Now define X on S by $X(a,b) := a$, so that we count only the number of red things selected. Then

$$p(X=x) = \frac{\binom{n_1}{x}\binom{n_2}{r-x}}{\binom{n}{r}}.$$

This gives rise to the <u>hypergeometric distribution</u>.

<u>Definition</u>

For a random variable X (discrete), the function

$$f(x) := p(X=x)$$

is the <u>probability density function</u> (or p.d.f.) of X.

<u>Examples</u>

- Toss a coin until the 1st head appears, then $S = \{1, 2, 3, \ldots\}$, and let X be defined on S by $X(a) := a$. Then

  $$p(X=1) = \tfrac{1}{2}, \quad p(X=2) = \left(\tfrac{1}{2}\right)^2, \ldots$$

  so

  $$f(x) = \left(\tfrac{1}{2}\right)^x$$

  is the p.d.f. for X. Now let

  $$F(x) := p(X \le a) = \sum_{x \le a} f(x)$$
  $$= \tfrac{1}{2} + \left(\tfrac{1}{2}\right)^2 + \cdots + \left(\tfrac{1}{2}\right)^x = 1 - \left(\tfrac{1}{2}\right)^x.$$

  This 'cumulative' function is called the <u>probability distribution function</u> for X, and in this example is a single-variate distribution.
  Notice that

  $$p(a \lneq X \le b) = \sum_{x \gneq a}^{x \le b} f(x) = F(b) - F(a).$$