

(9)

(Investigating what happens to the ratio x/n as $n \rightarrow \infty$ leads to "the law of large numbers", a standard statistical technique.)

E.g. In 'bridge', a 'hand' is the receipt of 13 cards,

$$\Rightarrow P(\text{no aces in hand}) = \frac{\binom{48}{13}}{\binom{52}{13}} \approx 0.304$$

Assuming perfect shuffling each time, repeat 5 times, then

$$P(\text{no aces in } x \text{ of } 5 \text{ 'hands'}) = \binom{5}{x} (0.304)^x (0.696)^{5-x}$$

If n is large, using Stirling's formula to approximate $r!$ can help the computation.

This can all be generalised to a multinomial distribution in an obvious way ...

Assume a Bernoulli experiment with A_1, \dots, A_r as all possible events (exhaustive and exclusive) with probabilities p_1, \dots, p_r .

Now repeat the experiment n times, then

$$S = \{ (x_1, \dots, x_r) \mid \sum x_i = n \}$$

is the sample space, and we define X on S

by $X(a_1, \dots, a_r) := (a_1, \dots, a_r)$. Then

$$P(X = (x_1, \dots, x_r)) = \binom{n}{x_1 \dots x_r} p_1^{x_1} \dots p_r^{x_r}$$

Again we have

$$\begin{aligned} F(n) &= \sum f(x_1, \dots, x_r) \\ &= (p_1 + \dots + p_r)^n = 1 \end{aligned}$$

hence the moniker "multinomial" (again there are really only $(r-1)$ variables involved).