There's a famous conjecture due to Ulam and Kelly ...

Suppose Γ_1 has nodes $v_1, \dots, v_p$
and Γ_2 has nodes $u_1, \dots, u_p$, with $p \geq 3$.

Then $\Gamma_1 - v_i \cong \Gamma_2 - u_i \implies \Gamma_1 \cong \Gamma_2$.

This has been proven when the Γ_i are either regular graphs, disconnected graphs, or trees.

Sometimes it's useful to be able to look at the larger picture of a graph. We say that Γ is bipartite if its node set $V = V_1 \dot{\cup} V_2$ (disjoint union) so that each edge of Γ connects a node in V_1 with a node in V_2. It's said to be complete if it contains every edge joining V_1 and V_2, and is often denoted $K_{m,n}$ where V_1 and V_2 have m and n nodes respectively. The special case of $K_{1,n}$ is called a star.

We can extend this to t-partite graphs whose nodes can be partitioned into the sets $V_1, V_2, \dots, V_t$ in such a way that any pair of nodes are neighbours only if they live in distinct sets V_i. Completeness holds if neighbourliness occurs if and only if nodes live in distinct V_i.

König's Theorem: Γ is bipartite iff all its cycles are even.

This follows quite simply; for if Γ has any odd cycles it cannot be bipartite; and if all cycles are even and we define V_1 and V_2 by picking a node v, putting it and all even distance nodes in V_1 with $V_2 = V - V_1$, then any edge in V_1 would create an oddcycle!