- A die has 3 faces with 1, 2 with 2 and 1 with 3.
Event: Roll die 5 times recording

$$x_1 = \#\,1\text{'s}, \quad x_2 = \#\,2\text{'s}, \quad x_3 = \#\,3\text{'s}.$$

Then $S = \{(5,0,0), (4,1,0), (4,0,1), \ldots, (0,0,5)\}$.
Now let X be defined on S by $X(a,b,c) := (a,b,c)$, then $f(a,b,c) = p(X = (a,b,c))$ is a p.d.f. for X and

$$f(a,b,c) = \binom{5}{a\ b\ c}\left(\tfrac{3}{6}\right)^a \left(\tfrac{2}{6}\right)^b \left(\tfrac{1}{6}\right)^c.$$

Actually, c is determined by a and b, $c = 5-b-a$, so

$$f(a,b) = \frac{5!}{a!\,b!\,(5-a-b)!}\left(\tfrac{1}{2}\right)^a \left(\tfrac{1}{3}\right)^b \left(\tfrac{1}{6}\right)^{5-a-b}$$

and

$$F(a,b) = \sum_{x\le a}\sum_{y \le b} f(x,y)$$

is the corresponding multivariate distribution function.

- Now let A be an event for an experiment with

$$p(A) = p, \quad \text{so } p(\bar{A}) = q = 1-p.$$

Repeating the experiment n times (independently) and recording $x = \#\,A\text{'s}$ gives

$$S = \{0, 1, \ldots, n\}, \quad \text{and we let } X(a) := a.$$

Now if α is any particular sequence of A's and $\bar{A}$'s then $p(\alpha) = p^x q^{n-x}$ if α has x A's. Hence

$$p(\text{exactly } x \text{ } A\text{'s in } n \text{ repetitions}) = \binom{n}{x} p^x q^{n-x}.$$

Notice that
$$\begin{aligned}\sum_0^n f(x) &=: F(n)\\ &= q^n + npq^{n-1} + \binom{n}{2}p^2q^{n-2} + \ldots + p^n\\ &= (q+p)^n = ((1-p)+p)^n = 1\end{aligned}$$

hence $F(x)$ is called the <u>binomial distribution</u> and such an experiment is a <u>Bernoulli Experiment</u>.

If $n=1$ this is called a Bernoulli distribution.