CS280, Spring 2003: Prelim #1

The test is out of 50; the points for each question are marked. Don't forget to put your name and student number on each blue book that you use. You can answer the questions in any order, but mark your work clearly. Don't forget to show all your work. Give us a chance to give you partial credit!

Good luck.

1. [4 points] Give (a) the converse and (b) the contrapositive of the statement "If a graph is Eulerian then it is connected."

2. [6 points] If S is a set, consider the relation R on the power set of S such that $(X, Y) \in R$ iff $X \cap Y = \emptyset$. Is this relation (a) reflexive, (b) symmetric, and (c) transitive? In each case, if you think it satisfies the property, explain why. If you think it does *not* satisfy the property, give a counterexample.

3. [4 points] Let $f(x) = 3x + 1$.

   (a) If $f : N \to N$, is f injective? Is it surjective?

   (b) If $f : R \to R$, is f injective? Is it surjective?

   For both (a) and (b), if you don't think it is injective, or you don't think it is surjective, give a counterexample.

4. [2 points] What is (a) $8^{1/3}$ (b) $\lfloor -2.1 \rfloor$?

5. [5 points] Prove that $\Sigma_{i=1}^{n}(1+i)2^i = n2^{n+1}$ for $n \geq 1$.

6. [8 points] Define $A_1 \oplus A_2 = (A_1 - A_2) \cup (A_2 - A_1)$. ($A_1 \oplus A_2$ is called the symmetric difference of A_1 and A_2.) Then, for $n \geq 2$, inductively define $A_1 \oplus \cdots \oplus A_{n+1} = (A_1 \oplus \cdots \oplus A_n) \oplus A_{n+1}$. Prove that $A_1 \oplus \cdots \oplus A_n$ consists of all the elements that appear in exactly an odd number of the sets $A_1, \cdots, A_n$, for $n \geq 2$. [Don't forget you can get partial credit here!]

7. [6 points] Let S be the smallest set of positive integers such that (a) $3 \in S$ and (b) if $x \in S$ and $y \in S$, then $x + y \in S$. Show that S consists of all multiples of 3.

8. [3 points] Is there an undirected graph with 5 vertices all of which have degree 3? (If you think there is such a graph, draw it. If not, explain why not. Just a "yes" or "no" gets no credit.)

9. [3 points] Can a graph that contains a triangle be bipartite? (Again, if you think it is possible, draw an example. If not, explain why not. Just a "yes" or "no" gets no credit.)

10. (a) [2 points] Which vertices in the graph on the left below have degree 3?

    (b) [3 points] Is the following pair of graphs isomorphic? (If so, describe the isomorphism. If not, explain why not. Just a "yes" or "no" gets no credit.)

11. [4 points] Find the adjacency matrix of the graph below. Describe in words the transitive closure of the graph.