Examples

- Bucket of 10 things has 3 defectives. Examining things successively, find p(last defective is 5^{th} thing examined).

  Let A = "5^{th} thing examined is defective"

  B = "exactly 2 defectives found in 1^{st} 4 examined"

  We want $p(A \wedge B)$.

  $$p(B) = \frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}} \quad \text{and} \quad p(A|B) = \frac{1}{6}$$

  only 1 defective left!

  Hence $p(A \wedge B) = p(A|B)\,p(B) = \frac{1}{6}\,\frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}}$.

- Have 2 buckets ... α has 5 blue and 4 white balls

  β " 4 " " 5 " "

  Now transfer 1 ball at random from α to β.

  Find p(now pick blue ball from β).

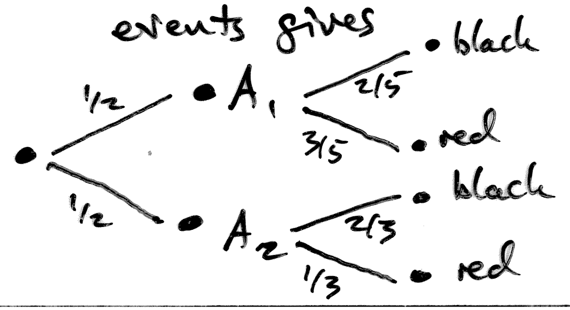
  Let αB = "pick blue from α", αW = "pick white from α",

  βB = " " " " β", βW = " " " " β".

  Then
  $$p(\beta B) = p(\beta B|\alpha B)\,p(\alpha B) + p(\beta B|\alpha W)\,p(\alpha W)$$
  $$= 5/10 \cdot 5/9 + 4/10 \cdot 4/9$$

- Have buckets A_1, A_2 with A_1 having 2 black and 3 red balls, and A_2 having 2 black and 1 red ball. Choose bucket at random then pick ball at random, then find p(red ball chosen).

  A first intuition might be $p(\text{red}) = \frac{3+1}{8} = \frac{1}{2}$ however this would be wrong. Diagramming the events gives



  $$\Rightarrow p(\text{red}) = \frac{1}{2}\cdot\frac{3}{5} + \frac{1}{2}\cdot\frac{1}{3} = \frac{7}{15}$$