CS280, Spring 2000: Prelim #1

The test is out of 30; the points for each question are marked. Don't forget to put your name and student number on each blue book that you use. You can answer the questions in any order, but mark your work clearly.

1. [1 points] What is a relation on S?

2. [4 points] Let R_1 and R_2 be relations on S. If R_1 and R_2 are both symmetric, is $R_1 \cup R_2$ a symmetric relation on S? (If you think it is, prove it; if not, provide a counterexample.)

3. [4 points] Let $S = \{1, 2, 3, 4, 5\}$ and $T = \{a, b, c, d\}$. For each question below, either give an example or prove that there are no examples.

    (a) Are there any one-to-one functions from S to T?

    (b) Are there any onto functions from S to T?

4. [1 point] What is the converse of "If pigs had wings, then they could fly."

5. Inductively define $a_0 = a_1 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$ for $n \geq 2$.

    (a) [1 point] What is a_4?

    (b) [4 points] Prove that a_n is odd for all $n \in N$.

6. [5 points] Show that $\sum_{i=n}^{2n-1}(2i + 1) = 3n^2$ for $n \geq 1$. (Note the summation starts at n, not 1.)

7. Consider the following directed graph:

    (a) [1 point] What is its adjacency matrix?

    (b) [2 point] What is the degree of v_2? How can you compute it from the adjacency matrix?

    (c) [3 points] Use the adjacency matrix to compute how many paths of length 2 there are from v_1 to v_2.

    (d) [4 points] Explain what an Eulerian path is and find one in the graph. Then explain what a Hamiltonian path is and fine one in the graph.