

We've talked about paths in graphs, but we can make finer distinctions:

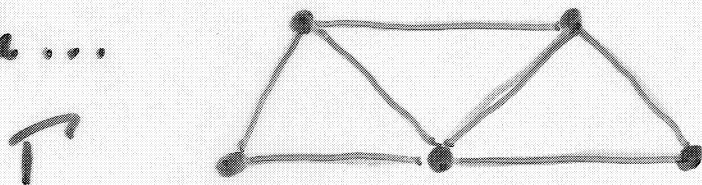
- a walk is an alternating sequence of nodes and edges.
- the length of a walk is its number of edges.
- a trail is a walk whose edges are distinct.
- a path is a walk whose nodes are distinct (\Rightarrow trail).
- a walk is closed if it ends where it began.
- a cycle is a closed path of length ≥ 3 , assuming the graph has no pair of nodes joined by ≥ 2 edges.

These definitions allow us to begin to give measurements of graphs:

- the girth of Γ , $g(\Gamma)$, is the length of its shortest cycle, and is undefined if \nexists cycles.
- the circumference of Γ , $c(\Gamma)$, is the length of a maximal cycle, and is undefined if \nexists cycles.
- a geodesic between v_1 and v_2 is a minimal path from v_1 to v_2 , and its length is their distance.
- the diameter of Γ , $d(\Gamma)$, is the length of a maximal geodesic — Γ must be connected for this.

use
to
define
metric
on Γ

For example ...



$$\begin{aligned}g(\Gamma) &= 3, \\c(\Gamma) &= 5, \\d(\Gamma) &= 2.\end{aligned}$$

We define a triangle to be any cycle of length 3. All the graph numbers we've mentioned so far are values which are isomorphism-invariant, i.e., they are the same for graphs which are isomorphic to one another.

If Γ is a graph and v is a node of Γ , then $\Gamma - v$ is a node-deleted subgraph of Γ (remove also edges meeting v).