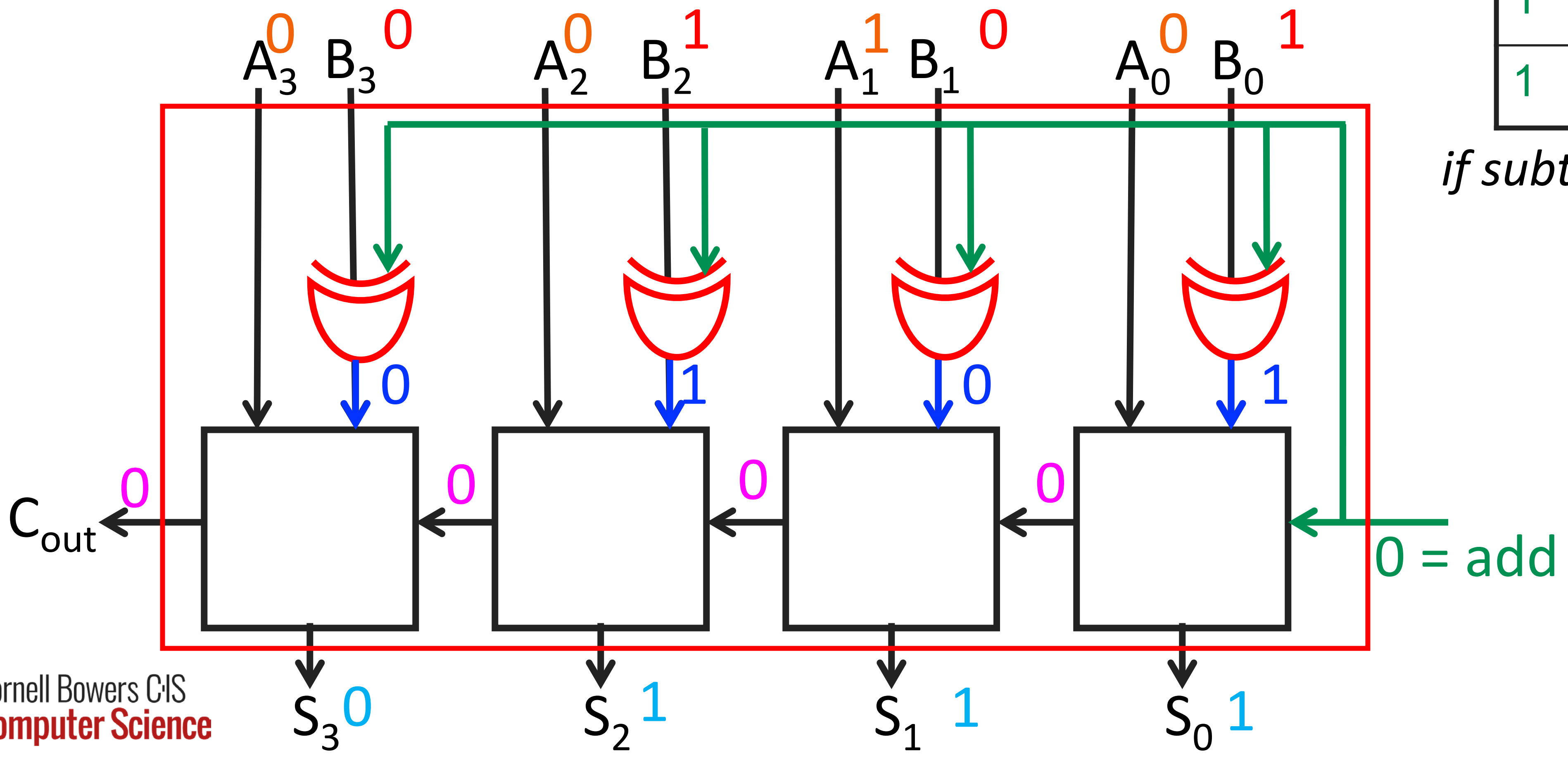
## Two's Complement Subtraction

- Subtraction is addition with a negated operand
    - Addition still works! E.g. 2 + 5 = 7

**Add or subtract with XOR gate**

| sub? | $B_0$ | new$B_0$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

*if subtracting, invert $B_0$*



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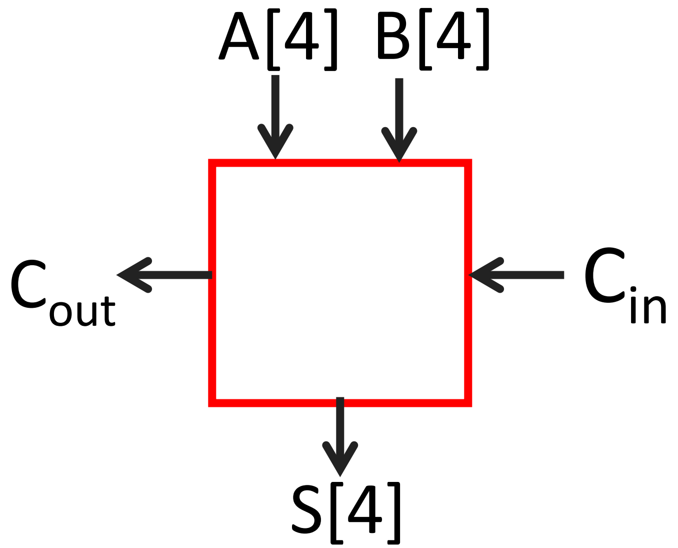
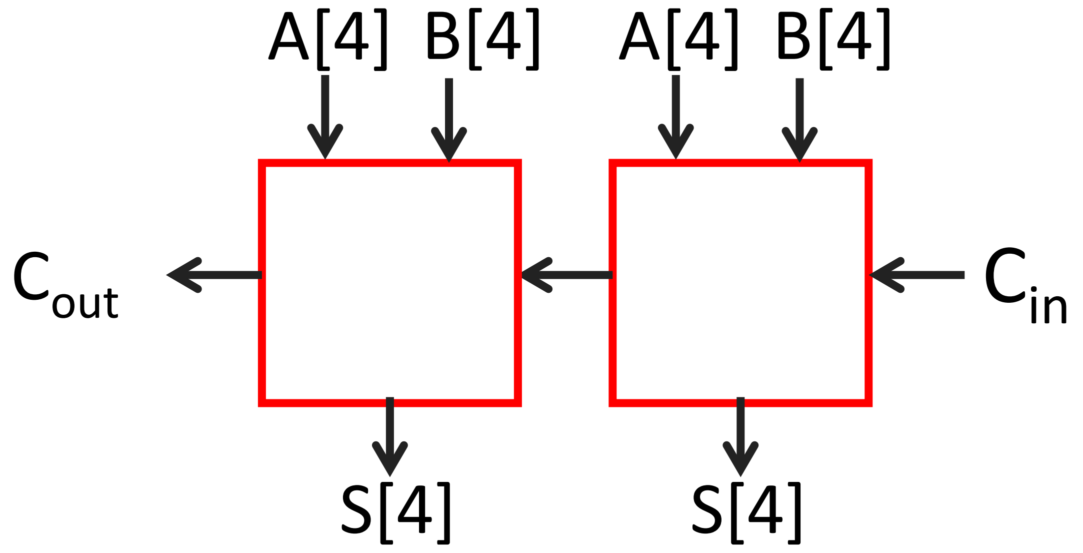
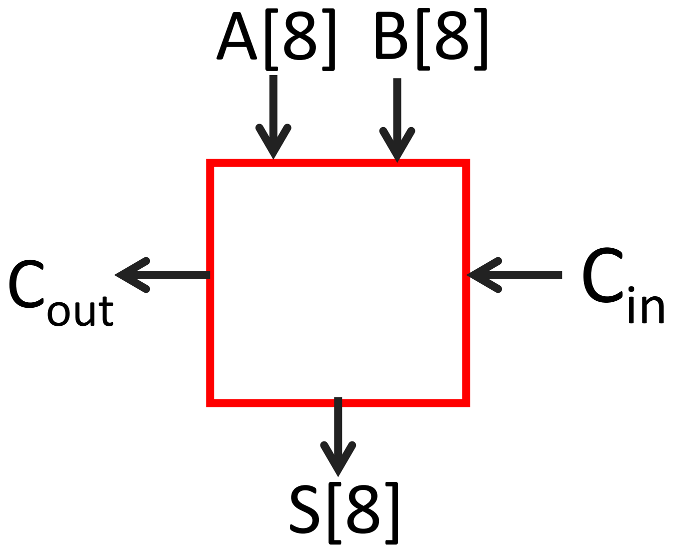
# 4-bit Adder with Two’s Complement



# 8-bit Adder with Two's Complement



# 8-bit Adder with Two's Complement



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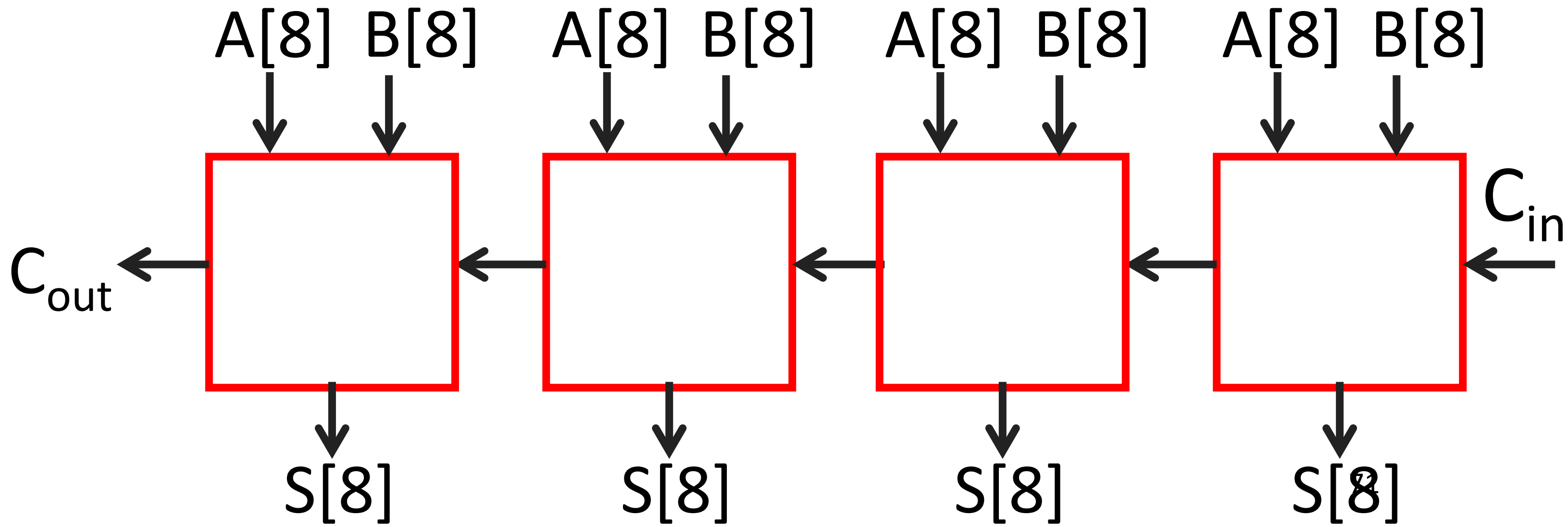
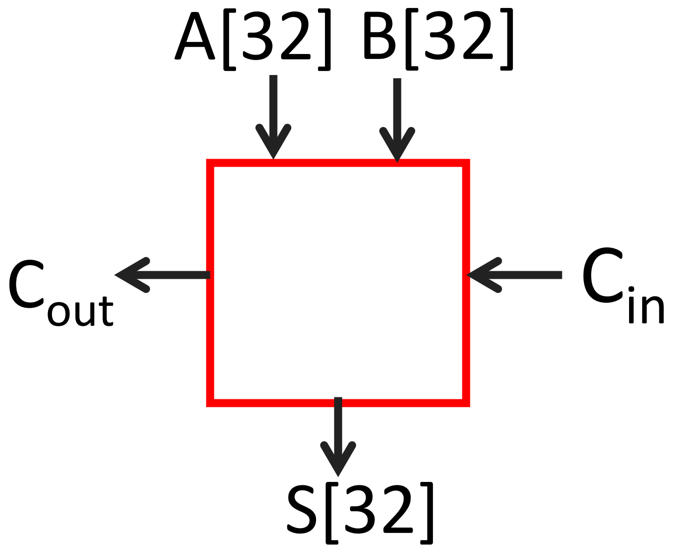
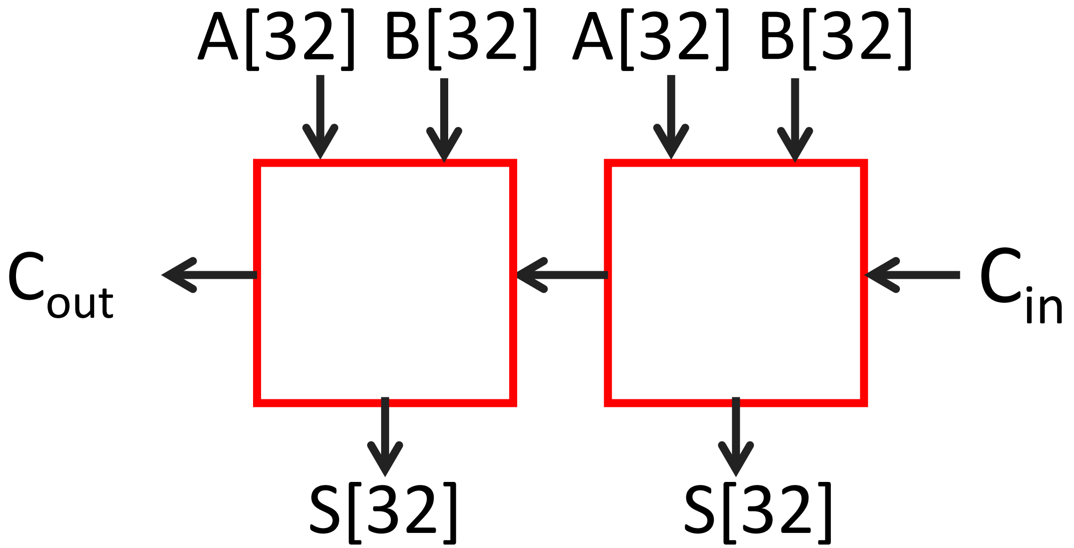
# 32-bit Adder with Two’s Complement



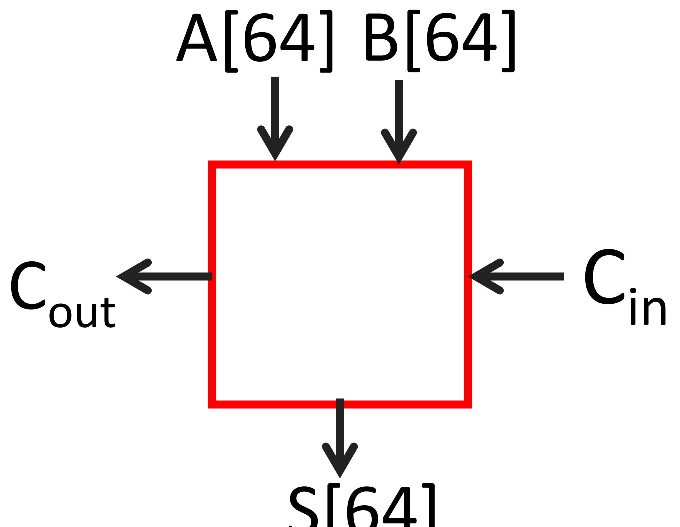
# 32-bit Adder with Two's Complement



## 64-bit Adder with Two's Complement



# 64-bit Adder with Two’s Complement



# Takeaway

Digital computers are implemented via logic circuits and thus represent *all* numbers in binary (base 2).

We (humans) often write numbers as decimal and hexadecimal for convenience, so need to be able to convert to binary and back (to understand what computer is doing!).

Adding two 1-bit numbers generalizes to adding two numbers of any size since 1-bit full adders can be cascaded.

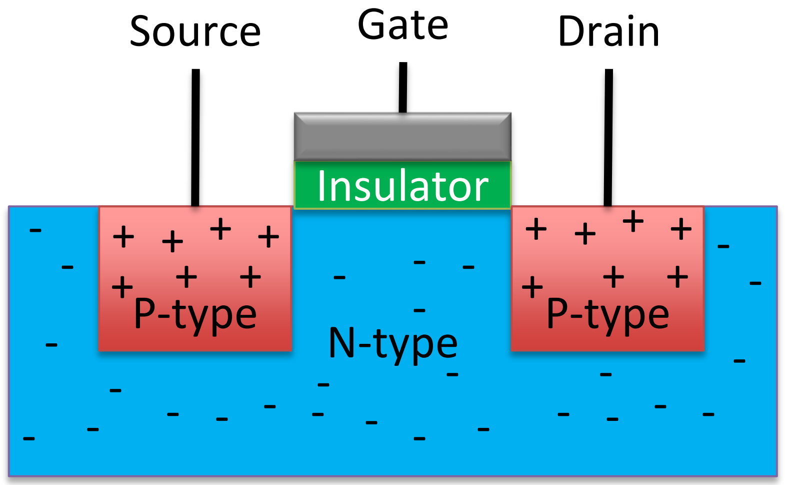
## Goals for Today: Bottom Up!

- From Switches to Logic Gates to Logic Circuits
- Logic Gates
  - From switches
  - Truth Tables
- Logic Circuits
  - From Truth Tables to Circuits (Sum of Products)
  - Identity Laws
- Binary Operations
  - One- and four-bit adders
  - Addition (two's complement)
- Transistors (electronic switch)

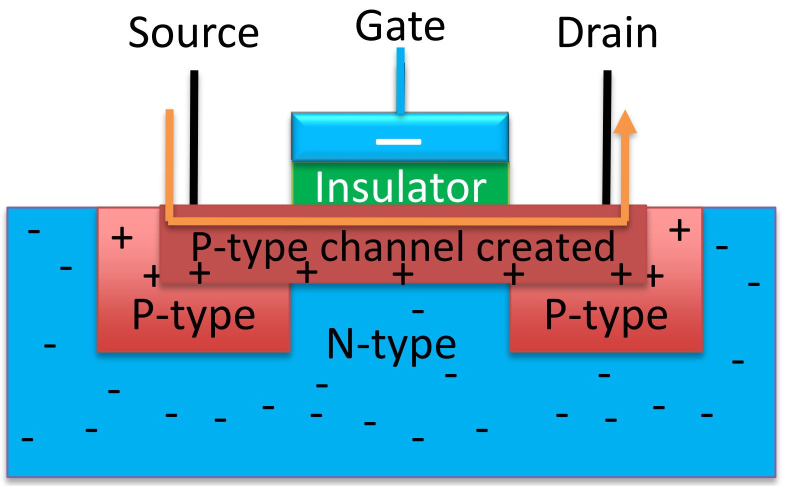
## Silicon Valley & the Semiconductor Industry

- Transistors:
- Youtube video “How does a transistor work”
  https://www.youtube.com/watch?v=IcrBqCFLHIY
- Break: show some Transistor, Fab, Wafer photos

# Transistors 101



P-Transistor **Off**



P-Transistor **On**

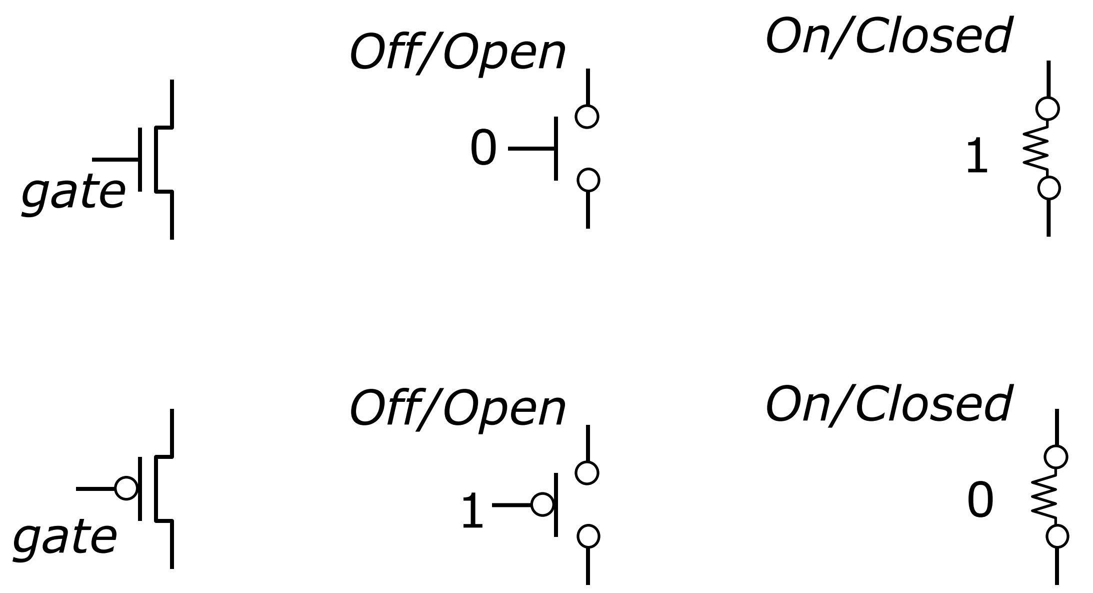
**N-Type Silicon:** negative free-carriers (electrons)
**P-Type Silicon:** positive free-carriers (holes)
**P-Transistor:** negative charge on gate generates electric field that creates a (+ charged) p-channel connecting source & drain
**N-Transistor:** works the opposite way
Metal-Oxide Semiconductor (Gate-Insulator-Silicon)

- Complementary MOS = **CMOS** technology uses both p- & n-type transistors

# CMOS Notation

N-type

P-type



Gate input controls whether current can flow between the other two terminals or not.

*Hint:* the "o" bubble of the p-type tells you that this gate wants a 0 to be turned on

## PollEV Question #6

Which of the following statements is false?

(A) P- and N-type transistors are both used in CMOS designs

(B) As transistors get smaller, the frequency of your processor will keep getting faster

(C) As transistors get smaller, you can fit more and more of them on a single chip

(D) Pure silicon is a semi conductor

(E) Experts believe that Moore's Law will soon end

## Which of the following statements is false?

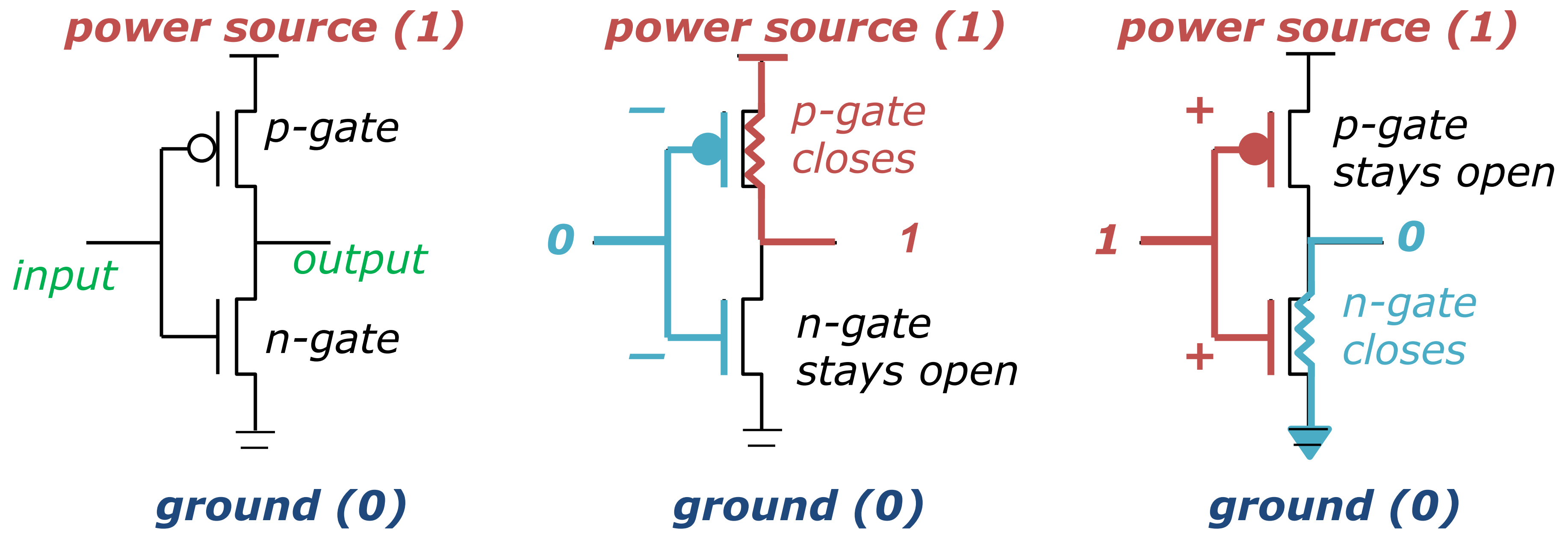
0

| | |
|---|---|
| P- and N-type transistors are both used in CMOS designs | 0% |
| As transistors get smaller, the frequency of your processor will keep getting faster | 0% |
| As transistors get smaller, you can fit more and more of them on a single chip | 0% |
| Pure silicon is a semi conductor | 0% |
| Experts believe that Moore’s Law will soon end | 0% |

# 2-Transistor Combination: NOT

- Logic gates are constructed by combining transistors in complementary arrangements
- Combine p&n transistors to make a NOT gate:

*CMOS Inverter :*



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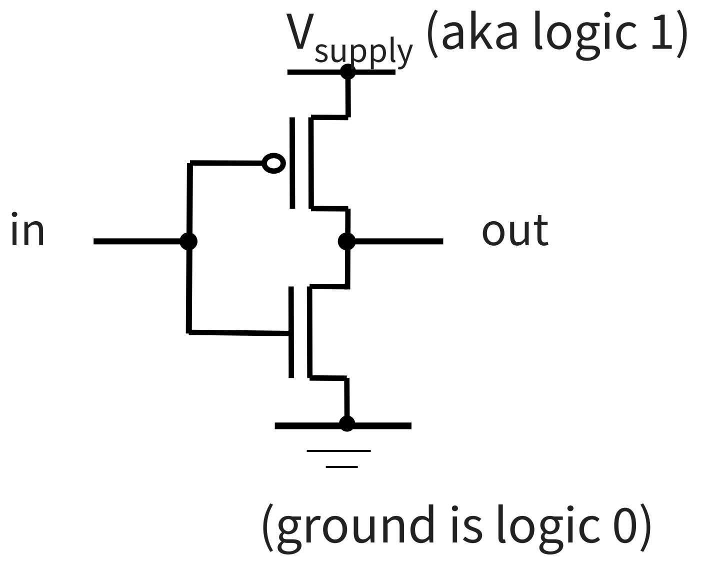
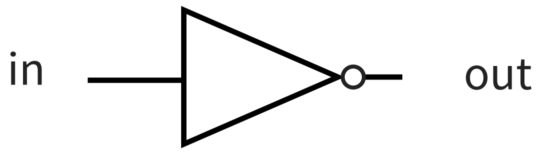
## Inverter



Function: NOT

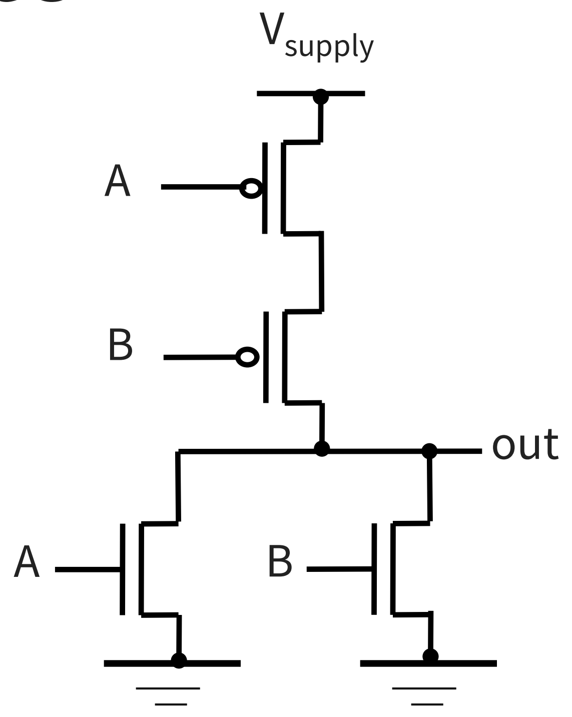
Symbol:



Truth Table:

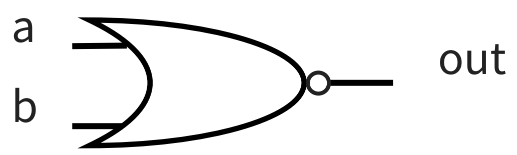
| In | Out |
|---|---|
| 0 | 1 |
| 1 | 0 |

## NOR Gate



Function: NOR

Symbol:

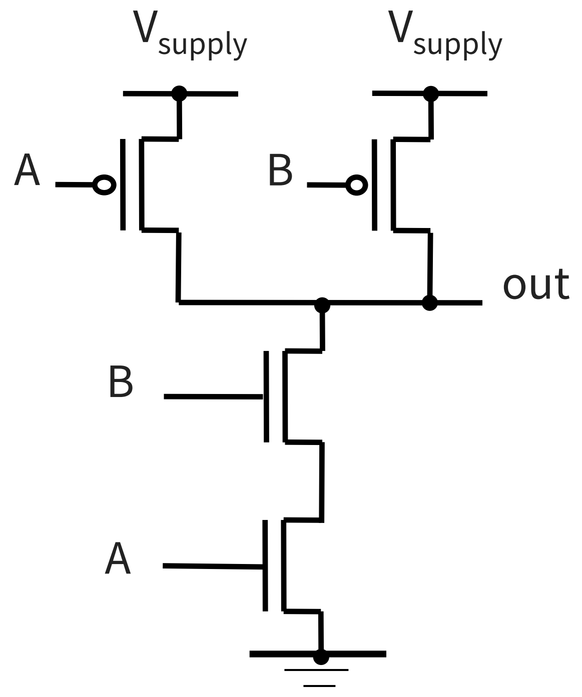


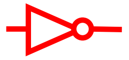
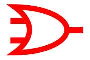
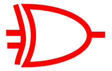
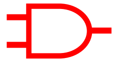
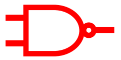
Truth Table:

| A | B | out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

## Which Gate is this?

## PollEV Question #7



(A) NOT 
(B) OR 
(C) XOR 
(D) AND 
(E) NAND 

Function:

Symbol:

Truth Table:

| A | B | out |
|---|---|---|
| 0 | 0 | |
| 0 | 1 | |
| 1 | 0 | |
| 1 | 1 | |

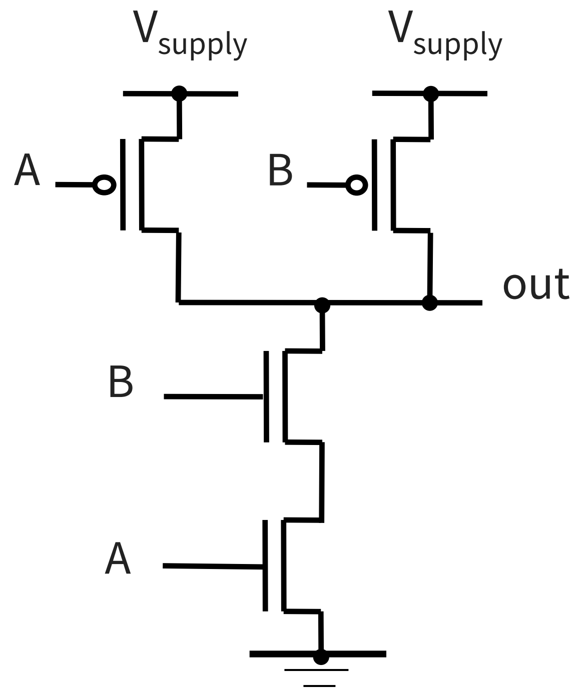
## Which Gate is this? (Take 2)

0

| | |
|---|---|
| NOT | 0% |
| OR | 0% |
| XOR | 0% |
| AND | 0% |
| NAND | 0% |

# Which Gate is this?

## PollEV Question #7



(A) NOT
(B) OR
(C) XOR
(D) AND
[✓ (E) NAND]

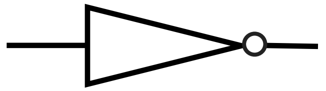
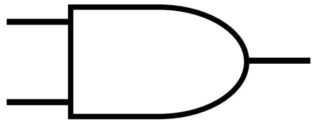
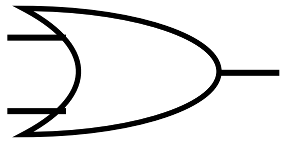
Function:

Symbol:

Truth Table:

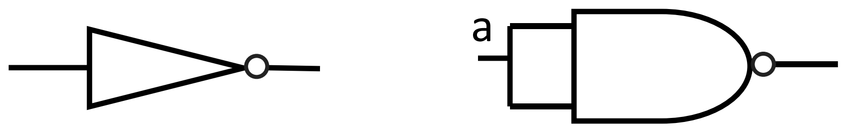
| A | B | out |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

## Building Functions (Revisited)

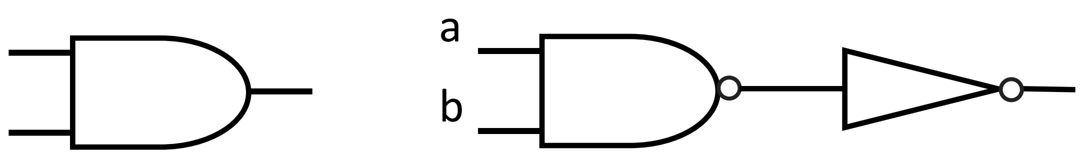
- NOT: 
- AND: 
- OR: 
- NAND and NOR are universal
  - Can implement ***any*** function with NAND or just NOR gates
  - useful for manufacturing

# Building Functions (Revisited)

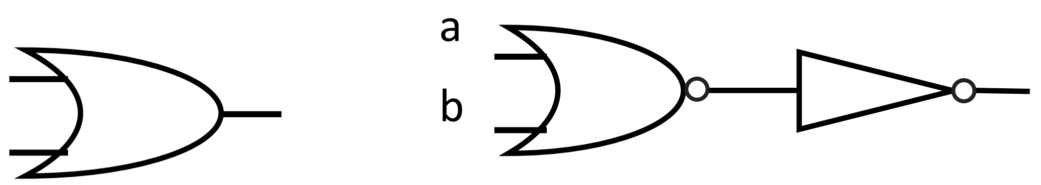
- NOT:



- AND:



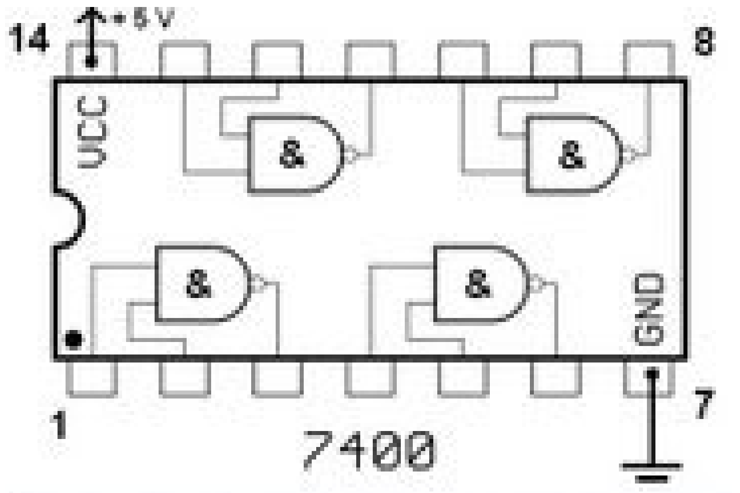
- OR:



- NAND and NOR are universal
  - Can implement ***any*** function with NAND or just NOR gates
  - useful for manufacturing

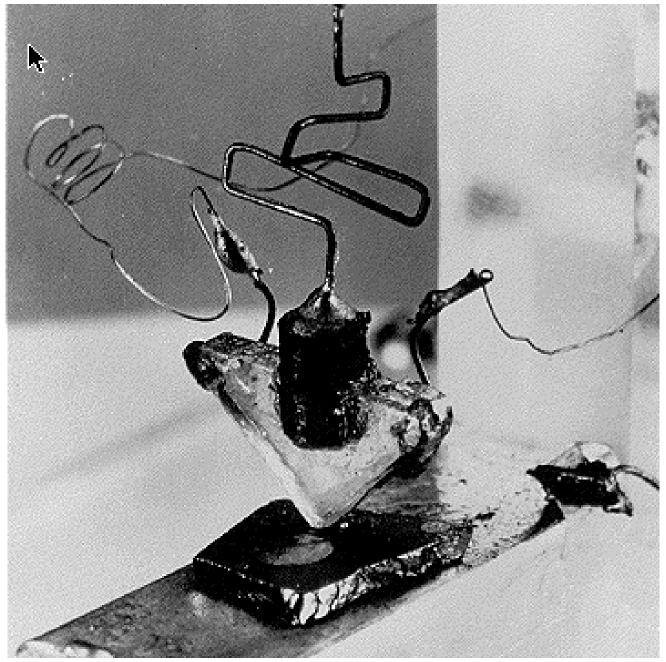
- Build your own computer!  See Nandgame https://nandgame.com/

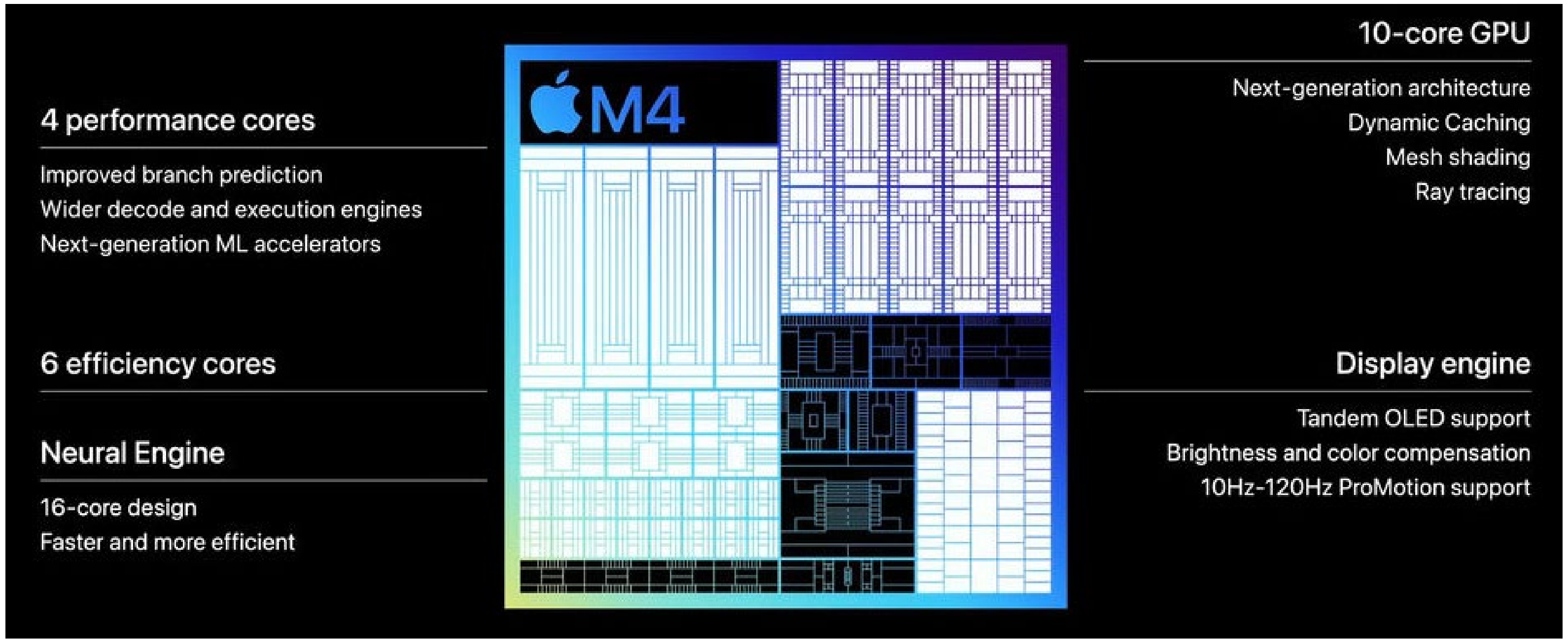
# Logic Gates



- One can buy gates separately
  - ex. 74xxx series of integrated circuits
  - cost ~$1 per chip, mostly for packaging and testing
- Cumbersome, but possible to build devices using gates put together manually

## Then and Now




4 performance cores
Improved branch prediction
Wider decode and execution engines
Next-generation ML accelerators
6 efficiency cores
Neural Engine
16-core design
Faster and more efficient
M4
10-core GPU
Next-generation architecture
Dynamic Caching
Mesh shading
Ray tracing
Display engine
Tandem OLED support
Brightness and color compensation
10Hz-120Hz ProMotion support

https://en.wikipedia.org/wiki/Apple_M4
https://en.wikipedia.org/wiki/Transistor_count

### The first transistor

- One workbench at AT&T Bell Labs
- 1947
- Bardeen, Brattain, and Shockley

### Apple M4

- 28 billion transistors, 3nm
- 177 square millimeters
- 4x-10x performance, 4x-6x efficiency, 8x-40x GPU, 16x Neural processing cores

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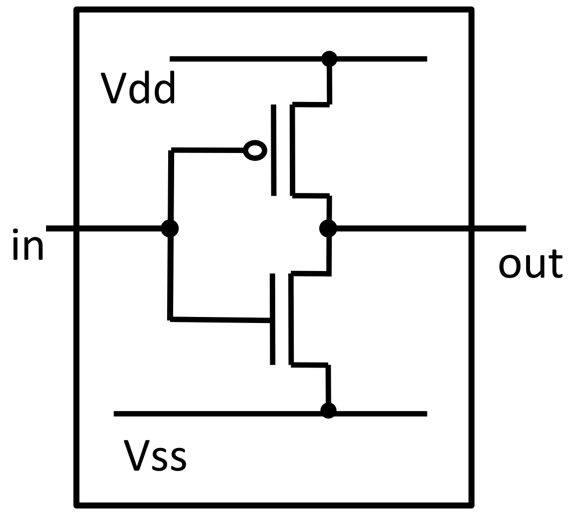
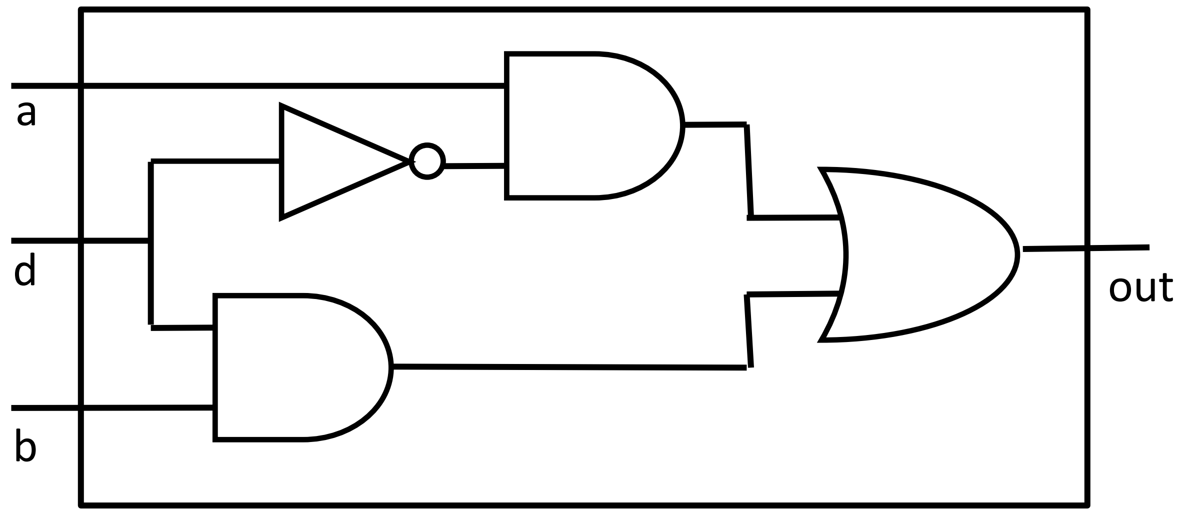
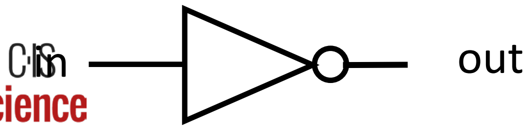
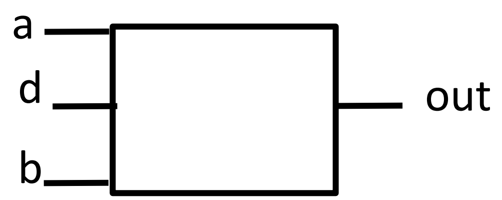
# Big Picture: Abstraction

- Hide complexity through simple abstractions
  - Simplicity
    - Box diagram represents inputs and outputs
  - Complexity
    - Hides underlying NMOS- and PMOS-transistors and atomic interactions









## Summary

- Most modern devices made of billions of transistors
  - You will build a processor in this course!
  - Modern transistors made from semiconductor materials
  - Transistors used to make logic gates and logic circuits

- We can now implement any logic circuit
  - Use P- & N-transistors to implement NAND/NOR gates
  - Use NAND or NOR gates to implement the logic circuit
  - *Efficiently*: use K-maps to find required minimal terms